## Effect of misalignements on Energy calibration and polarization Alain Blondel for the EPOL group from a presentation at FCC-ee optics tuning and alignment mini-workshop

-1- The importance of Energy calibration and Polarization
-2- Impact of alignement imperfections on spin motion
depolarization and interference with energy determination
$\rightarrow$ vertical orbit and vertical dispersion
-3- Specific polarization corrections
-- a possible exemple: $2 \pi$ vertical orbit bumps and harmonic spin matching
$-4-$ Ground motion and need for continuous corrections
-5- Collision effects
-6- List of recommendations as of today
TODAY: Will concentrate on collision offsets, work by Wenninger, Shatilov, Oide, AB

I have left the other slides for reference

## Works packages

A- Simulations of spin-tune to beam energy relationship
-- EPFL group obtained funding from CHART for a student and a postdoc (stdies started -- Yi Wu)
-- Ivan Koop now concentrating on res. dep at WW threshold (Qs is now 0.075, *good*!)
B. Simulation of the relationship between beam energies and centre-of-mass energy.
-- Impact of energy losses (Jacqueline Keintzel)
-- control of offsets and vertical dispersion (Wenninger, Oide, Shatilov, AB)
-- Studied the beamstrahlung monitor but does not work in a circular machine (Shatilov)
-- Studies will continue to implement beam deflection scans (AB-Oide-Shatilov-Wenninger)
C. Polarimeter desing and performance
-- now working to build a global collaboration (IJCLAB (Martens), BINP (Muchnoi), CERN (Lefevre), -- others?)
-- Aim to provide integration of polarimeters, wigglers, RF kickers in FCC-ee
-- conceptual design and cost estimate of polarimeter for FCC FS
D. Measurements in Particle Physics Experiments
-- not much work done beyond design study, needs to restart soon, very precious information from dimuons
E. Monochromatization

Angeles Faus, Jorg Wenninger, Pantaleo Raimondi, Frank Zimmermann, Dmitry Shatilov
-- new ideas for monochromatization in other dimensions than horizontal (x) axis. (time, $\mathbf{z}$ )
-- what its the limit?

## From beam energy to $\mathrm{E}_{\mathrm{CM}}$

## opposite sign dispersion

## $E_{e^{-}}$


$E e^{+}$No effect on ECM NB energy spread is reduced.

## Experience from LEP: Vernier scans

Relative position of beams measured to +- 80 nanometers from one scan

precision requires going far from maximum
$\rightarrow$ loose beam?


Try beam-beam deflection?

## vernier scans

7.2 Dispersion at the IP

For beams colliding with an offset at the IP, the CM energy spread and shift are affected by the local dispersion at the IP. For a total IP separation of the beams of $2 u_{0}$ the expressions for the CM energy shift and spread are [72]

$$
\begin{gather*}
\Delta \sqrt{s}=-2 u_{0} \frac{\sigma_{E}^{2}\left(D_{\mathrm{u} 1}-D_{\mathrm{u} 2}\right)}{E_{0}\left(\sigma_{B 1}^{2}+\sigma_{B 2}^{2}\right)}  \tag{90}\\
\sigma_{\sqrt{s}}^{2}=\sigma_{E}^{2}\left[\frac{\sigma_{\varepsilon}^{2}\left(D_{\mathrm{u} 1}+D_{\mathrm{u} 2}\right)^{2}+4 \sigma_{u}^{2}}{\sigma_{B 1}^{2}+\sigma_{B 2}^{2}}\right] \tag{91}
\end{gather*}
$$

$D_{\mathrm{u} 1}$ and $D_{\mathrm{u} 2}$ represent the dispersion at the IP for the two beams labelled by 1 and 2. $\sigma_{E}$ is the beam energy spread assumed here to be equal for both beams and $\sigma_{\epsilon}=\sigma_{E} / E$ is the relative energy spread. $\sigma_{B i}$ is the total transverse size of beam (i) at the IP,

$$
\begin{equation*}
\sigma_{B i}^{2}=\sigma_{u}^{2}+\left(D_{u i} \sigma_{\epsilon}\right)^{2} \tag{92}
\end{equation*}
$$

with $\sigma_{u}$ the betatronic component of the beam size.
If the beam sizes at the IP are dominated by the betatronic component which is rather likely, the energy shift simplifies to

$$
\begin{equation*}
\Delta \sqrt{s}=-u_{0} \frac{\sigma_{E}^{2} \Delta D^{*}}{E_{0} \sigma_{u}^{2}} \tag{93}
\end{equation*}
$$

where $\Delta D^{*}=D_{u 1}-D_{u 2}$ is the difference in dispersion at the IP between the two beams. This effect applies to both planes ( $u=x, y$ ). In general due to the very flat beam shapes the most critical effect arises in the vertical plane.

For FCC-ee at the $Z$ we have in vertical direction:

- Parasitic dispersion of e+ and e-beams at IP 10um the difference is $\Delta D_{y}^{*}=14 \mu m$.
- Sigma_y is 28 nm
- Sigma_E is $0.132 \% * 45000 \mathrm{MeV}=60 \mathrm{MeV}$
- Delta_ECM is therefore 1.4 MeV for a 1 nm offset
- Note that we cannot perform Vernier scans like at LEP, we can only displace the two beams by ~10\%sigma_y
- Assume each Vernier scan is accurate to $1 \%$ sigma_y, we get a precision of 400 keV .
the process should be simulated
- we need 100 beams scans to get an $\mathrm{E}_{\mathrm{CM}}$ accuracy of 40keV suggestion: vernier scan every hour or more.
- It is likely that Vernier scans will be performed regularly at least once per hour or more. ( $\rightarrow 100$ per week) we end up with an uncertainty of $\sim 10 \mathrm{keV}$ over the whole running period. (provided no systematic effects show up)
- The dispersion must be measured as well; this can be done by using the vernier scans with offset RF frequency
- this would lead to lots of Vernier scans!


## beam-beam deflection scans were already used at SLC, KEK and LEP

Luminosity Optimisation Using Beam-beam Deflections at LEP
C. Bovet, M.D. Hildreth, M. Lamont, H. Schmickler, J. Wenninger, CERN, Geneva, Switzerland

CERN-SL-96-025
https://inspirehep.net/literature/420668

Uncertainty on $\Delta y_{\text {opt }}=-5.6 \pm 0.1 \mu \mathrm{~m}$ is $1 / 40$ of the vertical beam size $3.8 \pm 0.2 \mu \mathrm{~m}$ which was itself measured in the process

beam-beam deflection measurement at FCC-ee as if in « squished perspective » looking from behind detectors endcaps

U-BPM
upstream electron beam position monitor located between final focus quads and compensating solenoid

U+BPM

BPM precision over $10^{8}$ bunch passages is ${ }^{\sim} 1 \mu \mathrm{~m}$

downstream positron beam position monitor
located between final focus quads and compensating solenoid

BPM in arc magnets

$\square \square \quad$ REFERENCE

1. beams collide head on
-- or at low current
1'. pilot bunches (not colliding) all the time $1^{\prime \prime}$ can be calibrated with low current vernier scan $1^{\prime \prime \prime}$ or occasional vernier scan



## COLLISION OFFSET

2. offset by $\delta_{y}=0.1 \sigma_{y}(=3.5 \mathrm{~nm})$
$\rightarrow$ opposite kick by $4 \mu \mathrm{rad}$
(Shatilov) in opposite directions for e+ and e-
$\rightarrow$ movement in the BPMs by
$\pm 2 \mu \mathrm{rad} \times 2.1 \mathrm{~m}= \pm 4.2 \mu \mathrm{~m}$
(x1000 demagnification due to optics)
with a very specific pattern of movements

Vertical beam size at the IP: $\sim 35 \mathrm{~nm}$ (at Z pole). Vertical offset of $0.1 \sigma_{y}$ leads to additional orbit angles about $\pm 2 \mu$ rad for the nominal bunch population $2.5 \mathrm{E}+11$. (D. Shatilov, simulation)


## Measurements of offsets and Opposite Sign Vertical Dispersion (OSVD)

## Purely statistical and preliminary arguments:

## OFFSETS:

Four measurements of 4.2 micron displacement with 1 micron precision can be made with $10^{8}$ bunch passages
(assume 10000 bunches in each beam)
$\rightarrow$ every 3 seconds
$\rightarrow$ measurement of beam beam offset with precision of $0.1 * 35 \mathrm{~nm} / 4.2 / \sqrt{ } 4=1 / 80$ of beam size or $\sim 0.4 \mathrm{~nm}$
NB no need of a scan in principle if a good and stable reference can be demonstrated. CAN WE USE THE PILOT BUNCHES? LEP did not have pilot bunches, but maybe we can use them? (there is a debate on this) Pilot bunches would provide $10^{\wedge} 8$ bunch measurements in 2 minutes (only 250 bunches of each beam)

## OSVD

we cannot really measure the dispersion at IP directly, but the beams will move in opposite directions upon a change of RF frequency
$\rightarrow$ we measure the opposite sign vertical dispersion (OSVD) this way!
Assuming that a relative momentum change of 10-3 is feasible, this measurement corresponds to a measurement of opposite sign vertical dispersion $D^{*} y(e+)-D^{*} y(e-)$ with a precision of 0.4 micrometer.

Plugging this into the equations of the earlier page this leads to a measurement of the possible shift in energy with a precision of $\pm \mathbf{2 0} \mathbf{~ k e V}$ each time the dispersion measurement is done. THIS IS VERY PROMISING because in particular it requires very little scanning across the beam.

## FCC-ee Energy Calibration and Polarization



Recent CDF: $\mathrm{m}_{\mathrm{W}}(\mathrm{MeV})=80^{\prime} 433.5 \pm 6.4_{\text {stat }} \pm 6.9_{\text {syst }} \quad$ ( $10^{-4}$ precision)
-- « could hint at new physics» and surely created a buzz!
-- precision measurements as broad exploration of new physics in quantum corrections, or mixing (SUSY, Heavy neutrinos, etc..)
(-- questions because inconsistent with previous measurements)

CDF measurement is remarkable in two ways:

1. (after 10 years of work)
systematic errors similar to statistical precision
2. relies for the precise calibration on $J / \psi, \Upsilon, Z$ masses all measured in e+e- colliders...
using resonant depolarization!


## Resonant depolarization is the cornerstone of the precision programme of FCC-ee



Improvement by factor 10-1000 on a long list of precision measurements.
e.g. W mass down to $\pm 250 \mathrm{keV}, Z$ mass and width $\pm 4 \mathrm{keV}, \sin ^{2} \theta_{\mathrm{W}}$ eff $^{2} \pm 2.10^{-6}$ etc. explore new physics at 10-100 TeV scale, or $10^{-5}$ mixing with known particles.
factor 500 more precise than LEP

First set of results obtained in the FCC Design Study:

Polarization and Centre-of-mass Energy Calibration at FCC-ee, arXiv:1909.12245

Table 15: Calculated uncertainties on the quantities most affected by the center-of-mass energy uncertainties, under the final systematic assumptions.

| Quantity | statistics | $\begin{aligned} & \hline \Delta \mathrm{E}_{\mathrm{CMabs}} \\ & 100 \mathrm{keV} \end{aligned}$ | $\begin{gathered} \Delta \mathrm{E}_{\text {CMSyst-ptp }} \\ \mathbf{4 0} \mathbf{~ k V V} \end{gathered}$ | calib. stats. $\left.200 \mathrm{keV} / \sqrt{( } N^{i}\right)$ | $\begin{gathered} \sigma E_{C M} \\ (84) \pm \mathbf{0 . 0 5} \mathrm{MeV} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{m}_{\mathrm{Z}}(\mathrm{keV})$ | 4 | 100 | 28 | 1 | - |
| $\Gamma_{\mathrm{Z}}(\mathrm{keV})$ | 4 | 2.5 | 22 | 1 | 10 |
| $\sin ^{2} \theta_{W}^{\mathrm{eff}} \times 10^{6}$ from $A_{F B}^{\mu \mu}$ | 2 | - | 2.4 | 0.1 | - |
| $\frac{\Delta \alpha_{Q E D}\left(\mathrm{M}_{\mathrm{Z}}\right)}{\alpha_{Q E D}\left(\mathrm{M}_{\mathrm{Z}}\right)} \times 10^{5}$ | 3 | 0.1 | 0.9 | - | 0.05 |
| $\mathrm{m}_{\mathrm{w}}(\mathrm{MeV})$ | 0.250 | -- 0.300 -- |  |  |  |

Next challenges for the feasibility study.
-- Ascertain the above with integrated simulations
-- Match systematic errors with statistics.
most relevant errors : the point-to-point systematics

- these are effects that would lead to a deviation from relation between
-- the spin tune as measured by resonant depolarization
-- and the center-of-mass energy.
-- examples: 1. interference between depoarizing resonances and the induced depolarizing resonance because the spin tune varies with energy.

2. effects due to collision offsets folded by opposite sign dispersion

## targets and procedures

1. Center-of-mass energy precision of $< \pm 100 \mathrm{keV}$ ( $<10 \mathrm{keV}$ ptp) around the $Z$ peak
2. Center-of-mass energy precision of $< \pm 200 \mathrm{keV}$ at W pair threshold
3. For the $Z$ peak-cross-section and width, require energy spread uncertainty $\Delta \sigma_{\mathrm{E}} / \sigma_{\mathrm{E}}=0.2 \%$ NB: at $2.310^{36} / \mathrm{cm}^{2} / \mathrm{s} / \mathrm{IP}$ : full LEP statistics $10^{6} \mu \mu 2.10^{7} \mathrm{qq}$ in 6 minutes in each expt determine energy spread and boost of ECM $\rightarrow$ beam and beamstrahlung energy loss
-- use resonant depolarization as main measuring method
-- use pilot bunches to calibrate during physics data taking: 100 calibrations per day each $10^{-6}$ rel. -- long lifetime at $Z$ requires the use of wigglers at beginning of fills
$\rightarrow$ take data at points where self-polarization is expected

$$
\mathrm{v}_{\mathrm{s}}=\frac{g-2}{2} \frac{E_{b}}{m_{e}}=\frac{E_{b}}{0.4406486(1)} \approx N+(0.5 \pm 0.1) \quad \mathrm{E}_{\mathrm{CM}}=(N+(0.5 \pm 0.1)) \times 0.8812972 \mathrm{GeV}
$$

Given the $Z$ and W widths of 2 GeV , this is easy to accommodate with little loss of statistics. It might be more difficult for the Higgs 125.09+-0.2 corresponds to $v_{s}=141.94+-022$


LEP (1989-2000) first observation of $P_{\perp}$ in 1990 first resonant depolarization in 1991

$$
\tau_{p}=\left(\frac{5 \sqrt{3}}{8} \frac{\hbar r_{e} E_{\text {beam }}^{5}}{\boldsymbol{m}_{e}^{6} \rho^{3}}\right)^{-1} \quad \begin{aligned}
& =\sim 5 \text { hours at LEP } \\
& \text { but at FCC-ee } \\
& \sim 256 \text { hrs at } Z \text { pole }
\end{aligned}
$$

~14 hrs at WW thresh.

$$
10 \% \text { of that time for } P=9 \%
$$

$$
\begin{aligned}
P_{\infty} & =0.924 \times \frac{1}{1+\frac{\tau_{P}}{\tau_{d}}} \quad \frac{1}{\tau_{p}} \propto \sum_{j}\left|B_{j}\right|^{3} L_{j} \\
\tau_{p}^{e f f} & =\tau_{p} \times \frac{1}{1+\frac{\tau_{P}}{\tau_{d}}} \cdot \frac{1}{\tau_{d}} \propto \sum_{j}\left|B_{j}\right|^{3} L_{j} \frac{11}{18}\left|\Gamma_{j}\right|^{2} \\
\nu & =a_{e} \gamma=\frac{g_{e}-2}{2} \frac{E_{B e a m}}{m_{e} c^{2}}=\frac{E_{B e a m}}{0.4406486(1)}
\end{aligned}
$$

Derbenev-Kondratenko
« spin-orbit coupling
= dependence of equilibrium
"spin » on particle energy

Spin tune at the $Z$ peak : 103.5
The scan points 99.5 / 103.5 / 106.5 are perfect optimum for $Z$ width and $\alpha_{\text {QED }}$ meast Spin tune for W threshold 183.5
$\frac{1}{\tau_{p}} \propto \sum_{j}\left|\mathbf{B}_{j}\right|^{3} L_{j} \propto I_{3}$,
$\frac{1}{\tau_{d}} \propto \sum_{j} \frac{11}{18}\left|\mathbf{B}_{j}\right|^{3} L_{j}\left|\mathbf{\Gamma}_{j}\right|^{2}$.
can be improved by increasing the sum of $|\mathrm{B}|^{3}$ (Wigglers)
the sources of depolarization can be separated into harmonics
(the integer resonances) and/or into the components of motion:
horizontal betatron: $\quad\left|\boldsymbol{\Gamma}_{x}\right|^{2} \propto \delta \eta^{2} \delta \mathrm{n}^{2}$
vertical betatron: $\quad\left|\Gamma_{y}\right|^{2} \propto \delta \eta^{2}$,
synchrotron: $\quad\left|\Gamma_{z}\right|^{2} \propto A \delta \eta^{2}+B \delta \mathbf{n}^{2}$,
$\delta \eta$ vertical dispersion $\delta n$ average angle between 'closed orbit spin' and magnetic field
recipes:
-- reduce the emittance (esp. $\varepsilon_{\mathrm{y}}$ ) and vertical dispersion $\delta \eta$
$\rightarrow$ this is the same as for luminosity optimization!
-- reduce the vertical spin motion $\delta \boldsymbol{n} \rightarrow$ harmonic spin matching
-- do not increase the energy spread

## SPIN PRECESSION

RESONANT DEPOLARIZATION

$$
\begin{aligned}
& (v \text { is the spin tune) } \\
& \delta \theta_{\text {spin }} \\
& =(\mathrm{g}-2) / 2 \cdot \mathrm{E}_{\text {beam }} / \mathrm{m}_{\mathrm{e}} \delta \theta_{\text {trajectory }} \\
& \delta \theta_{\text {spin }}
\end{aligned}=v \cdot \delta \theta_{\text {trajectory }} .
$$

## AMPLIFICATION

$\rightarrow$ high precision
$\rightarrow$ sensitivity to misalignements
-- depolarization
-- spurious spin resonances


Once the beams are polarized, an RF kicker at the spin precession frequency (fractional part thereof) will provoke a spin rotation and depolarization Simulation of FCC-ee by I. Kopp:


FCCee_z_530_nosol_23_al.sad


100 microrad orbit kick gets compensated by the pi bump but generates a lasting 25 mrad spin kick

the pi bump generates a spin component rotation of the spin in the $x-z$ direction. The largest rotation is created by the QD quadrupole (focus in vertical plane)

Simulations of self-polarization
Orbit correction leading to similar values for vertical dispersion and vertical emittance than for the luminosity optimization

## E. Gianfelice

@ Z

significant impact of spin resonance from vertical orbit @Z
@WW

It might kill polarization completely @W
-- Sufficient level of polarization at $Z$ for machine that is optimized for luminosity.
-- Additional correction of dispersion and harmonic spin matching is necessary at W
-- Effect of resonant depolarization vs beam energy unknown
-- These studies will be repeated with simulation on same machine of lumi/polarization

## From resonant depolarization to Center-of-mass energy - from spin tune to beam energy--

The spin tune may not be en exact measurement of the average of the beam energy along the magnetic trajectory of particles. Additional spin rotations may bias the issue. Anton Bogomyagkov and Eliana Gianfelice have made many estimates.

| synchrotron oscillations | $\Delta \mathrm{E} / \mathrm{E}$ | $-210^{-14}$ |
| :--- | :---: | :---: |
| Energy dependent momentum compaction | $\Delta \mathrm{E} / \mathrm{E}$ | $10^{-7}$ |
| Solenoid compensation |  | $210^{-11}$ |
| Horizontal betatron oscillations | $\Delta \mathrm{E} / \mathrm{E}$ | $2.510^{-7}$ |
| Horizontal correctors*) | $\Delta \mathrm{E} / \mathrm{E}$ | $2.510^{-7}$ |
| Vertical betatron oscillations ${ }^{* *}$ ) | $\Delta \mathrm{E} / \mathrm{E}$ | $2.510^{-7}$ |
| Uncertainty in chromaticity correction $\mathrm{O}\left(10^{-6}\right) \Delta \mathrm{E} / \mathrm{E}$ | $510^{-8}$ |  |
| invariant mass shift due to beam potential |  | $410^{-10}$ |

[^0]

Deterministic Harmonic spin matching :
measure orbit, decompose in harmonics, cancel components near to spin tune.
(:) NO FIDDLING AROUND.
This worked very well at LEP-Z
and should work even better at FCC-ee-Z,W if orbit is measured better.

LEP TidExperiment


Figure 23: Beam energy variations measured over 24 hours compared to the expectation from the tidal LEP deformation.
ground motion (here earth tides) affects the beam energy by changing the ring circumference against a given RF frequency.
-- Tides can be calculated
-- The effect can be seen in the BPMs
-- the effect corresponds to a swing of up to +- 120 MeV in 6 hours at the $Z$ pole! At max rate almost $1 \mathrm{MeV} /$ minute needs correction at that level for ee->H experiment

Other sources of motion: Geneva lake level, rain or snow on mountains, etc have been observed, at longer time scales.

This must be corrected at appropriate intervals by varying the RF frequency or by other methods

Such variations must be carefully recorded and the records organized on a long lasting data base: these parameters enter the centre-of-mass determination and will in fine be part of the physics results

## Recommandations

0. the running mode at $Z$ and WW (and even more for ee-> H) will involve important activity for ECM calibration

1. The measurements and corrections of vertical orbit and vertical dispersion are crucial
2. they should be available for pilot bunches ( $<10^{10} \mathrm{e}+/ \mathrm{e}-/$ bunch, short bunches) as well as for lumi bunches
3. spin correction bumps should be foreseen (e.g. two pi-bumps in the arcs in 8 locations ( 2 around each IP))
4. Ground motion should be corrected regularly (minutes) by RF changes or otherwise
5. correction and monitoring of collision offsets and opposite sign dispersion should be devised
$=======================================================================================$
6. finally since this the ECM calibration will enter the physics results of experiments directly,
$\rightarrow$ careful and continuous monitoring and logging of all relevant parameters should be foreseen

## FCC-ee feasibility study


statistical precision at the $Z$
centre-of-mass energy errors:

$$
\begin{align*}
& \frac{\Delta m_{\mathrm{Z}}}{m_{\mathrm{Z}}}=\left\{\frac{\Delta \sqrt{s}}{\sqrt{s}}\right\}_{\text {abs }} \oplus\left\{\frac{\Delta\left(\sqrt{s_{+}}+\sqrt{s_{-}}\right)}{\sqrt{s_{+}}+\sqrt{s_{-}}}\right\}_{\text {ptp-syst }} \oplus_{i}\left\{\frac{\Delta \sqrt{s_{ \pm}^{2}}}{\sqrt{s_{ \pm}^{2} N_{ \pm}^{2}}}\right\}_{\text {sampling }}, \\
& \frac{\Delta \Gamma_{\mathrm{Z}}}{\Gamma_{\mathrm{Z}}}=\left\{\frac{\Delta \sqrt{s}}{\sqrt{s}}\right\}_{\text {abs }} \oplus\left\{\frac{\Delta\left(\sqrt{s_{+}}-\sqrt{s_{-}}\right)}{\sqrt{s_{+}}-\sqrt{s_{-}}}\right\}_{\text {ptp-syst }} \oplus i\left\{\frac{\Delta \sqrt{s_{ \pm}^{2}}}{\sqrt{s_{ \pm}^{i} N_{ \pm}^{2}}}\right\}_{\text {sampling }}, \\
& \Delta A_{\mathrm{FB}}^{\mu \mu}(\text { pole })=\frac{\partial A_{\mathrm{FB}}^{\mu \mu}}{\partial \sqrt{s}}\left\{\Delta\left(\sqrt{s_{0}}-0.5\left(\sqrt{s_{+}}+\sqrt{s_{-}}\right)\right)\right\}_{\mathrm{ptp}-\mathrm{syst}} \oplus i \frac{\partial A_{\mathrm{FB}}^{\mu \mu}}{\partial \sqrt{s}}\left\{\frac{\Delta \sqrt{s_{0, \pm}^{2}}}{\sqrt{N_{0, \pm}^{i}}}\right\}_{\text {sampling }},  \tag{3.1}\\
& \frac{\Delta \alpha_{\mathrm{QED}}\left(m_{Z}^{2}\right)}{\alpha_{\mathrm{QED}}\left(m_{\mathrm{Z}}^{2}\right)}=\left\{\frac{\Delta \sqrt{s}}{\sqrt{s}}\right\}_{\mathrm{abs}} \oplus\left\{\frac{\Delta\left(\sqrt{s_{+}}-\sqrt{s_{-}}\right)}{\sqrt{s_{+}}-\sqrt{s_{-}}}\right\}_{\mathrm{ptp}-\text { syst }} \oplus_{i}\left\{\frac{\Delta \sqrt{s_{ \pm}^{2}}}{\sqrt{s_{ \pm}^{2} N_{ \pm}^{2}}}\right\}_{\text {sampling }},
\end{align*}
$$

with $\frac{\partial A_{\mathrm{FB}}^{\mu \mu}}{\partial \sqrt{s}} \simeq 0.09 / \mathrm{GeV}$.
Three categories:

- Absolute dominate for $Z$ and $W$ mass
- ptp Point-to-point dominate for $\Gamma_{Z} \& A_{F B}{ }^{\mu \mu}$ (peak and off-peak)
- Due to sampling - turns out to be negligible for 1 meast $/(15 \mathrm{~min}=1000 \mathrm{~s}) \rightarrow 10^{4}$ measts


[^0]:    *) $2.510^{-6}$ if horizontal orbit change by $>0.8 \mathrm{~mm}$ between calibration is unnoticed or if quadrupole stability worse than 5 microns over that time. consider that 0.2 mm orbit will be noticed $\left.{ }^{* *}\right) 2.510^{-6}$ for vertical excursion of 1 mm . Consider orbit can be corrected better than 0.3 mm .

