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# FCC circumference constraints from the injectors and the RF system

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# Outline

- Introduction
- Starting point
- Baseline FCC-hh harmonic @ 400 MHz
- Alternative FCC-hh harmonic @ 500 MHz
- Summary

# Introduction

## Synchronization principle for hadron synchrotrons

- Basic principle: velocity during transfer is the same.

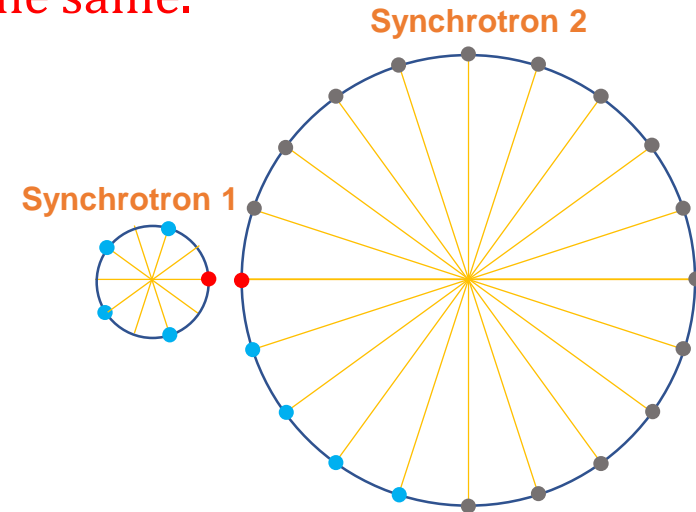
$$\Rightarrow \frac{C_2}{C_1} = \frac{v \cdot T_{rev,2}}{v \cdot T_{rev,1}} = \frac{f_{rev,1}}{f_{rev,2}} = \frac{n_1}{n_2}$$

$$f_{rev} = \frac{f_{RF}}{h} \xrightarrow[\text{Assume } f_{RF,1} = f_{RF,2}]{\text{Assume}} \frac{h_2}{h_1} = \frac{C_2}{C_1} = \frac{n_1}{n_2}$$

- $n_{1,2}$  integers should be kept as small as possible
- Transfer is allowed every  $n_2 \cdot T_{rev,2}$

### Examples

- SPS to LHC transfer  $27 \cdot T_{rev,SPS} = 7 \cdot T_{rev,LHC}$



$T_{rev,1,2}$  -- revolution periods  
 $h_{1,2}$  -- harmonic numbers  
 $C_{1,2}$  -- circumference

### Previous presentations see:

- ✓ [148th FCC-ee Optics Design Meeting & 19th FCCIS WP2.2 Meeting](#) (I. Karpov, H. Damerau)
- ✓ [FCC-ee parameters meeting # 08](#) (H. Damerau, I. Karpov, L.H. Zhang)
- ✓ [FCC-ee parameters meeting # 09](#) (H. Damerau, I. Karpov, L.H. Zhang)
- ✓ [Discussion on the FCC circumference constraints from the injectors and the RF system](#) (H. Damerau, I. Karpov, L.H. Zhang)

# Starting point

- ❑ Keep the possibility of SPS & LHC as potential injectors
- ❑ **Proposed FCC circumference: 91172.7 m (PA31-1.0)**
- ❑ **LHC RF frequency ~400.79 MHz** was initial baseline option for FCC-hh

	Circumference (m)	Harmonic number (400.79 MHz)
FCC	91172.7 (intended)	$2 \times 4 \times 4 \times 13 \times 293 = 121888$
LHC	$2\pi \cdot 11 \cdot 100 \cdot 27/7 = 26658.7$	$2 \times 3 \times 4 \times 5 \times 11 \times 3 \times 9 = 35640$
SPS	$2\pi \cdot 11 \cdot 100 = 6911.5$	$2 \times 3 \times 4 \times 5 \times 11 \times 7 = 4620 \times 2$

- **Why is 121888 unfavourable harmonic number?**
  - No continuous bunch clock for 25 ns spacing
  - Not suited for synchronous transfer ( $h_{\text{FCC}}/h_{\text{LHC}} = 15236/4455$ )
- Note that the ratio of harmonic numbers of LHC and SPS is 27/7 (fixed), which comes from the ratio of circumferences of LHC and SPS
- Note that the factor 11 in the harmonic numbers of SPS and LHC comes from the ratio of circumference of SPS and PS, which will be not a constraint assuming PS replacement as FCC-hh pre-injector

# Baseline FCC-hh harmonic

## ➤ Fixed RF frequency: 400.79 MHz

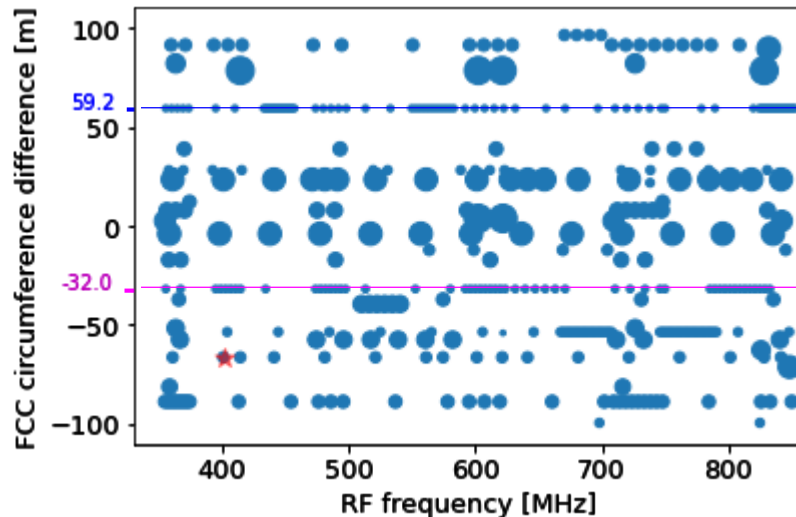
	Circumference (m)	Harmonic number (400.79 MHz)
FCC	91106.187 (shorted by 66.5 m)	$2 \times 3 \times 4 \times 5 \times 5 \times 7 \times 29 = 121800$
LHC	$2\pi \cdot 11 \cdot 100 \cdot 27/7 = 26658.7$	$2 \times 3 \times 4 \times 5 \times 11 \times 3 \times 9 = 35640$
SPS	$2\pi \cdot 11 \cdot 100 = 6911.5$	$2 \times 3 \times 4 \times 5 \times 11 \times 7 = 4620 \times 2$

## ➤ The pros and cons of $h_{\text{FCC}} = 121800 @ 400.79\text{MHz}$

Pros	<ul style="list-style-type: none"> <li>• Keep the same RF frequency as LHC</li> <li>• Continuous bunch clock for experiments for bunch spacings of 2.5, 5.0, 7.5, 10, 12.5, 15, 17.5, 25 ns</li> <li>• PS can still be possible as FCC-hh pre-injector</li> </ul>
Cons	<ul style="list-style-type: none"> <li>• LHC-to-FCC transfer wait time may be a little long (297 turns in FCC, corresponding to ~90.2 ms)</li> <li>• The prime number of 29 in the division ratios relatively large</li> </ul>

# Why stick to $f_{RF} = 400.79$ MHz?

- Scanned  $h_{LHC}$  from 30000 to 76000,  $f_{RF}$ : 337 ~ 854 MHz
  - Scaling laws:  $h_{LHC} = 35640 + 27 \cdot n$ ,  $h_{SPS} = 2 \cdot 4620 + 7 \cdot n$
- Requirements and assumptions:
  - $h_{FCC}$  dividable by 2 (sufficient for 4 IPs);
  - $\Delta C_{FCC} < \pm 100$  m;
  - Largest prime factor in the factorization of FCC/LHC/SPS harmonic number  $< 200$ ;
  - Denominator in  $h_{FCC} / h_{LHC}$  and  $h_{FCC} / h_{SPS} < 300$
  - Maximum bunch spacing less or close to 25 ns



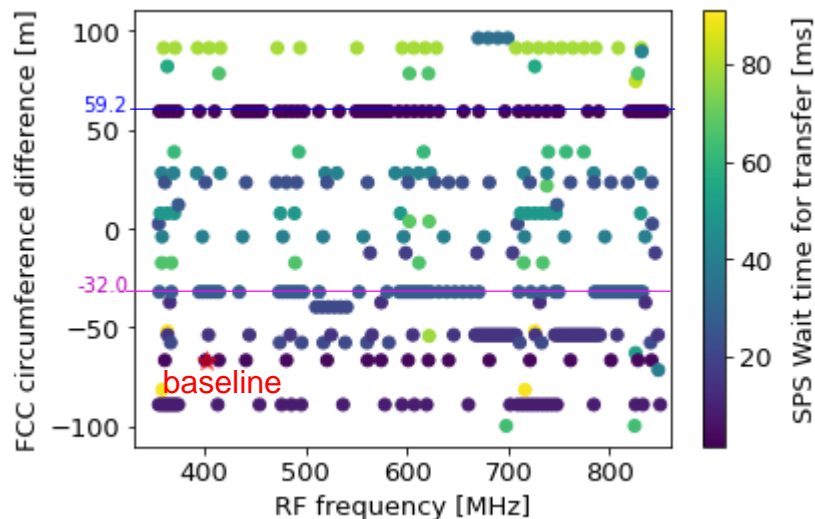
Flexible option for RF frequency

Small largest prime factor

Point size indicates the largest prime factor

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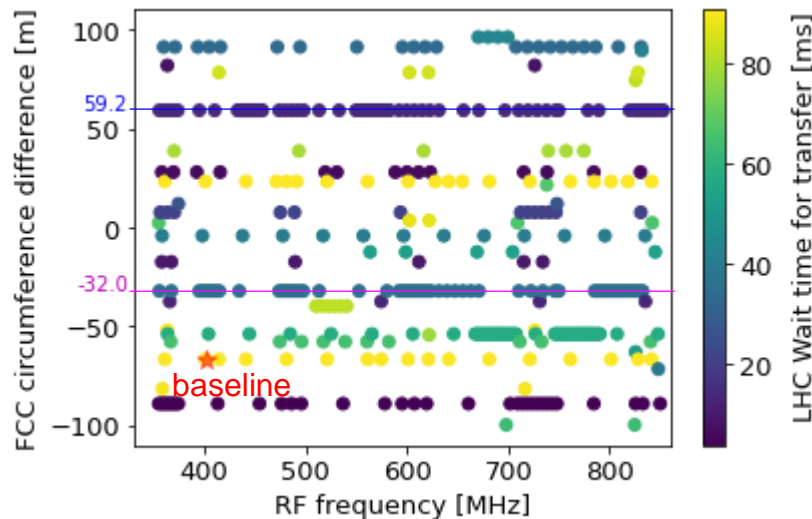


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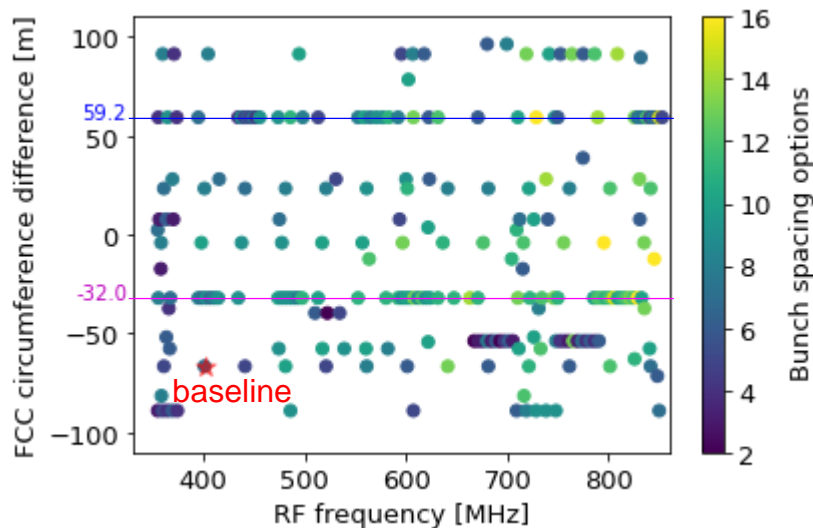
Small largest prime factor

Short wait time for transfer



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Flexible option for RF frequency

Small largest prime factor

Short wait time for transfer

Many bunch spacings possible

# Attractive alternative

	Circumference (m)	Harmonic number ( <b>497.34 MHz</b> )
FCC	<b>91140.710</b> (shorted by 32.0 m)	$2 \times 3^3 \times 7 \times 4 \times 5 \times 4 \times 5 = \mathbf{151200}$
LHC	$2\pi \cdot 11 \cdot 100 \cdot \mathbf{27/7} = 26658.7$	$2 \times 3^2 \times 7 \times \mathbf{13} \times 3^3 = 44226$
SPS	$2\pi \cdot 11 \cdot 100 = 6911.5$	$2 \times 3^2 \times 7 \times \mathbf{13} \times 7 = 5733 \times 2$

## ➤ The pros and cons of $h_{\text{FCC}}=151200$ @ 497.34 MHz

Pros	<ul style="list-style-type: none"> <li>• More flexible native bunch spacings 2.01, 4.02, 6.03, 8.04, 10.05, 12.06, 14.07, 16.09, 18.10, 20.11, 24.13 ns</li> <li>• LHC-to-FCC or SPS-to-FCC transfer wait times are favorably small: 35.5 ms or 27.6 ms</li> <li>• The corresponding circumference allows many other RF frequencies</li> <li>• The largest prime factor (13) small</li> <li>• <b>Robust optimum, also for larger max. bunch spacing</b> (e.g. 25 → 50 ns)</li> </ul>
Cons	<ul style="list-style-type: none"> <li>• <b>Requires new RF systems for SPS and/or LHC</b></li> <li>• <b>Exact bunch spacings like 25 ns proposed in FCC-hh CDR need to vary</b></li> <li>• <b>PS needs to be replaced as FCC-hh pre-injector</b></li> </ul>

# 500 MHz RF system

## ➤ Advantage of 500 MHz

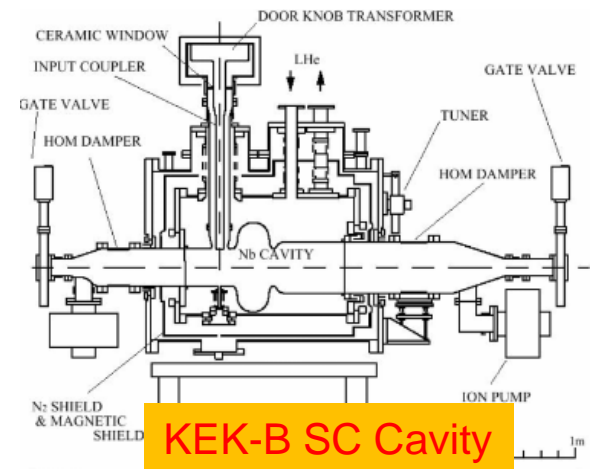
- Smaller cavity size
- More beneficial to single bunch stability (for FCC-hh) [Discussion on the FCC circumference constraints from the injectors and the RF system \(April 26, 2022\) · Indico \(cern.ch\)](#)
- Higher break down voltage, higher gradient

## ➤ Consequence:

- Need new RF systems in SPS/LHC

## ➤ Examples using 500 MHz RF system

- Mainly in Electron Storage Ring
- [TPS](#), [SLS](#), [BESSY](#)... (synchrotron radiation light sources)
- [CESR](#), [KEK-B](#), [BEPCII](#)...(electron-positron colliders)



# Summary

Scheme	Baseline	Alternative
RF frequency [MHz]	400.79	497.34
$C_{FCC}$ [m]	91106.187	91140.710
$h_{FCC}$	121800	151200
RF system	based on LHC	new
Bunch spacing [ns]	2.5, 5.0, 7.5, 10, 12.5, 15, 17.5, 25 (proposed in FCC-hh CDR)	2.01, 4.02, 6.03, 8.04, 10.05, 12.06, 14.07, 16.09, 18.1, 20.11, 24.13 (More flexible)
LHC-to-FCC transfer	297 revolutions in FCC	117 revolutions in FCC
SPS-to-FCC transfer	11 revolutions in FCC	91 revolutions in FCC
FCC-hh pre-injector	allows PS (1959!)	new
Largest prime factor	29	13

- Baseline scheme of FCC-hh circumference and RF frequency basically meets all requirements proposed in CDR
- Attractive alternative option could offer more flexibility

Thanks for your attention!

Spare slides

# Impact of RF frequency on beam dynamics

➤ From **single bunch instability** point of view:

- Loss of Landau damping<sup>1</sup>:

$$N_p = -\frac{V_0 \cos \phi_{s0}}{q \omega_0 \text{Im} Z/k} \frac{\pi \phi_{max}^4}{32 \mu (\mu + 1) h k_{max}} \propto \frac{(h\tau)^4}{h} = h^3 \tau^4$$

- $N_p \propto h^3 \tau^4$  for a constant bunch length
- $N_p \propto h^2$  for a constant longitudinal emittance

- Longitudinal microwave instability<sup>2</sup>:

$$N_p = \frac{3\pi}{q \omega_0} \frac{h V_0}{|(Z_L/n)_{eff}|} \left( \frac{L}{2\pi R} \right)^3 \propto h \tau^3$$

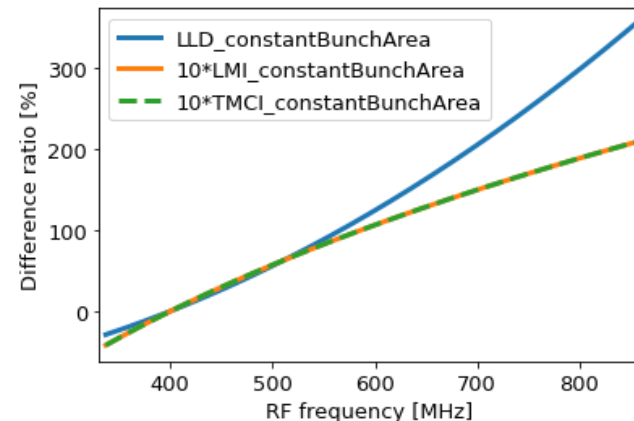
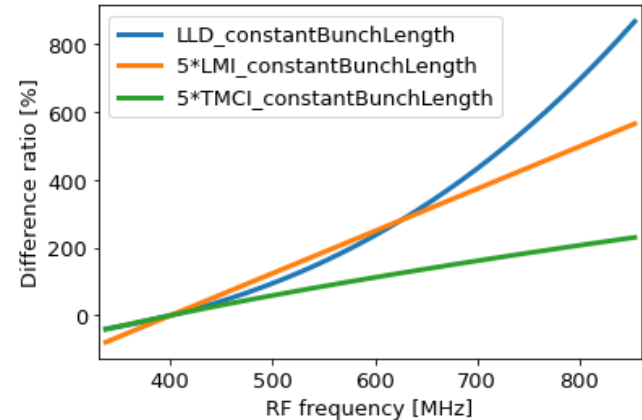
- $N_p \propto h \tau^3$  for a constant bunch length
- $N_p \propto h^{1/4}$  for a constant longitudinal emittance

- Transverse mode coupling instability<sup>2</sup>

$$N_p = \frac{4\pi}{q \omega_0} \frac{Q_s E/q}{\beta_{av} \text{Im}(Z_T)_{eff}} \frac{L}{R} \propto h^{1/2} \tau$$

- $N_p \propto h^{1/2} \tau$  for a constant bunch length
- $N_p \propto h^{1/4}$  for a constant longitudinal emittance

⇒ Higher harmonic number seems more beneficial to a higher bunch intensity

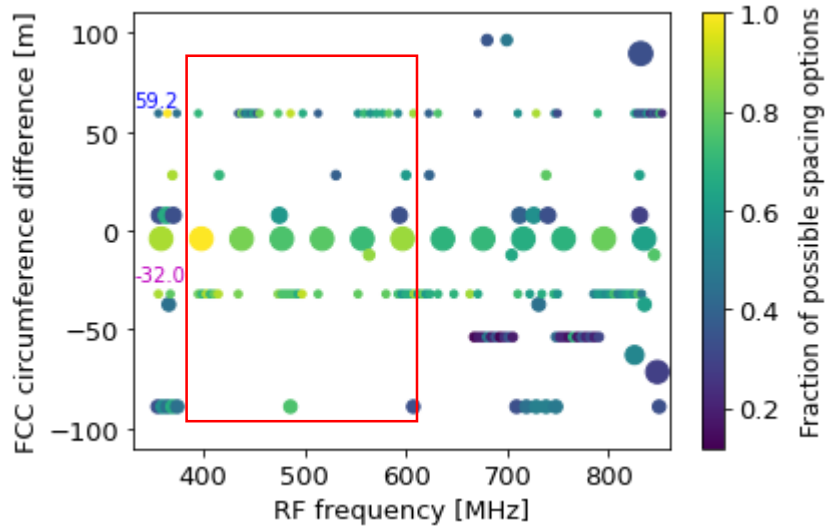


<sup>1</sup> Ivan Karpov, Theodoros Argyropoulos, and Elena Shaposhnikova, Phys. Rev. Accel. Beams **24**, 011002 (2021)

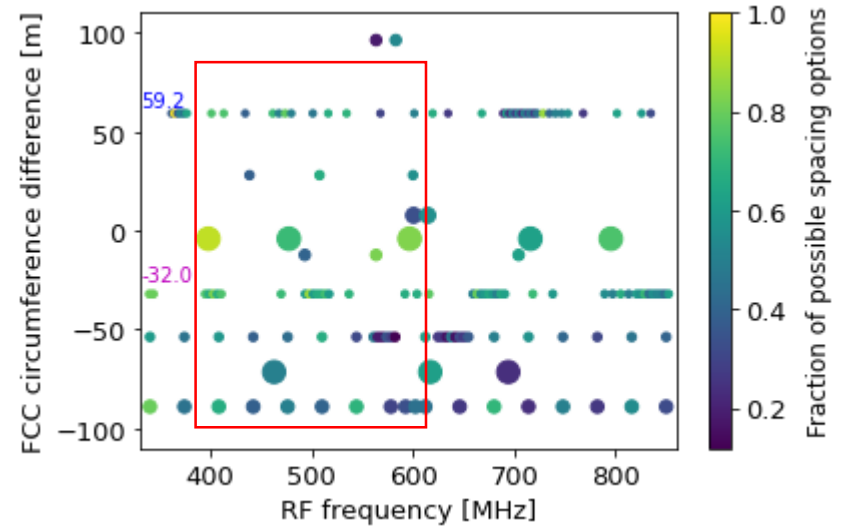
<sup>2</sup> Francesco Ruggiero, Single-Beam Collective Effects in the LHC, CERN SL/95-09 (AP)

# Results for different $\tau_{\text{spacing,max}}$

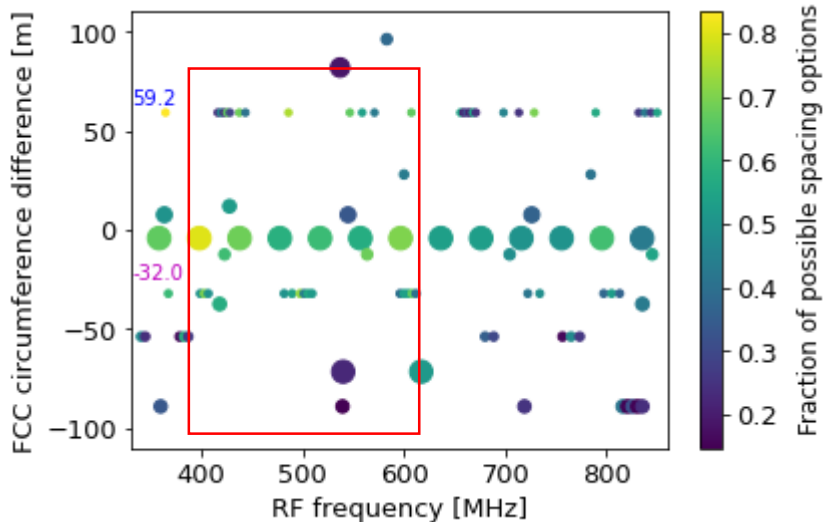
Max. bunch spacing < 25.5 ns



Max. bunch spacing < 30.5 ns



Max. bunch spacing < 50.5 ns



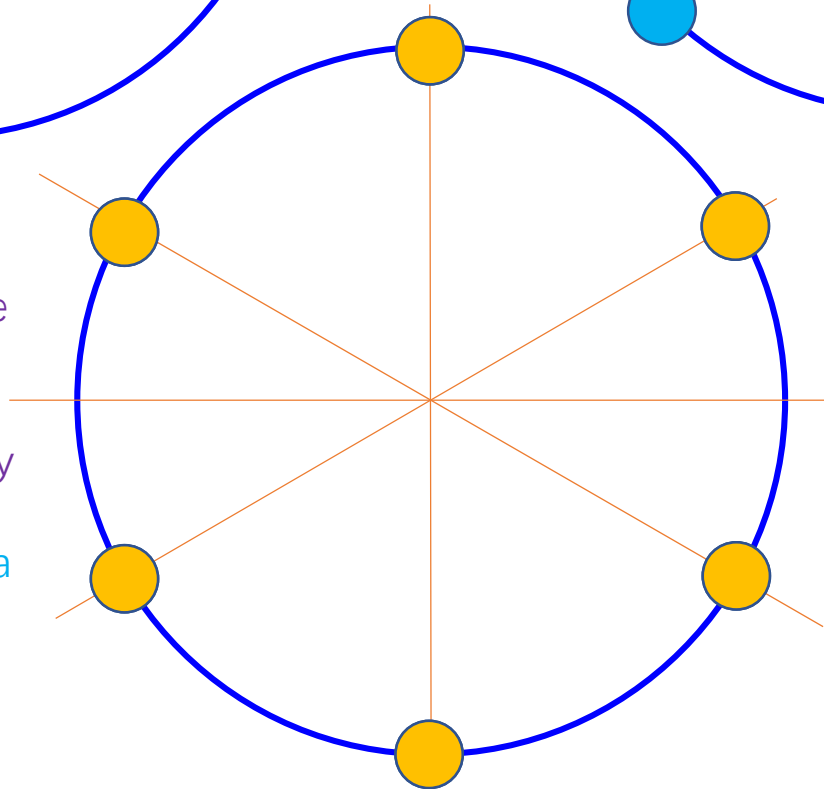
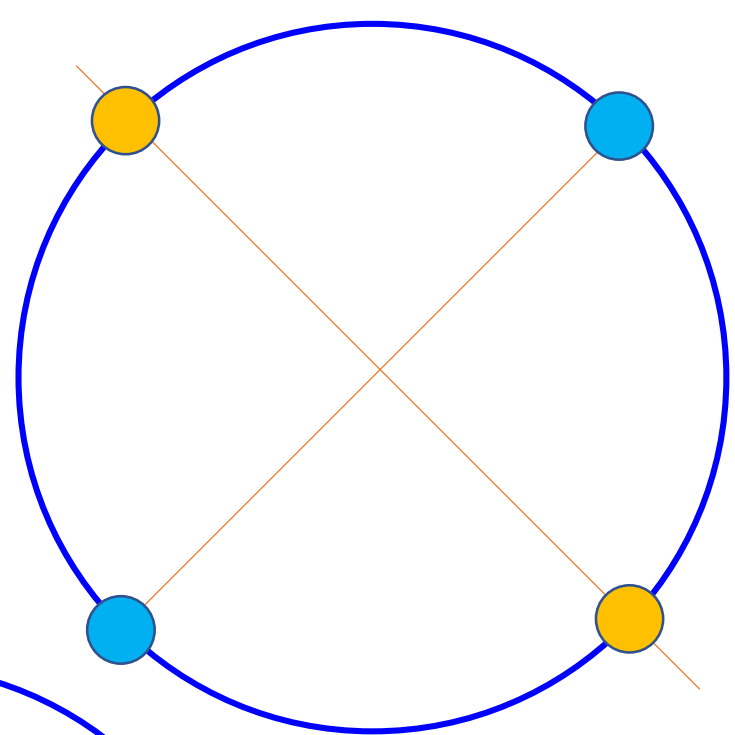
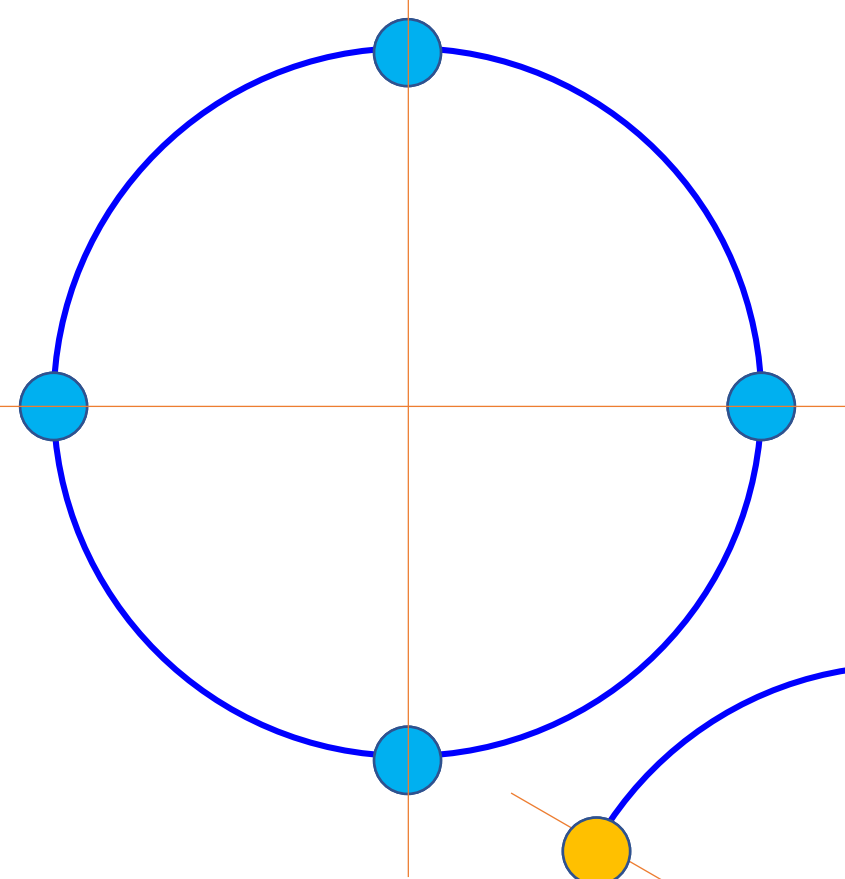
From 'Fraction of possible bunch spacing options' point of view, preferable frequency range:

400 ~ 600 MHz



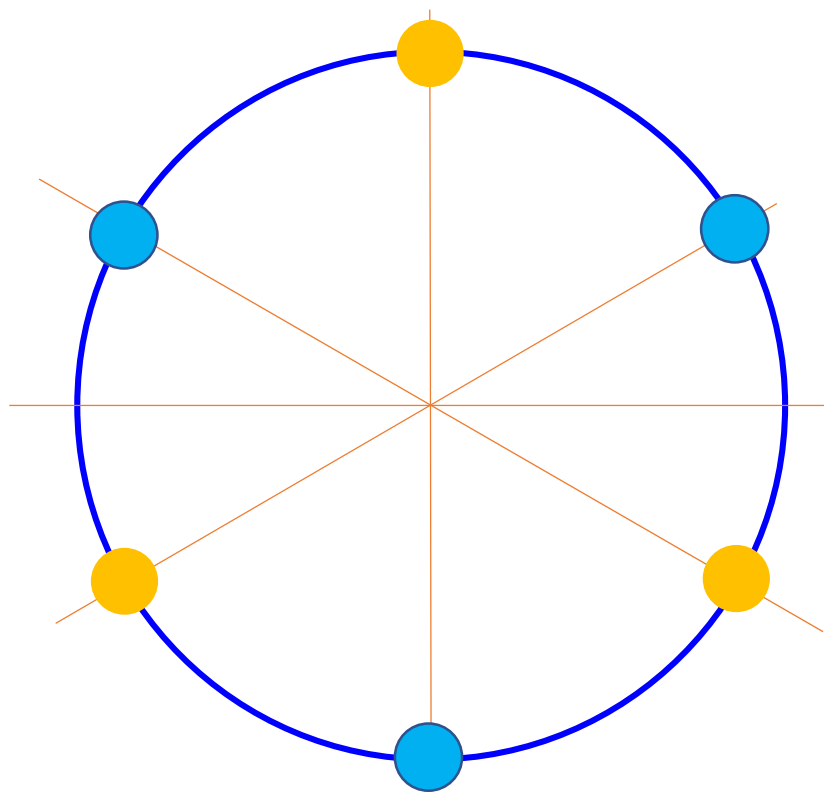
# Comparison for different $\tau_{\text{spacing,max}}$

RF frequency (MHz)	Bunch spacings $\leq 25.5$ [ns]	Percentage of possible spacings	Bunch spacings $\leq 30.5$ [ns]	Percentage of possible spacings	Bunch spacings $\leq 50.5$ [ns]	Percentage of possible spacings
364.35	2.74, 5.49, 8.23, 10.98, 13.72, 16.47, 19.21, 21.96, 24.70	<b>9/9 = 100%</b>	2.74, 5.49, 8.23, 10.98, 13.72, 16.47, 19.21, 21.96, 24.7, 27.45, 30.19]	<b>11/11 = 100%</b>	[2.74, 5.49, 8.23, 10.98, 13.72, 16.47, 19.21, 21.96, 24.7, 27.45, 30.19, 32.94, 38.42, 41.17, 49.4]	<b>15/18 = 83.3%</b>
400.79	2.5, 5.0, 7.5, 12.5, 15, 17.5, 25	<b>7/10 = 70%</b>	2.5, 5.0, 7.5, 12.5, 15, 17.5, 25	<b>7/12 = 58.3%</b>	[2.5, 4.99, 7.49, 12.48, 14.97, 17.47, 24.95, 34.93, 37.43]	<b>9/20 = 45%</b>
<b>497.34</b>	<b>[2.01 4.02 6.03 8.04 10.05 12.06 14.07 16.09 18.1 20.11, 24.13]</b>	<b>11/12 = 91.7%</b>	<b>[2.01, 4.02, 6.03, 8.04, 10.05, 12.06, 14.07, 16.09, 18.1, 20.11, 24.13, 28.15, 30.16]</b>	<b>13/15 = 86.7%</b>	<b>[2.01, 4.02, 6.03, 8.04, 10.05, 12.06, 14.07, 16.09, 18.1, 20.11, 24.13, 28.15, 30.16, 36.19, 40.21, 42.22, 48.26, 50.27]</b>	<b>18/25 = 72%</b>
563.54	[1.77, 3.55, 5.32, 7.1, 8.87, 10.65, 12.42, 14.2, 15.97, 17.74, 21.29, 24.84]	<b>12/14 = 85.7%</b>	[1.77, 3.55, 5.32, 7.1, 8.87, 10.65, 12.42, 14.2, 15.97, 17.74, 21.29, 24.84, 26.62, 30.17]	<b>14/17 = 82.3%</b>	[1.77, 3.55, 5.32, 7.1, 8.87, 10.65, 12.42, 14.2, 15.97, 17.74, 21.29, 24.84, 26.62, 30.17, 31.94, 35.49, 37.26, 42.59, 49.69]	<b>19/28 = 67.9%</b>
607.26	[1.65, 3.29, 4.94, 6.59, 8.23, 9.88, 11.53, 13.17, 16.47, 18.11, 19.76, 23.05, 24.7]	<b>13/15 = 86.7%</b>	[1.65, 3.29, 4.94, 6.59, 8.23, 9.88, 11.53, 13.17, 16.47, 18.11, 19.76, 23.05, 24.7]	<b>13/18 = 72.2%</b>	[1.65, 3.29, 4.94, 6.59, 8.23, 9.88, 11.53, 13.17, 16.47, 18.11, 19.76, 23.05, 24.7, 32.93, 34.58, 36.23, 39.52, 41.17, 46.11, 49.4]	<b>20/30 = 66.7%</b>
607.86	[1.65, 3.29, 4.94, 6.58, 8.23, 9.87, 11.52, 13.16, 16.45, 18.1, 19.74, 23.03, 24.68]	<b>13/15 = 86.7%</b>	[1.65, 3.29, 4.94, 6.58, 8.23, 9.87, 11.52, 13.16, 16.45, 18.1, 19.74, 23.03, 24.68]	<b>13/18 = 72.2%</b>	[1.65, 3.29, 4.94, 6.58, 8.23, 9.87, 11.52, 13.16, 16.45, 18.1, 19.74, 23.03, 24.68, 32.9, 34.55, 36.19, 39.48, 41.13, 46.06, 49.35]	<b>20/30 = 66.7%</b>
728.71	[1.37, 2.74, 4.12, 5.49, 6.86, 8.23, 9.61, 10.98, 12.35, 13.72, 15.1, 16.47, 19.21, 20.58, 21.96, 24.7]	<b>16/18 = 88.8%</b>	[1.37, 2.74, 4.12, 5.49, 6.86, 8.23, 9.61, 10.98, 12.35, 13.72, 15.1, 16.47, 19.21, 20.58, 21.96, 24.7, 27.45, 28.82, 30.19]	<b>19/22 = 86.4%</b>	[1.37, 2.74, 4.12, 5.49, 6.86, 8.23, 9.61, 10.98, 12.35, 13.72, 15.1, 16.47, 19.21, 20.58, 21.96, 24.7, 27.45, 28.82, 30.19, 32.93, 38.42, 41.17, 45.29, 48.03, 49.4]	<b>25/36 = 69.4%</b>
805.22	[1.24, 2.48, 3.73, 4.97, 6.21, 7.45, 9.94, 11.18, 12.42, 14.9, 18.63, 19.87, 21.11, 22.35, 24.84]	<b>15/20 = 75%</b>	[1.24, 2.48, 3.73, 4.97, 6.21, 7.45, 9.94, 11.18, 12.42, 14.9, 18.63, 19.87, 21.11, 22.35, 24.84, 29.81]	<b>16/24 = 66.7%</b>	[1.24, 2.48, 3.73, 4.97, 6.21, 7.45, 9.94, 11.18, 12.42, 14.9, 18.63, 19.87, 21.11, 22.35, 24.84, 29.81, 31.05, 37.26, 42.22, 44.71, 49.68]	<b>21/40 = 52.5%</b>
828.91	[1.21, 2.41, 3.62, 4.83, 6.03, 7.24, 8.44, 9.65, 10.86, 12.06, 14.48, 16.89, 18.1, 21.72, 24.13, 25.33]	<b>16/21 = 76.2%</b>	[1.21, 2.41, 3.62, 4.83, 6.03, 7.24, 8.44, 9.65, 10.86, 12.06, 14.48, 16.89, 18.1, 21.72, 24.13, 25.33, 28.95, 30.16]	<b>18/25 = 72%</b>	[1.21, 2.41, 3.62, 4.83, 6.03, 7.24, 8.44, 9.65, 10.86, 12.06, 14.48, 16.89, 18.1, 21.72, 24.13, 25.33, 28.95, 30.16, 33.78, 36.19, 42.22, 43.43, 48.26]	<b>23/41 = 56.1%</b>



Bunch (trains) must have 4-fold symmetry to make particle collision occurring simultaneously in the 4 IPs; Two-fold symmetry only ensures a pair of IPs out of 4 IPs exist collisions at the same time

Question:  
The 4-fold symmetry of bunch trains must require the 4-fold symmetry of the harmonics (or dividable by 4)?



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