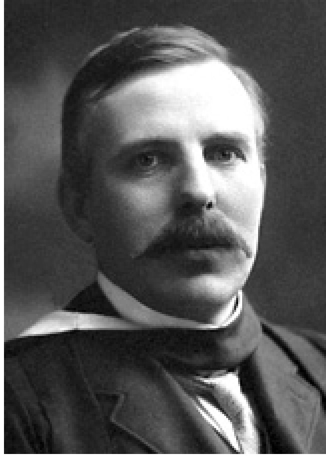
ESIPAP 2022

Gas-based detectors, how they work,

course 1,
laboratory training session 4,
Friday February 4th 13h30-14h30,
in part pre-recorded

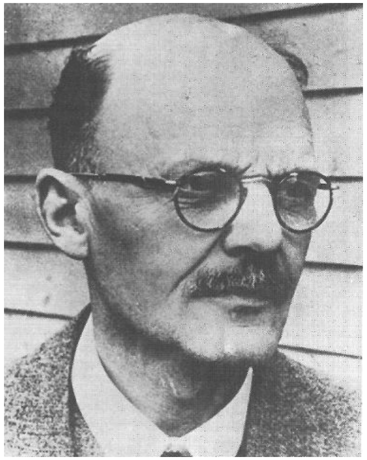
Gas-based detectors, a brief history

E(a)rnest Rutherford
(1871-1937)

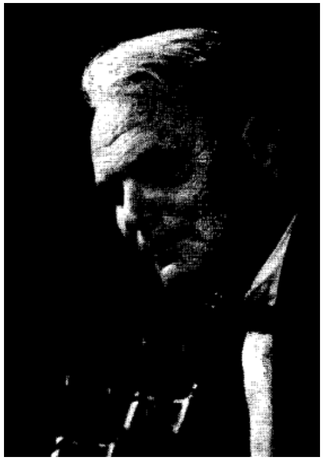


Geiger counter

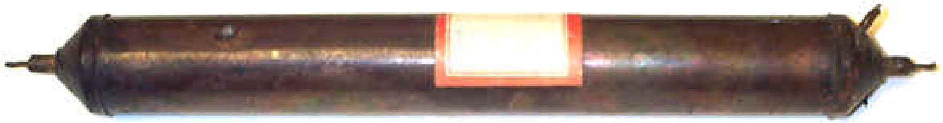
- Detects radiation by discharge;
- can count α, β and γ particles (at low rates ...);
- no tracking capability.
- 1908: Ernest Rutherford and Hans Geiger
- 1928: Hans Geiger and Walther Müller



Hans Geiger
(1882-1945)



Walt(h)er Müller
(1905-1979)

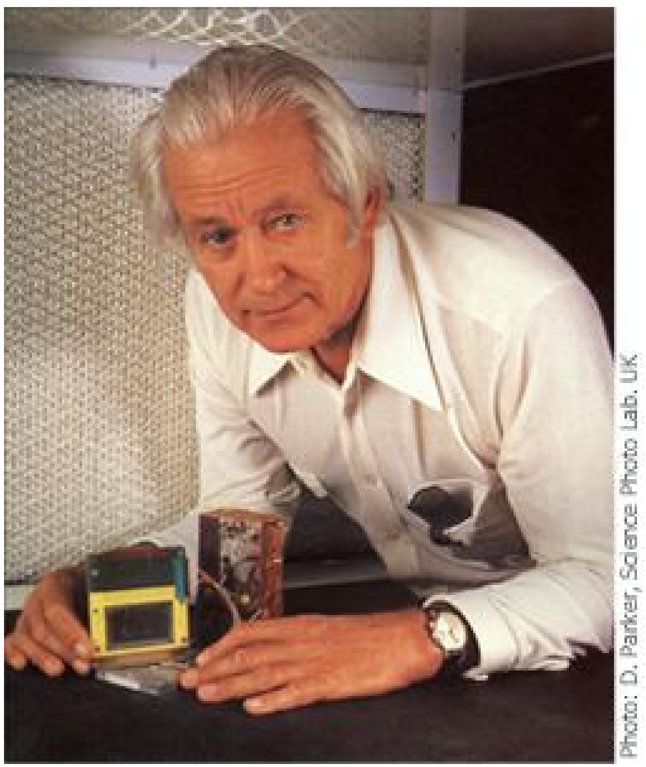


A Geiger-Muller counter built in 1939 and used in the 1947-1950 for cosmic ray studies in balloons and on board B29 aircraft by Robert Millikan et al.

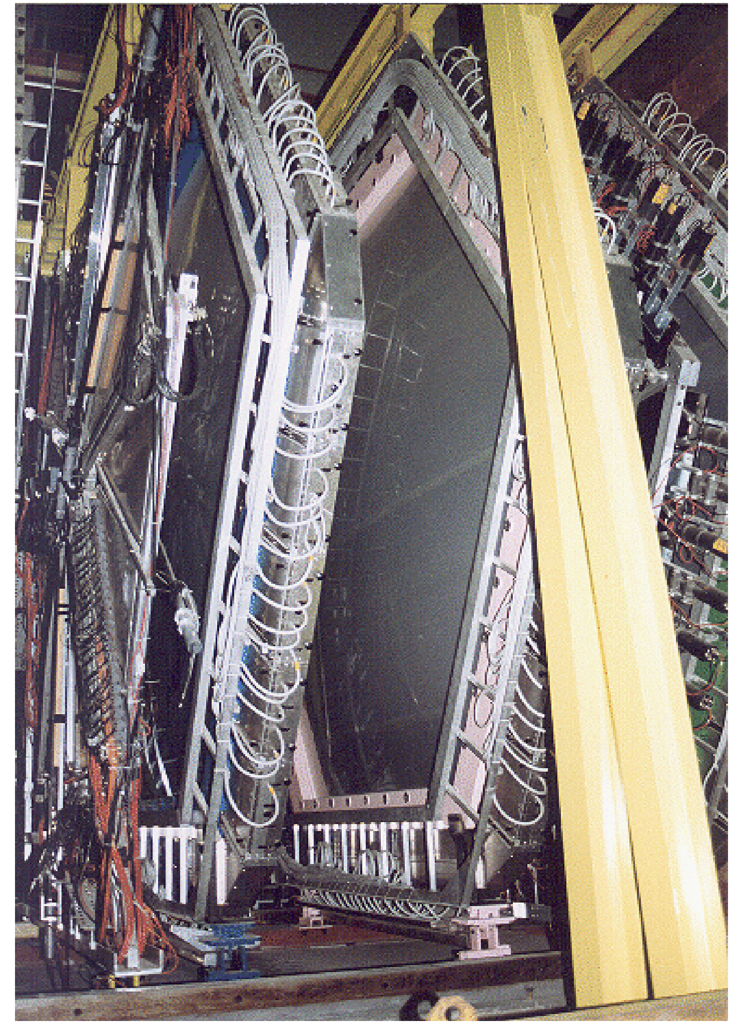
Made of copper, 30 cm long

MWPC

- First gaseous tracking device
- 1968: Georges Charpak



Georges Charpak
(1924-1992-2010)

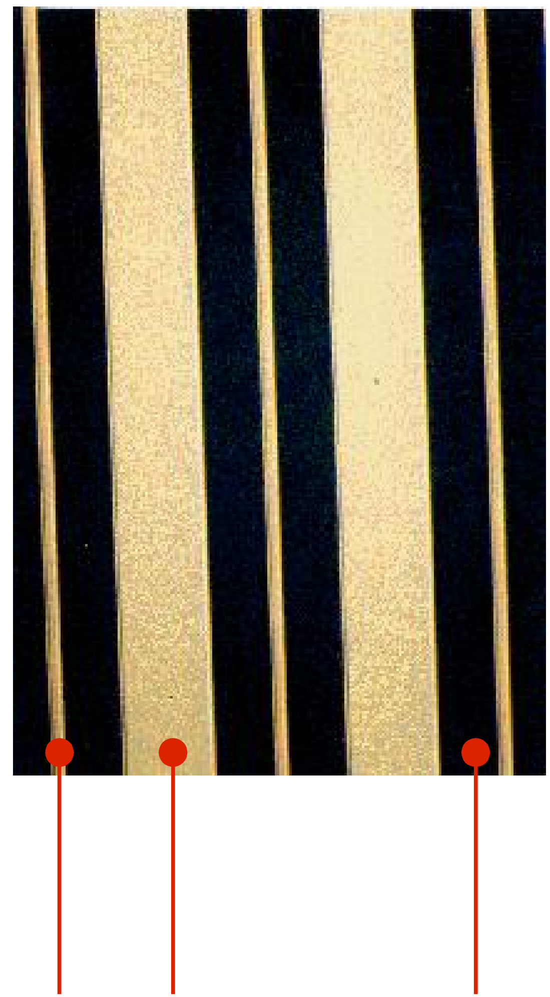


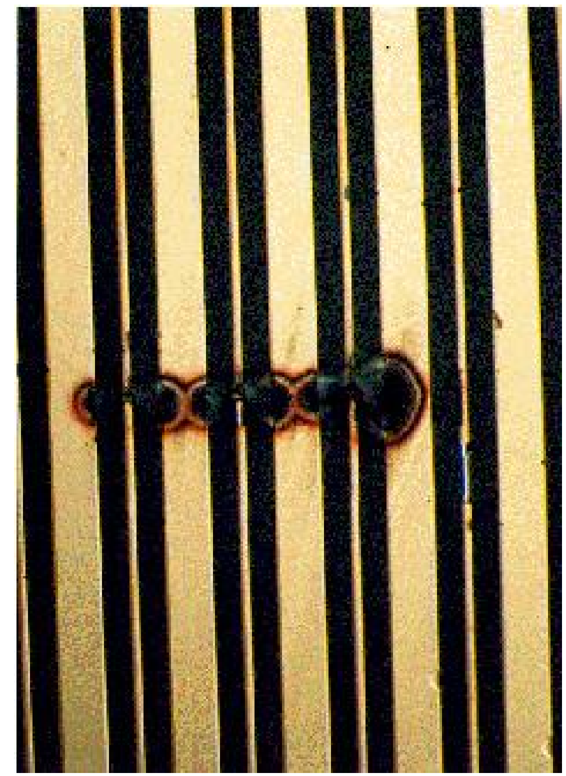
One of the NA60 muon chambers

MSGC: an early MPGD

- Built using solid-state techniques;
- good resolution;
- poor resistance to high rates.

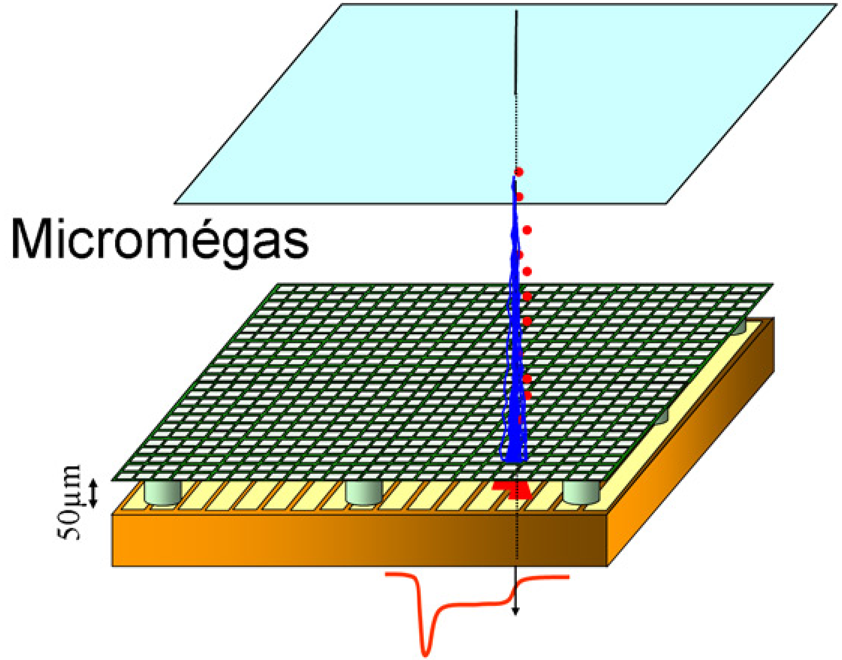
- 1988: Anton Oed

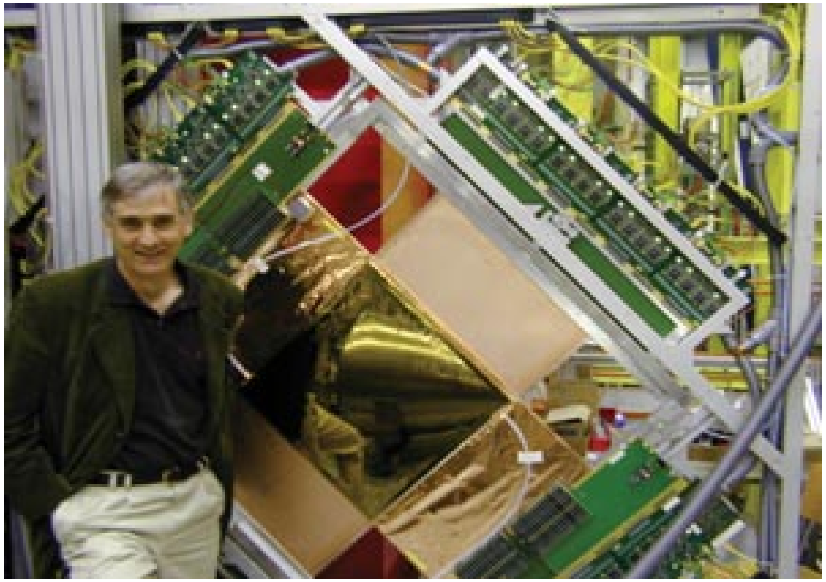




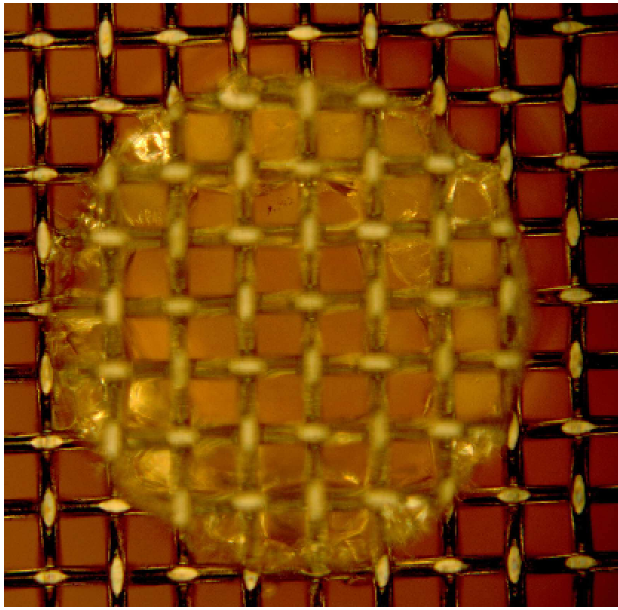
Micromégas

- Fast, rate tolerant tracking device.
- 1994: Yannis Giomataris and Georges Charpak.





Yannis Giomataris

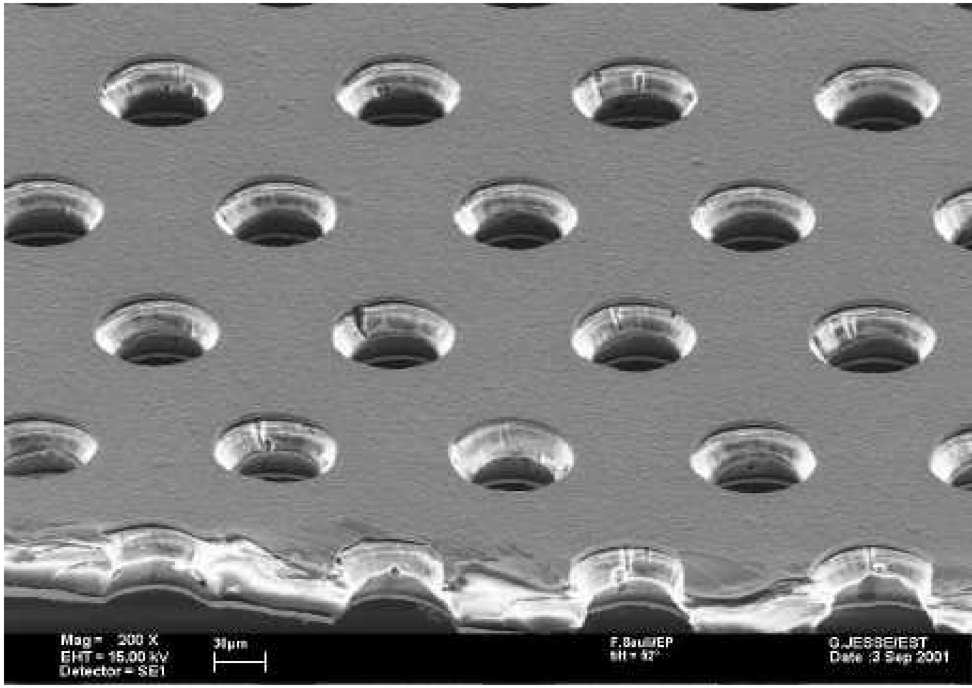


Wire diameter: 18 µm,
Pitch: 63 µm, Gap: 192 µm

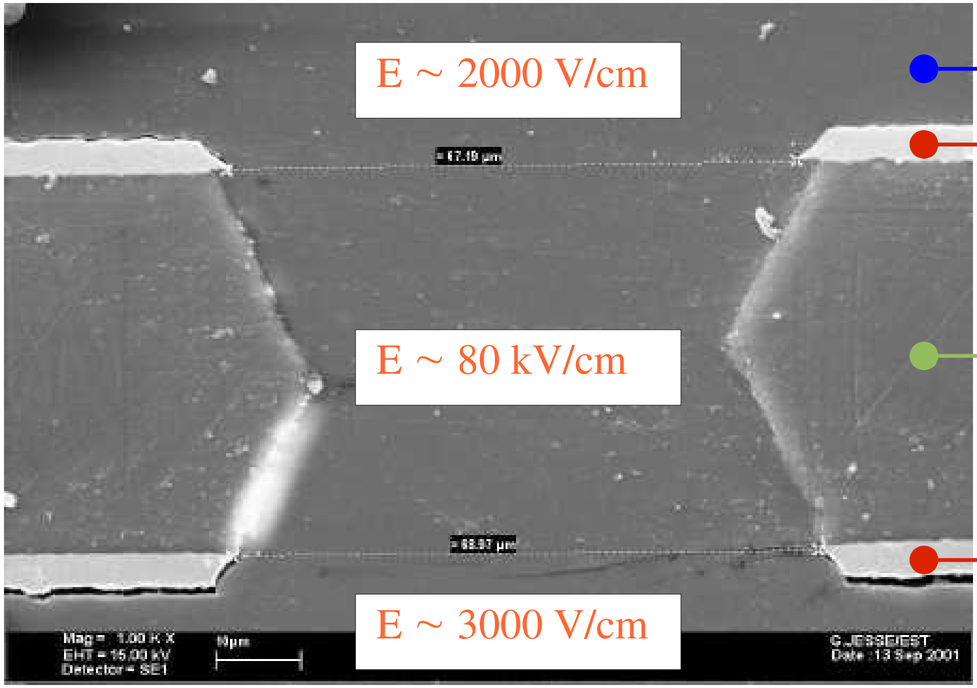
[Purba Bhattacharya et al., 10.1016/j.nima.2013.07.086; ILC NewsLine]

GEM

- Originally, a “pre-amplifier”.
- 1996: Fabio Sauli



A few electrons enter here

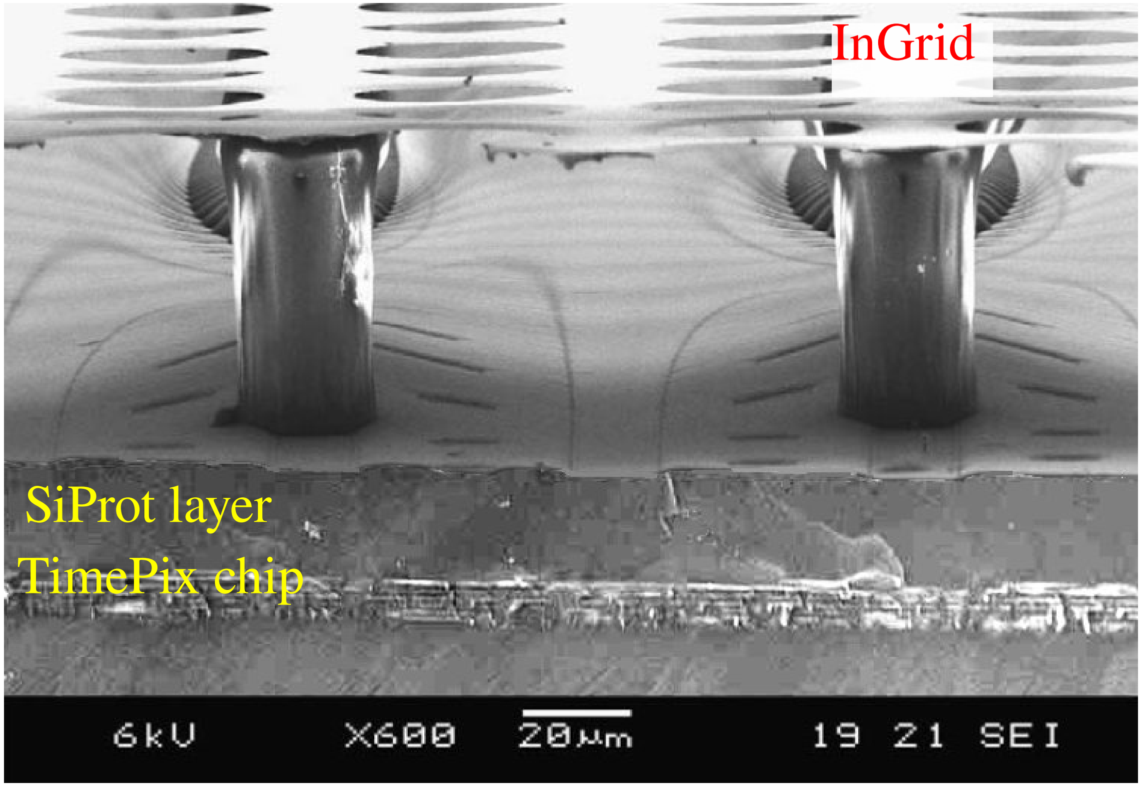


Many electrons exit here

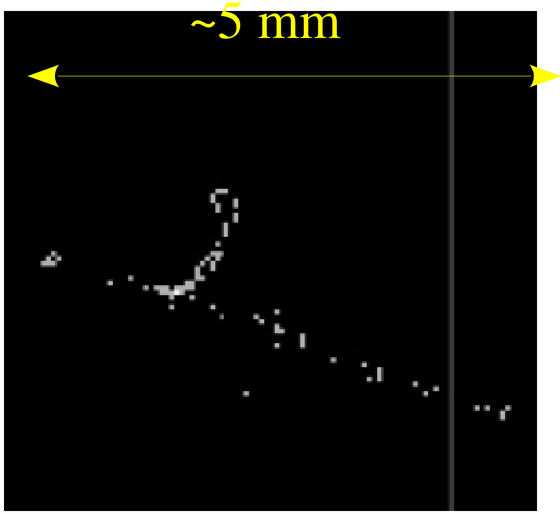
Fabio Sauli

Gossip

- The “electronic bubble chamber”.



Harry van der Graaf (r)



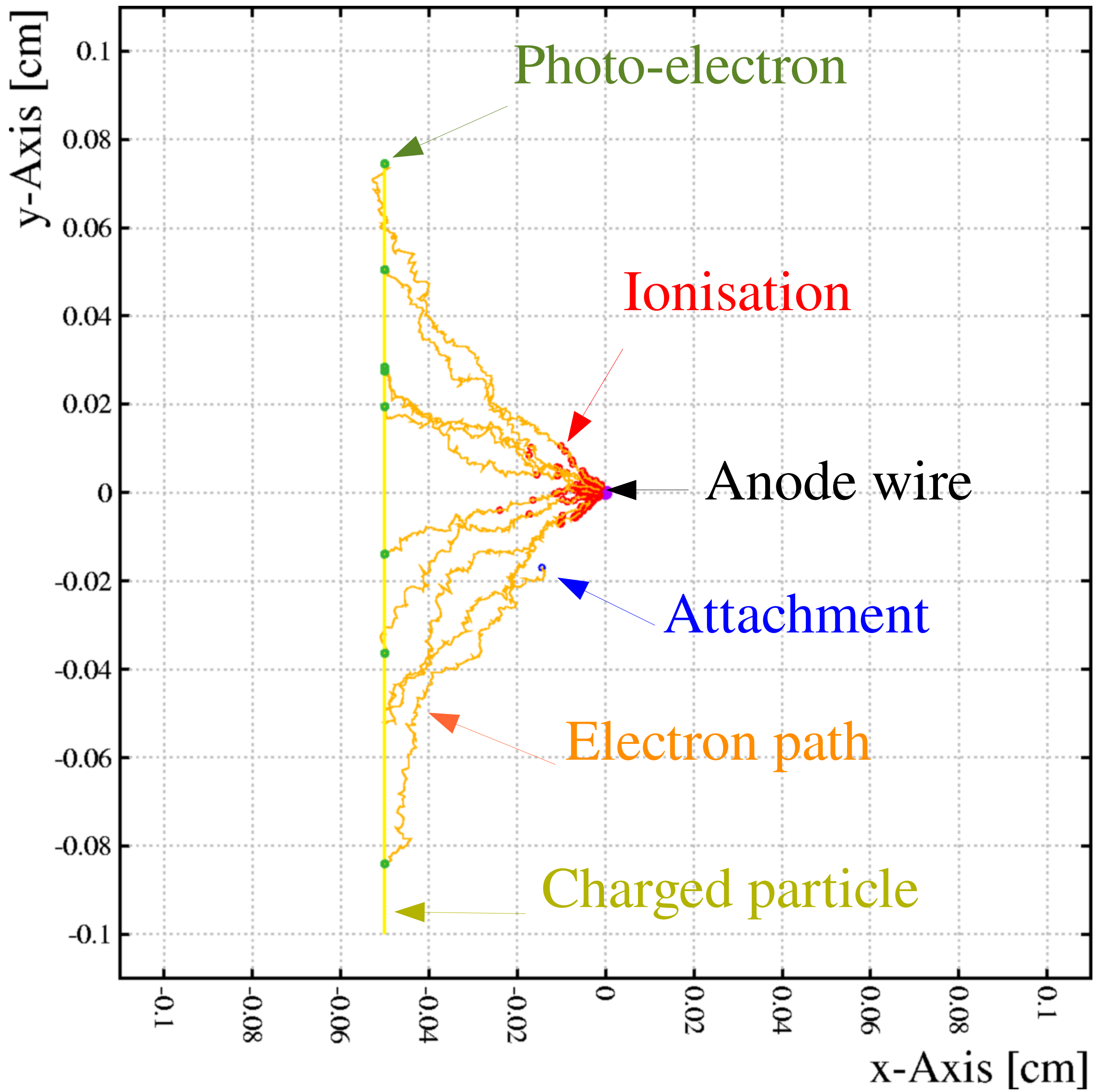
δ-electrons made visible in He/iC_4H_{10}, using a modified MediPix, ~2004.

How they work

- Gas-based detectors all work according to much the same principles:
  - a charged particle passing through the gas ionises a few gas molecules;
  - the electric field in the gas volume transports the ionisation electrons and provokes multiplication;
  - the movement of electrons and ions leads to induced currents in electrodes;
  - the signals are processed and recorded.

At the 100 µm scale

- Example:
  - CSC-like structure,
  - Ar 80 % CO_2 20 %,
  - 10 GeV µ.
- Electron are shown every 100 collisions, but have been tracked rigorously.
- Ions are not shown.



Ionisation

[Four Curies: Pierre, Marie, Irène and Pierre's father, around 1904 at the BIPM]

1896: Ionisation by radiation

- Early in the study of radioactivity, ionisation by radiation was recognised:

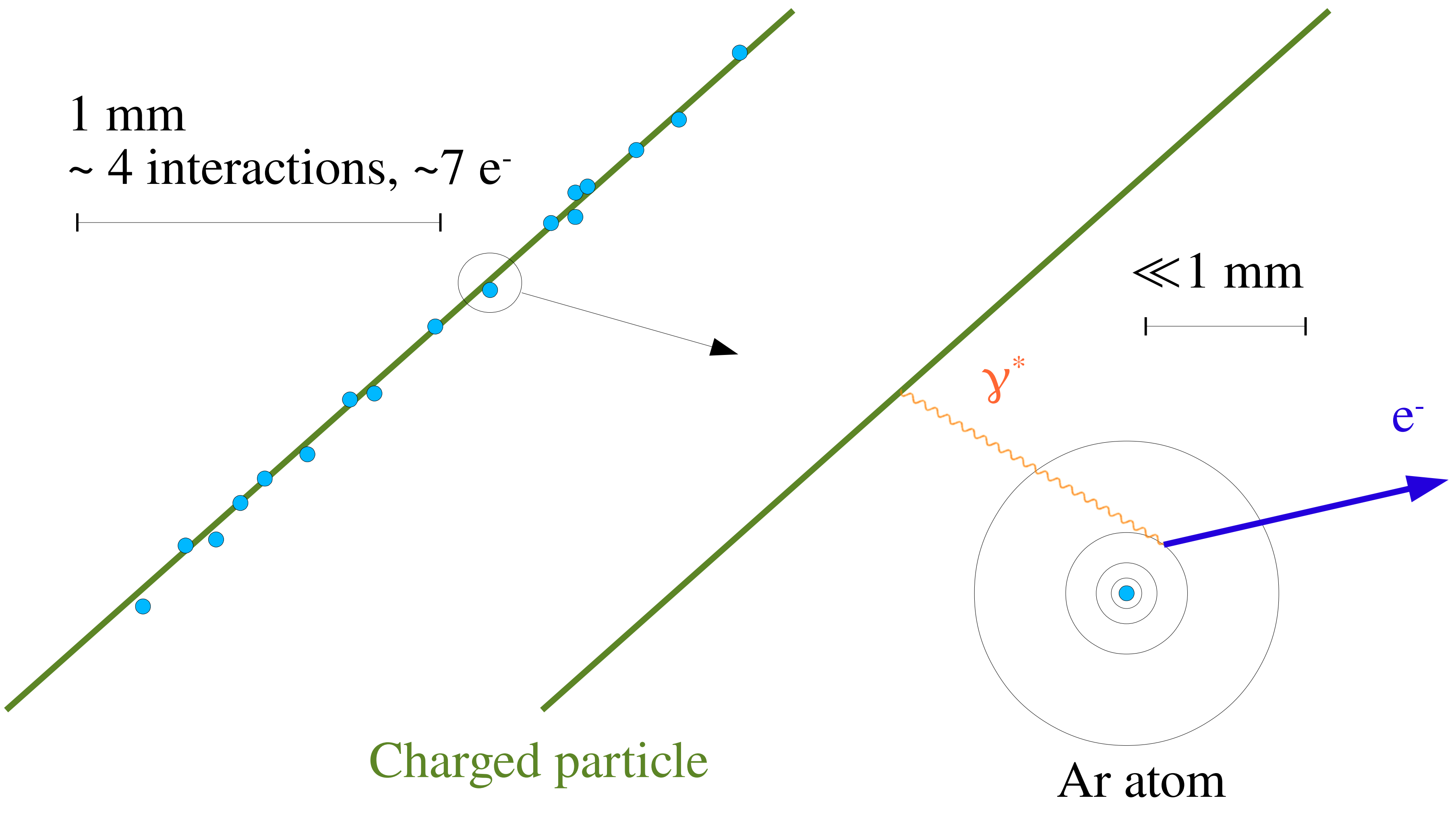
” Becquerel discovered in 1896 the special radiating properties of uranium and its compounds. Uranium emits very weak rays which leave an impression on photographic plates. These rays pass through black paper and metals; they make air electrically conductive. “

[Pierre Curie, Nobel Lecture, June 6th 1905]

“A sphere of charged uranium, which discharges spontaneously in the air under the influence of its own radiation, retains its charge in an absolute vacuum. The exchanges of electrical charges that take place between charged bodies under the influence of the new rays, are the result of a special conductivity imparted to the surrounding gases, a conductivity that persists for several moments after the radiation has ceased to act.”

[Antoine Henri Becquerel, Nobel Lecture, December 11th 1903]

Virtual photon exchange



Beauty of ionisation

- gas is light:
  - energy loss is small – non-destructive.

- ionisation happens often at low energy,
  - high density of electron deposition,
  - high resolution tracking.

Core formulae PAI model


Wade Allison


John Cobb

- Key: photo-absorption cross section $\sigma_\gamma(E)$

$$\frac{\beta^2\pi}{\alpha}\frac{d\sigma}{dE}=\frac{\sigma_\gamma(E)}{E}\log\left(\frac{1}{\sqrt{(1-\beta^2\epsilon_1)^2+\beta^4\epsilon_2^2}}\right)+$$ Relativistic rise

Cross section to transfer an energy E in a single collision of an incident charged particle with an atom.

$$\frac{1}{N\hbar c}\left(\beta^2-\frac{\epsilon_1}{|\epsilon|^2}\right)\theta+$$ Черенков radiation

$$\frac{\sigma_\gamma(E)}{E}\log\left(\frac{2m_e c^2\beta^2}{E}\right)+$$ Resonance region

$$\frac{1}{E^2}\int_0^E \sigma_\gamma(E_1)\,dE_1$$ Rutherford scattering

With:

$$\epsilon_2(E)=\frac{N_e\hbar c}{EZ}\sigma_\gamma(E)$$

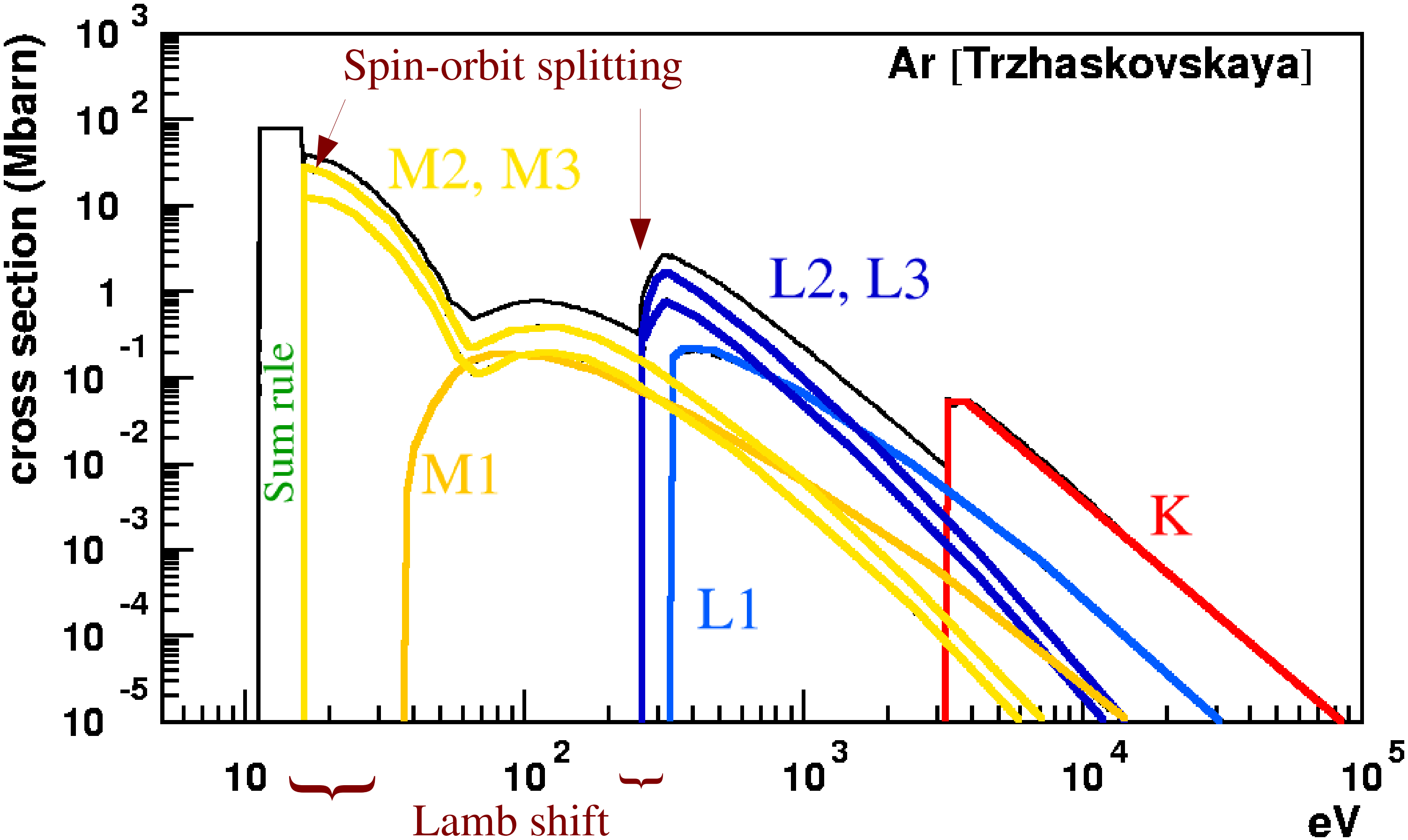
$$\epsilon_1(E)=1+\frac{2}{\pi}\mathrm{P}\int_0^\infty\frac{x\,\epsilon_2(x)}{x^2-E^2}\,dx$$

$$\theta=\arg(1-\epsilon_1\beta^2+i\,\epsilon_2\beta^2)=\frac{\pi}{2}-\arctan\frac{1-\epsilon_1\beta^2}{\epsilon_2\beta^2}$$

Photo-absorption in Ar (Heed)

Igor Smirnov

- Argon has 3 shells, hence 3 groups of lines:



K = 1s

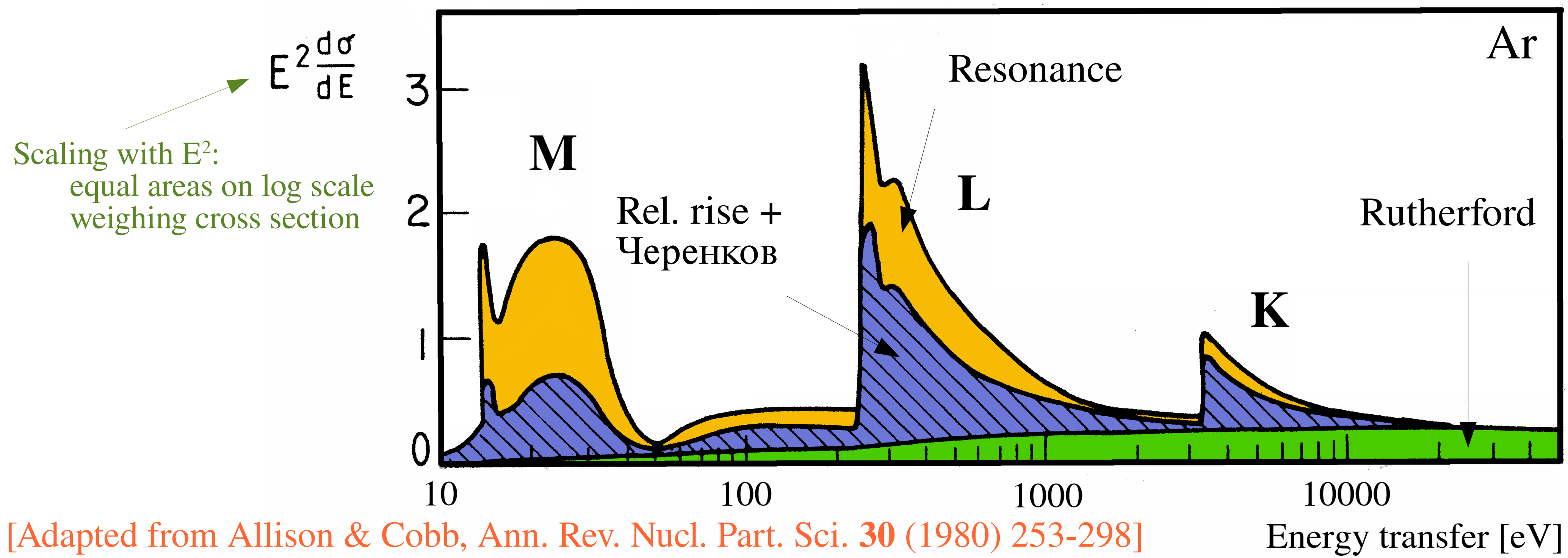
L1 = 2s
L2 = 2p 1/2
L3 = 2p 3/2

M1 = 3s
M2 = 3p 1/2
M3 = 3p 3/2

[Plot from Igor Smirnov]

Importance of the PAI model terms

- All electron orbitals (shells) participate:
  - outer shells: frequent interactions, few electrons;
  - inner shells: few interactions, many electrons.
- All terms in the formula are important.



[Adapted from Allison & Cobb, Ann. Rev. Nucl. Part. Sci. **30** (1980) 253-298]

Electric fields

William Gilbert
(1544-1603)

1600: “Electric force”

- 1544: William Gilbert born in Colchester
- 1600: De magnete, magneticisque corporibus,
  et de magno magnete tellure.
- Concluded that the Earth is a magnet; and he is credited with the first use of the term “electric force”:

> vim illam electricam nobis placet appella-
> re quæ ab humore prouenit

- 1601: Physician to Elizabeth I and James I.

[Guilielmi Gilberti, *De magnete* ..., excudebat Petrus Short anno MDC, Londini, courtesy Universidad Complutense de Madrid and Google books]

Field calculation techniques

- Closed expressions, "analytic method":
  - almost all 2d structures of wires, planes + periodicities;
  - dielectrics and space/surface charge are laborious;
  - fast and precise, if applicable.

- Finite element method:
  - 2d and 3d structures, with or without dielectrics;
  - several major intrinsic shortcomings.

- Integral equations or Boundary element methods:
  - equally comprehensive as FEM, without the intrinsic flaws;
  - technically challenging and emerging;
  - consumes more CPU time than FEM, but catching up.

- Finite differences method:
  - used for iterative, time-dependent calculations.

1814: Cauchy-Riemann equations

Augustin Louis Cauchy
(Aug 21st 1789 – May 23rd 1857)

- Express the existence of a derivative of a complex analytic function $f = u + \mathrm{i}\, v$:

$$f'(z) = \frac{\partial f}{\partial x} = \frac{\partial u}{\partial x} + \mathrm{i}\frac{\partial v}{\partial x}$$
$$= \frac{\partial f}{\partial \mathrm{i}\, y} = -\mathrm{i}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}$$

Georg Friedrich Bernhard Riemann
(Sep 17st 1826 – Jul 20th 1866)

- implies that the real part u is harmonic:

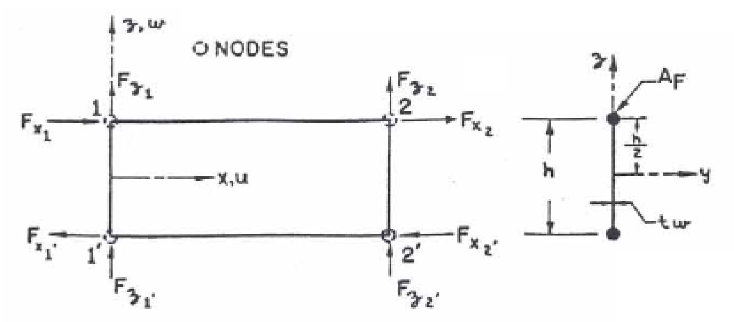
$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 v}{\partial x \partial y} = \frac{\partial^2 v}{\partial y \partial x} = -\frac{\partial^2 u}{\partial y^2} \quad \rightarrow \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \nabla^2 u = 0$$

Reference: A.L. Cauchy, *Sur les intégrales définies* (1814). This *mémoire* was read in 1814, but only submitted to the printer in 1825. Riemann was born a year later.

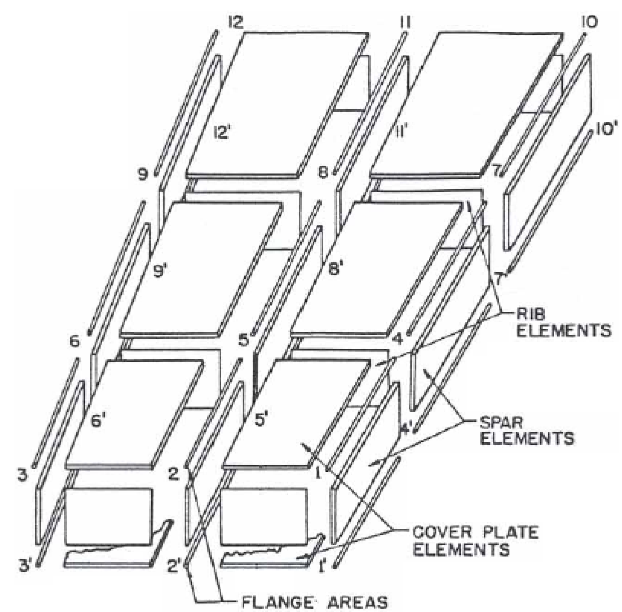
Ιωάννης Αργύρης
(1913-2004)

Aircraft wings – finite elements

- "*Stiffness and Deflection Analysis of Complex Structures*", a study in the use of the finite element technique (then called "direct stiffness method") for aircraft wing design.



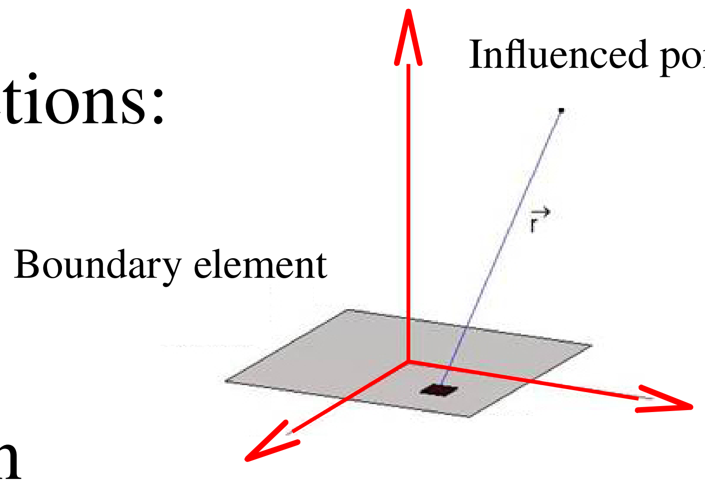
$$[K] = \frac{6EI}{Lh^2(1+4n)} \begin{array}{c} \begin{array}{cccccc} u_1 & v_1 & w_1 & u_2 & v_2 & w_2 \end{array} \\ \begin{bmatrix} (4/3)(1+n) & & & & & \\ 0 & 0 & & & & \\ -(h/L) & 0 & h^2/L^2 & & & \\ (2/3)(1-2n) & 0 & -(h/L) & (4/3)(1+n) & & \\ 0 & 0 & 0 & 0 & 0 & \\ h/L & 0 & -(h^2/L^2) & h/L & 0 & h^2/L^2 \end{bmatrix} \end{array}$$

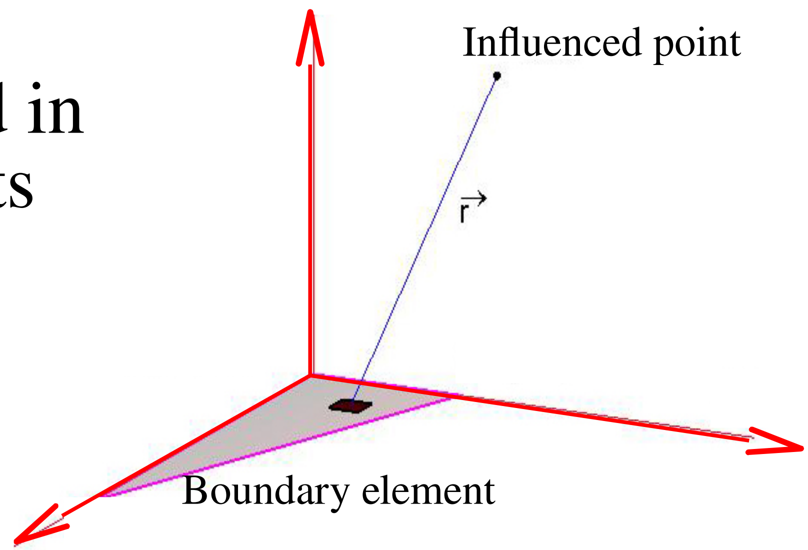


[M.J. Turner, R.W. Clough, H.C. Martin and L.J. Topp, *Stiffness and Deflection Analysis of Complex Structures*, J. Aero. Sc. **23** (1956), 805-824. MJT & LJT with Boeing.]

neBEM's Green's functions

- neBEM has only 3 Green's functions:
  - rectangle;
  - right-angled triangle;
  - line segment.
- The Green's functions have been computed by integrating a uniform charge distribution across the element.
- This avoids the nodal charges found in several BEM methods. But the joints between elements still have a jump.





Electron transport

Mean free path in argon

- Literature will tell you:
  - e^- cross section Ar atom: $\sigma \approx 1.5\ 10^{-16}$ cm^2
  - atoms per unit volume: $n_0 \approx 2.7\ 10^{19}$ atoms/cm^3
- Mean free path for an electron ?
  - An electron hits all atoms of which the centre is less than a cross section σ radius from its path;
  - over a distance L, the electron hits $n_0\ \sigma\ L$ atoms;
  - mean free path = distance over which it hits 1 atom;

$$\lambda_e = 1/(\sigma\, n_0) \approx 2.5\ \mu\text{m}$$

  - much larger than:
    - 4 nm — distance between atoms, and
    - 140-600 pm — typical gas molecule diameters.

Drift velocity in electric fields

- Imagine that an electron stops every time it collides with a gas molecule and then continues along E.

- To cover a distance λ_e it will need a time t:

$$\frac{1}{2}\frac{qE}{m_e}t^2 = \lambda_e, \qquad t = \sqrt{\frac{2\lambda_e m_e}{qE}}, \qquad \bar{v} = \frac{\lambda_e}{t} = \sqrt{\frac{\lambda_e qE}{2m_e}}$$

- which gives:

$$\bar{v} \approx 13\,\mathrm{cm}/\mu\mathrm{s} \quad \text{for} \quad E = 1\,\mathrm{kV/cm}$$

Drift velocity in argon

- Compare with a Magboltz calculation for pure argon:
  - $\sqrt{E}$ dependence is not too far off, although linearly proportional is more common at low field,

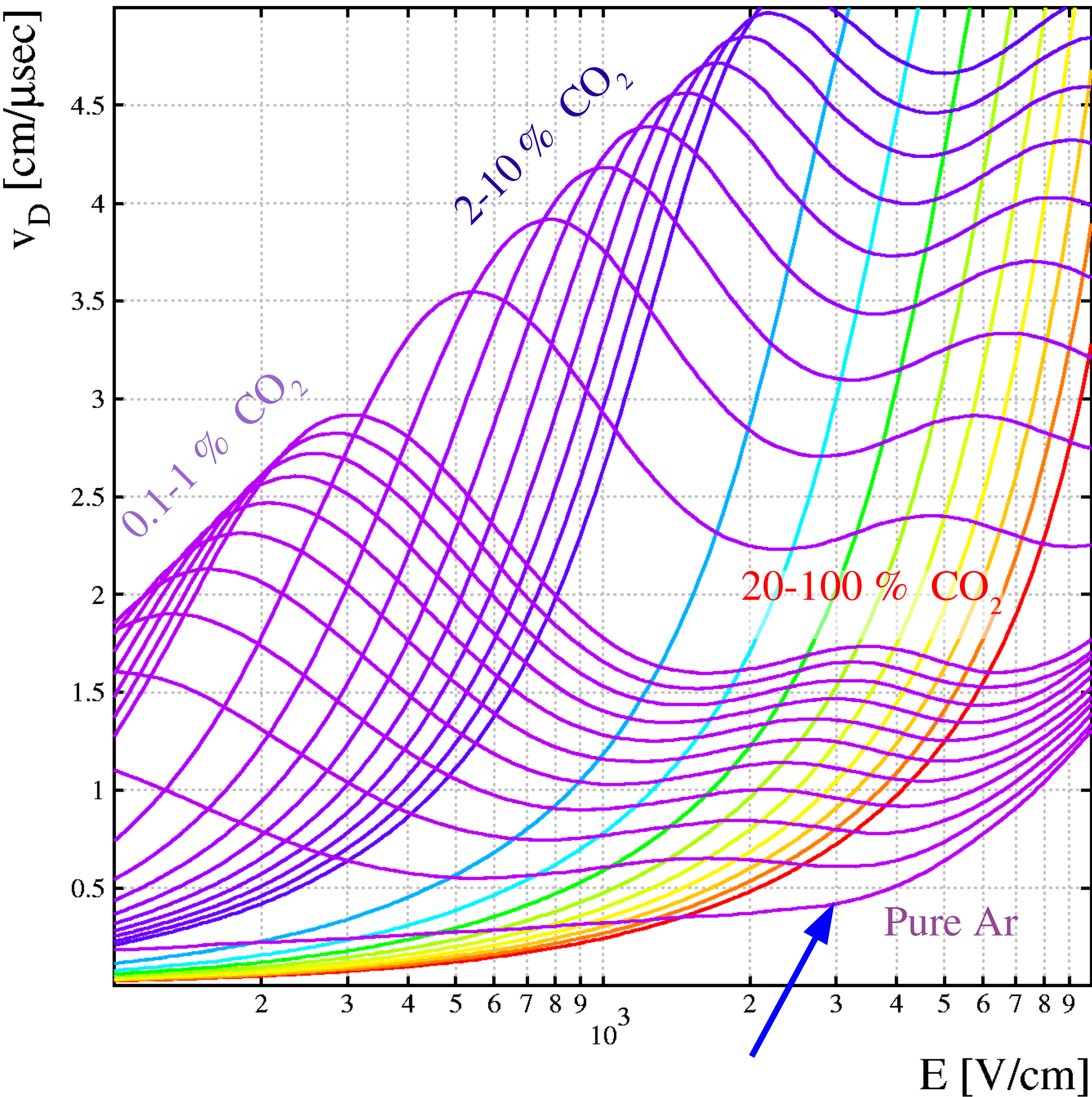
    BUT

  - the velocity is *vastly* overestimated ! Magboltz finds a velocity that is *30 times* smaller ...

    WHY ?

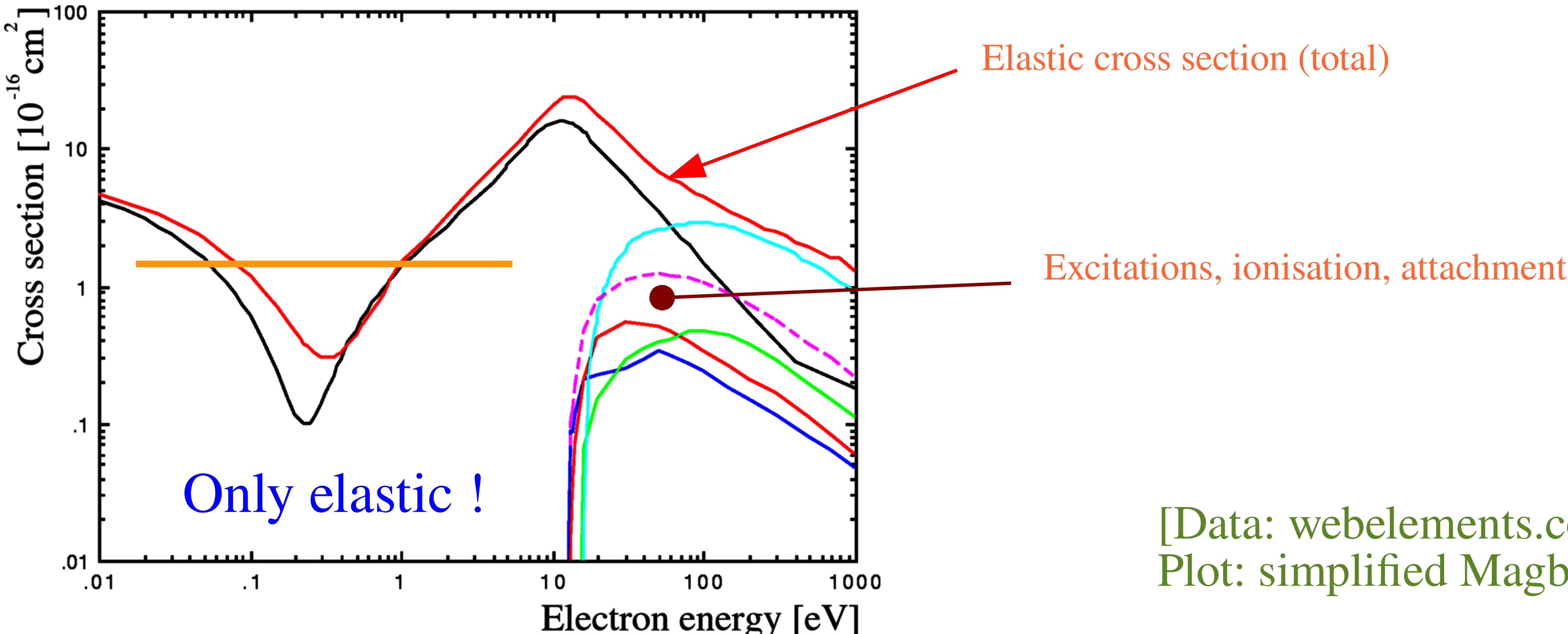
Adding CO_2

- CO_2 makes the gas faster, dramatically.
- Additives like CO_2 are called “quenchers” or “admixtures”.
- Drift velocities calculated by Magboltz for Ar/CO_2 at 3 bar.
  (Note where the arrow is !)



Cross section of argon

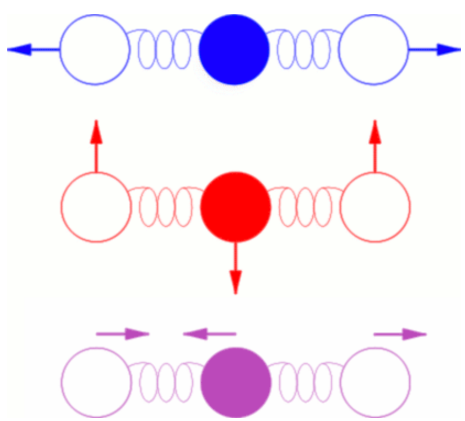
- Cross section in a hard-sphere model:
  - Radius: ~70 pm
  - Surface: $\sigma = \pi\left(70\ 10^{-10}\,\mathrm{cm}\right)^2 \approx 1.5\ 10^{-16}\,\mathrm{cm}^2 = 150\,\mathrm{Mb}$

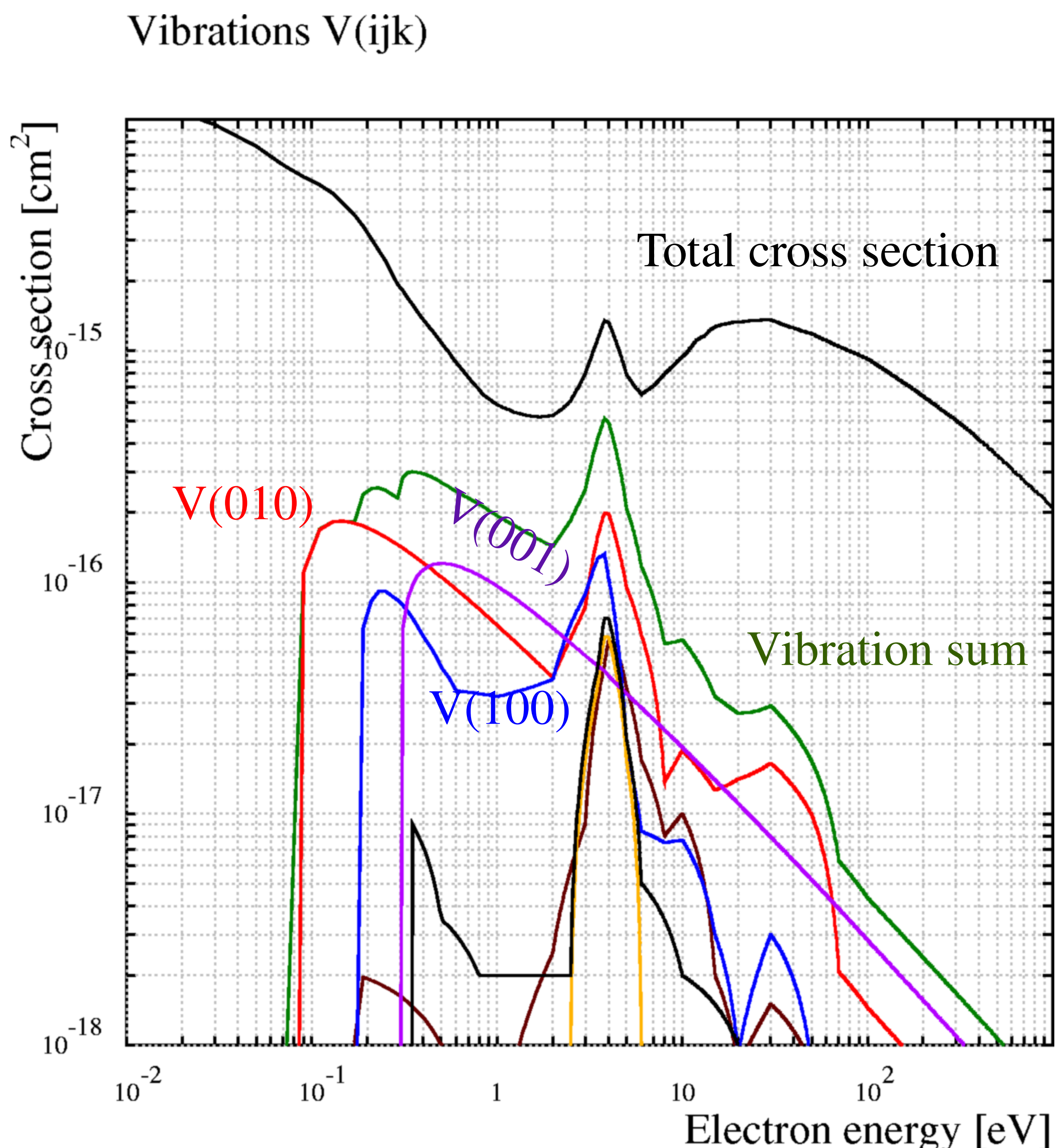


[Data: webelements.com
Plot: simplified Magboltz]

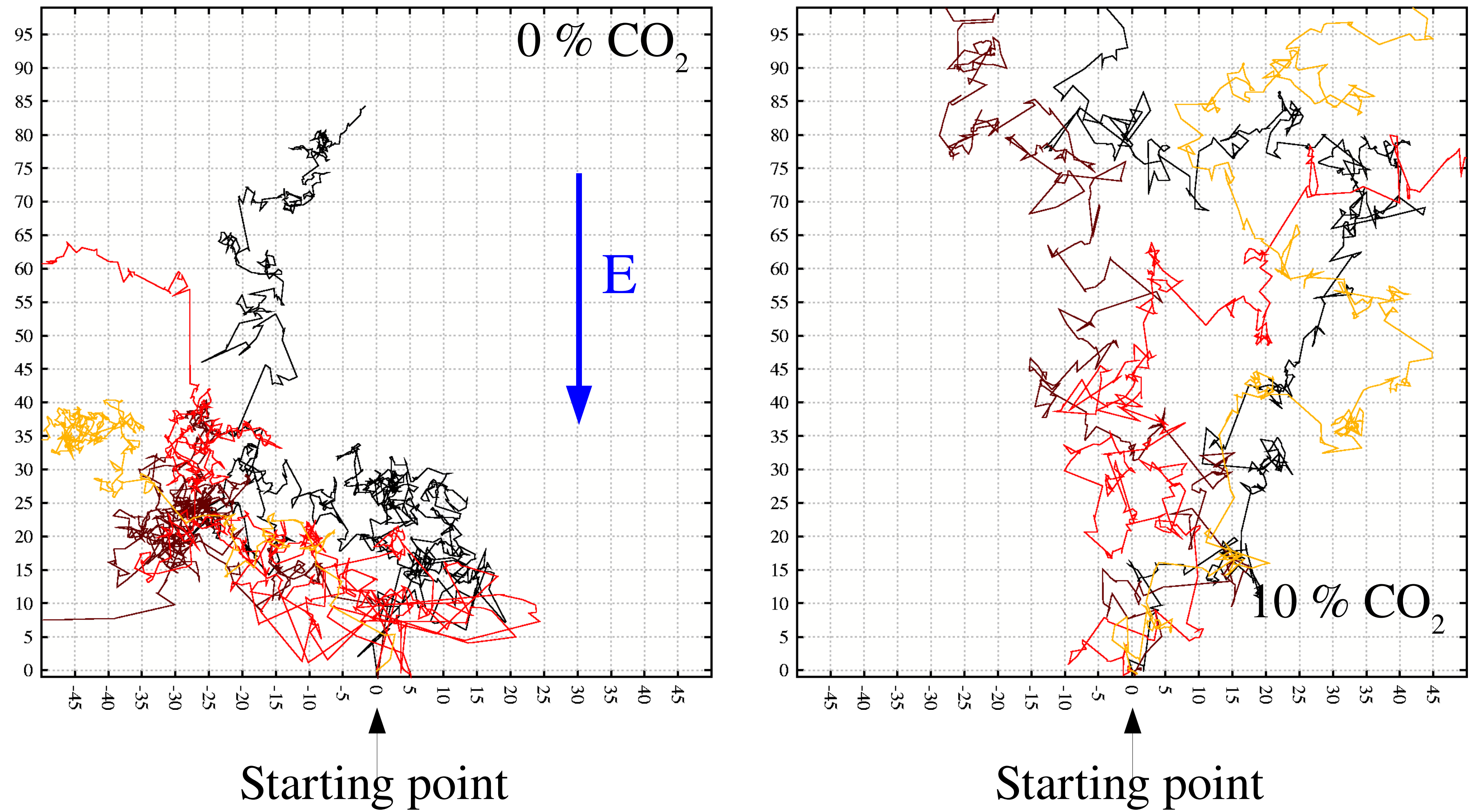
CO_2 – vibration modes

- CO_2 is linear:
  - O – C – O
- Vibration modes are numbered V(*ijk*)
  - *i*: symmetric,
  - *j*: bending,
  - *k*: anti-symmetric.





Electrons in Ar/CO_2 at E=1 kV/cm



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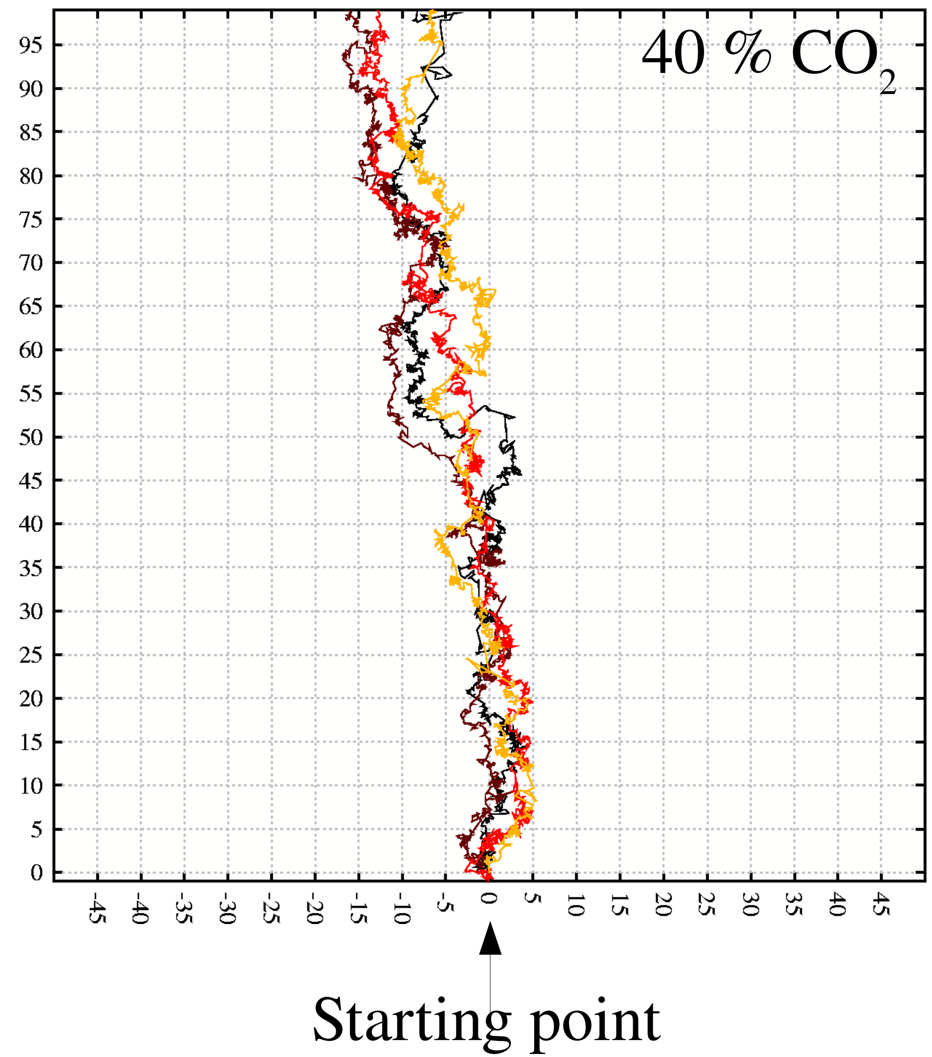
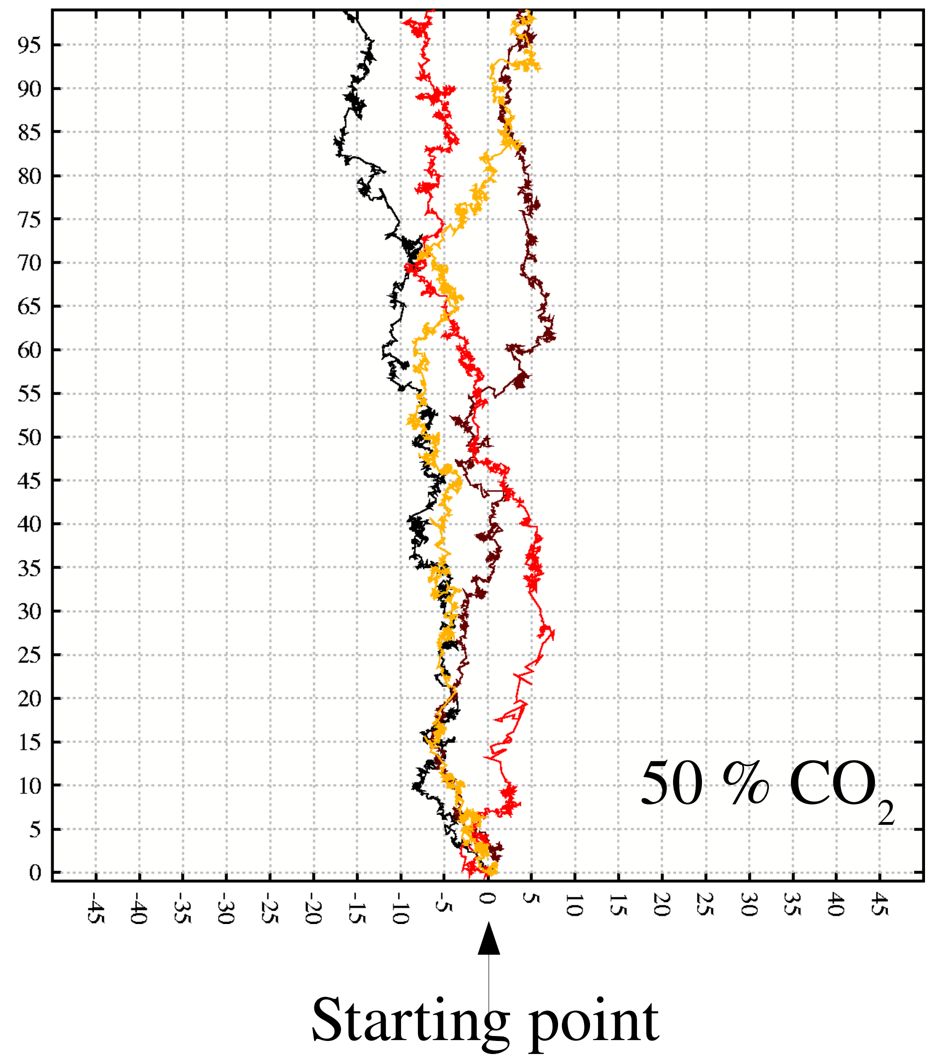


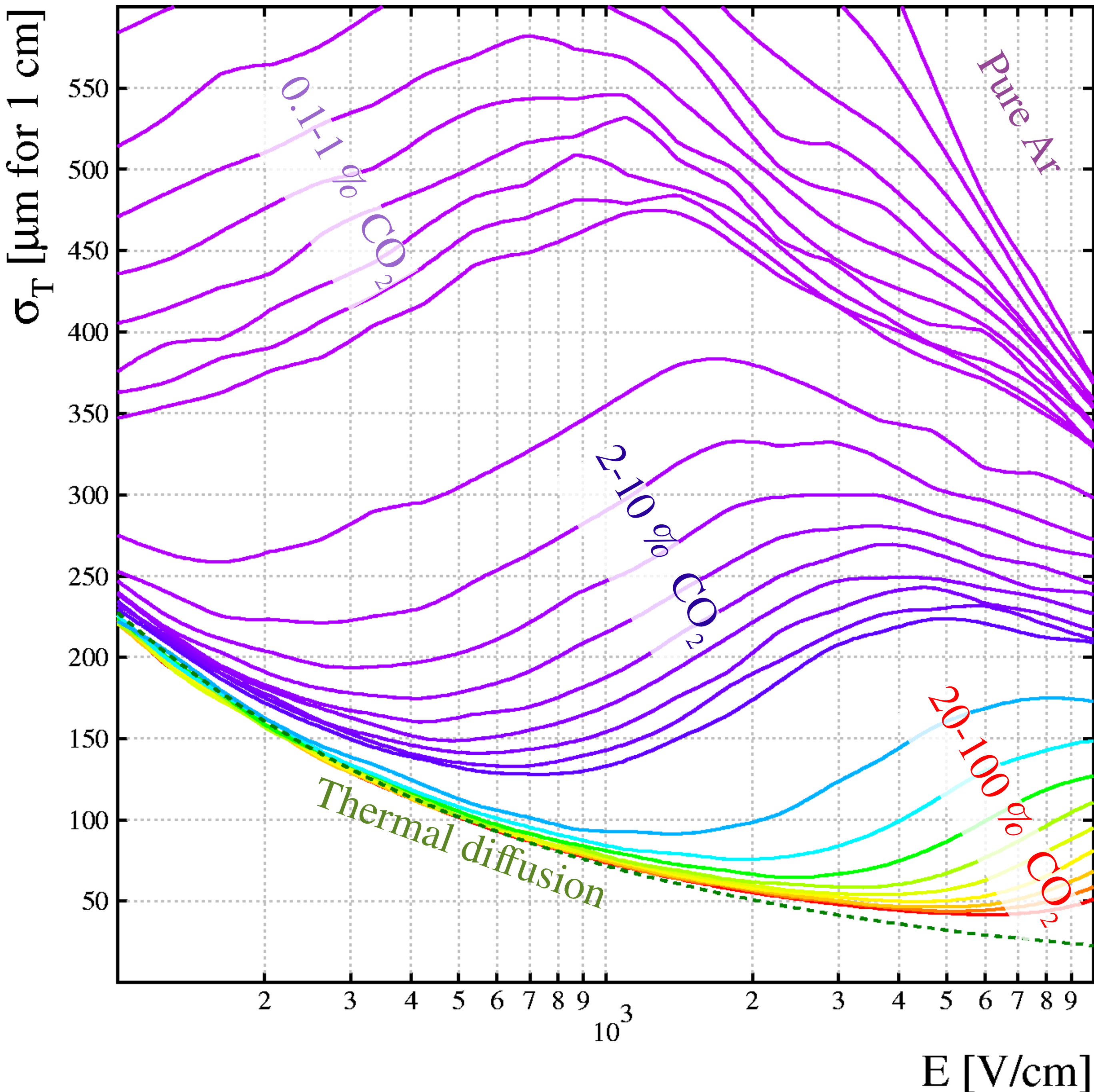


Diffusion

- The combination of a high velocity and low drift velocity implies that the electrons scatter a lot.
- Diffusion = RMS of the difference between the actual and the average movement

Adding CO_2

- Transverse diffusion is much reduced by CO_2.
- Calculated by Magboltz for Ar/CO_2 at 3 bar.



Attachment

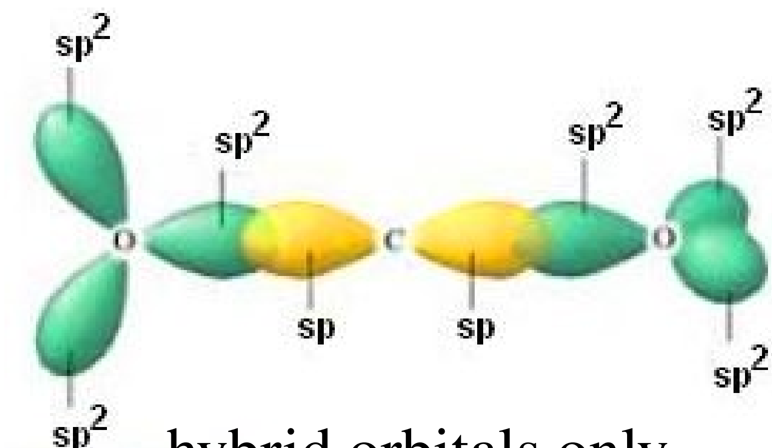
- Some quencher gases can attach electrons.

- Energy-momentum conservation requires:
  - 3-body interaction or
  - dissociation.

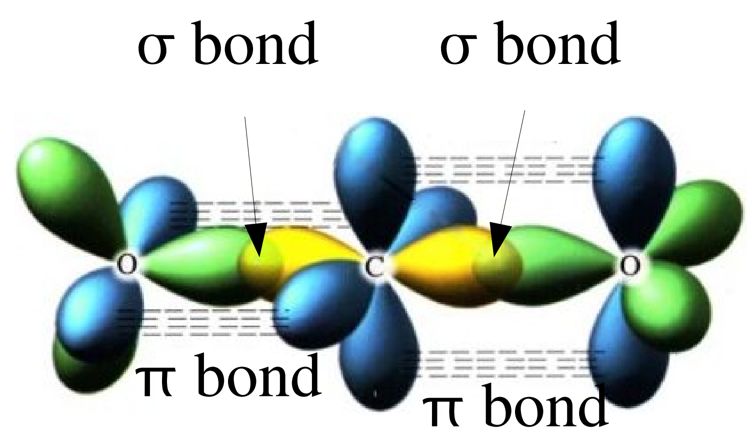
- Examples:
  - O_2: mostly 3-body O_2^- and at higher ϵ 2-body dissociative;
  - H_2O: $[H_2O]_n$ has positive electron affinity, H_2O probably not;
  - CF_4: mostly dissociative $F^- + CF_3$, $F + CF_3^-$ (below 10 eV);
  - SF_6: SF_6^{-*} < 0.1 eV, then $F^- + SF_n^-$ (n=3, 4, 5)
  - CS_2: negative ion TPC;
  - CO_2: O^-, $[CO_2]_n^-$ but no CO_2^- (4 eV and 8.2 eV).

Attachment in CO_2

- CO_2 is a linear molecule:



hybrid orbitals only,
p-orbitals not shown



[Source: presumably SS Zumdahl, Chemistry (1983) DC Heath and Company.]

Arthur V. Phelps
(1923 - 2012)

1962: Numerical e^- transport

- Iterative approach, allowing for inelastic cross section terms:
  - educated guess of cross sections (elastic & inelastic);
  - numerically solve the Boltzmann equation (no moments);
  - compare calculated and measured mobility and diffusion;
  - adjust cross sections.

"... more than 50,000 transistors plus extremely fast magnetic core storage. The new system can simultaneously read and write electronically at the rate of 3,000,000 bits of information a second, when eight data channels are in use. In 2.18 millionths of a second, it can locate and make ready for use any of 32,768 data or instruction numbers (each of 10 digits) in the magnetic core storage. The 7090 can perform any of the following operations in one second: 229,000 additions or subtractions, 39,500 multiplications, or 32,700 divisions. " (IBM 7090 documentation)

[L.S. Frost and A.V. Phelps, *Rotational Excitation and Momentum Transfer Cross Sections for Electrons in* H_2 *and* N_2 *from Transport Coefficients*, Phys. Rev. **127** (1962) 1621–1633.]

Magboltz: microscopic e^- transport

- A large number of cross sections for 60 molecules...
  - Numerous organic gases, additives, *e.g.* CO_2:
    - elastic scattering,
    - 44 inelastic cross sections (5 vibrations and 30 rotations + super-elastic and 9 polyads),
    - attachment,
    - 67 excited states and
    - 11 ionisations.
  - noble gases (He, Ne, Ar, Kr, Xe):
    - elastic scattering,
    - 44 excited states and
    - 7 ionisations.

LXcat

- LXcat (pronounced *elecscat*) is an open-access website for collecting, displaying, and downloading ELECtron SCATtering cross sections and swarm parameters (mobility, diffusion coefficient, reaction rates, etc.) required for modeling low temperature plasmas. [...]”

[http://www.lxcat.laplace.univ-tlse.fr/]

Gas gain

Sir John Sealy Edward Townsend
(1868-1957)

1901: Gas multiplication

- John Townsend:

Let a force X be applied to N_0 negative ions in a gas at pressure p and temperature t. Let N be the total number of negative ions after the N_0 ions have travelled a distance x. The new negative ions travel with the same velocity as the original N_0 ions, so that all the negative ions will be found together during the motion. The number of negative ions produced by N ions travelling through a distance dx will be $\alpha N dx$; where α is a constant depending on X, p, and t.

Then

$$dN = \alpha N dx.$$

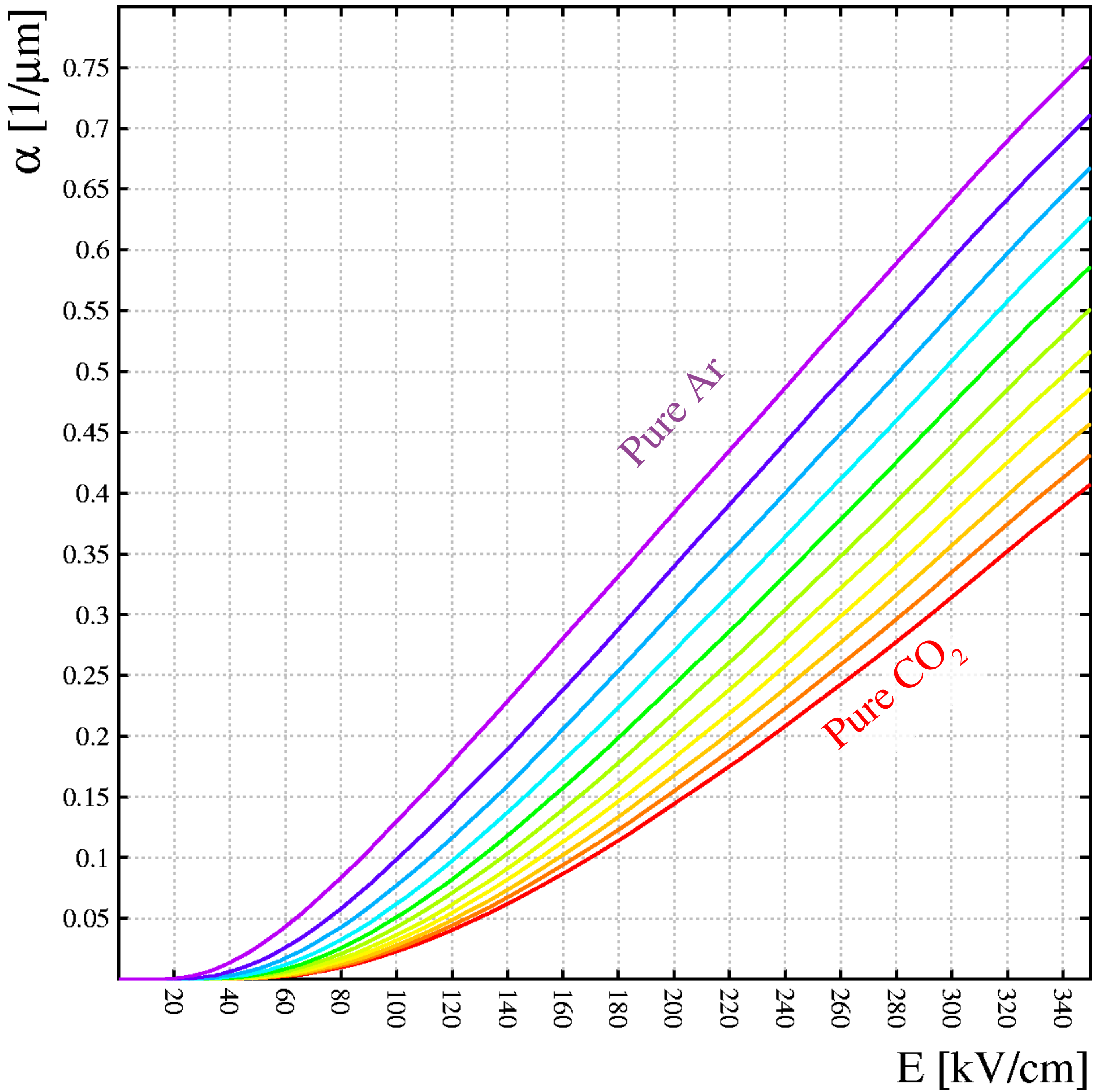
Hence

$$N = N_0 \epsilon^{\alpha x}$$

[J.S. Townsend, "*The conductivity produced in gases by the motion of negatively charged ions*", Phil. Mag. **6-1** (1901) 198-227. If access to the Philosophical Magazine is restricted, then consult a German-language abstract at http://jfm.sub.uni-goettingen.de/.]

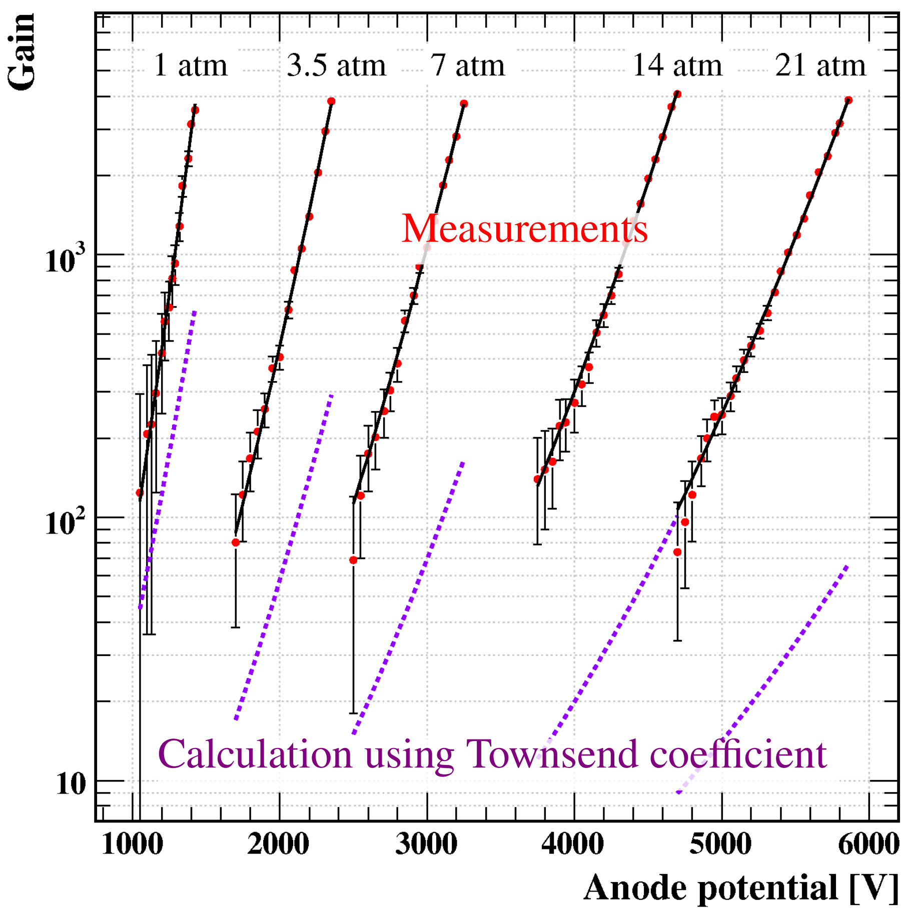
$\alpha(\text{Ar-}CO_2)$

- α = number of e^- an avalanche e^- creates per cm.
- Adding CO_2 reduces the gain.
- Calculated by Magboltz for Ar/CO_2 at 3 bar.

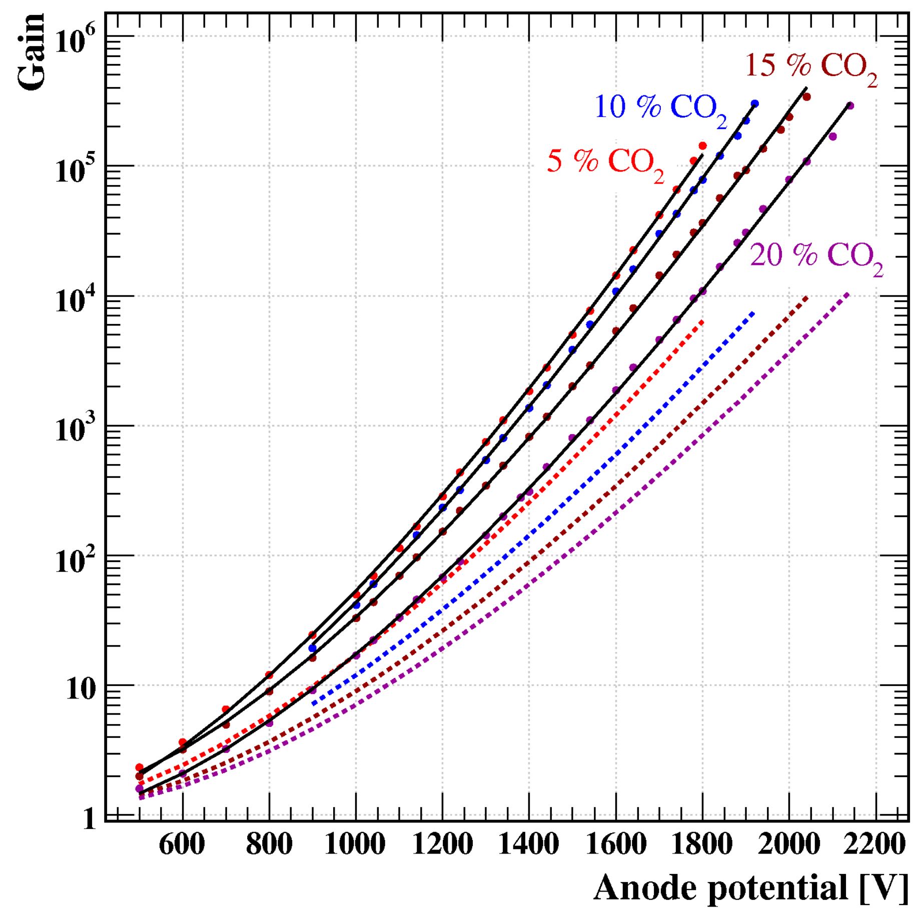


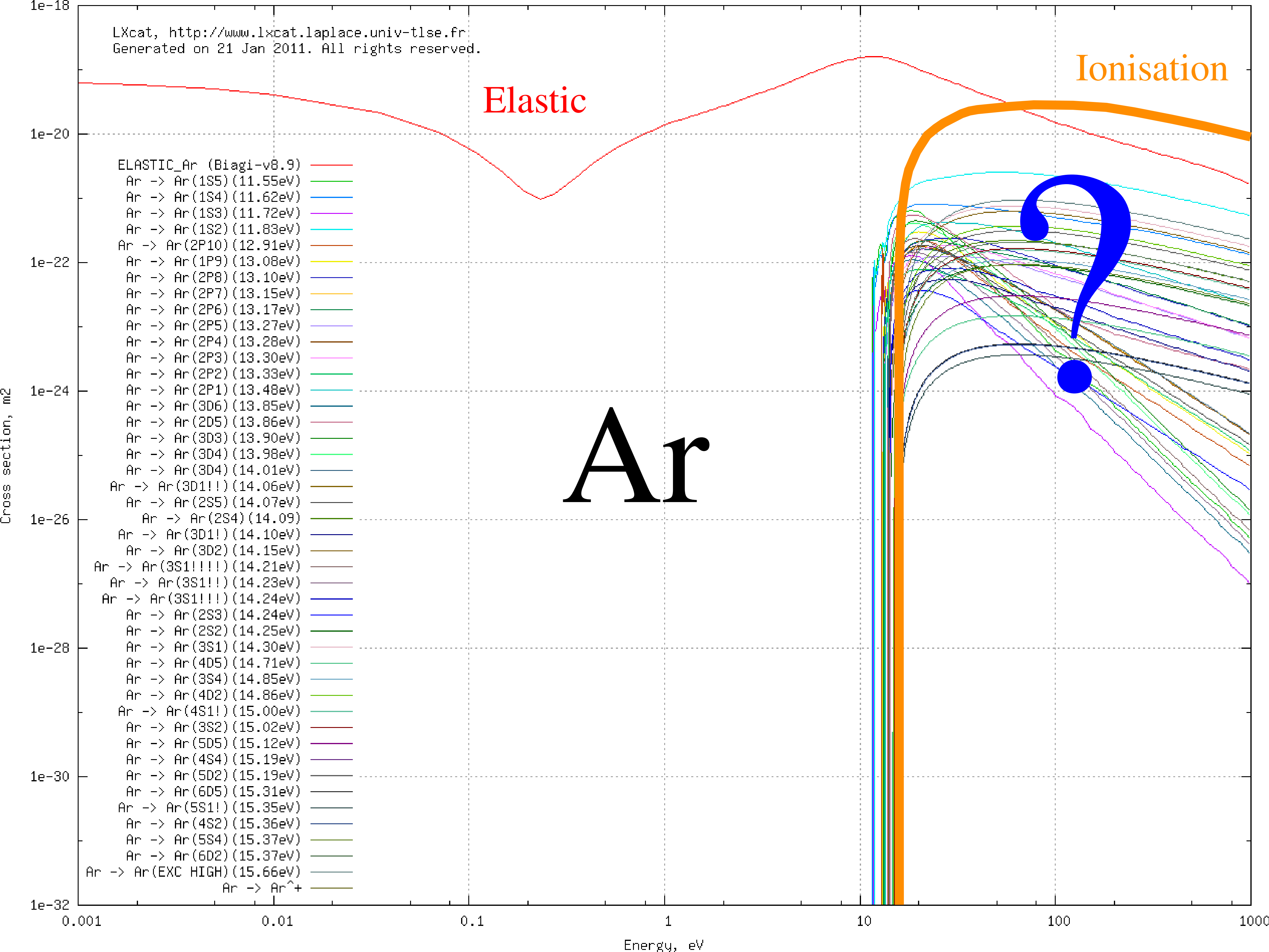
Does this reproduce the measurements ?

- Ar - CH_4

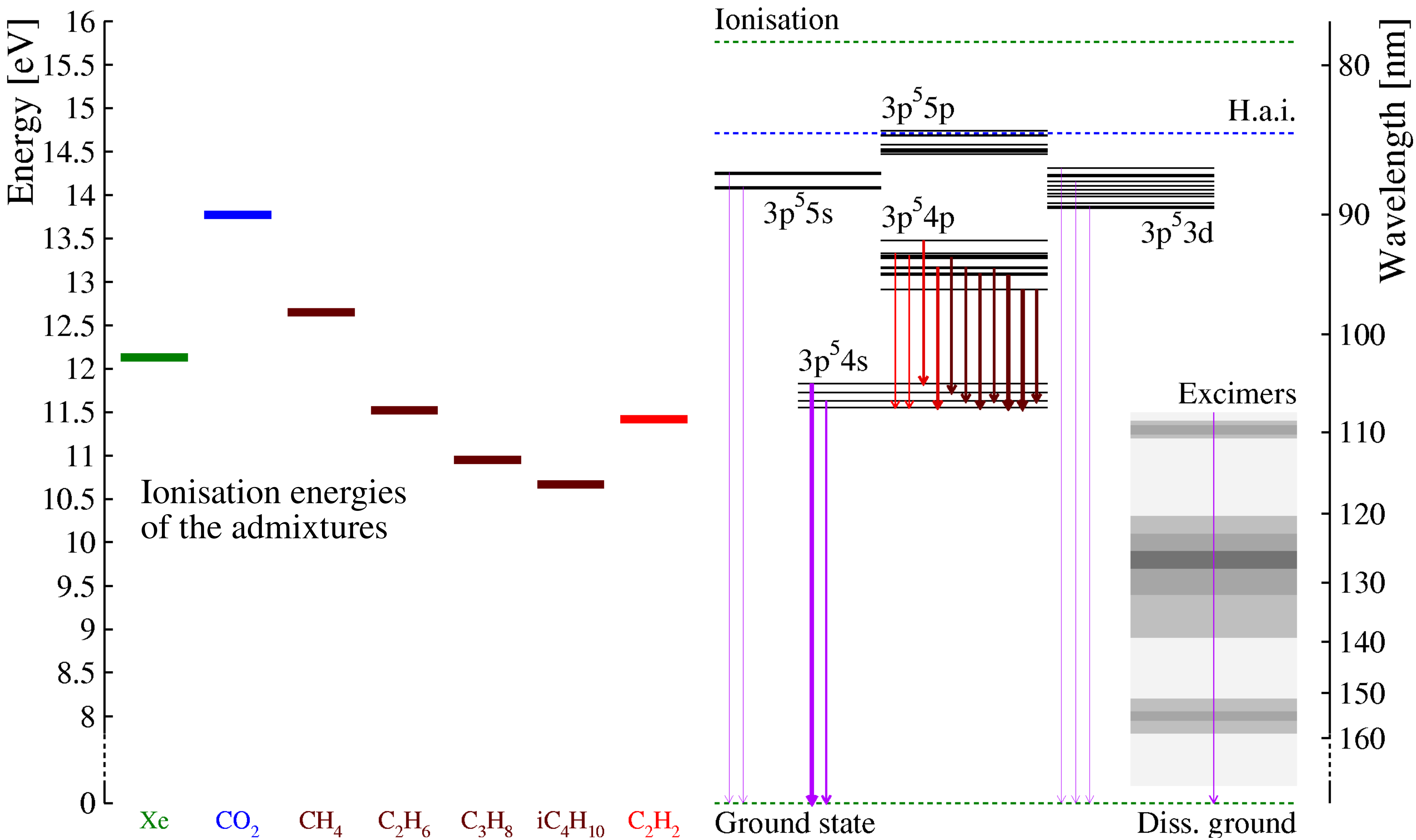


- Ar - CO_2




LXcat, http://www.lxcat.laplace.univ-tlse.fr
Generated on 21 Jan 2011. All rights reserved.
Elastic
Ionisation
?
Ar
ELASTIC_Ar (Biagi-v8.9)
Ar -> Ar(1S5)(11.55eV)
Ar -> Ar(1S4)(11.62eV)
Ar -> Ar(1S3)(11.72eV)
Ar -> Ar(1S2)(11.83eV)
Ar -> Ar(2P10)(12.91eV)
Ar -> Ar(1P9)(13.08eV)
Ar -> Ar(2P8)(13.10eV)
Ar -> Ar(2P7)(13.15eV)
Ar -> Ar(2P6)(13.17eV)
Ar -> Ar(2P5)(13.27eV)
Ar -> Ar(2P4)(13.28eV)
Ar -> Ar(2P3)(13.30eV)
Ar -> Ar(2P2)(13.33eV)
Ar -> Ar(2P1)(13.48eV)
Ar -> Ar(3D6)(13.85eV)
Ar -> Ar(2D5)(13.86eV)
Ar -> Ar(3D3)(13.90eV)
Ar -> Ar(3D4)(13.98eV)
Ar -> Ar(3D4)(14.01eV)
Ar -> Ar(3D1!!)(14.06eV)
Ar -> Ar(2S5)(14.07eV)
Ar -> Ar(2S4)(14.09)
Ar -> Ar(3D1!)(14.10eV)
Ar -> Ar(3D2)(14.15eV)
Ar -> Ar(3S1!!!!)(14.21eV)
Ar -> Ar(3S1!!)(14.23eV)
Ar -> Ar(3S1!!!)(14.24eV)
Ar -> Ar(2S3)(14.24eV)
Ar -> Ar(2S2)(14.25eV)
Ar -> Ar(3S1)(14.30eV)
Ar -> Ar(4D5)(14.71eV)
Ar -> Ar(3S4)(14.85eV)
Ar -> Ar(4D2)(14.86eV)
Ar -> Ar(4S1!)(15.00eV)
Ar -> Ar(3S2)(15.02eV)
Ar -> Ar(5D5)(15.12eV)
Ar -> Ar(4S4)(15.19eV)
Ar -> Ar(5D2)(15.19eV)
Ar -> Ar(6D5)(15.31eV)
Ar -> Ar(5S1!)(15.35eV)
Ar -> Ar(4S2)(15.36eV)
Ar -> Ar(5S4)(15.37eV)
Ar -> Ar(6D2)(15.37eV)
Ar -> Ar(EXC HIGH)(15.66eV)
Ar -> Ar^+
1e-18
1e-20
1e-22
1e-24
1e-26
1e-28
1e-30
1e-32
Cross section, m2
0.001
0.01
0.1
1
10
100
1000
Energy, eV

Level diagram argon and admixtures



Determining the Penning parameter

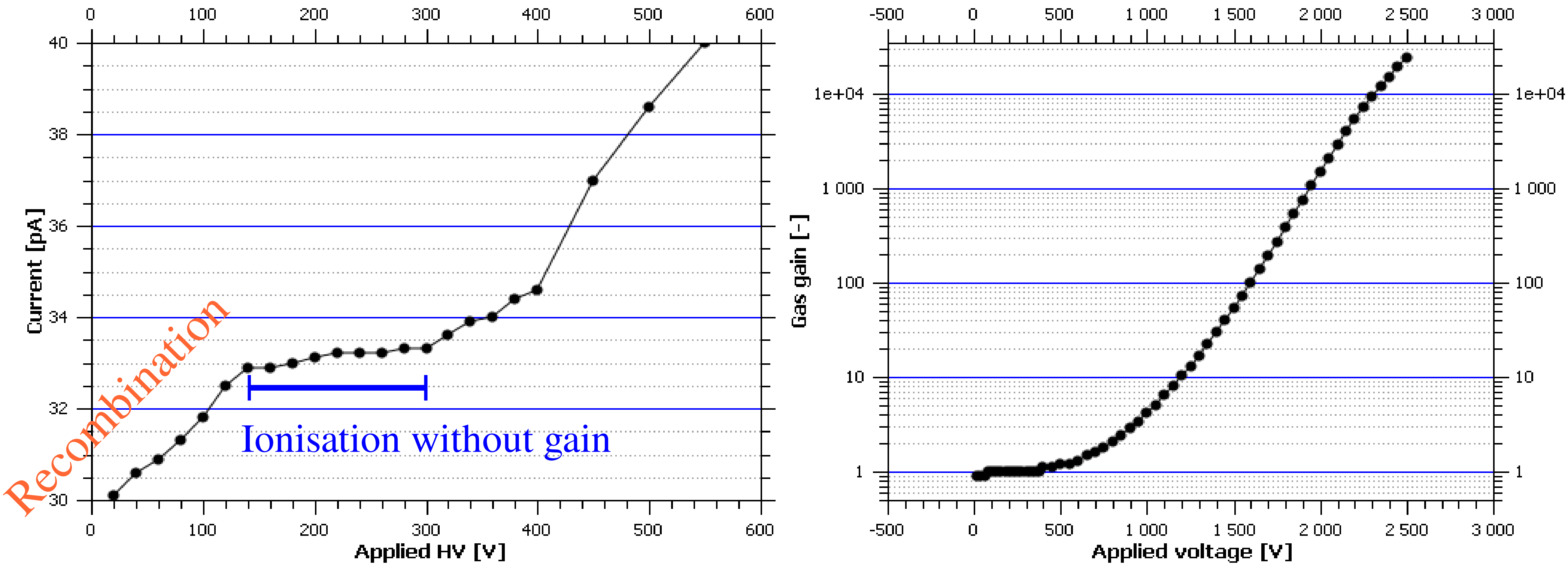
- The Penning transfer rate r_P is measured by finding the fraction of the excitations to be added to α so that the measured gain is reproduced:

$$G = \exp \int \alpha \left(1 + r_P \frac{\nu_{\mathrm{exc}}}{\nu_{\mathrm{ion}}} \right)$$

- r_P depends on gas choice, quencher fraction and density.
- Ideally, one would like to determine a separate r_P for each excitation, but for now, we do not have the data for that.

Data covers 5 orders of magnitude !

- Current reference is taken at the ionisation level.
- Main source of error: ~5 %.

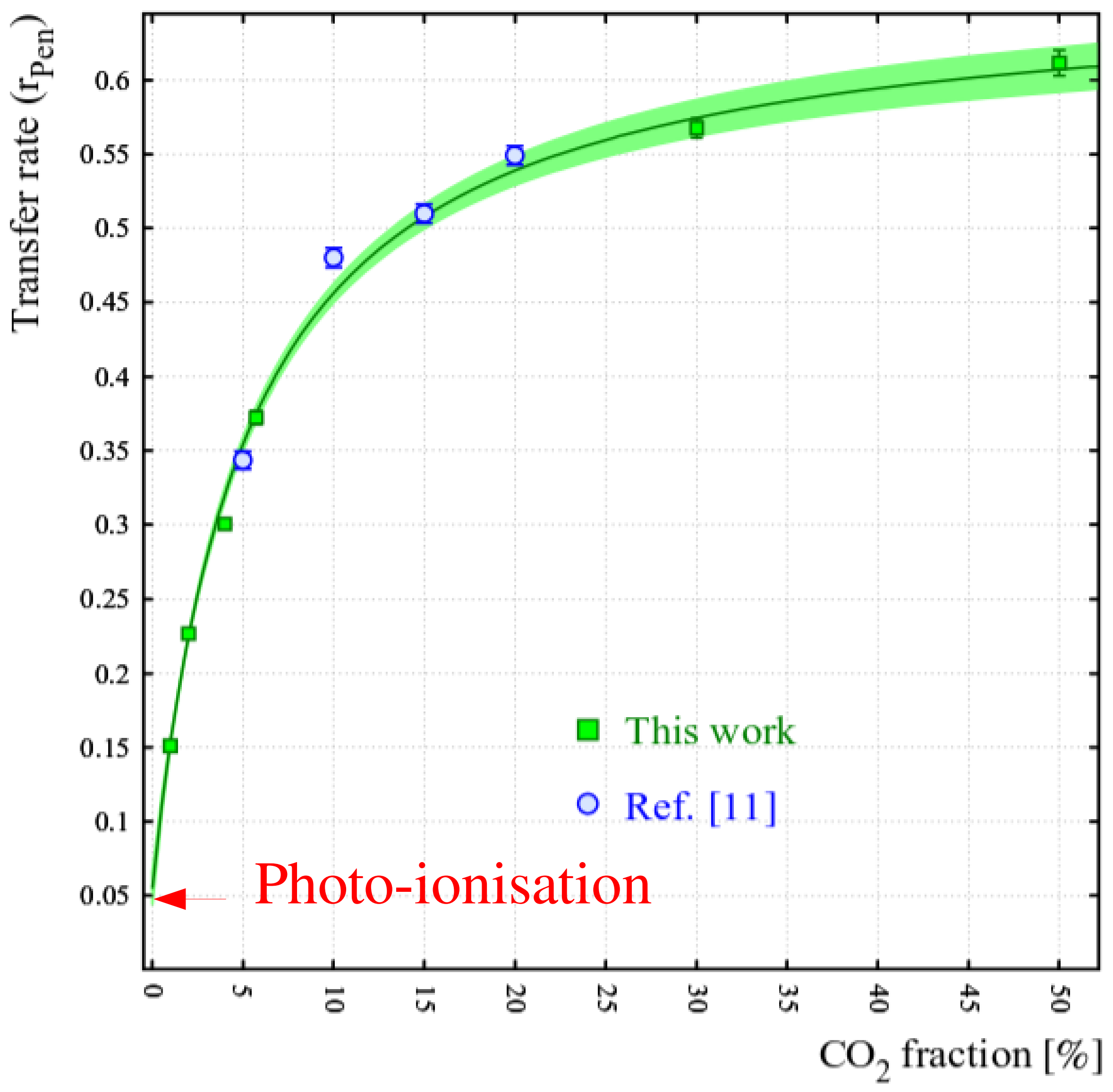


Ar-CO_2 transfer rates

- Penning parameter fits with data from Tadeusz Kowalski et al. 1992 and 2013.
- At p = 1070 hPa.

[10.1016/0168-9002(92)90305-N, 10.1016/j.nima.2014.09.061]

Loss of excitation



Gain calculations

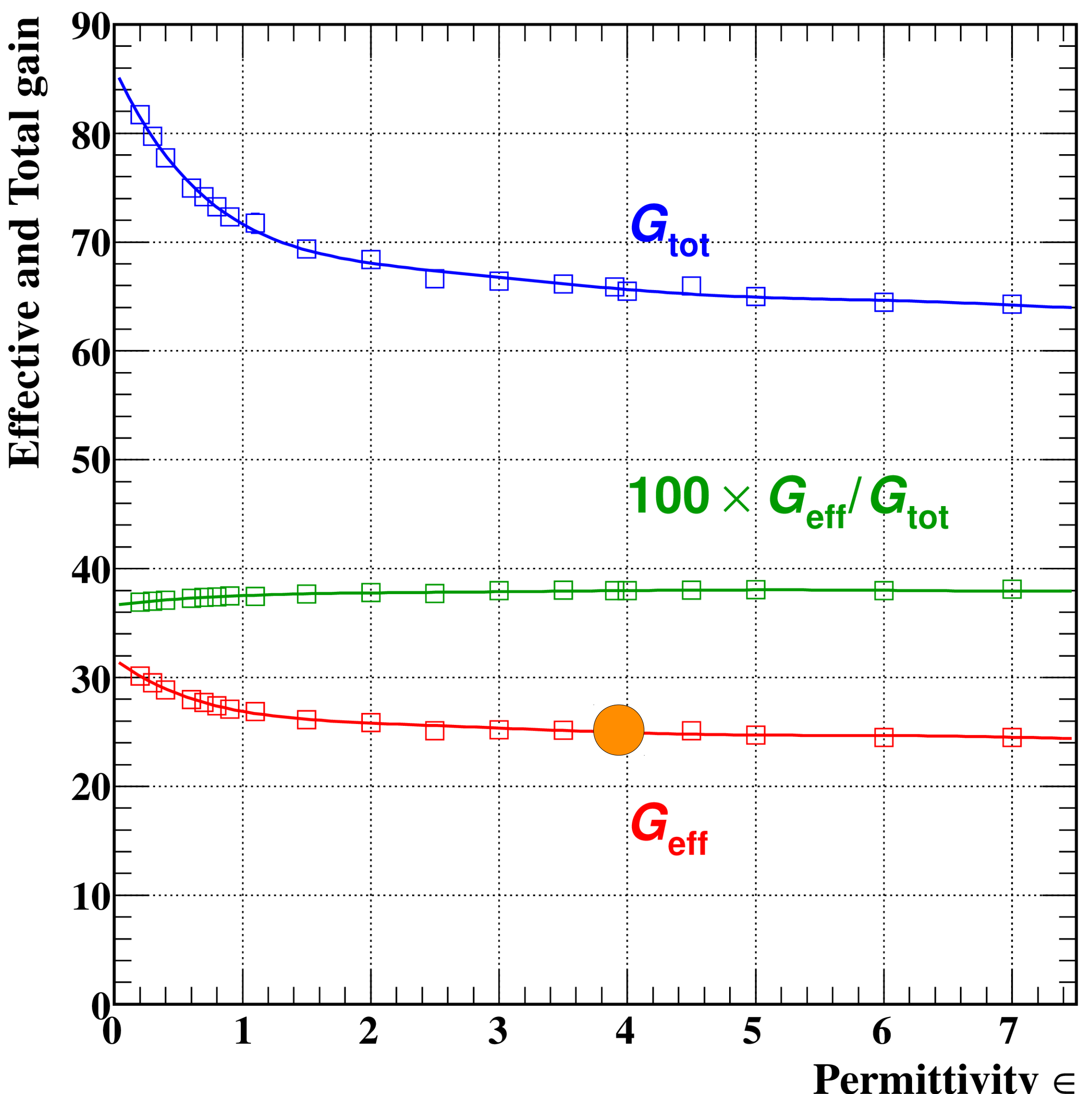
Total gain vs effective gain in a GEM

- Total gain: G_{tot}
  - total number of electrons produced by the average avalanche
- Effective gain: G_{eff}
  - number of electrons produced by the average avalanche and that reach the GEM read-out structure
  - the other electrons land on the PI, modifying the field, or on the bottom GEM electrode.

Varying the permittivity

- Reference geometry:
  - inner diam $d = 50$ µm,
  - outer diam $D = 70$ µm,
  - pitch $T = 140$ µm,
- Material reference:
  - permittivity ε = 3.9.

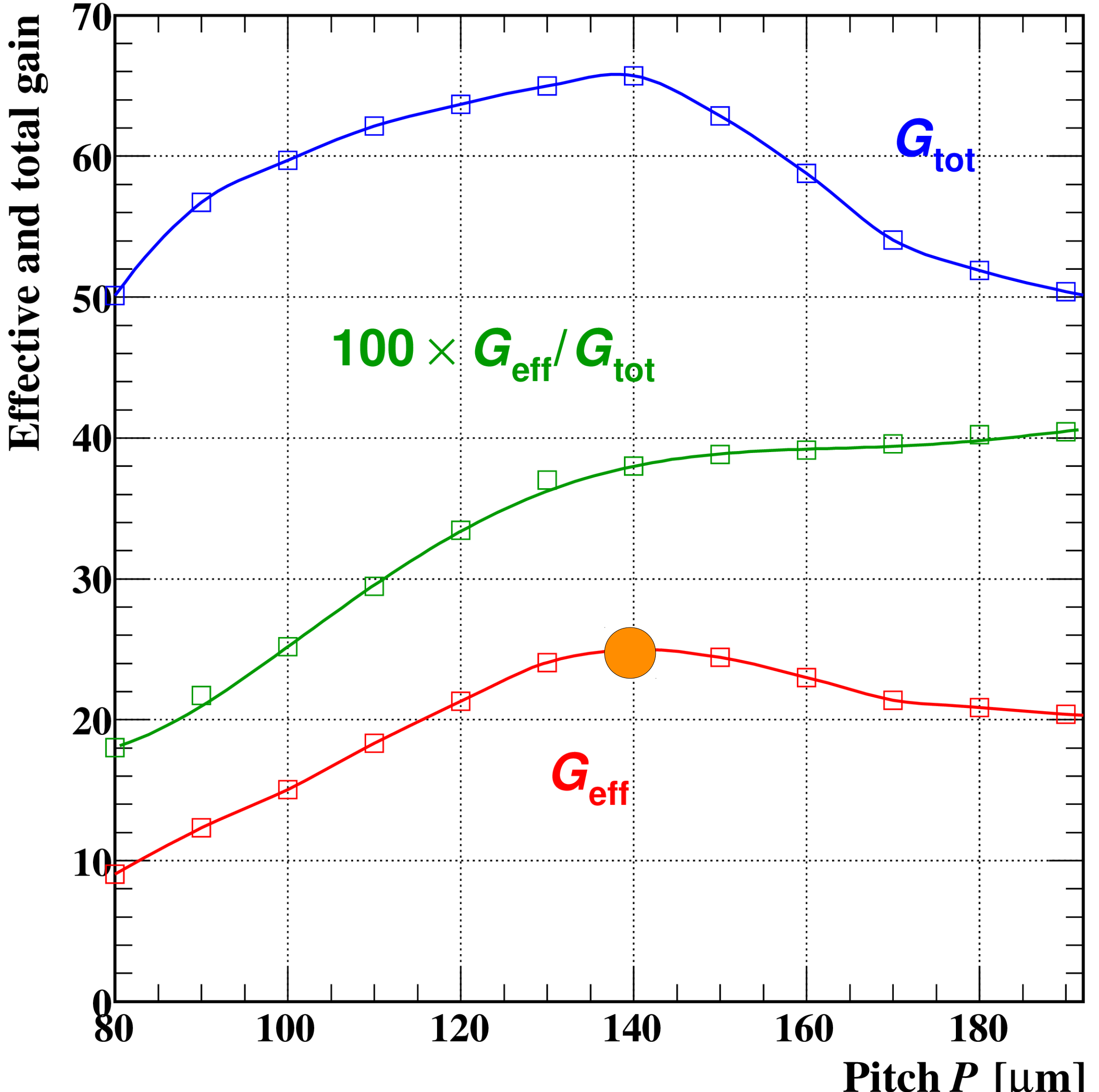
- Permittivity affects the gain at low ε.



Varying the pitch

- Reference geometry:
  - inner diam d = 50 µm,
  - outer diam D = 70 µm,
  - pitch T = 140 µm,
- Material reference:
  - permittivity ε = 3.9.

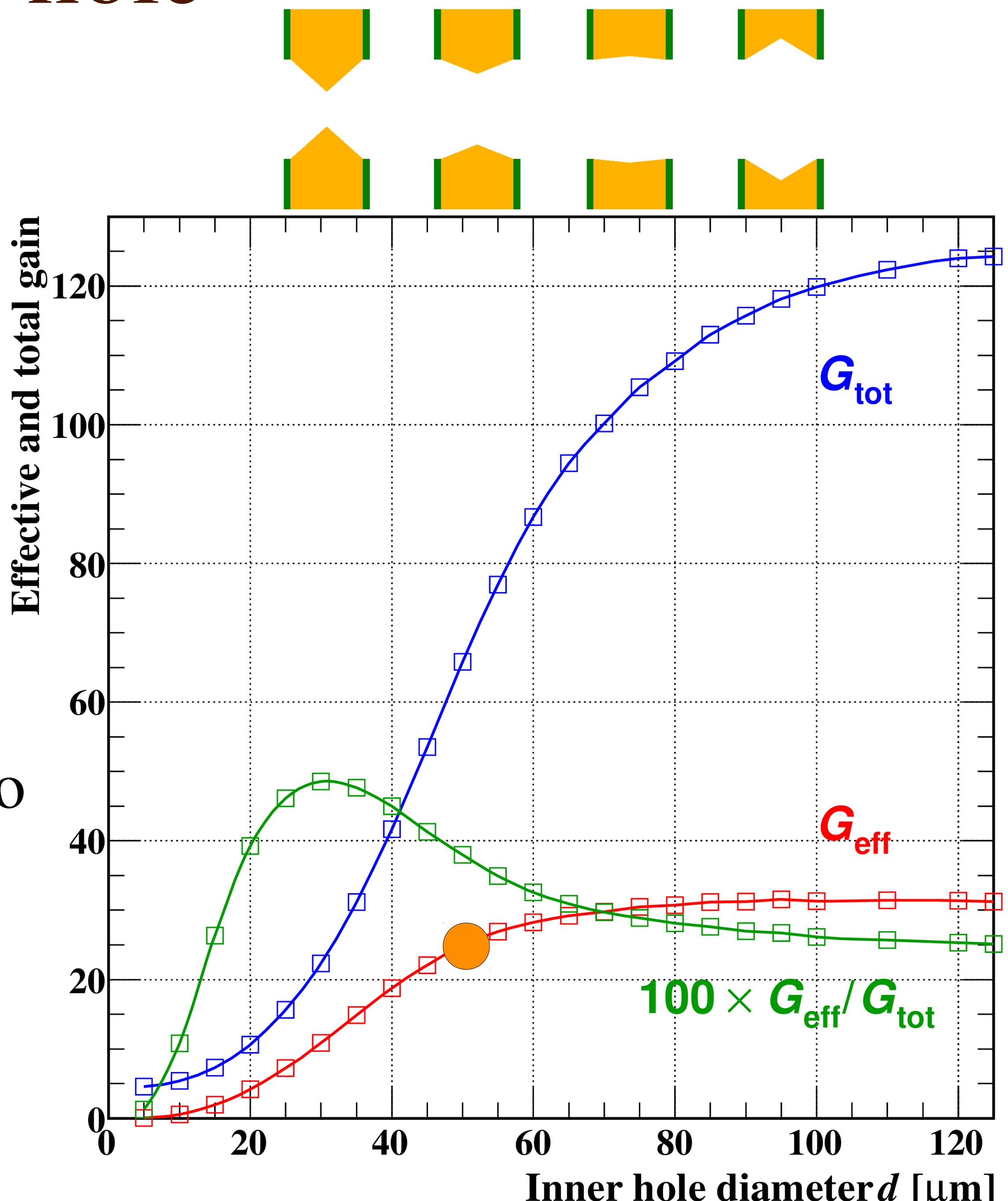
- Usual pitch maximises the gain.



Varying the inner hole diameter

- Reference geometry:
  - inner diam d = 50 µm,
  - outer diam D = 70 µm,
  - pitch T = 140 µm,
- Material reference:
  - permittivity ε = 3.9.

- At small d, electrons hit the PI near the tip;
- G_{eff} increases with d up to cylindrical, then flattens;
- over-etching does cause G_{tot} to keep increasing.



Ion Transport

Ion transport

- In this laboratory, we look into electron transport, not into ion transport.
- We do this because electrons are responsible for the signals in GEMs.
- Beware though that the signals in
  - wire chambers,
  - Micromegas
  - and others,
- are generated by ion movement
- Ion transport is a rich field which we can discuss in a small group if desired: ion reactions, cluster formation ...

Next

- Josh Renner has prepared exercises to simulate a LEM.
- Beware … GEM and LEM look similar at first sight, but there are important differences !

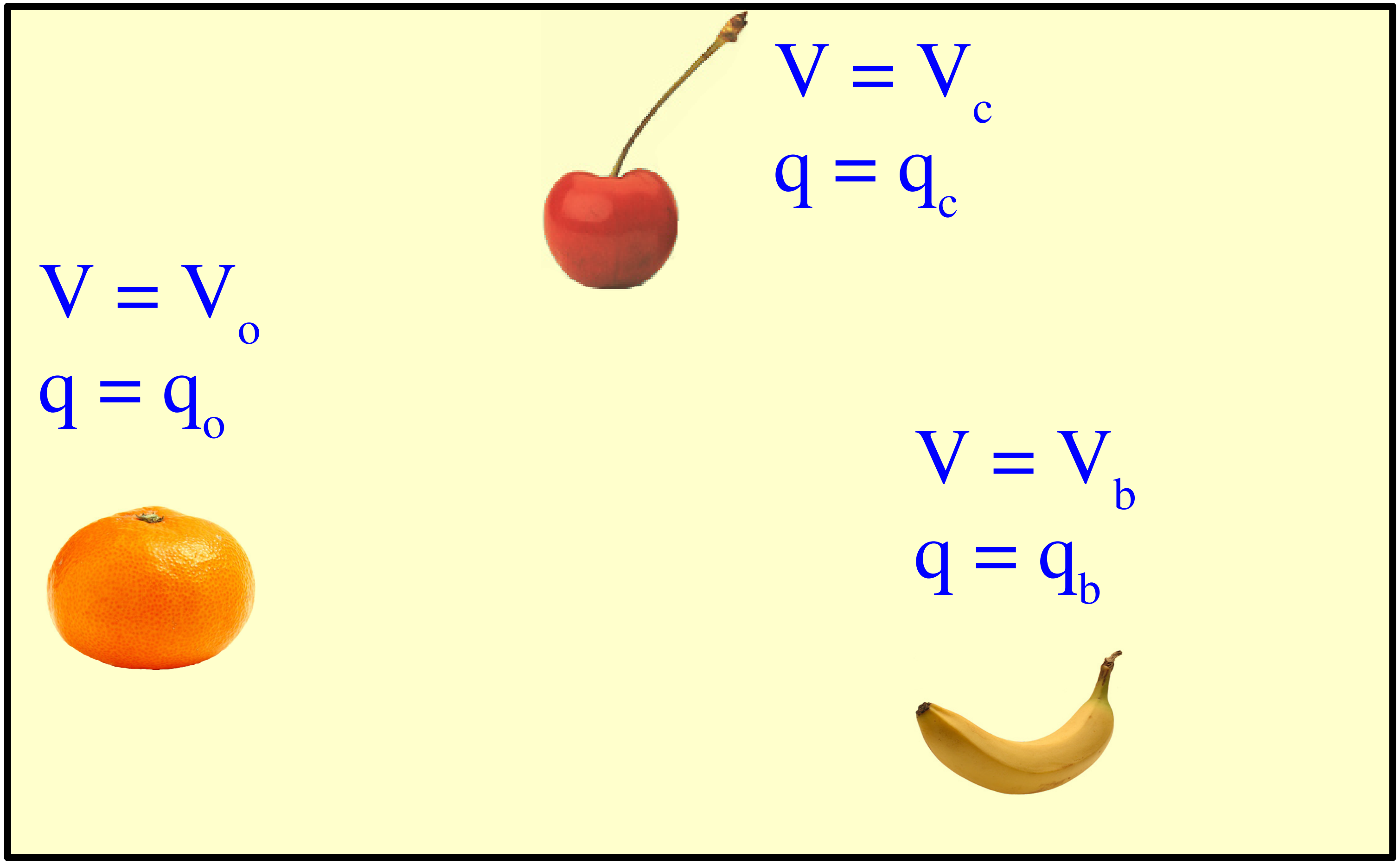
Not used

Signals

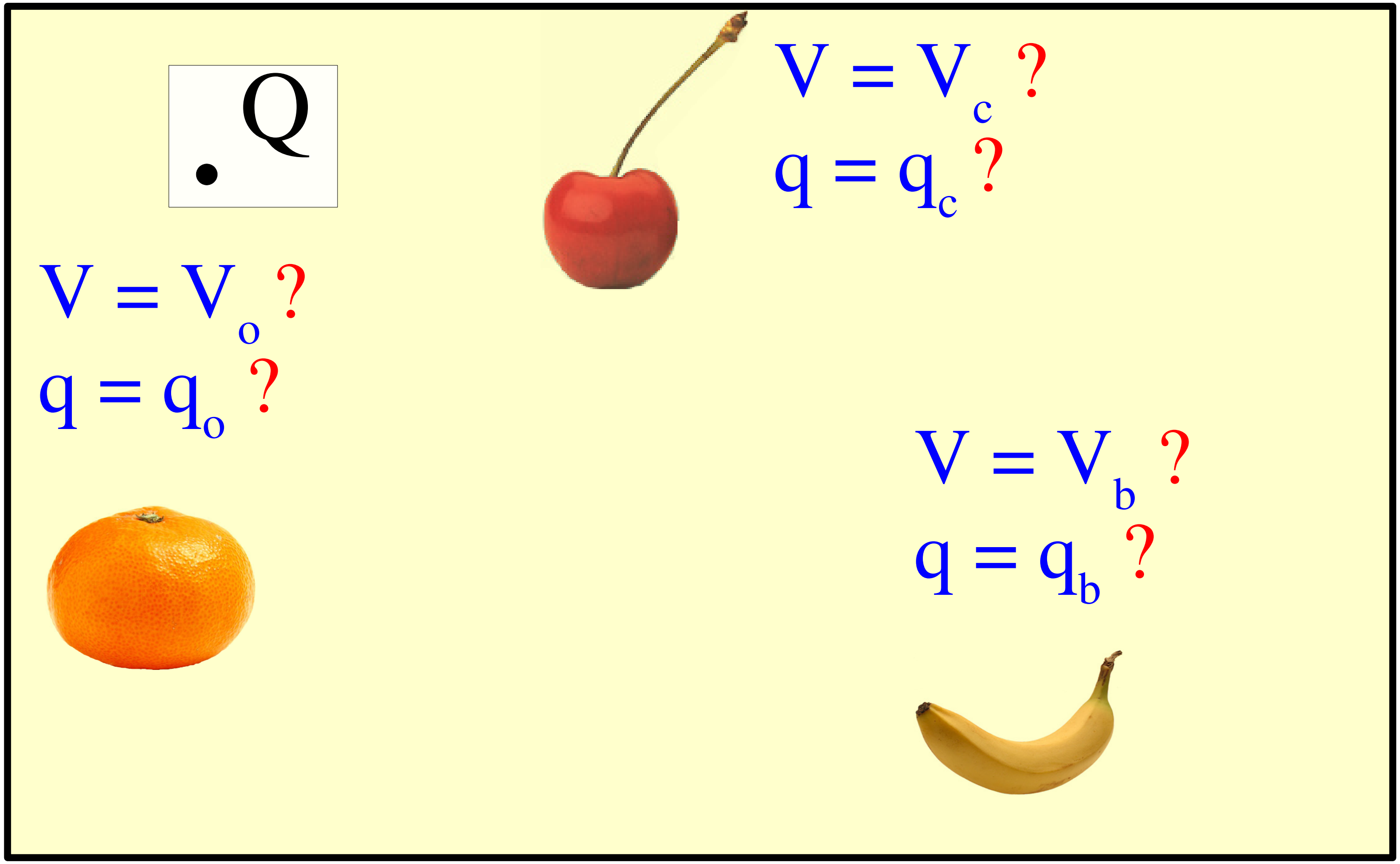
Signals

- Remains reading the signals induced by the electrons and ions moving around in the chamber.
- The charge of the electrons and ions tries to change the voltage of the electrodes.
- The electronics compensates for this by supplying charge.

Current induction



Current induction



Current induction

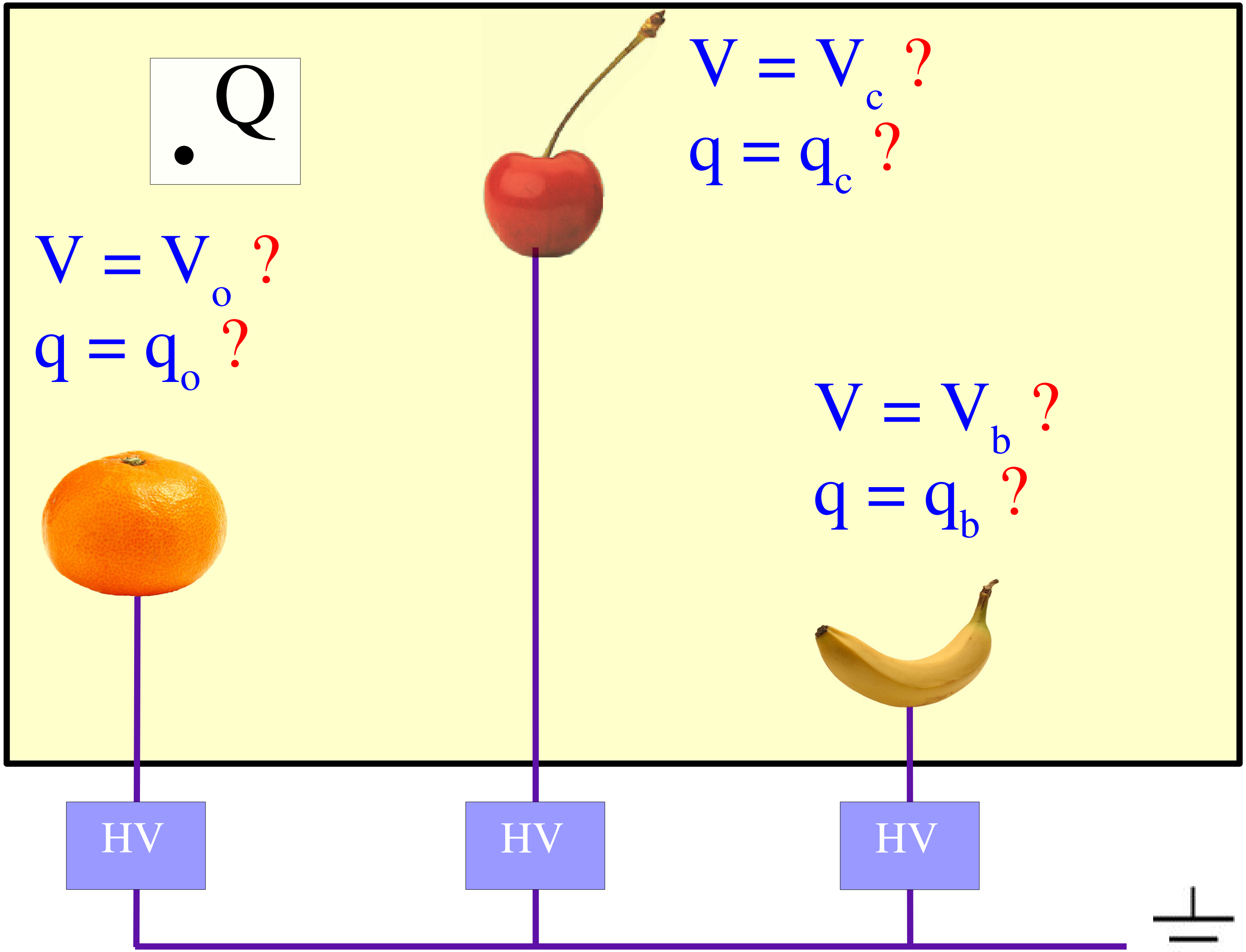
$\cdot Q$

$$V = V_c + \Delta V_c$$
$$q = q_c$$

$$V = V_o + \Delta V_o$$
$$q = q_o$$

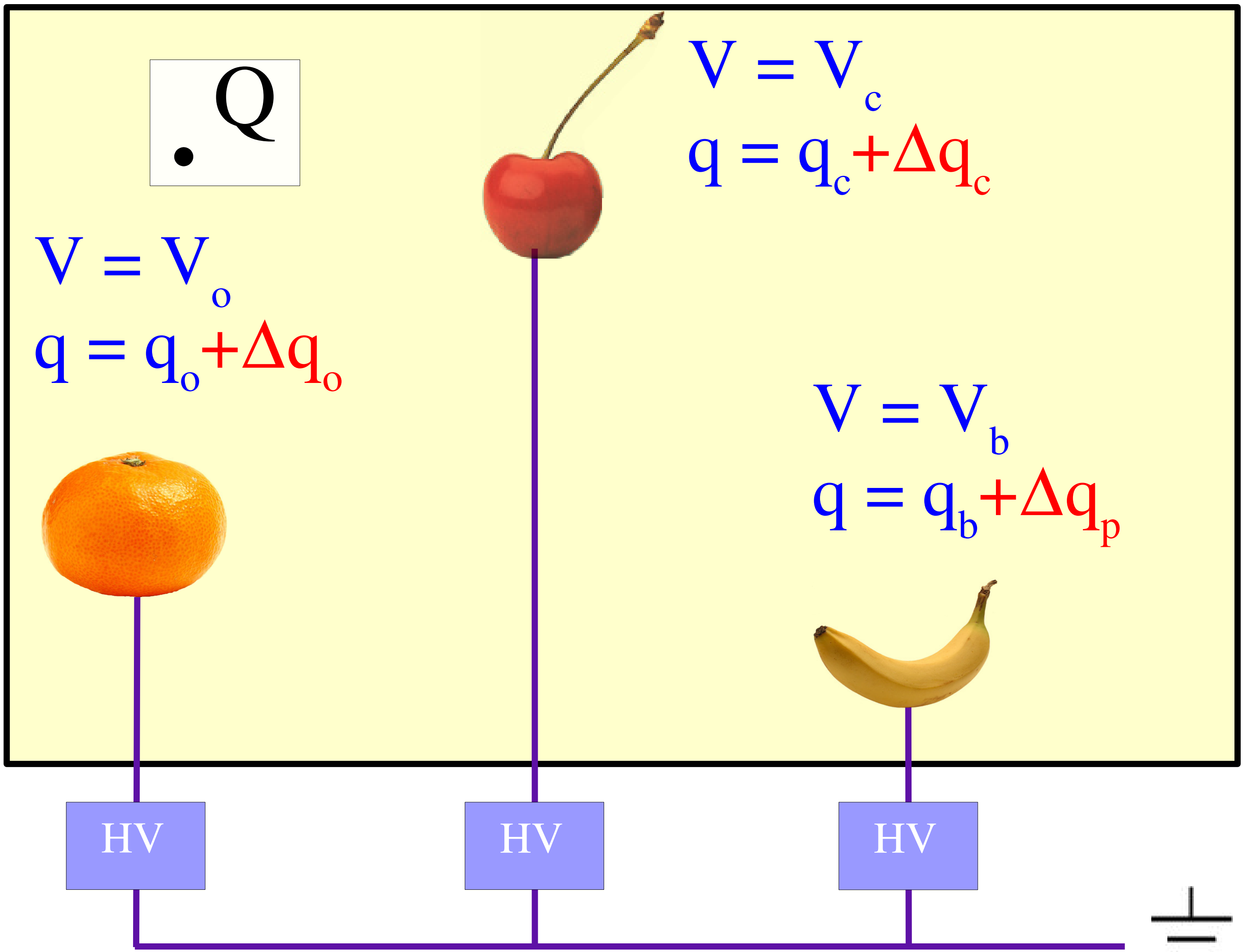
$$V = V_b + \Delta V_p$$
$$q = q_b$$

Current induction



Current induction

No charge creation:
$\Delta q_o + \Delta q_c + \Delta q_p = 0$



Trying to guess the signals ...

- Properties of the current induced in an electrode:
  - proportional to the charge Q;
  - proportional to the velocity of the charge $\vec{v}_{\mathrm{d}}$;
  - dependent on the geometry.

- This leads to the following ansatz:

$$I = -Q\,\vec{v}_{\mathrm{d}} \cdot \vec{E}_{\mathrm{W}}$$

(the sign is mere convention, see next slide)

- The geometry is contained in $\vec{E}_{\mathrm{W}}$, necessarily a vector, the *weighting field*:
  - each electrode has its own weighting field;
  - unit of the weighting field ?

Signs of Current and Weighting field

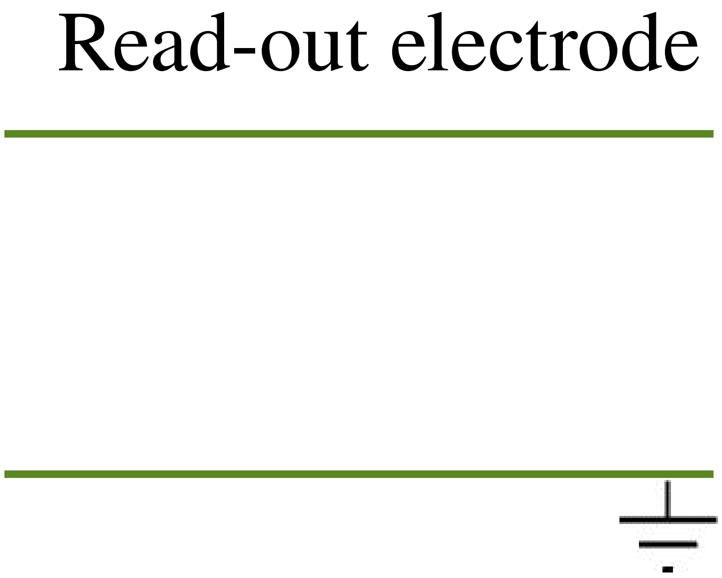
- Sign of the current:
  - Signal current is (by convention) positive if positive charge flows from the read-out electrode to ground (via HV).

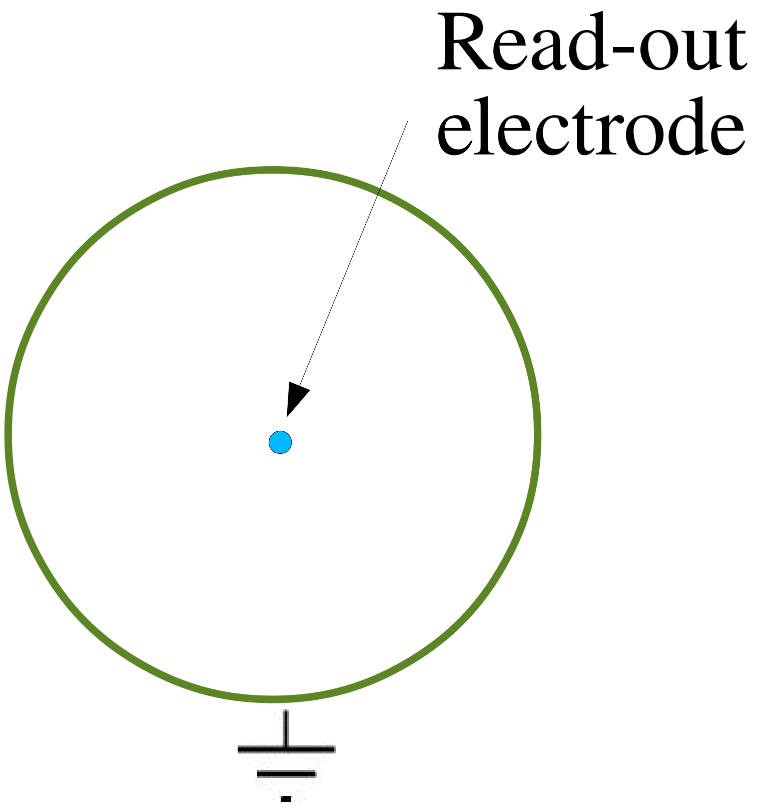
- Orientation of $\vec{E}_{\mathrm{W}}$:
  - place a positive charge Q on the surface of the read-out;
  - move it away from the read-out;
  - positive charge flows in to compensate: negative current;
  - to make the signs match, $\vec{v}_{\mathrm{d}} \cdot \vec{E}_{\mathrm{W}}$ needs to be positive;
  - $\vec{E}_{\mathrm{W}}$ points *away* from the electrodes being read out;
  - $\vec{E}_{\mathrm{W}}$ points *into* all other electrodes.

- Many people invert the signs – which is perfectly fine.

Weighting field – examples

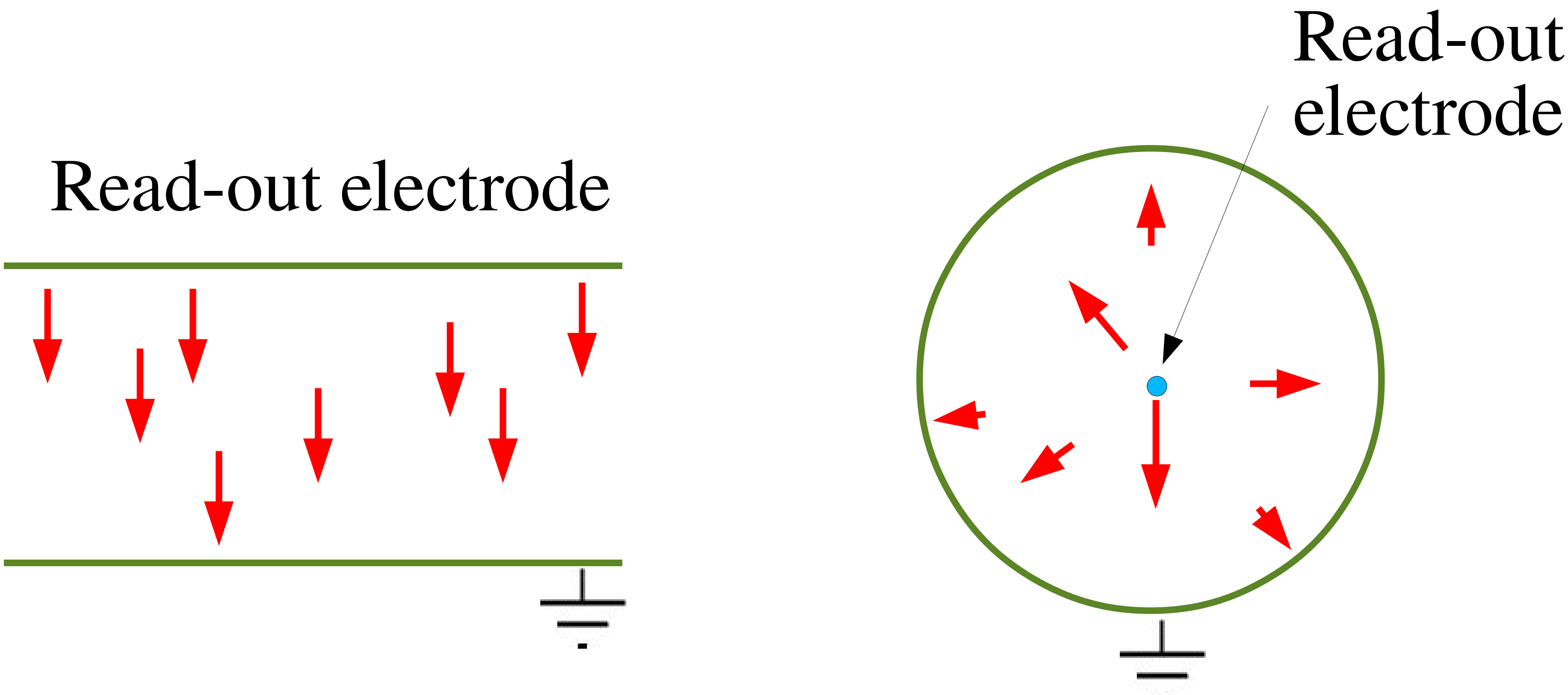
- The weighting field is often easy to guess:





Weighting field – examples

- The weighting field is often easy to guess:



Weighting fields – more in general

- Claim: $\vec{E}_{\mathrm{w}}$ for a given read-out electrode can be computed the same way as a potential:
  - read-out electrode set to 1;
  - all other electrodes set to 0;
  - note ... 0 and 1, not 0 V and 1 V !

  - the resulting potential is called "weighting potential";
- This is plausible considering examples, and is proven using Green's reciprocity.

George Green's father's mill (Nottingham)

1828: George Green's work

- The basic techniques to solve electrostatics problems, still used today, were published by George Green in: "An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism".

  "(...) it was written by a young man, who has been obliged to obtain the little knowledge he possesses, at such intervals and by such means, as other indispensable avocations which offer but few opportunities of mental improvement, afforded."

- Now available from http://arxiv.org/pdf/0807.0088v1, originally only 53 copies were printed, only for the subscribers.

[Original printed for the author by T Wheelhouse, Nottingham (1828). Facsimile Mayer & Müller, Berlin (1889), scanned by Google books.]

Integrating the current

- Net charge over a trajectory $z(t)$:

$$
\begin{aligned}
Q_{\text{net}} &= \int_{t_{\text{start}}}^{t_{\text{end}}} I(t) \\
&= -\int_{t_{\text{start}}}^{t_{\text{end}}} Q \; E_{\text{W}}(z(t)) \cdot v_{\text{drift}}(z(t))\,\mathrm{d}t \\
&= \int_{t_{\text{start}}}^{t_{\text{end}}} Q \; \frac{\mathrm{d}V_{\text{W}}(z)}{\mathrm{d}z} \cdot \frac{\mathrm{d}z}{\mathrm{d}t} \; \mathrm{d}t \\
&= Q \left(V_{\text{W}}(z_{\text{end}}) - V_{\text{W}}(z_{\text{start}}) \right)
\end{aligned}
$$

$$E_{\text{W}} = -\nabla V_{\text{W}}$$

- By construction, all electrodes have $V_{\text{w}} = 0$ or $V_{\text{w}} = 1$.
- Hence, the integral of the current between electrodes can only be -1, 0 or 1.

Sum of all currents

- Summing the current on all electrodes, observe that

$$E_{\mathrm{W}}^{\mathrm{tot}} = \sum_i E_{\mathrm{W}}^i$$

- is computed by placing a unit weighting potential on every electrode, exactly once.

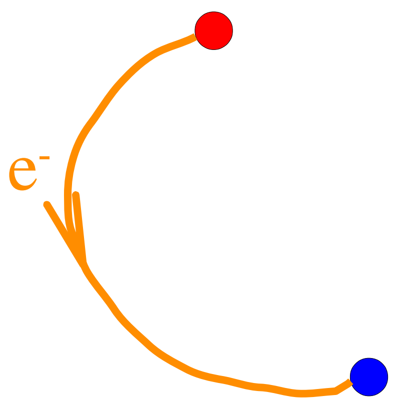
- The weighting potential is then constant and the weighting field vanishes:

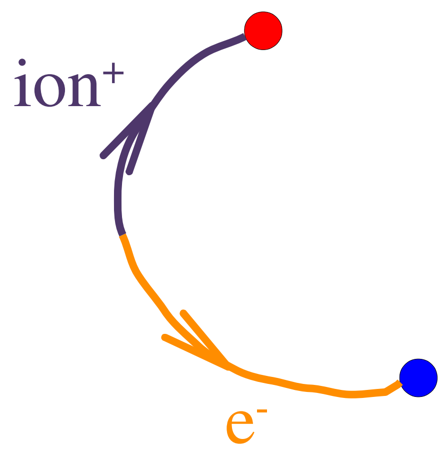
$$\begin{aligned} I_{\mathrm{tot}}(t) &= \sum_i I_i(t) = \sum_i -Q\, v_{\mathrm{d}} \cdot E_{\mathrm{W}}^i = -Q\, v_{\mathrm{d}} \cdot \sum_i E_{\mathrm{W}}^i \\ &= 0 \end{aligned}$$

- Thus, the sum of all currents is zero at all times.

Changing sign of charge and velocity

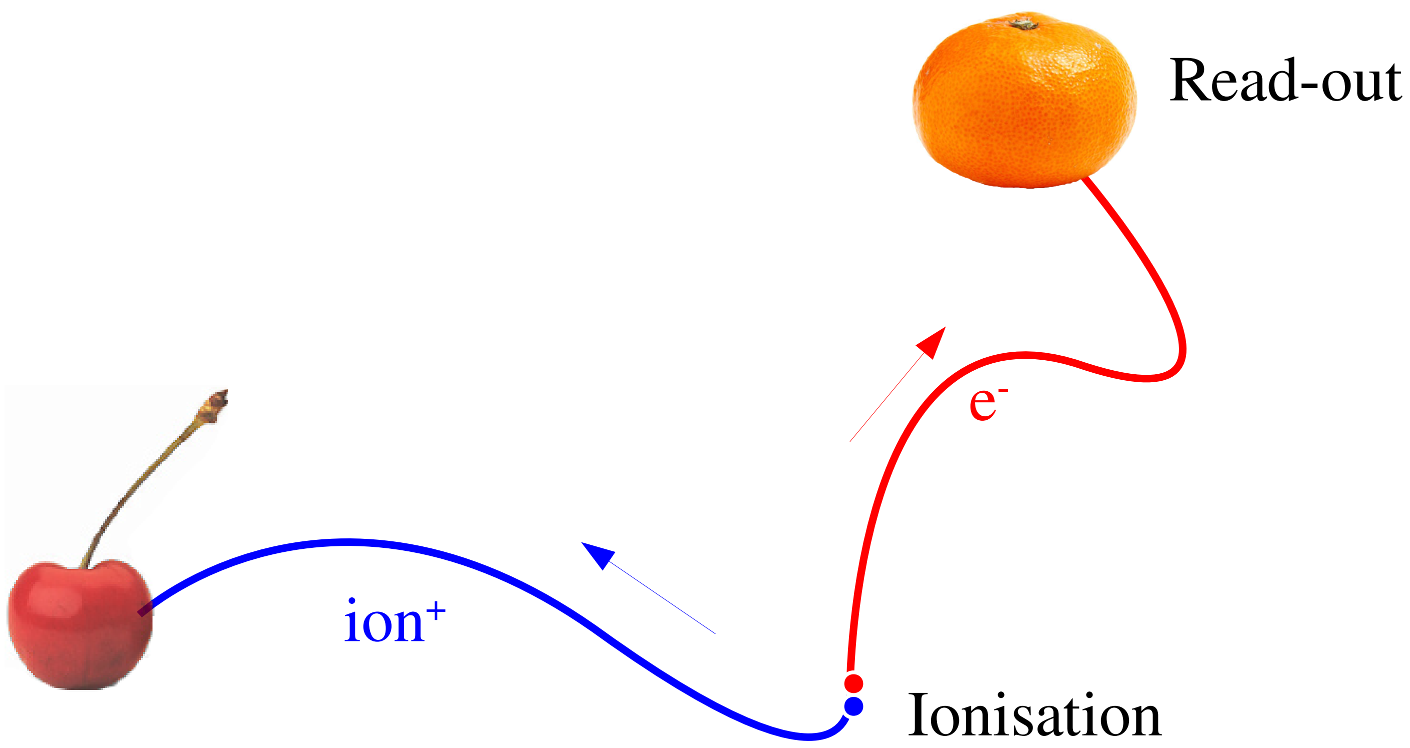
- Observe that the following are equivalent in terms of *total* charge induced – the time dependence will differ:





Combined e^- - ion^+ current

- How about the total current induced by an e^- - ion^+ pair ?



Stigler's law

“no scientific discovery is named after its original discoverer”

Jean le Rond d'Alembert (Nov 16^{th} 1717 –Oct 29^{th} 1783)

1749: 2d flow of liquids

- Jean le Rond d'Alembert takes part in a hydrodynamics contest in Berlin. Euler gives the prize to Jaques Adami.
- d'Alembert and Euler don't speak for 10 years, but:

59. On peut encore trouver M & N par la méthode ſuivante qui eſt un peu plus ſimple. Puiſque $\frac{dp}{dz} = -\frac{dq}{dx}$ & $\frac{dp}{dx} = \frac{dq}{dz}$, donc $q\,dx + p\,dz$ & $p\,dx - q\,dz$ ſeront des différentielles complettes.

J. le Rond d'Alembert, "*Theoria resistentiae quam patitur corpus in fluido motum, ex principiis omnino novis et simplissimus deducta, habita ratione tum velocitatis, figurae, et massae corporis moti, tum densitatis compressionis partium fluidi*" (1749). Manuscript at the Berlin-Brandenburgische Akademie der Wissenschaften as document I-M478.

J. le Rond d'Alembert, "*Essai d'une nouvelle théorie de la résistance des fluides*" (1752) Paris. Available from Gallica BnF.

Caspar Wessel (1745-1818)
Jean-Robert Argand (1768-1822)
Johann Carl Friedrich Gauss (1777-1855)
Sir William Rowan Hamilton (1805-1865)
Charles Sanders Peirce (1839-1914)
Georg Frobenius (1849-1917)

Why not 3d ?

- The complex numbers $(\mathbb{R}^2,+,\times)$ form a field, like the real numbers $(\mathbb{R},+,\times)$, but $(\mathbb{R}^3,+,\times)$ does not. As a result, 2d arithmetic can be done with complex numbers, but there is no 3d equivalent for this.

- It can be proven that only $\mathbb{R}$ and $\mathbb{C}$ can form a commutative, associative division algebra.

- $(\mathbb{R}^4,+,\times)$ can be made into a non-commutative division algebra known as quaternions, but this does not help since $\nabla \cdot E$ links all dimensions.

MPGDs and the mean free path

- Recall:
  - Mean free path of electrons in Ar: 2.5 μm,

- Compare with:
  - Micromegas mesh pitch: 63.5 μm
  - GEM polyimide thickness: 50 μm
  - Micromegas wire diameter: 18 μm
  - GEM conductor thickness: 5 μm

- Hence:
  - mean free path approaches small structural elements;
  - such devices should be treated at a molecular level.

Calculating transport properties

- One can of course *measure* every mixture one needs …
- … but it would be far more efficient if one could *compute* the transport properties of arbitrary mixtures.

LXcat people

- Art Phelps,
- Leanne Pitchford – Toulouse,
- Klaus Bartschat – Iowa,
- Oleg Zatsarinny – Iowa,
- Michael Allan – Fribourg,
- Steve Biagi
- ...

Art Phelps

Leanne Pitchford

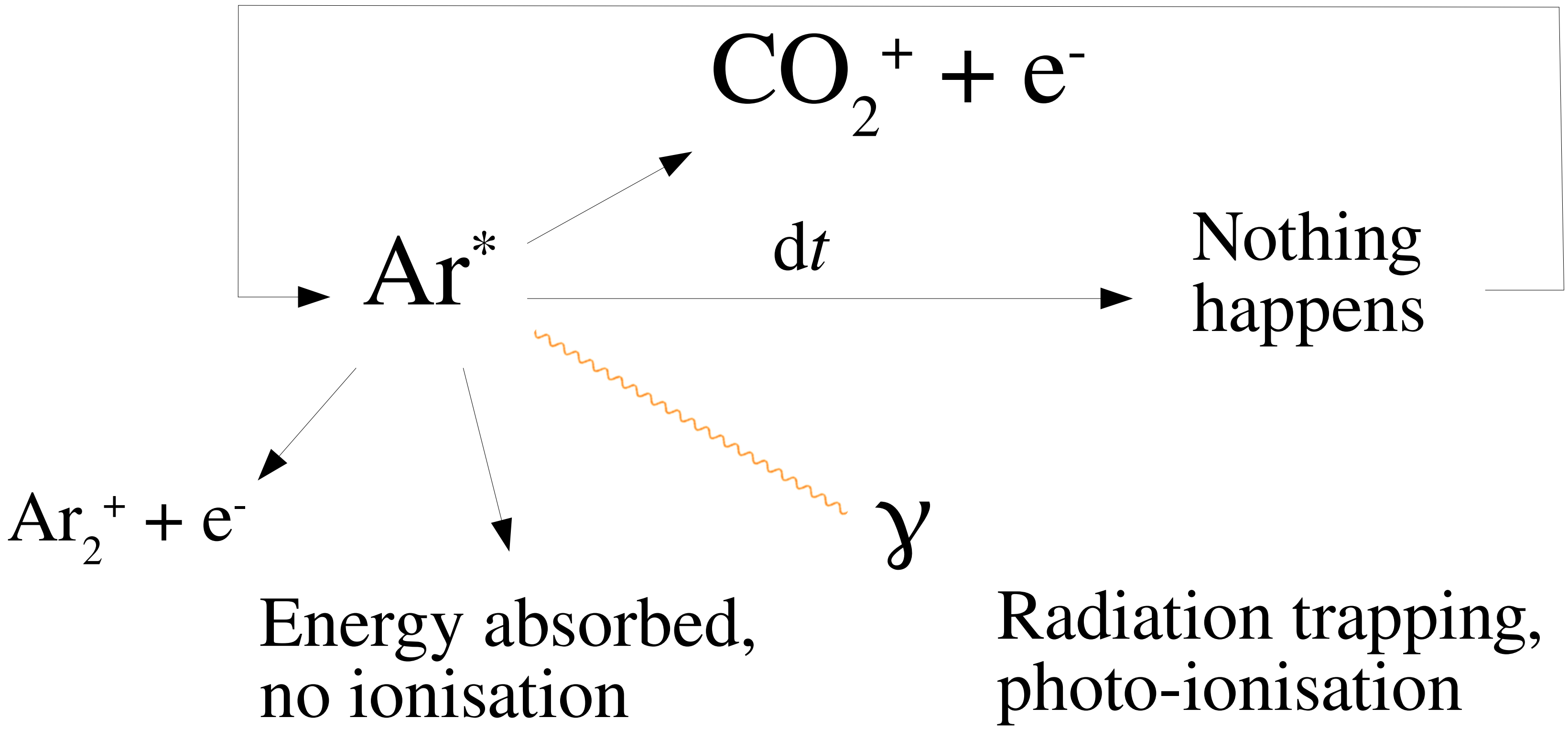
Michael Allan

Klaus Bartschat

Frans Michel Penning
(1894-1953)

Simplified Penning model

- Take small steps until the energy has been used up



Electron tracking methods

Scale $\gg$ mean free path (> 1 mm)

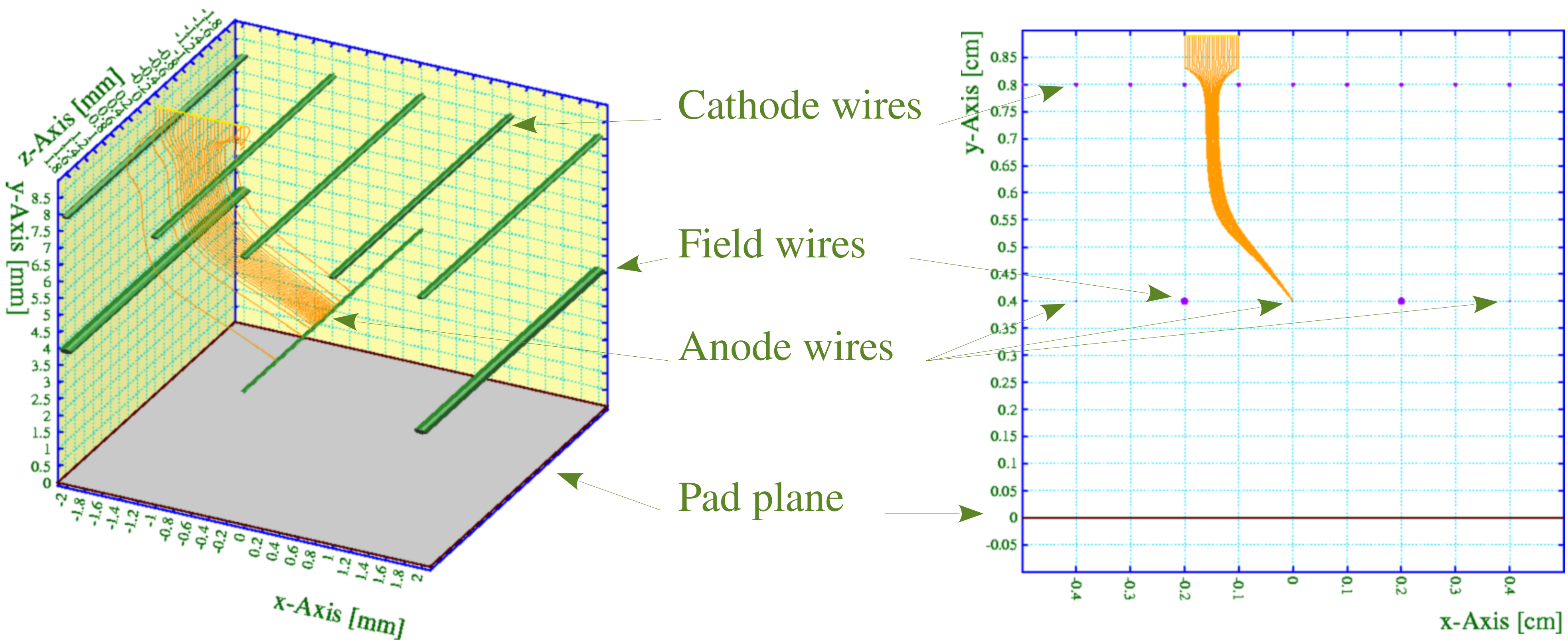
- For practical purposes, electrons from a given starting point reach the same electrode – but with a spread in time and gain.

- Electrons transport is treated by:
  - integrating the equation of motion, using the Runge-Kutta-Fehlberg method, to obtain the path;
  - integrating the diffusion and Townsend coefficients to obtain spread and gain.

- This approach is adequate for TPCs, drift tubes etc.

Runge-Kutta-Fehlberg integration

- Example: a TPC read-out cell



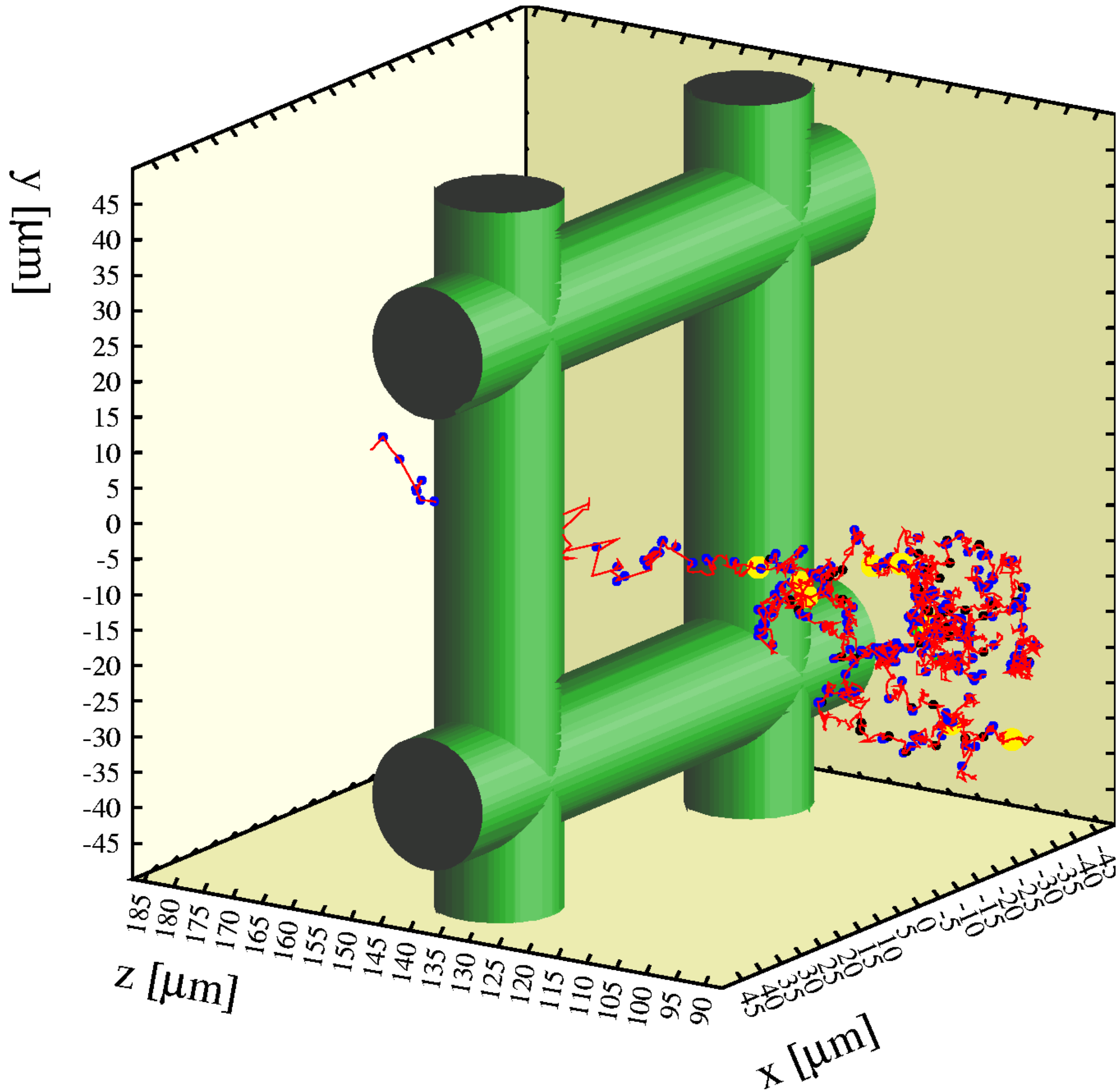
Scale ~ mean free path (1-100 μm)

- At this scale, where the mean free path approaches the characteristic dimensions of detector elements, free flight between collisions is no longer parabolic.

- The only viable approach seems to be a molecular simulation of the transport processes.

- Can be achieved by running Magboltz in the detector field, rather than in a constant field as is done when preparing classic transport tables.

Microscopic



- Legend:
  - – electron
  - inelastic
  - excitation
  - ionisation

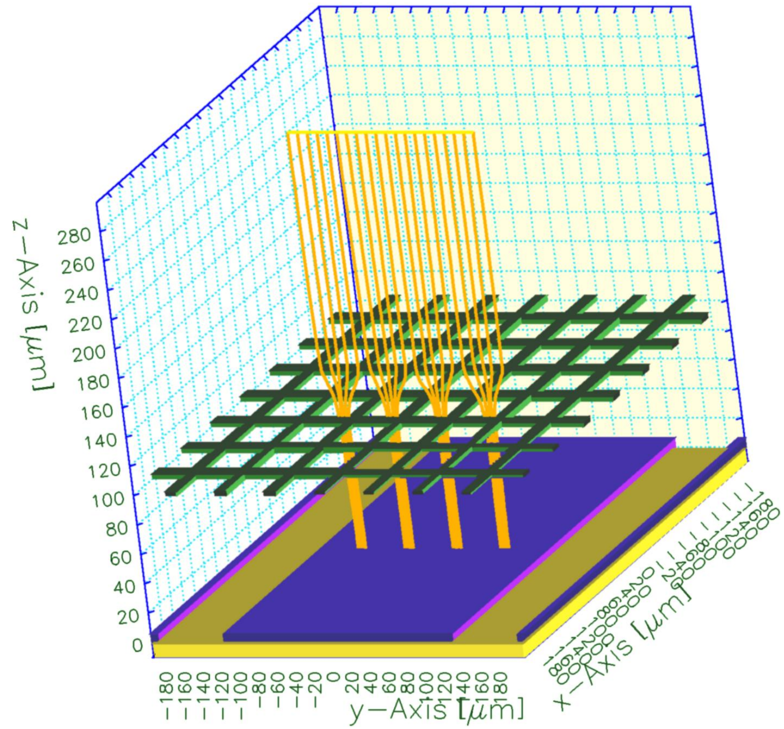
Scale > mean free path (100 μm - 1 mm)

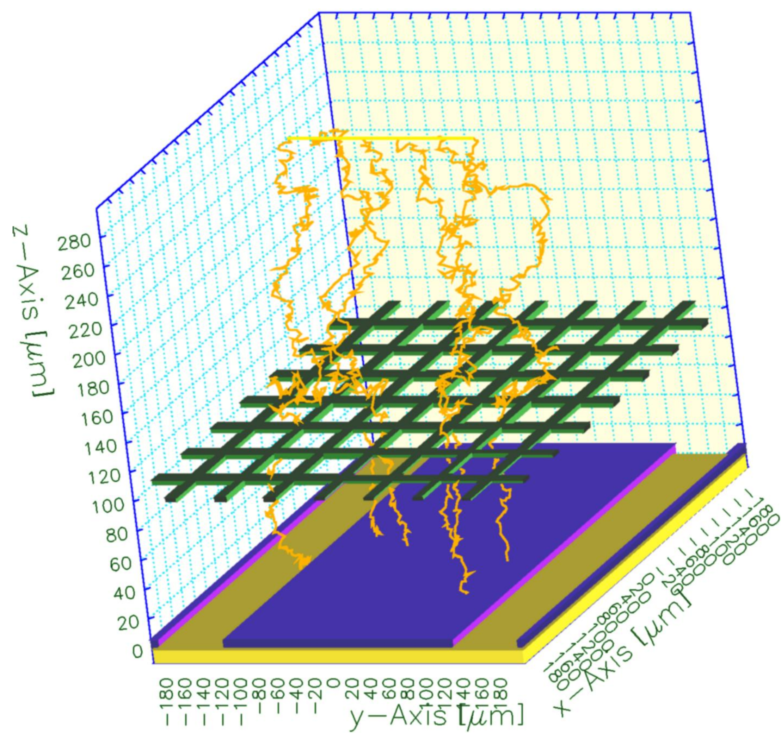
- Electrons from a single starting point may end up on any of several electrodes.
- Calculations use Monte Carlo techniques, based on the mean drift velocity and the diffusion tensor computed by microscopic integration of the equation of motion in a constant field. Gain depends on the path.
- This approach is adequate as long as the drift field is locally constant – a reasonably valid assumption in a Micromegas but less so in a GEM.

Analytic vs Monte Carlo

- Analytic integration:
  - Runge-Kutta-Fehlberg technique;
  - automatically adjusted step size;
  - optional integration of diffusion, multiplication and losses.

- Transport table-based Monte Carlo:
  - non-Gaussian in accelerating, divergent and convergent fields;
  - step size has to be set by user.
  - Replaced by molecular simulation.





[Figures made by Gilles Barouch, CEA]

Maximising the LEM/GEM gain

- You will do calculations on LEMs, here we show the principle on GEMs.
-
- Microscopic Monte Carlo shows that the standard GEM has a gain near the maximum.

Varying the outer hole diameter

- Reference geometry:
  - inner diam $d = 50$ µm,
  - outer diam $D = 70$ µm,
  - pitch $T = 140$ µm,
- Material reference:
  - permittivity ε = 3.9.

- At small D the electrons do not find the entrance;
- increasing D at constant d exposes PI and decreases G_{eff}.