ESIPAP 2021 - Module 1
Physics of particle and astroparticle detectors
Tracking

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Documents authorised: PDG booklet, lecture notes.
Whenever an explanation is required, try to support your answer with a formula and explain its various terms.
Duration: 1h30.

1 Determining the sign of the particle charge

6 points

In a tracking system with a magnetic field oriented along the beam axis, typical of colliders, the sign of particles charge is determined from the curvature of reconstructed tracks.
We remind that the absolute value of the curvature C of a trajectory is inversely proportional to the radius R and that the sign of C is defined by the orientation of the trajectory in the 2D plane transverse to the beam.

1.1 By mean of drawings, where the magnetic field and the trajectory are displayed, explain how the charge can actually be determined.

Solution
The charge drives the direction in which the trajectory is curved according to $\vec{F} = q\vec{v} \times \vec{B}$.

1.2 Justify roughly the formula $p = 0.3BR$, where p is the momentum and $B = 4$ T the magnetic field. Then explain why it is more difficult to determine the sign if the momentum is very high.

Solution
Following $\vec{F} = q\vec{v} \times \vec{B}$ and using the relation $F = \frac{\mathrm{d}p}{\mathrm{d}t} = m\frac{v^2}{R}$ which holds for a circular momentum, one gets $mv = qBR$. The factor 0.3 can be computed from imposing p in GeV/c, B in T and R in m.

1.3 The uncertainty on the curvature measurement is given by $\sigma_C^2 = K\frac{\sigma^2}{L^4}$, where $K = 25$ is a constant, $L = 1$ m is the length over which the trajectory is measured and finaly $\sigma = 20$ μm is the position measurement uncertainty for each point along the track.
If we require that the curvature amplitude is larger than 3 times the uncertainty (on the curvature), find an upper limit on the momentum depending on the quantities B, L, K, σ. Compute the corresponding numerical value and comment.

Solution
We impose $C > 3\sigma_C$ wich translates into (after squaring) $\frac{0.09B^2}{p^2} > 9K\frac{\sigma^2}{L^4}$ and finally $\frac{0.01B^2L^4}{K\sigma^2} > p^2$. The numerical constraint ends up to $p < 4000$ GeV/c.

2 Multilayer track fit

7 points

We consider a tracker made of six equivalent layers with same spatial resolution and same material budget b. The system covers a length L. A uniform magnetic field of strength B bends particle trajectories in order to measure particle momenta through the track curvature R. The question arises about the best layer arrangement to optimize the resolution on the momentum.
Three geometries are proposed as depicted on figure 1.

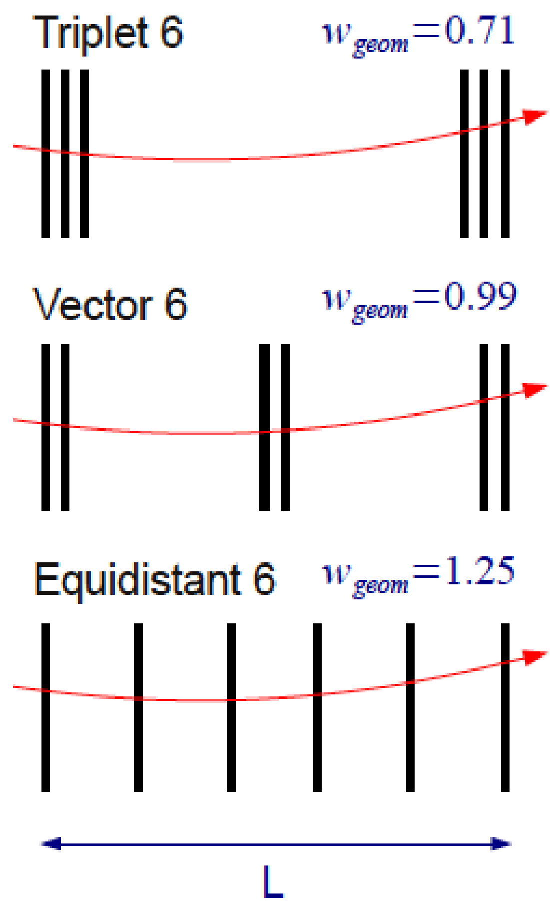

Figure 1: Possible layout geometries with six layers. The particle charge is assumed to be +1.

When multiple scattering is a dominant effect, the relative resolution on the curvature can be parametrized with:

$$\frac{\sigma_R}{R} = w_{geom}\,\frac{2b}{BL}.$$

The values of the parameter w_{geom} are given in the previous figure.

2.1 From the trajectories displayed in the figure, give the orientation of the magnetic field with respect to the detector.
Remind what is mathematical relation between momentum and curvature.

Solution

The deviation is oriented upward hence the force. From the relation $\vec{F} = q\vec{v} \times \vec{B}$, with $q = +1$, the magnetic field is oriented inward the figure. We remind that $p = 0.3BR$.

2.2 What can be the expression of b, in function of parameters that you will propose and considering that a single detection layer is made of several material?

Solution

The parameter b is derived from the standard-deviation of the multiple scattering angular deviation (Highland formula). It is essentially proportional to $\frac{1}{p}\sqrt{\frac{t}{X_{eff}}}$, where t is the thickness of one layer and X_{eff} an effective radiation length taken into account that each layer is made of various material.

2.3 Explain why the factor b appears at the numerator and the factor BL appears at the denominator.

Solution

The parameter b is proportional to the fluctuation of the angular deviation, which is detrimental to the momentum resolution. Consequently it appears at the numerator.

With larger magnetic field B the curvature R will be smaller and easier to measure. Also with larger lever arm L, it is easier to measure a given curvature. Both parameters play in favor of the resolution and have to appear at the denominator.

2.4 Explain why w_{geom} is worst for the arrangement where layers are equidistant. You can use a drawing for your discussion.

Solution

When individual detectors are divided into two stacks, one can consider that there is only one deviation affecting the trajectory. That is the deviation from the last detector of the first stack. Within one stack, sensors are close enough to mitigate strongly the effect of the angular deviation.

In contrast, for the equidistant arrangement, the angular deviation from each detector is equally important, leading to a larger impact of multiple scattering on the resolution (hence larger b).

3 Momentum resolution for muons in ATLAS

7 points

The ATLAS experiment reconstruct muon trajectories with two types of detector. The Inner Detector (ID) is located close to the interaction point and extend over 1 m in radius. Outside the calorimeter, the Muon Spectrometer (MS) is a very large system and extends in radius from about 5 m to 11 m.

The relative resolution on the muon momentum is displayed in figure 2 for both systems.

3.1 Explain the overall behavior of the momentum resolution.

Solution

At low momentum the resolution is dominated by multiple scattering effect, so the overall resolution decreases with the momentum. This trend stops at some point because the higher the momentum the more difficult it is to measure the curvature, hence the momentum. One can then observe a rise of the resolution at high momentum.

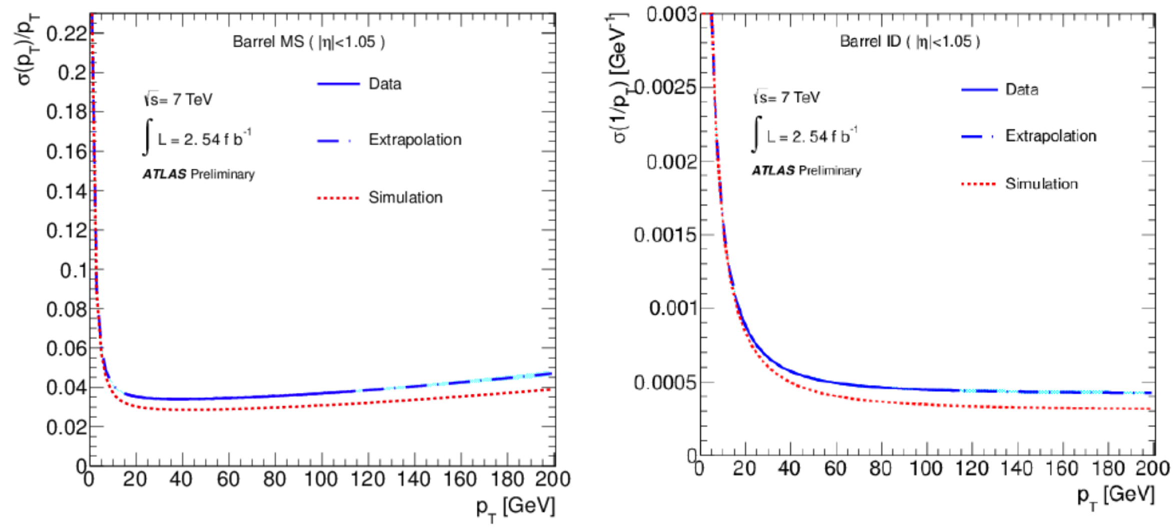

Figure 2: Mometum relative resolution for muons detected in ATLAS (consider only on the solid blue line). The left plot corresponds to measurements done with the Muon Spectrometer (MS) while the right plot to measurements done with the Inner Detector (ID).

3.2 If you were to build such a detector, suggest which technologies you will choose for the ID and for the MS.

Solution

Being located close to the interaction point, the inner detector (ID) has to be compact and shall features excellent spatial resolution for trajectory extrapolations. Silicon sensors are best suited for the ID.
On the contrary, the muon spectrometer (MS) extends over a large radius and hence cover a large area. The lever arm will drive the momentum resolution so the intrinsic spatial resolution is less critical. Hence gaseous detectors will fit the MS. Since muons are the last particles to be detected in the outside layers of the detector, which also serve as magnetic return yoke, simple drift tubes, micro-pattern gaz chambers or resistive plate chambers will fit the need of the MS..

3.3 Can you explain why the measurement with the ID is much more precise than the measurement with the MS? Also explain why the relative resolution gets worst at high momentum for the MS?

Solution

The relative momentum resolution in a cylindrical geometry, typical of collider, is proportional to the factor $\frac{\sigma}{BL^2\sqrt{K+6}}$ where K is the number of layers and σ the average intrinsic resolution per layer. The MS has a measurement lever arm (L) 5 times larger than the ID as indicated in the text. The other parameters (B, σ and K) are not specified. But qualitatively they benefit the ID, which has more layers, higher magnetic field and better precision. One can only assume their values over-compensate for the larger level arm in the MS.