Astroparticles - Tutorial

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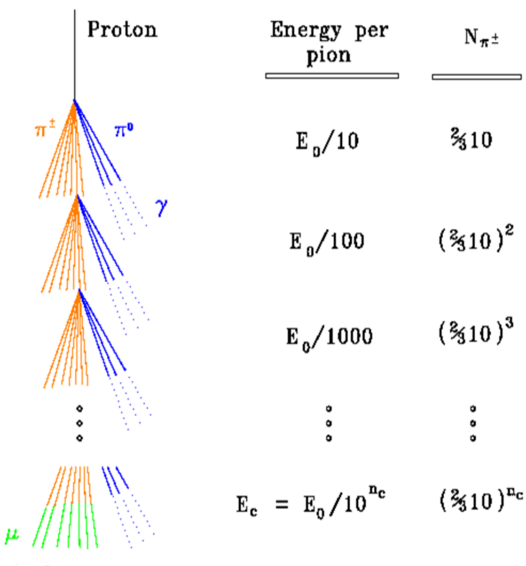
Menu:

A classical problem based on the Heitler model of shower development.

1 Heitler model for EM showers



Using a simplified model à la Heitler, we want to model the electromagnetic (EM) and the muonic component of a shower induced by a proton with an energy $E_0 = 10^{18}$eV and to study its longitudinal development.
For the sake of simplicity, we will assume the values of two fixed parameters, the critical energy for pions $\varepsilon_\pi = 100$ GeV and their interaction length $\lambda_\pi = 120$ g/cm^2.
We will also do the following hypotheses:

1. At each fixed interaction length $\lambda = \lambda_\pi$, all the non decayed hadrons with energy $> \varepsilon_\pi$ interact with a nucleus of the atmosphere.

2. Each interaction produces secondary hadrons (we assume here only pions) with multiplicity m, 2/3 of them are $\pi^\pm$ and 1/3 are π^0. One assume that the multiplicity is fixed and its value is $m = 10$.

3. Each of the m pions produced takes away a fraction of the parent energy $1/m$.

4. π^0 decay immediately in two γ that will feed the EM component.

5. When charged pions reach their critical energy ε_π, they all decay and produce each one muon eac that propagates to the ground. We will assume that the muons take away 1/2 of the critical energy.

6. Both the hadronic and the EM component reach their respective critical energy at about the same depth. After this point, charged pions will decay before they interact producing muons and the hadronic shower development will stop. As for the EM component, ionisation losses of e^+, e^- start to dominate over the radiation processes and the development will also stop. The overall shower will decrease in terms of number of particle. The depth at which both components reach critical energy is also where the number of particles in the shower reaches a maximum. This depth is called X_p^{max} and the number of particles at that point is called N_p^{max} (also called the shower size).

Remark about the EM component development: the EM component, which is fed at each step of the hadronic shower by the $\pi_0 \to \gamma\gamma$ decays, is going to develop as well on its own (but instead with multiplicity 2 and interaction length X_0). As $\lambda_\pi \approx 4 \times X_0$, after every λ_π step of roughly $4 \times X_0$ the EM components energy is divided into $2^4 = 16$ parts which is not very different than the assumed pion multiplicity. Hence, it makes sens to assume that the hadronic and EM components reach their respective critical energy at about the same depth. Note that this coincidence can also be described by the following numerical equivalence:

$$\frac{\lambda_\pi}{X_0} \approx \frac{\ln(10)}{\ln(2)}.$$

Try to answer the following questions:

1. Find the number of interaction lengths n_c before the hadronic component reaches its critical energy and give an approximate value using the given parameters.
2. Show that
$$X_p^{max} = \lambda_\pi \log_{10}\left(\frac{E_0}{\varepsilon_\pi}\right).$$
3. By how much does the depth of maximum X_p^{max} change when the primary energy E_0 changes by a $\times 10$?

Let's assume that the superposition principle holds, namely that a shower initiated by a nucleus with energy E_0 consisting of A nucleons can be described as the superposition of $A\times$ proton-induced showers each having an initial energy E_0/A.

4. Give the expression for the depth of the maximum development X_A^{max} of a shower initiated by a nucleus of mass A as a function of the interaction length λ_π and the depth of maximum for a proton shower X_p^{max}.
5. What would be the statistical difference between the depth of maximum of iron showers and of proton showers?
6. Given that the resolution on the X^{max} as measured by the Pierre Auger Observatory using the fluorescence telescopes techniques reaches $\Delta X^{max} = 20$ g/cm^2, what is the resolution achieved on $\ln(A)$?

Note that in reality, the X^{max} measurements are affected by important shower to shower fluctuations (mostly due to the broad distribution of the depths of first-interactions) that spoil a bit the resolution on $\ln(A)$ but also makes it difficult to measure it on an event per event basis.

Let now study the energy balance between muons and in the EM component.

7. Compute what is the fraction of the total energy transferred to the EM component after n_c steps.
8. Compute the number of muons produced at the end of the development. What fraction of energy do they take away.
9. Compare the EM, muonic and initial energy. Do you observed missing energy? What is taking this missing energy away?
10. Suppose now that the incident particle is an iron nucleus with the same total energy than the proton. Assuming superposition principle holds, what is the muons energy fraction in that case?

    Conclude if this muons energy fraction can be used to deduce the nature of the incident particle.

A few useful numerical values and formulae:
$(2/3)^7 \approx 6 \times 10^{-2}$
$\log_{10}(x) = \ln(x)/\ln(10)$
$\log_2(x) = \ln(x)/\ln(2)$
$\ln(10) \approx 2.3$
$\ln(2) \approx 0.7$
$\log_{10}(56) \approx 1.75$
$\log_2(56) \approx 5.8$
$\sum_{k=1}^{n} r^k = r\frac{1-r^k}{1-r}$
$X_0 = 36.7$ g/cm^2 radiation length in air