

# Experimental particle. physics

**esipap...**

European School of Instrumentation  
in Particle & Astroparticle Physics

3.

particle interactions  
in particle detectors

*lots of material borrowed from the excellent course of H.-C. Schultz-Coulon*  
<http://www.kip.uni-heidelberg.de/~coulon/Lectures/Detectors/>

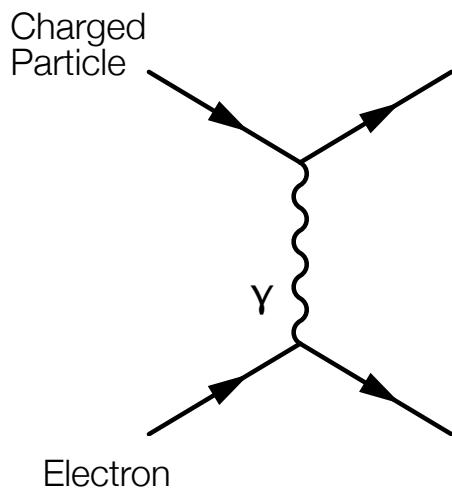
# How do we detect particles?

- In order to detect a particle, it must:
  - ✓ interact with the material of the detector
  - ✓ transfer energy in some recognizable fashion (signal)
- Detection of particles happens via their energy loss in the material they traverses

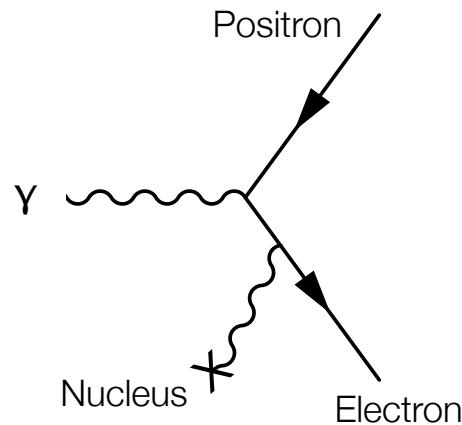
Charged particles	Ionization, Bremsstrahlung, Cherenkov, ...	multiple interactions
Photons	Photo/Compton effect, pair production	single interactions...
Hadrons	Nuclear interactions	multiple interactions
Neutrinos	Weak interactions	

# Example of particle interactions

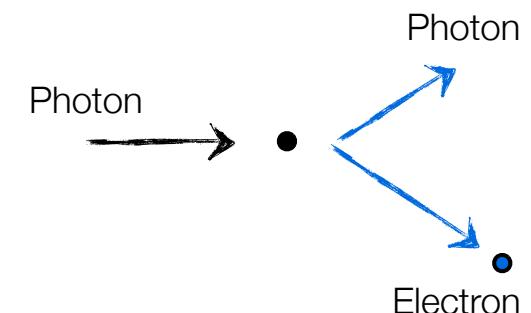
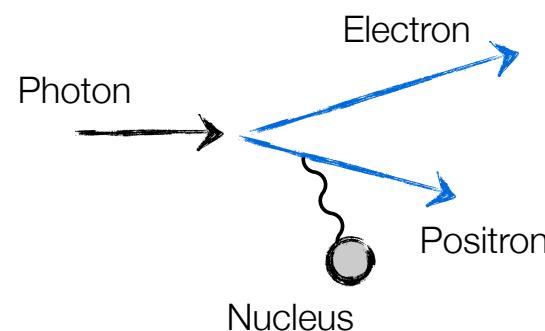
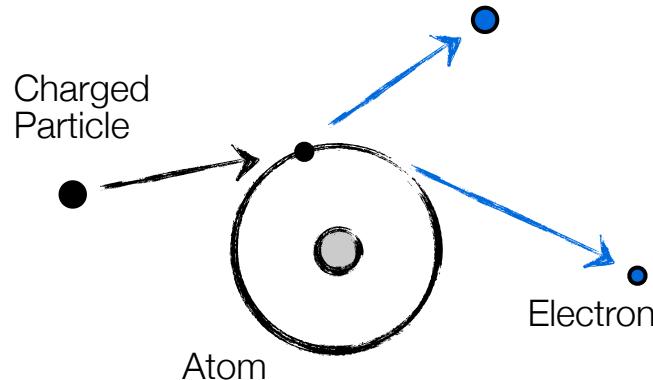
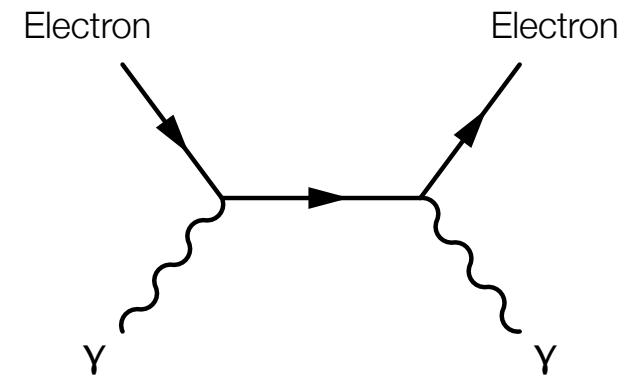
Ionization:



Pair production:



Compton scattering:

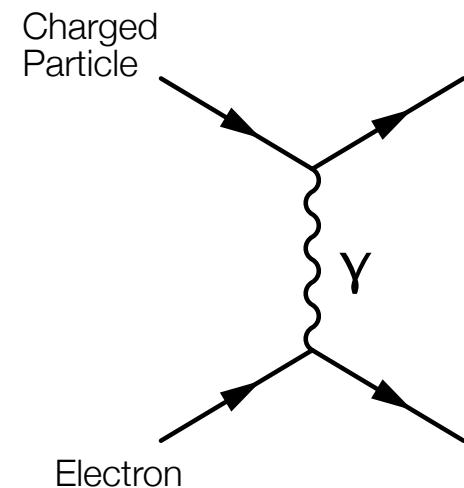


# Energy loss by ionization: Bethe-Bloch formula

For now assume:  $Mc^2 \gg m_e c^2$

i.e. energy loss for heavy charged particles  
[ $dE/dx$  for electrons more difficult ...]

Interaction dominated  
by elastic collisions with electrons ...



Bethe-Bloch Formula

$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[ \frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

$\propto 1/\beta^2 \cdot \ln(\text{const} \cdot \beta^2 \gamma^2)$

# Bethe-Bloch formula for heavy particles

[see e.g. PDG 2010]

$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[ \frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

[· p]

density

$$K = 4\pi N_A r_e^2 m_e c^2 = 0.307 \text{ MeV g}^{-1} \text{ cm}^2$$

$$T_{\max} = 2m_e c^2 \beta^2 \gamma^2 / (1 + 2\gamma m_e/M + (m_e/M)^2)$$

[Max. energy transfer in single collision]

$z$  : Charge of incident particle

$M$  : Mass of incident particle

$Z$  : Charge number of medium

$A$  : Atomic mass of medium

$I$  : Mean excitation energy of medium

$\delta$  : Density correction [transv. extension of electric field]

$$N_A = 6.022 \cdot 10^{23}$$

[Avogardo's number]

$$r_e = e^2 / 4\pi\epsilon_0 m_e c^2 = 2.8 \text{ fm}$$

[Classical electron radius]

$$m_e = 511 \text{ keV}$$

[Electron mass]

$$\beta = v/c$$

[Velocity]

$$\gamma = (1 - \beta^2)^{-1/2}$$

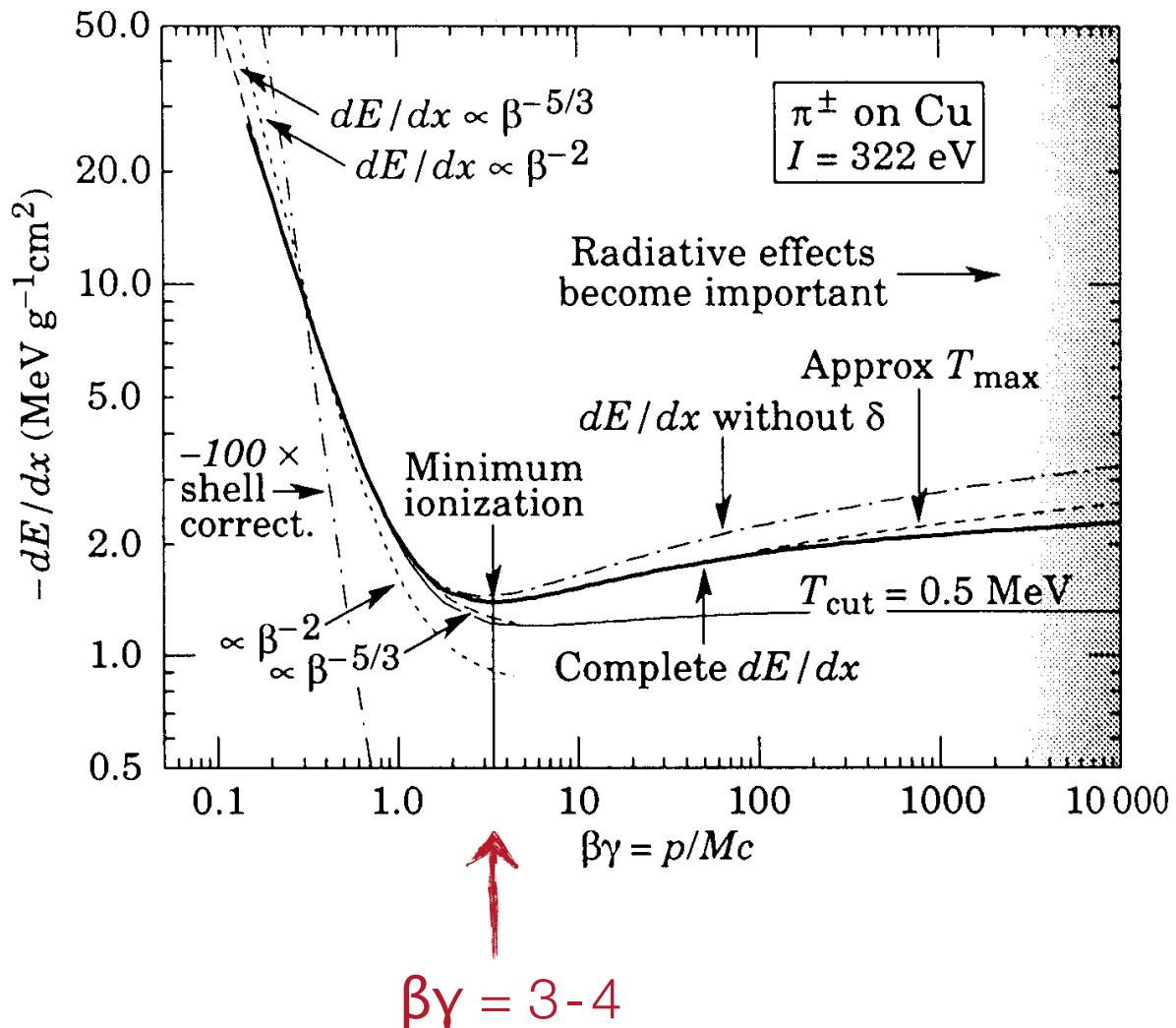
[Lorentz factor]

Validity:

$$0.05 < \beta\gamma < 500$$

$$M > m_\mu$$

# Energy loss of pions in Cu



Minimum ionizing particles (MIP):  $\beta\gamma = 3-4$

$dE/dx$  falls  $\sim \beta^{-2}$ ; kinematic factor  
[precise dependence:  $\sim \beta^{-5/3}$ ]

$dE/dx$  rises  $\sim \ln(\beta\gamma)^2$ ; relativistic rise  
[rel. extension of transversal E-field]

Saturation at large  $(\beta\gamma)$  due to density effect (correction  $\delta$ )  
[polarization of medium]

Units:  $\text{MeV g}^{-1} \text{cm}^2$

MIP loses  $\sim 13 \text{ MeV/cm}$   
[density of copper:  $8.94 \text{ g/cm}^3$ ]

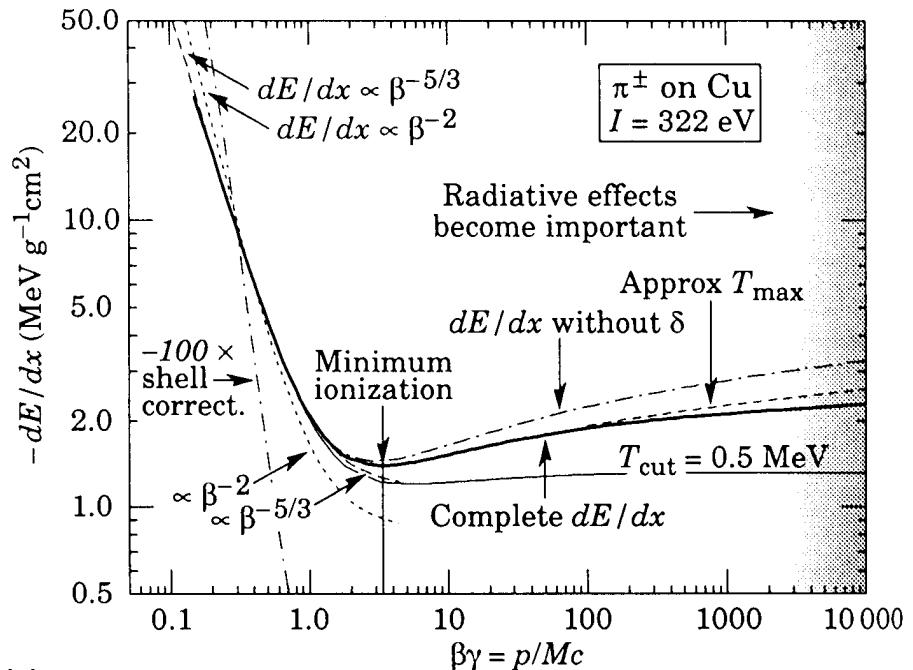
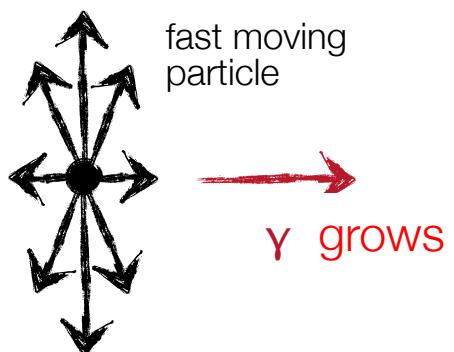
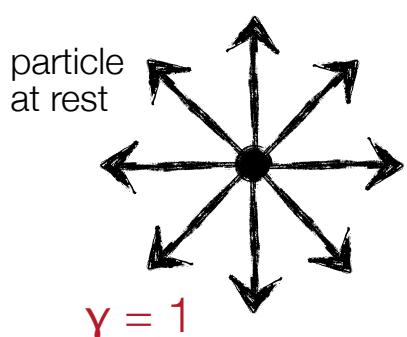
# Understanding Bethe-Bloch

1/ $\beta^2$ -dependence:

Slower particles fell electric force of atomic electrons for longer time ...

Relativistic rise for  $\beta\gamma > 4$ :

High energy particle: transversal electric field increases due to Lorentz transform;  $E_y \rightarrow \gamma E_y$ . Thus interaction cross section increases ...



Corrections:

- low energy : shell corrections
- high energy : density corrections

# Understanding Bethe-Bloch

## Density correction:

Polarization effect ...

[density dependent]

- Shielding of electrical field far from particle path; effectively cuts off the long range contribution ...

More relevant at high  $\gamma$  ...

[Increased range of electric field; larger  $b_{\max}$ ; ...]

For high energies:

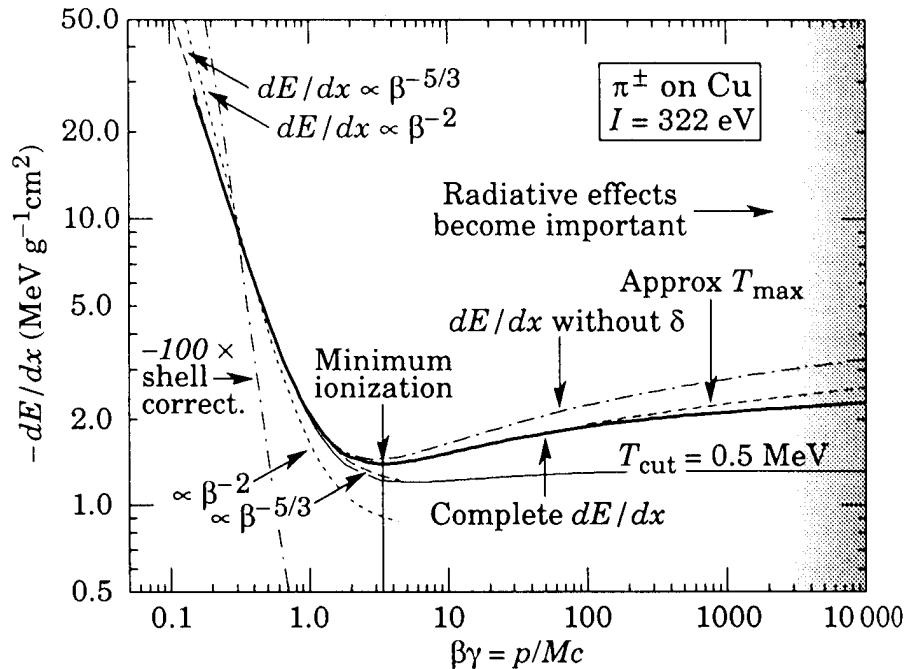
$$\delta/2 \rightarrow \ln(\hbar\omega/I) + \ln \beta\gamma - 1/2$$

## Shell correction:

Arises if particle velocity is close to orbital velocity of electrons, i.e.  $\beta c \sim v_e$ .

Assumption that electron is at rest breaks down ...

Capture process is possible ...



Density effect leads to saturation at high energy ...

Shell correction are in general small ...

# Energy loss of (heavy) charged particles

Dependence on

Mass A

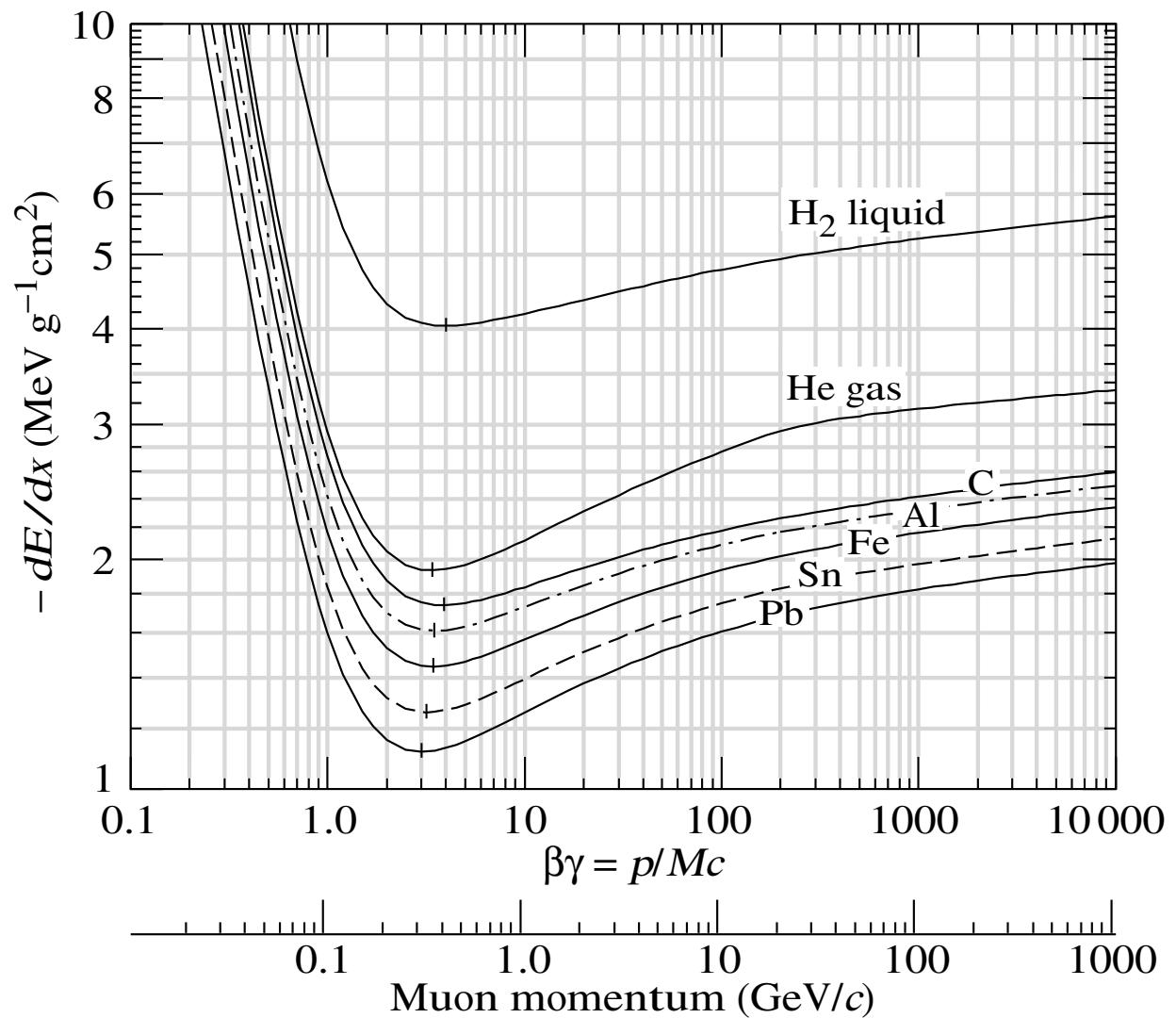
Charge Z

of target nucleus

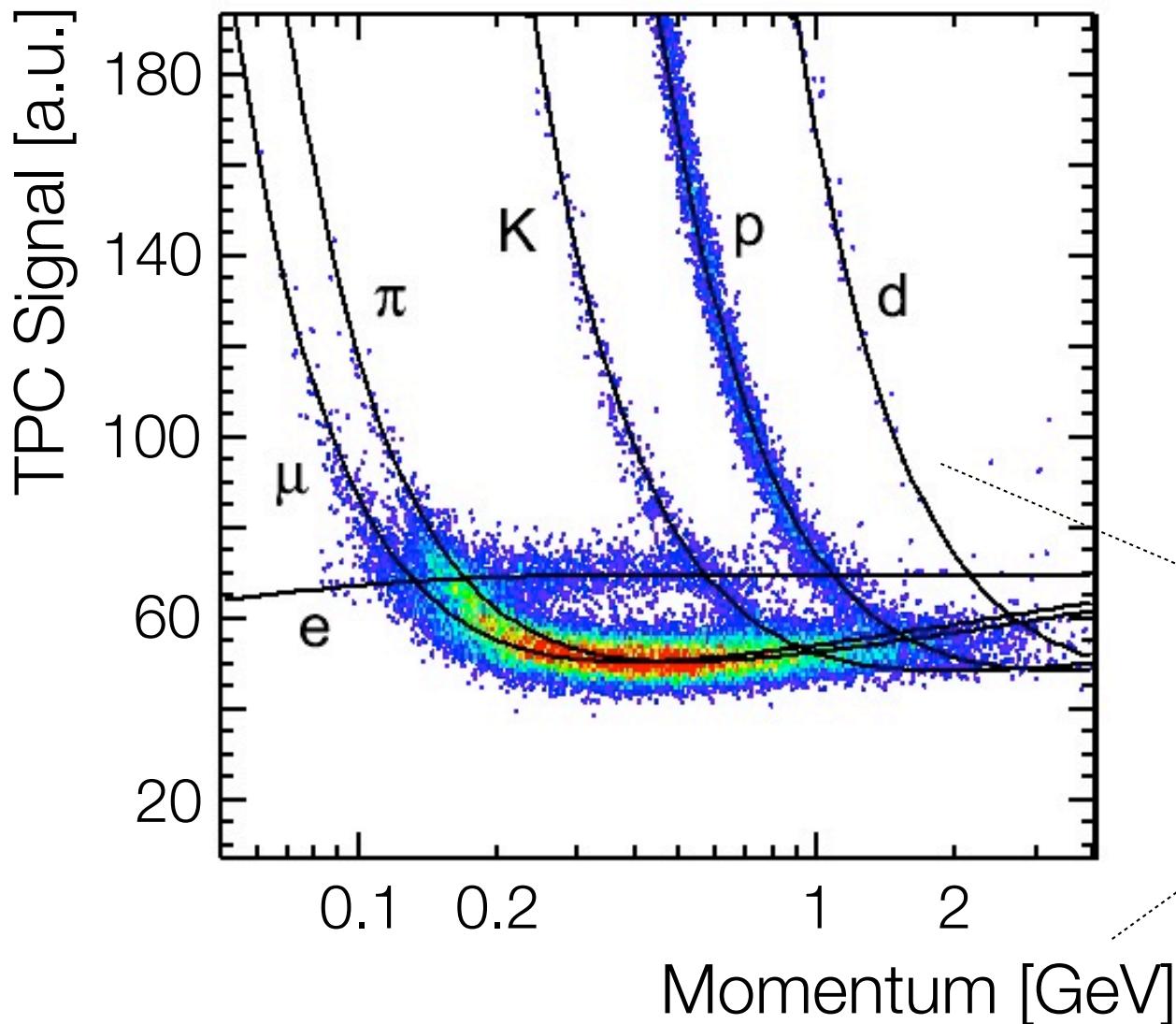
Minimum ionization:

ca. 1 - 2 MeV/g cm<sup>-2</sup>  
[H<sub>2</sub>: 4 MeV/g cm<sup>-2</sup>]

$$-\left\langle \frac{dE}{dx} \right\rangle \sim \frac{Z}{A}$$



# Identifying particles by $dE/dx$

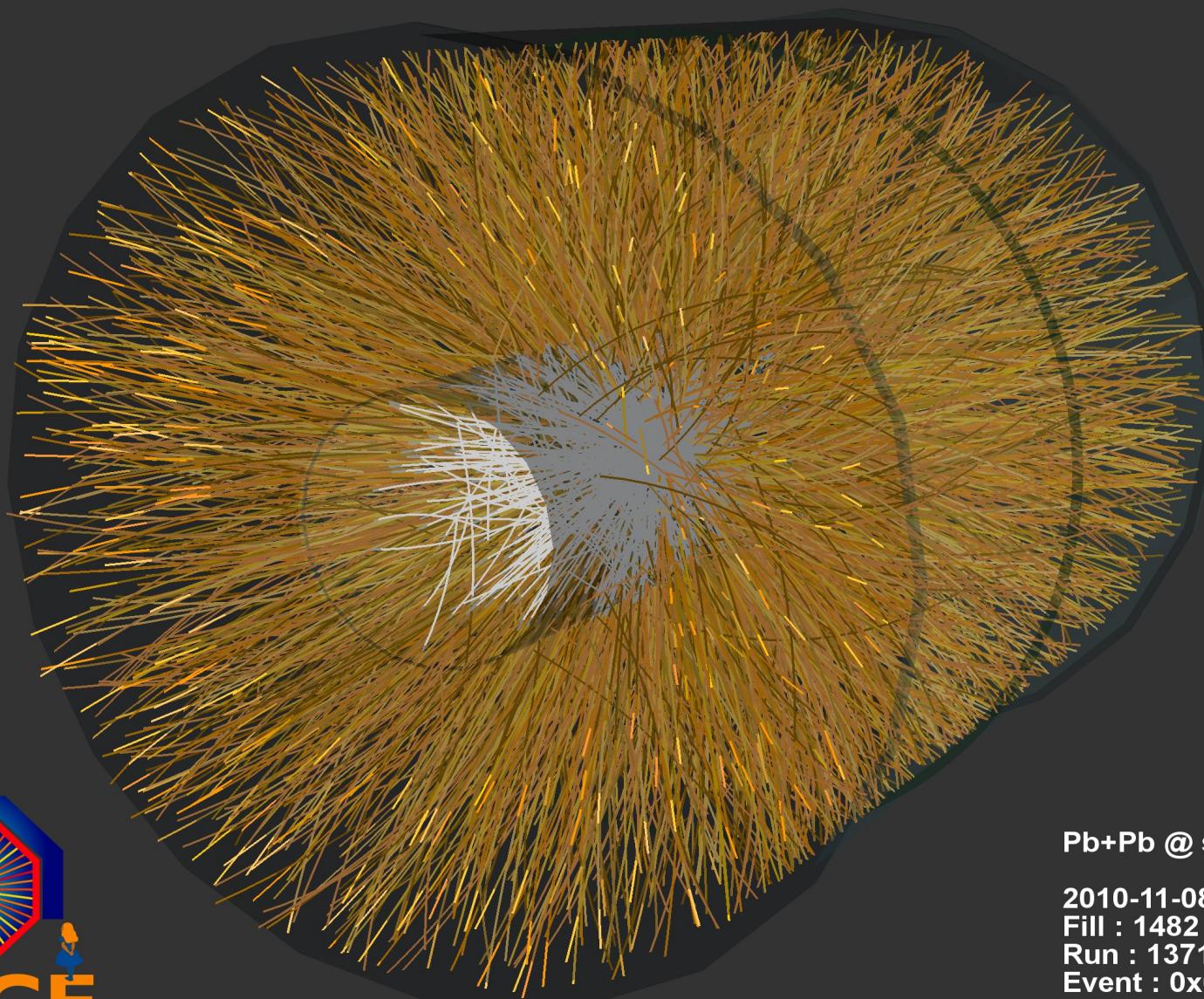


Measured  
energy loss  
[ALICE TPC, 2009]

Bethe-Bloch

Remember:  
 $dE/dx$  depends on  $\beta$ !

# ALICE TPC



Pb+Pb @  $\text{sqrt}(s) = 2.76 \text{ ATeV}$

2010-11-08 11:30:46

Fill : 1482

Run : 137124

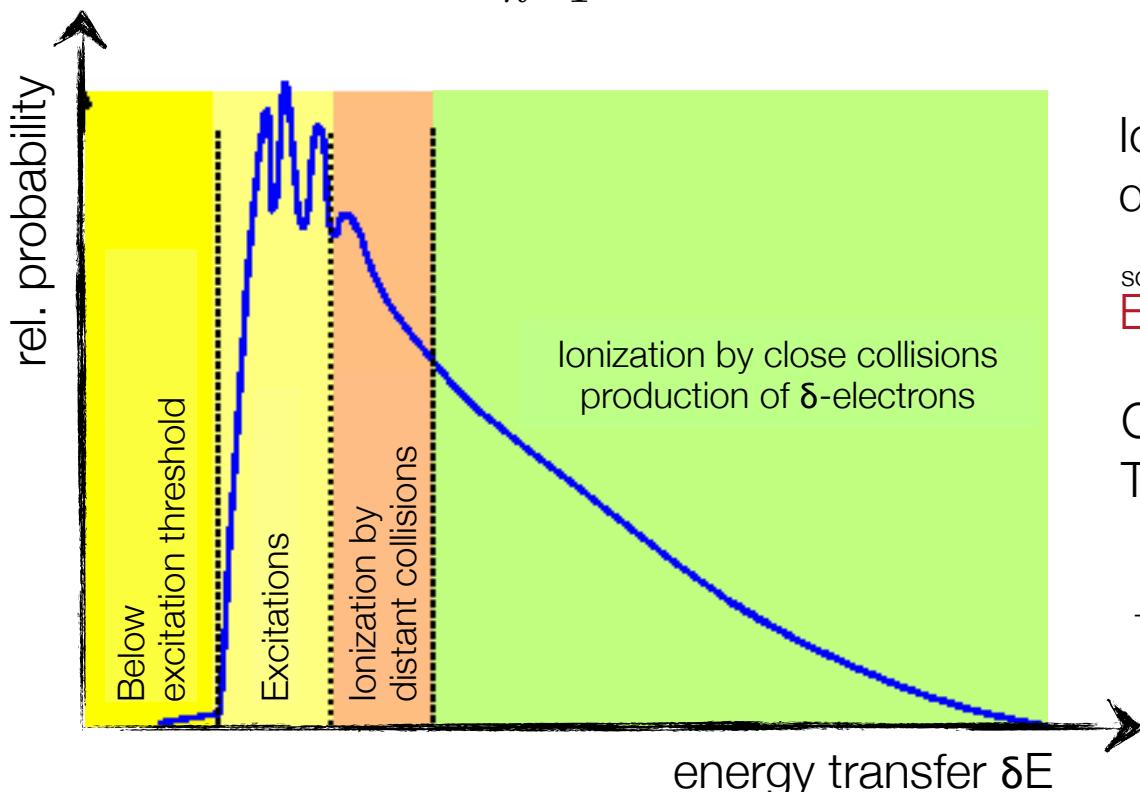
Event : 0x00000000D3BBE693

# $dE/dx$ fluctuations

Bethe-Bloch describes mean energy loss; measurement via energy loss  $\Delta E$  in a material of thickness  $\Delta x$  with

$$\Delta E = \sum_{n=1}^N \delta E_n$$

$N$  : number of collisions  
 $\delta E$  : energy loss in a single collision



Ionization loss  $\delta E$   
distributed statistically ...

so-called  
**Energy loss 'straggling'**

Complicated problem ...  
Thin absorbers: **Landau distribution**

Standard Gauss with mean energy loss  $E_0$   
+ tail towards high energies due to  $\delta$ -electrons

see also Allison & Cobb  
[Ann. Rev. Nucl. Part. Sci. 30 (1980) 253.]

# Mean particle range

Integrate over energy loss  
from E down to 0

$$R = \int_E^0 \frac{dE}{dE/dx}$$

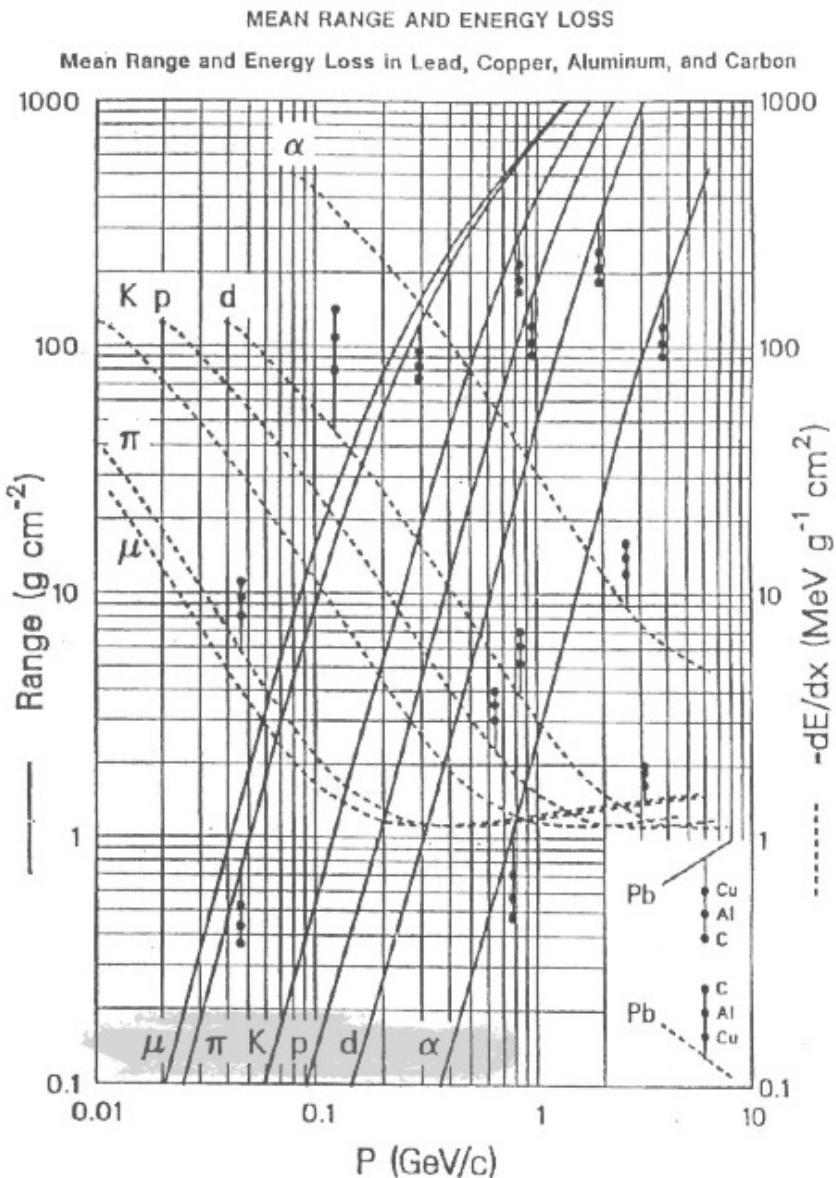
Example:

Proton with  $p = 1 \text{ GeV}$

Target: lead with  $\rho = 11.34 \text{ g/cm}^3$

$$R/M = 200 \text{ g cm}^{-2} \text{ GeV}^{-1}$$

$$\rightarrow R = 200/11.34/1 \text{ cm} \sim 20 \text{ cm}$$



# Energy loss of electrons

Bethe-Bloch formula needs modification

Incident and target electron have same mass  $m_e$   
Scattering of identical, undistinguishable particles

$$-\left\langle \frac{dE}{dx} \right\rangle_{\text{el.}} = K \frac{Z}{A} \frac{1}{\beta^2} \left[ \ln \frac{m_e \beta^2 c^2 \gamma^2 T}{2I^2} + F(\gamma) \right]$$

[T: kinetic energy of electron]

Remark: different energy loss for electrons and positrons at low energy as positrons are not identical with electrons; different treatment ...

# Bremsstrahlung and Radiation Length

Bremsstrahlung arises if particles are accelerated in Coulomb field of nucleus

$$\frac{dE}{dx} = 4\alpha N_A \frac{z^2 Z^2}{A} \left( \frac{1}{4\pi\epsilon_0} \frac{e^2}{mc^2} \right)^2 E \ln \frac{183}{Z^{\frac{1}{3}}} \propto \frac{E}{m^2}$$

i.e. energy loss proportional to  $1/m^2$  → main relevance for electrons ...

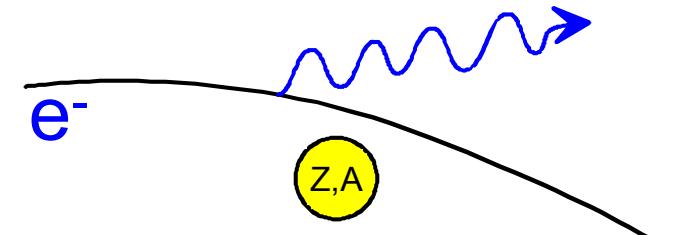
... or ultra-relativistic muons

Consider electrons:

$$\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 \cdot E \ln \frac{183}{Z^{\frac{1}{3}}}$$

$$\frac{dE}{dx} = \frac{E}{X_0} \quad \text{with} \quad X_0 = \frac{A}{4\alpha N_A Z^2 r_e^2 \ln \frac{183}{Z^{\frac{1}{3}}}}$$

[Radiation length in g/cm<sup>2</sup>]



$$\rightarrow E = E_0 e^{-x/X_0}$$

After passage of one  $X_0$  electron has lost all but  $(1/e)^{th}$  of its energy  
[i.e. 63%]

# Critical Energy

Critical energy:

$$\frac{dE}{dx}(E_c) \Big|_{\text{Brems}} = \frac{dE}{dx}(E_c) \Big|_{\text{Ion}}$$

Approximation:

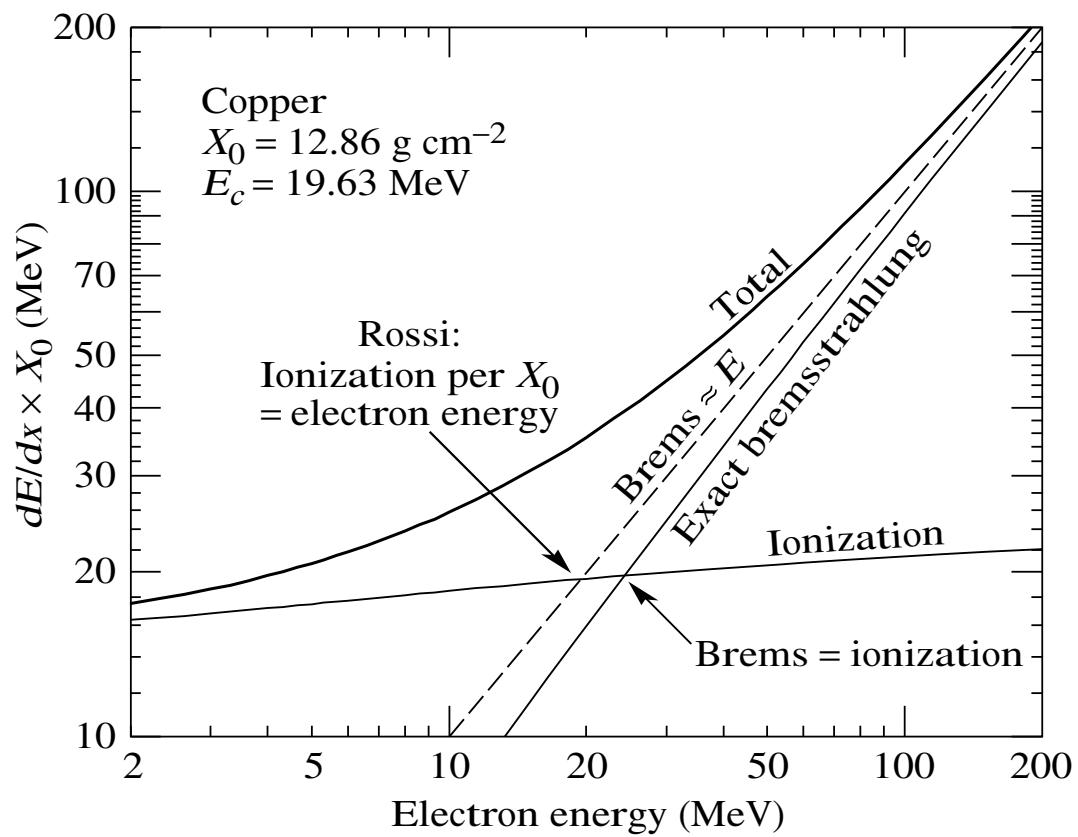
$$E_c^{\text{Gas}} = \frac{710 \text{ MeV}}{Z + 0.92}$$

$$E_c^{\text{Sol/Liq}} = \frac{610 \text{ MeV}}{Z + 1.24}$$

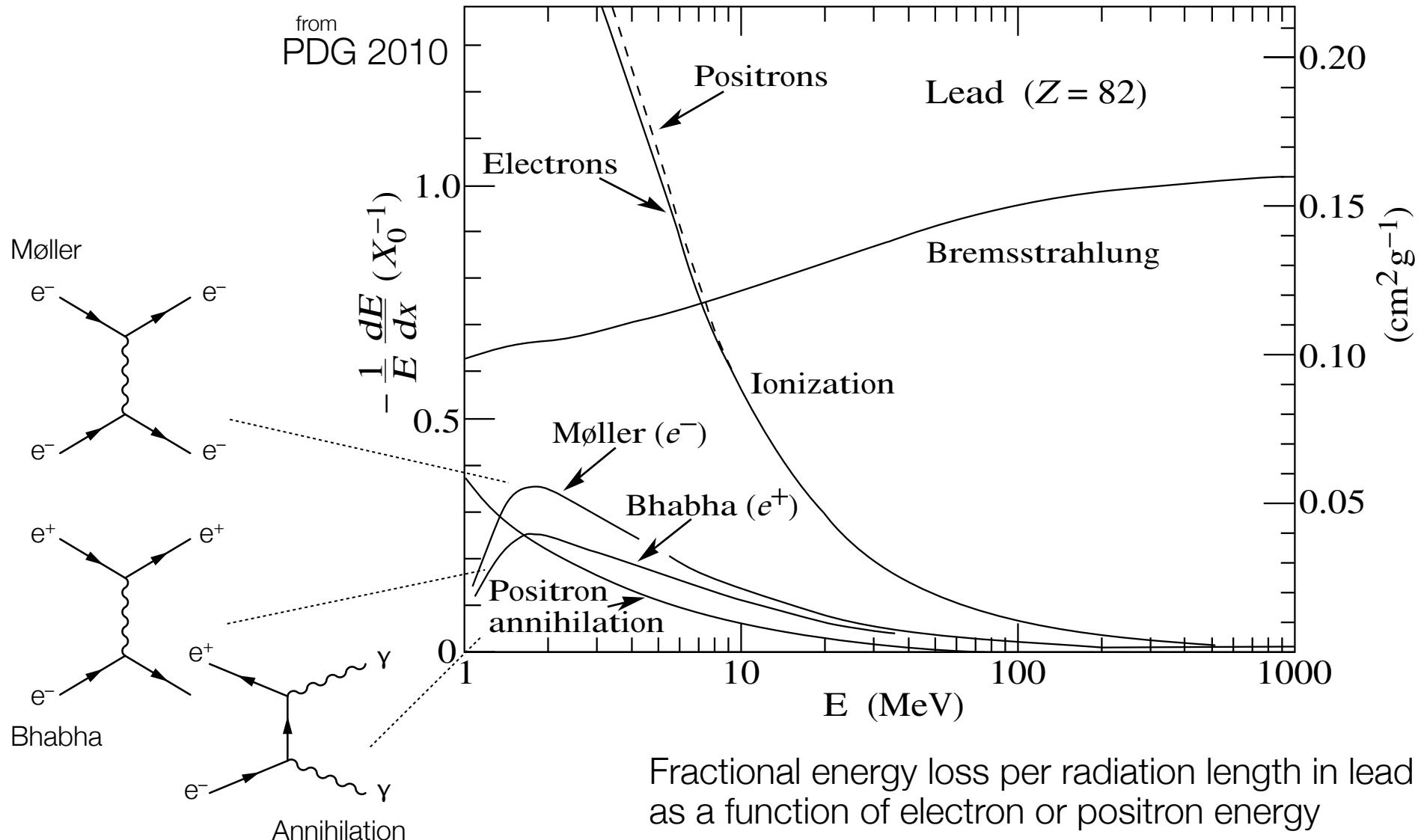
Example Copper:

$$E_c \approx 610/30 \text{ MeV} \approx 20 \text{ MeV}$$

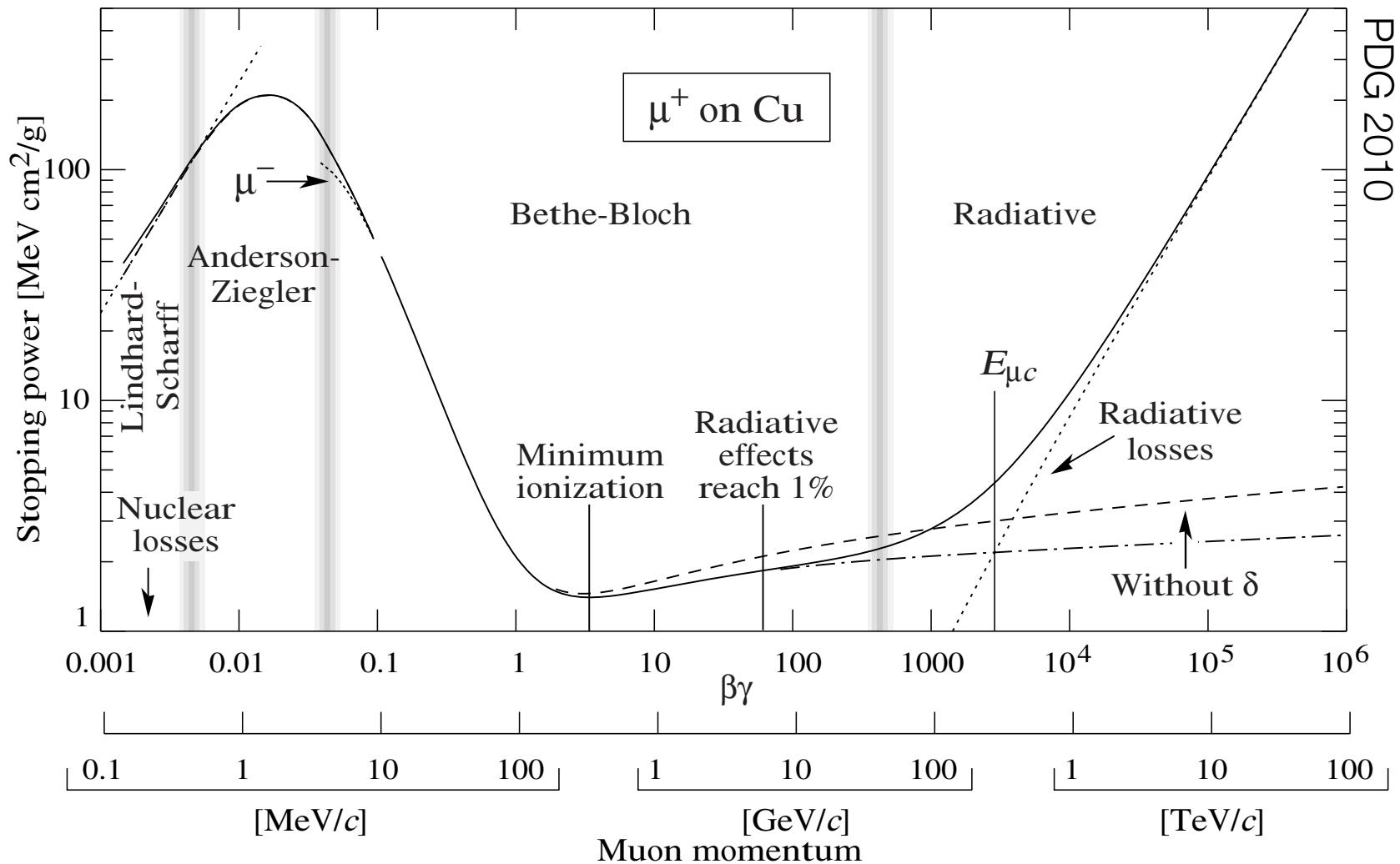
$$\left( \frac{dE}{dx} \right)_{\text{Tot}} = \left( \frac{dE}{dx} \right)_{\text{Ion}} + \left( \frac{dE}{dx} \right)_{\text{Brems}}$$



# Total Energy Loss of Electrons



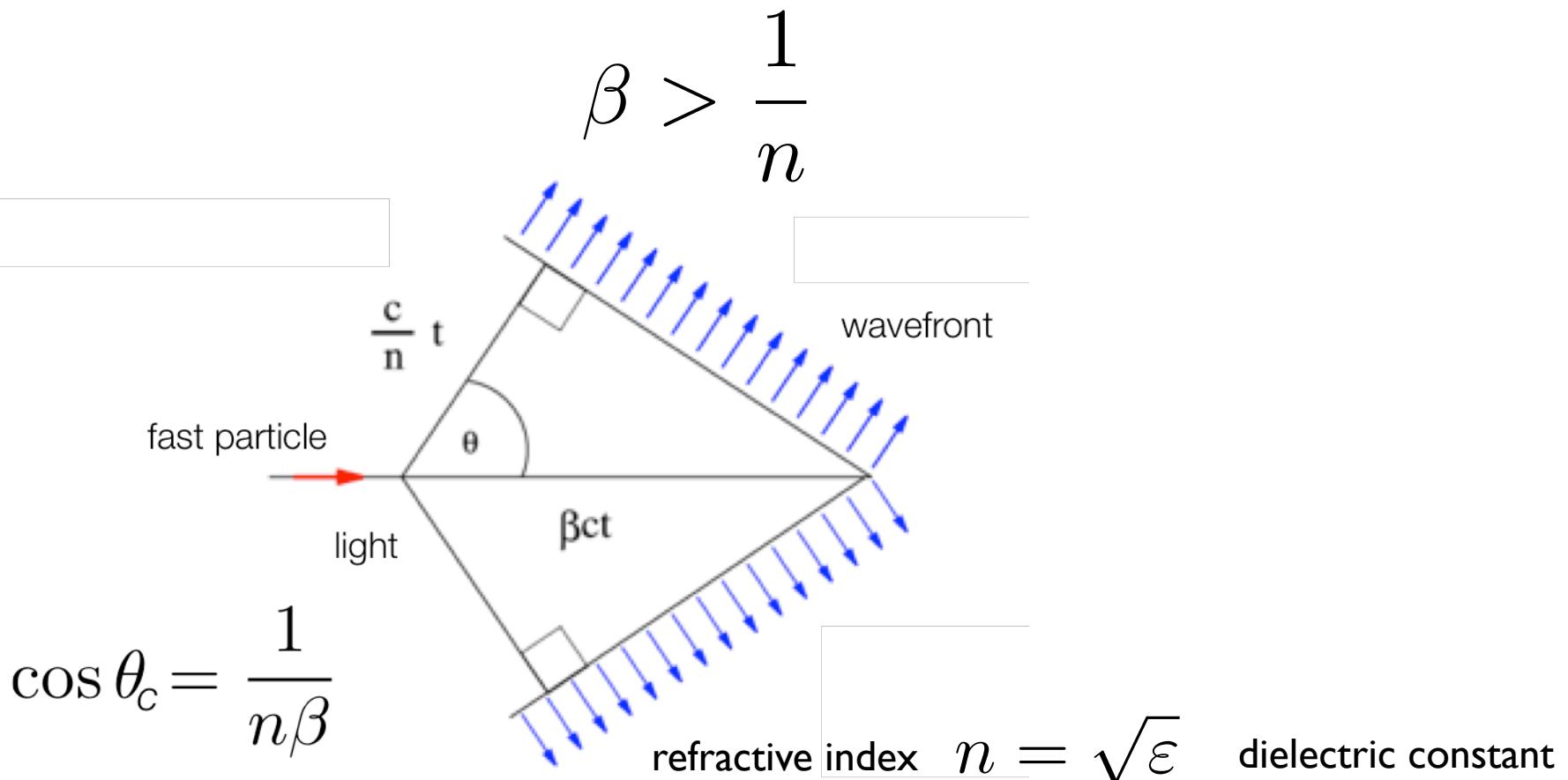
# Energy loss for muons



# Cherenkov radiation

Particles moving in a medium with speed larger than speed of light in that medium will loose energy by emitting electromagnetic radiation

- ✓ Charged particle polarize medium generating an electrical dipole varying in time
- ✓ Every point in trajectory emits a spherical EM wave, waves constructively interfere...



# Cherenkov radiation: radiators

## Parameters of Typical Radiator

Medium	n	$\beta_{\text{thr}}$	$\theta_{\max} [\beta=1]$	$N_{\text{ph}} [\text{eV}^{-1} \text{cm}^{-1}]$
Air	1.000283	0.9997	1.36	0.208
Isobutan	1.00127	0.9987	2.89	0.941
Water	1.33	0.752	41.2	160.8
Quartz	1.46	0.685	46.7	196.4

Note: Energy loss by Cherenkov radiation very small w.r.t. ionization (< 1%).

Example:

[Proton with  $E_{\text{kin}} = 1 \text{ GeV}$  passing through 1 cm water ]

$$\beta = p/E \approx 0.875; \cos\theta_C = 1/n\beta = 0.859 \rightarrow \theta_C = 30.8^\circ$$

$$d^2N/(dEdx) = 370 \sin^2\theta_C \text{ eV}^{-1} \text{ cm}^{-1} \approx 100 \text{ eV}^{-1} \text{ cm}^{-1}$$

$$\begin{aligned} \rightarrow \Delta E_{\text{loss}} &= \langle E \rangle d^2N/(dEdx) \Delta E \Delta x \\ &= 2.5 \text{ eV} \cdot 100 \text{ eV}^{-1} \text{ cm}^{-1} \cdot 5 \text{ eV} \cdot 1 \text{ cm} = 1.25 \text{ keV} \end{aligned}$$

Visible light only!  
[ $E = 1 - 5 \text{ eV}; \lambda = 300 - 600 \text{ nm}$ ]

]  
 $\Delta E_{\text{loss}} < 1.25 \text{ keV}$

# LHCb RICH

Measurement of Cherenkov angle:

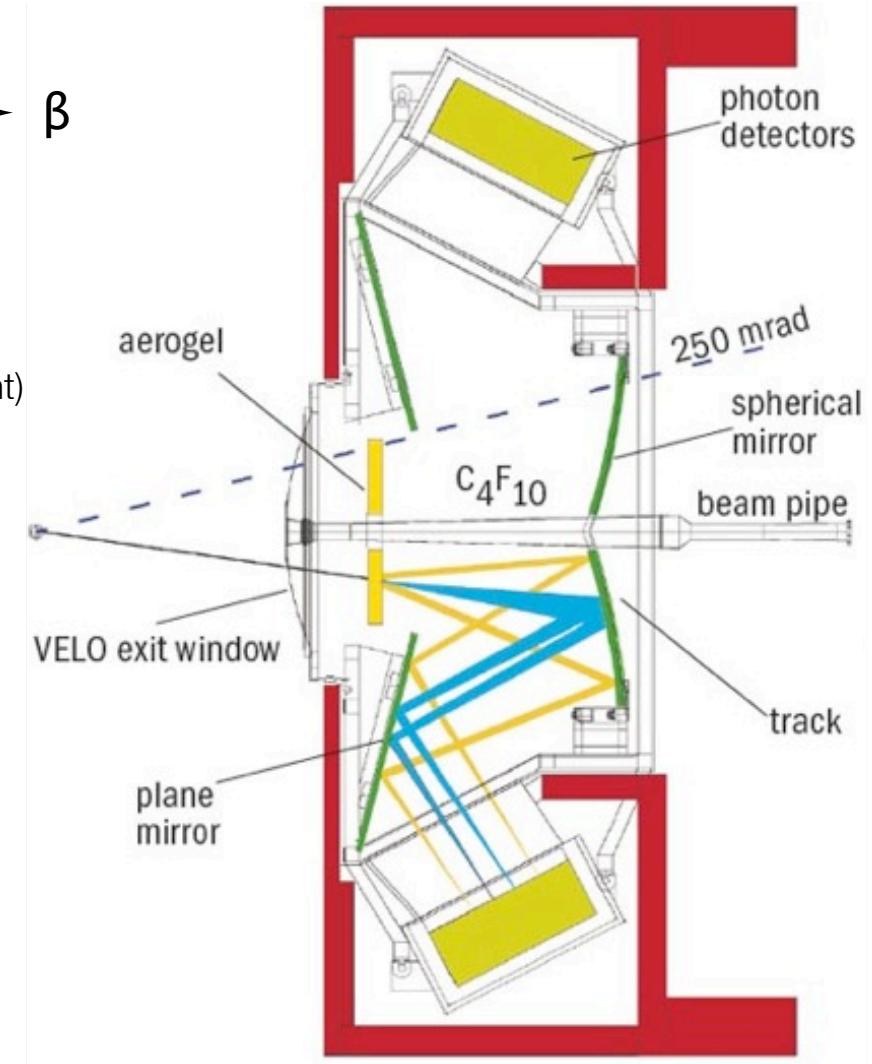
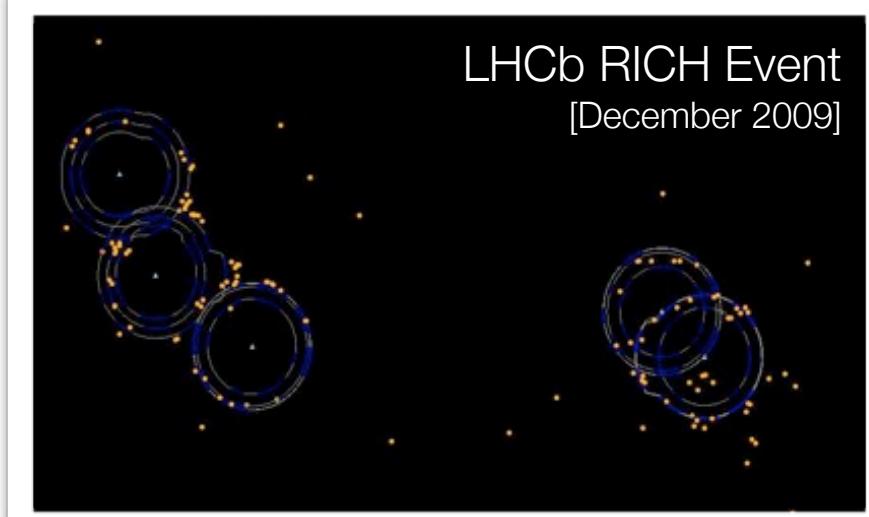
Use medium with known refractive index  $n \rightarrow \beta$

Principle of:

RICH (Ring Imaging Cherenkov Counter)

DIRC (Detection of Internally Reflected Cherenkov Light)

DISC (special DIRC; e.g. Panda)



LHCb RICH

# Interaction of photons with matter

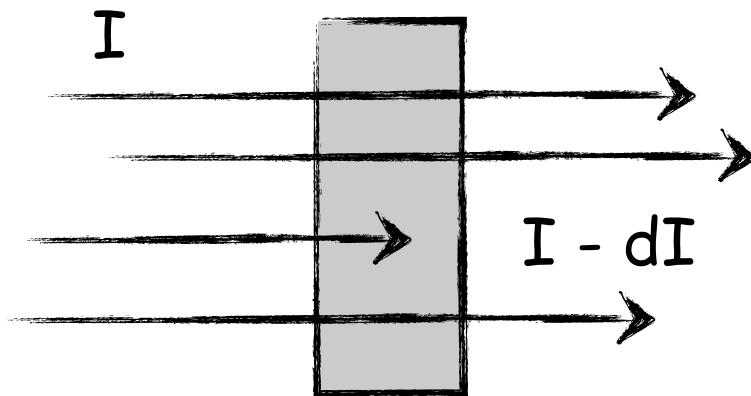
Characteristic for interactions of photons with matter:

A single interaction removes photon from beam !

Possible Interactions

Photoelectric Effect  
Compton Scattering  
Pair Production

Rayleigh Scattering ( $\gamma A \rightarrow \gamma A$ ; A = atom; coherent)  
Thomson Scattering ( $\gamma e \rightarrow \gamma e$ ; elastic scattering)  
Photo Nuclear Absorption ( $\gamma K \rightarrow pK/nK$ )  
Nuclear Resonance Scattering ( $\gamma K \rightarrow K^* \rightarrow \gamma K$ )  
Delbrück Scattering ( $\gamma K \rightarrow \gamma K$ )  
Hadron Pair production ( $\gamma K \rightarrow h^+h^- K$ )



$$dI = -\mu I dx$$

[  $\mu$ : absorption coefficient ]

depends on  
 $E, Z, \rho$

→ Beer-Lambert law:

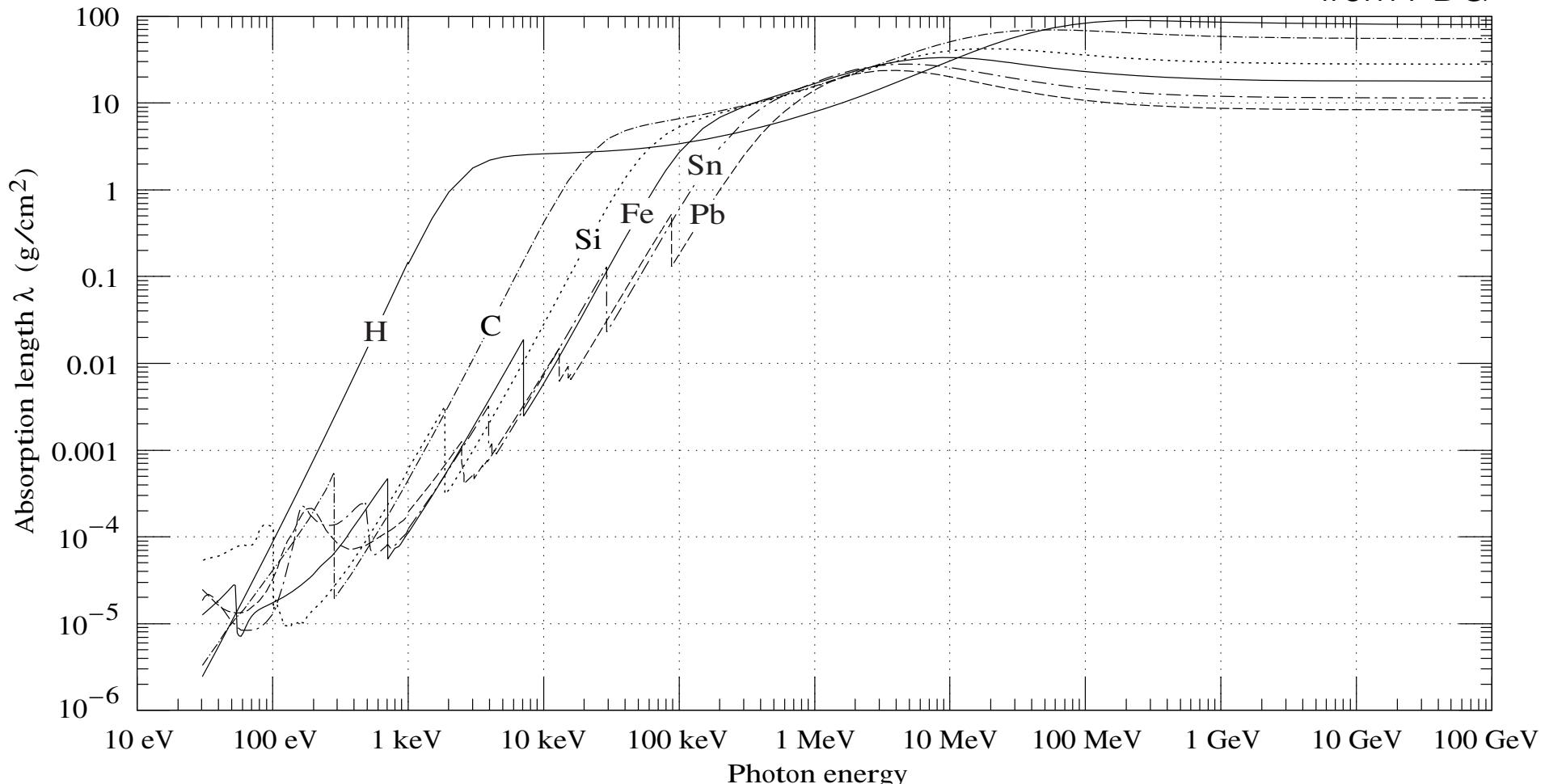
$$I(x) = I_0 e^{-\mu x}$$

with  $\lambda = 1/\mu = 1/n\sigma$

[ mean free path ]

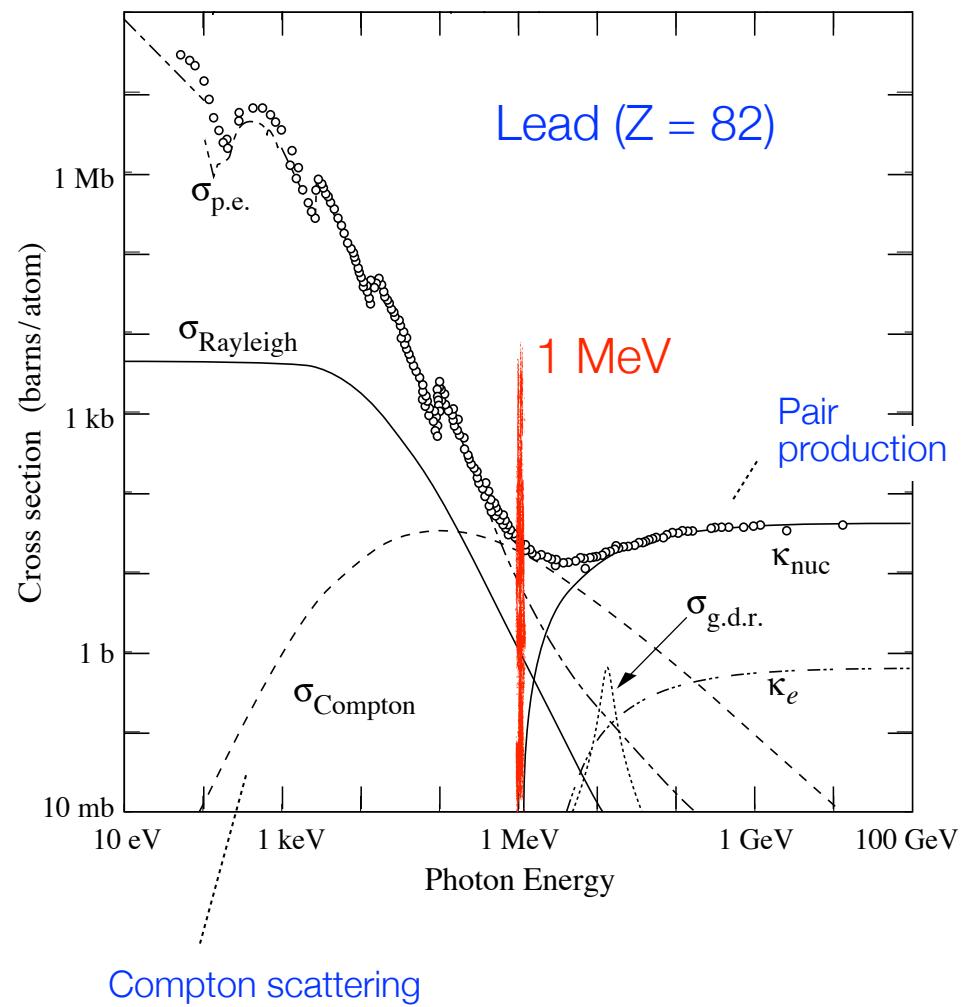
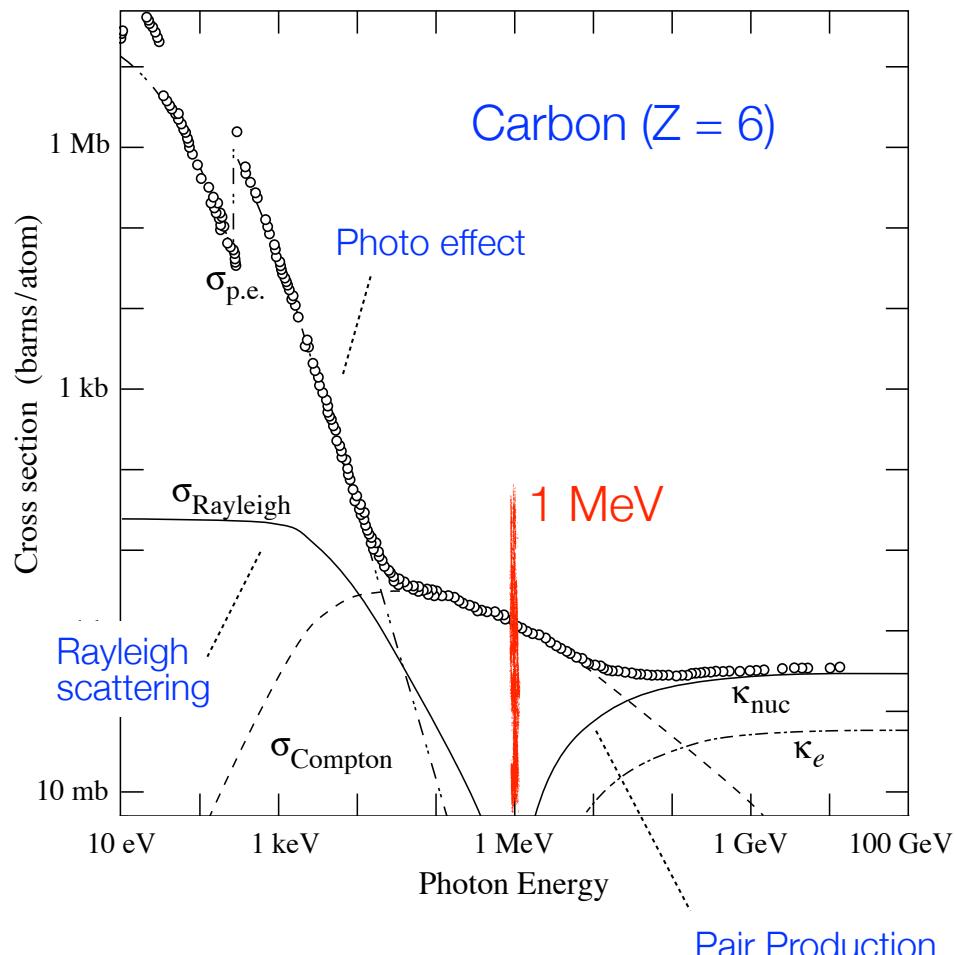
# Interaction of photons with matter

from PDG



# Interaction of photons with matter

Photon Total Cross Sections



# Pair production

Cross Section:  
[for  $E_\gamma \gg m_e c^2$ ]

$$\sigma_{\text{pair}} \approx \underbrace{\frac{7}{9} \left( 4 \alpha r_e^2 Z^2 \ln \frac{183}{Z^{\frac{1}{3}}} \right)}_{A/N_A X_0}$$

$A/N_A X_0$

[ $X_0$ : radiation length]  
[in cm or g/cm<sup>2</sup>]

Absorption coefficient:

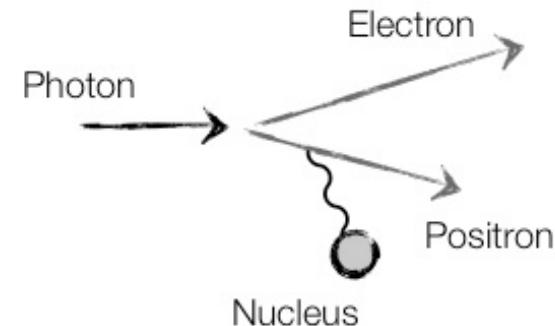
$$\mu = n \sigma \quad [\text{with } n: \text{particle density}]$$

$$\mu = \rho \cdot N_A / A \cdot \sigma_{\text{pair}}$$

$$= 7/9 \frac{1}{X_0}$$

[where now  $X_0$  is in cm]

$$I(x) = I_0 e^{-\mu x}$$



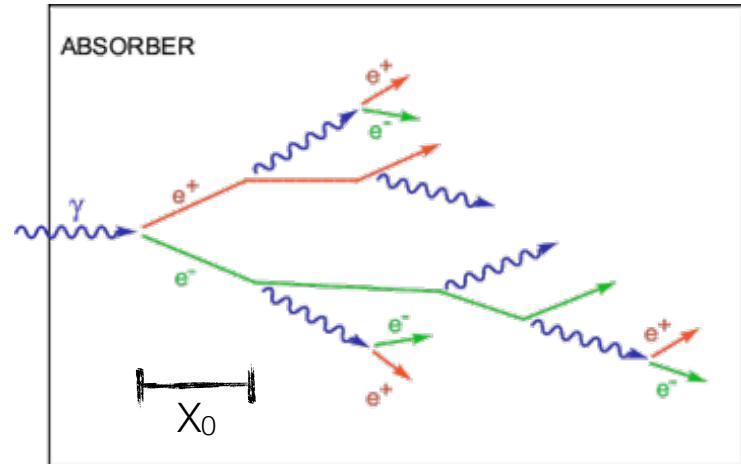
	$\rho$ [g/cm <sup>3</sup> ]	$X_0$ [cm]
H <sub>2</sub> [fl.]	0.071	865
C	2.27	18.8
Fe	7.87	1.76
Pb	11.35	0.56
Air	$1.2 \cdot 10^{-3}$	$30 \cdot 10^3$

# Electromagnetic showers

Reminder:

Dominant processes  
at high energies ...

Photons : Pair production  
Electrons : Bremsstrahlung



Pair production:

$$\begin{aligned}\sigma_{\text{pair}} &\approx \frac{7}{9} \left( 4 \alpha r_e^2 Z^2 \ln \frac{183}{Z^{\frac{1}{3}}} \right) \\ &= \frac{7}{9} \frac{A}{N_A X_0} \quad [X_0: \text{radiation length}] \quad [\text{in cm or g/cm}^2]\end{aligned}$$

Absorption  
coefficient:

$$\mu = n\sigma = \rho \frac{N_A}{A} \cdot \sigma_{\text{pair}} = \frac{7}{9} \frac{\rho}{X_0}$$

Bremsstrahlung:

$$\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 \cdot E \ln \frac{183}{Z^{\frac{1}{3}}} = \frac{E}{X_0}$$

$$\rightarrow E = E_0 e^{-x/X_0}$$

After passage of one  $X_0$  electron  
has only  $(1/e)^{\text{th}}$  of its primary energy ...  
[i.e. 37%]

# Electromagnetic showers

Further basics:

Critical Energy [see above]:

$$\left. \frac{dE}{dx}(E_c) \right|_{\text{Brems}} = \left. \frac{dE}{dx}(E_c) \right|_{\text{Ion}}$$

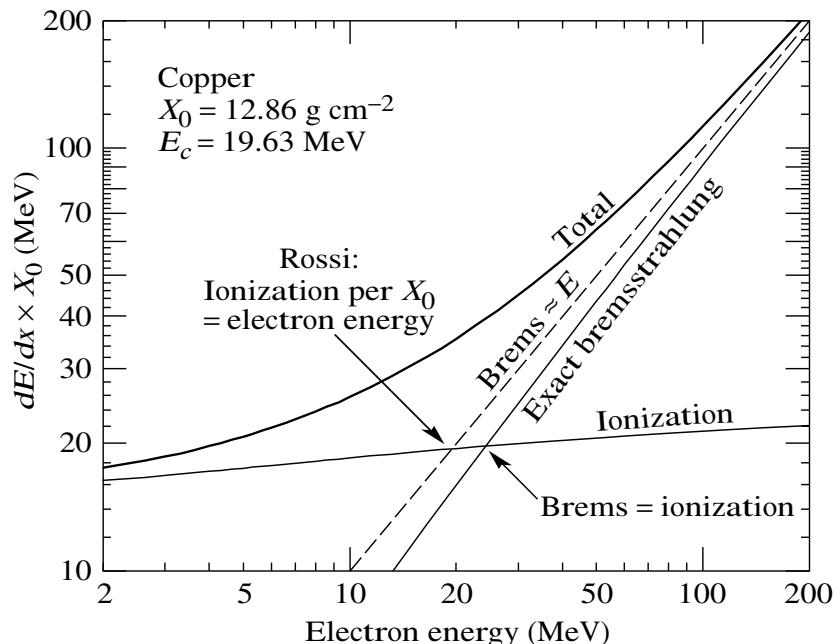
Approximations:

$$E_c^{\text{Gas}} = \frac{710 \text{ MeV}}{Z + 0.92} \quad \left[ E_c^{\text{Sol/Liq}} = \frac{610 \text{ MeV}}{Z + 1.24} \right]$$

$$\left( \frac{dE}{dx} \right)_{\text{Brems}} / \left( \frac{dE}{dx} \right)_{\text{Ion}} \approx \frac{Z \cdot E}{800 \text{ MeV}}$$

Transverse size of EM shower given by radiation length via Molière radius

[see also later]



with:

$$\left. \frac{dE}{dx} \right|_{\text{Brems}} = \frac{E}{X_0} \quad \& \quad \left. \frac{dE}{dx} \right|_{\text{Ion}} \approx \frac{E_c}{X_0} = \text{const.}$$

$$R_M = \frac{21 \text{ MeV}}{E_c} X_0$$

R<sub>M</sub> : Molière radius  
 E<sub>c</sub> : Critical Energy [Rossi]  
 X<sub>0</sub> : Radiation length

# Electromagnetic showers

Typical values for  $X_0$ ,  $E_c$  and  $R_M$  of materials used in calorimeter

	$X_0$ [cm]	$E_c$ [MeV]	$R_M$ [cm]
Pb	0.56	7.2	1.6
Scintillator (Sz)	34.7	80	9.1
Fe	1.76	21	1.8
Ar (liquid)	14	31	9.5
BGO	1.12	10.1	2.3
Sz/Pb	3.1	12.6	5.2
PB glass (SF5)	2.4	11.8	4.3

# EM shower longitudinal development

## Longitudinal profile

Parametrization:  
[Longo 1975]

$$\frac{dE}{dt} = E_0 t^\alpha e^{-\beta t}$$

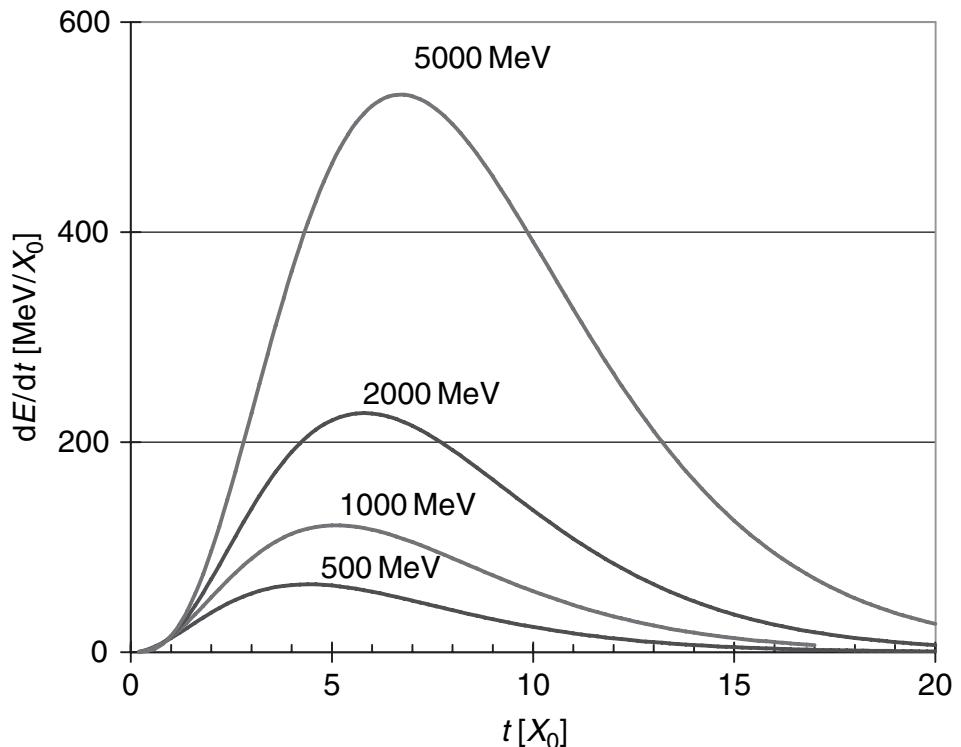
$\alpha, \beta$  : free parameters

$t^\alpha$  : at small depth number of secondaries increases ...

$e^{-\beta t}$  : at larger depth absorption dominates ...

Numbers for  $E = 2$  GeV (approximate):

$$\alpha = 2, \beta = 0.5, t_{\max} = \alpha/\beta$$



More exact  
[Longo 1985]

$$\frac{dE}{dt} = E_0 \cdot \beta \cdot \frac{(\beta t)^{\alpha-1} e^{-\beta t}}{\Gamma(\alpha)}$$

[ $\Gamma$ : Gamma function]

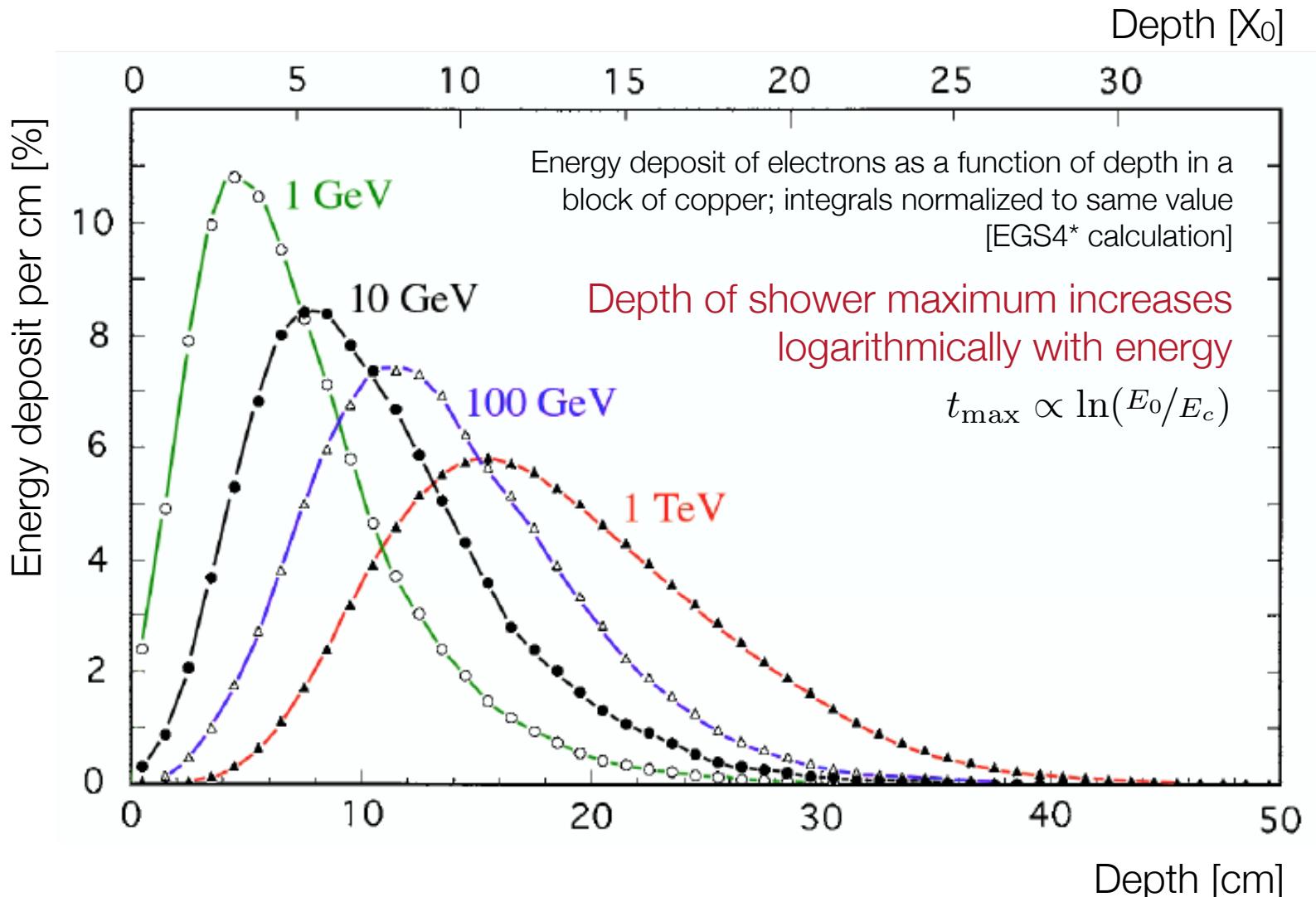
$$\rightarrow t_{\max} = \frac{\alpha - 1}{\beta} = \ln\left(\frac{E_0}{E_c}\right) + C_{e\gamma}$$

with:

$$C_{e\gamma} = -0.5 \quad [\gamma\text{-induced}]$$

$$C_{e\gamma} = -1.0 \quad [e\text{-induced}]$$

# EM shower longitudinal development



\*EGS = Electron Gamma Shower

# EM shower transverse development

Transverse shower development ...

Opening angle  
for bremsstrahlung and pair production

$$\langle \theta^2 \rangle \approx (m/E)^2 = 1/\gamma^2$$

Small contribution as  $m_e/E_c = 0.05$

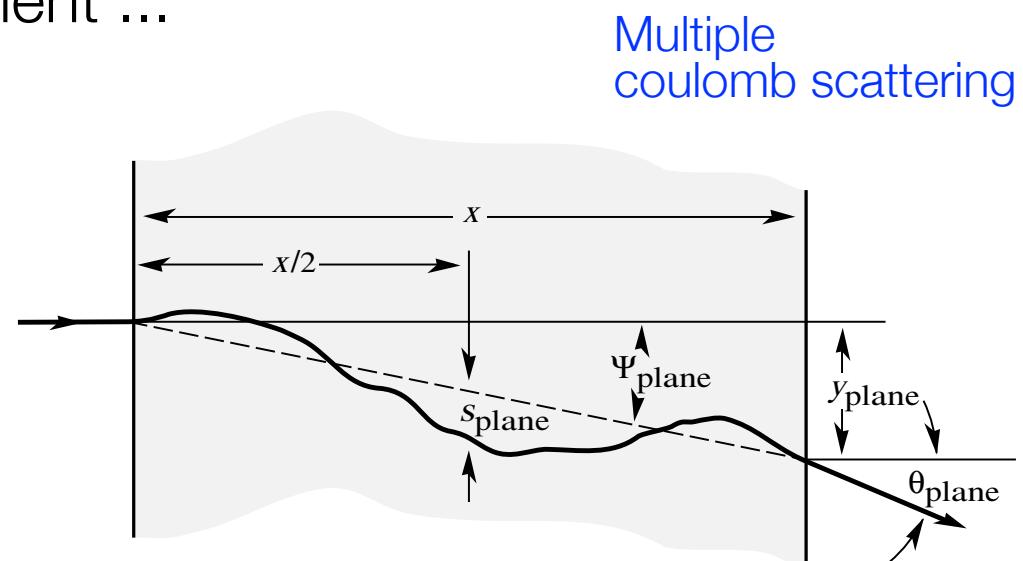
Multiple scattering  
deflection angle in 2-dimensional plane ...

$$\langle \theta_k^2 \rangle = \sum_{m=1}^k \theta_m^2 = k \langle \theta^2 \rangle$$

$$\sqrt{\langle \theta^2 \rangle} \approx \frac{13.6 \text{ MeV}/c}{p} \sqrt{\frac{x}{X_0}} \quad [\beta = 1]$$

In 3-dimensions extra factor  $\sqrt{2}$ :

$$\sqrt{\langle \theta^2 \rangle_{3d}} \approx \frac{19.2 \text{ MeV}/c}{p} \sqrt{\frac{x}{X_0}} \quad [\beta = 1]$$



Assuming the approximate range of electrons  
to be  $X_0$  yields lateral extension:  $R = \langle \theta \rangle \cdot X_0 \dots$

$$R_M = \langle \theta \rangle_{x=X_0} \cdot X_0 \approx \frac{21 \text{ MeV}}{E_C} X_0$$

Molière Radius;  
characterizes lateral shower spread ...

# EM shower transverse profile

## Transverse profile

Parametrization:

$$\frac{dE}{dr} = \alpha e^{-r/R_M} + \beta e^{-r/\lambda_{\min}}$$

$\alpha, \beta$  : free parameters

$R_M$  : Molière radius

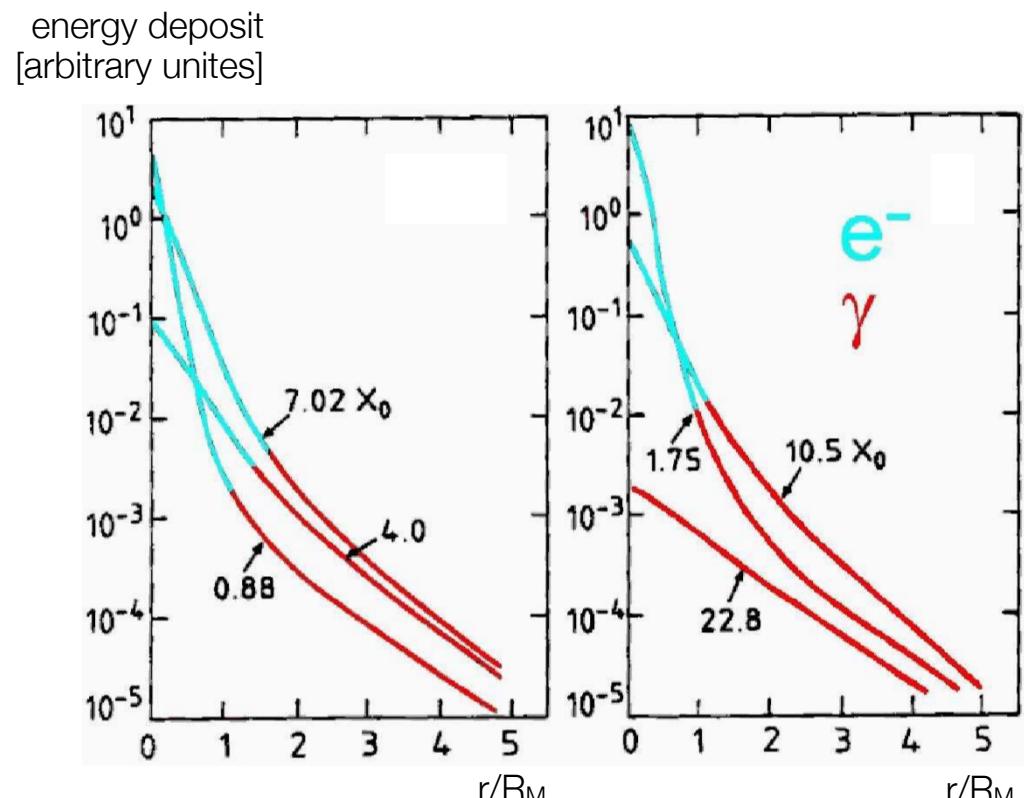
$\lambda_{\min}$ : range of low energetic photons ...

Inner part: coulomb scattering ...

Electrons and positrons move away from shower axis due to multiple scattering ...

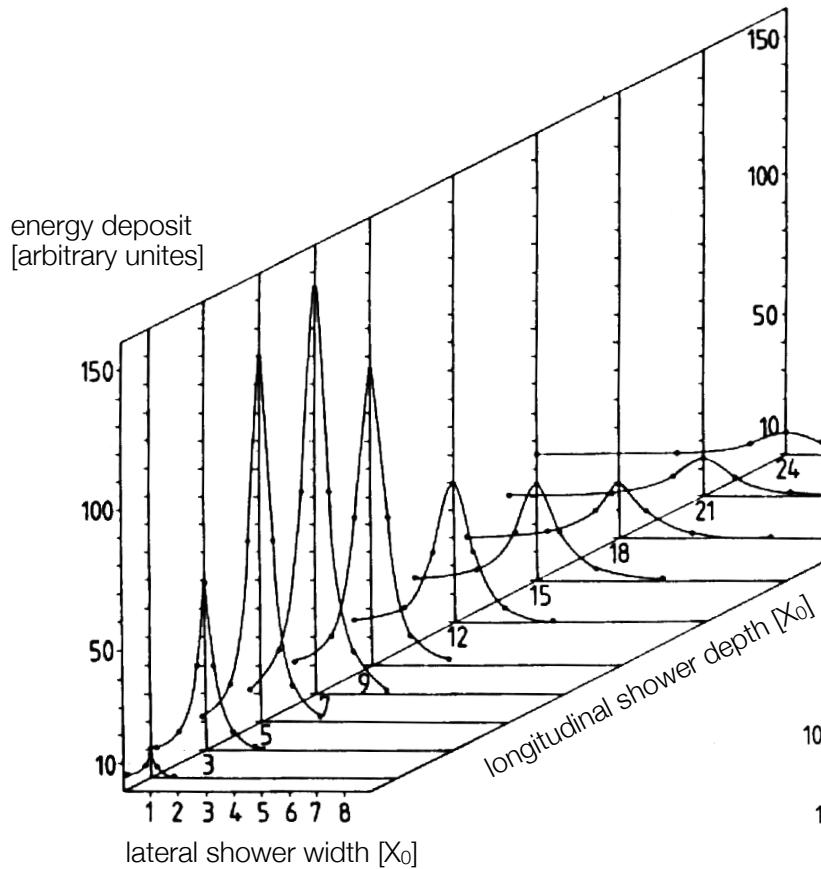
Outer part: low energy photons ...

Photons (and electrons) produced in isotropic processes (Compton scattering, photo-electric effect) move away from shower axis; predominant beyond shower maximum, particularly in high-Z absorber media...

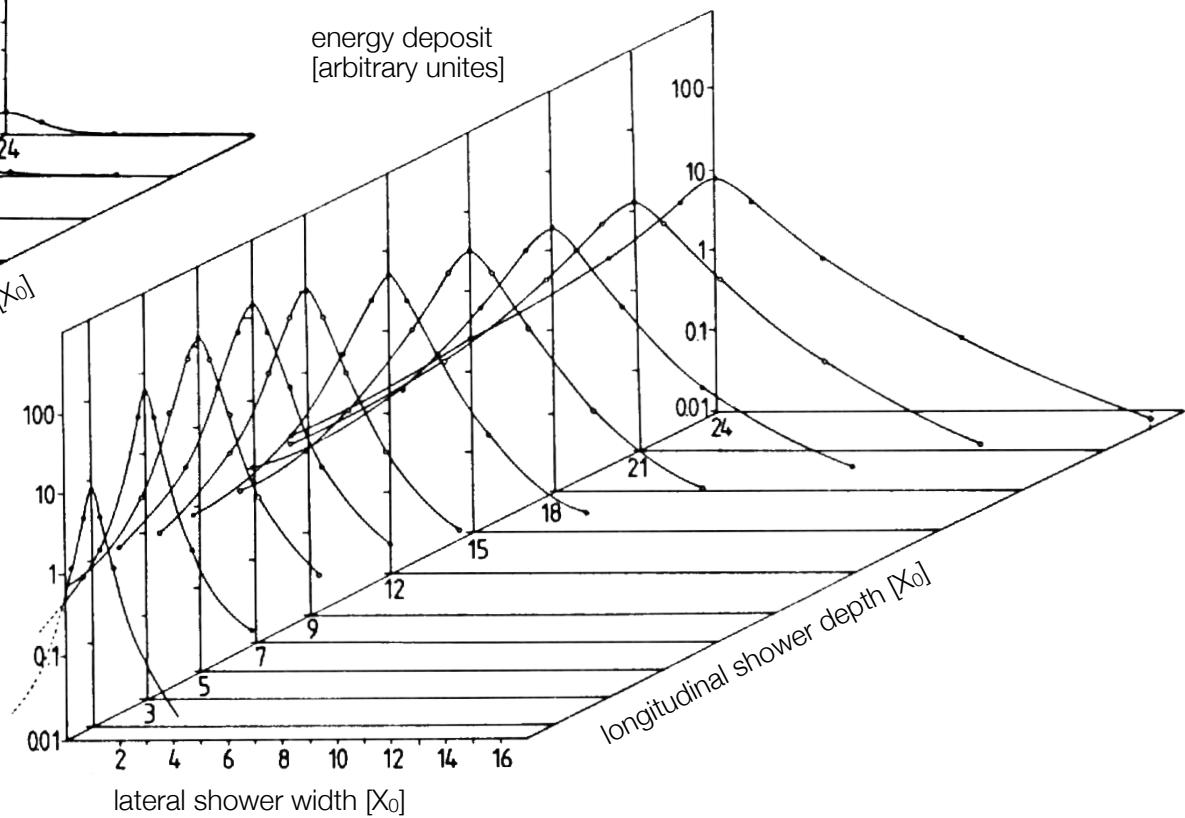


Shower gets wider at larger depth ...

# EM shower profiles



Longitudinal and transversal shower profile  
for a 6 GeV electron in lead absorber ...  
[left: linear scale; right: logarithmic scale]



# EM showers in a nutshell

Radiation length:

$$X_0 = 716.4 \text{ g cm}^{-2} \frac{A}{Z(Z+1) \ln \frac{287}{\sqrt{Z}}}$$

Critical energy:

$$E_c^{\text{Gas}} = \frac{710 \text{ MeV}}{Z + 0.92} \quad E_c^{\text{Sol/Liq}} = \frac{610 \text{ MeV}}{Z + 1.24}$$

Shower maximum:

$$t_{\max} = \ln \frac{E}{E_c} - \begin{cases} 1.0 & e^- \text{ induced shower} \\ 0.5 & \gamma \text{ induced shower} \end{cases}$$

Longitudinal  
energy containment:

$$L(95\%) = t_{\max} + 0.08Z + 9.6 [X_0]$$

Transverse  
Energy containment:

$$R(90\%) = R_M \quad R_M = \frac{21 \text{ MeV}}{E_c} X_0$$
$$R(95\%) = 2R_M$$

# Hadronic showers

Shower development:

1.  $p + \text{Nucleus} \rightarrow \text{Pions} + N^* + \dots$

2. Secondary particles ...

undergo further inelastic collisions until they fall below pion production threshold

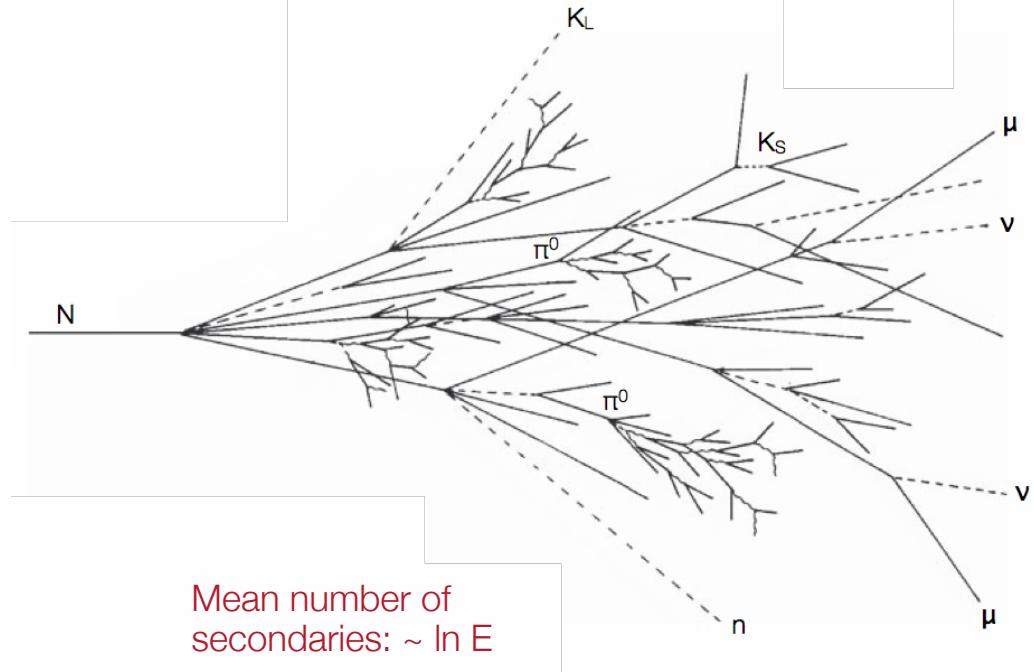
3. Sequential decays ...

$\pi^0 \rightarrow \gamma\gamma$ : yields electromagnetic shower

Fission fragments  $\rightarrow \beta\text{-decay}, \gamma\text{-decay}$

Neutron capture  $\rightarrow$  fission

Spallation ...



Mean number of secondaries:  $\sim \ln E$

Typical transverse momentum:  $p_t \sim 350 \text{ MeV}/c$

Substantial electromagnetic fraction

$$f_{em} \sim \ln E$$

[variations significant]

Cascade energy distribution:  
[Example: 5 GeV proton in lead-scintillator calorimeter]

Ionization energy of charged particles ( $p, \pi, \mu$ )

1980 MeV [40%]

Electromagnetic shower ( $\pi^0, \eta^0, e$ )

760 MeV [15%]

Neutrons

520 MeV [10%]

Photons from nuclear de-excitation

310 MeV [ 6%]

Non-detectable energy (nuclear binding, neutrinos)

1430 MeV [29%]

---

5000 MeV [29%]

# Hadronic showers

Hadronic interaction:

Cross Section:

$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{inel}}$$

at high energies  
also diffractive contribution

For substantial energies  
 $\sigma_{\text{inel}}$  dominates:

$$\sigma_{\text{el}} \approx 10 \text{ mb}$$

$$\sigma_{\text{inel}} \propto A^{2/3} \text{ [geometrical cross section]}$$

$$\therefore \sigma_{\text{tot}} = \sigma_{\text{tot}}(pA) \approx \sigma_{\text{tot}}(pp) \cdot A^{2/3}$$

[ $\sigma_{\text{tot}}$  slightly grows with  $\sqrt{s}$ ]

Hadronic interaction length:

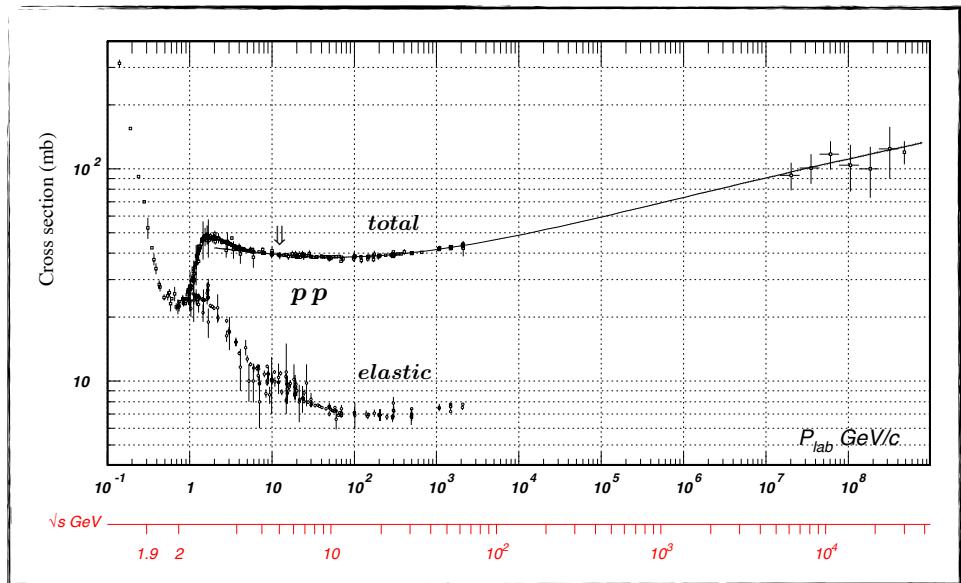
$$\lambda_{\text{int}} = \frac{1}{\sigma_{\text{tot}} \cdot n} = \frac{A}{\sigma_{pp} A^{2/3} \cdot N_A \rho} \sim A^{1/3} \quad [\text{for } \sqrt{s} \approx 1 - 100 \text{ GeV}]$$

$$\approx 35 \text{ g/cm}^2 \cdot A^{1/3}$$

which yields:

$$N(x) = N_0 \exp(-x/\lambda_{\text{int}})$$

Remark: In principle one should distinguish between collision length  $\lambda_w \sim 1/\sigma_{\text{tot}}$  and interaction length  $\lambda_{\text{int}} \sim 1/\sigma_{\text{inel}}$  where the latter considers inelastic processes only (absorption) ...



Total proton-proton cross section  
[similar for p+n in 1-100 GeV range]

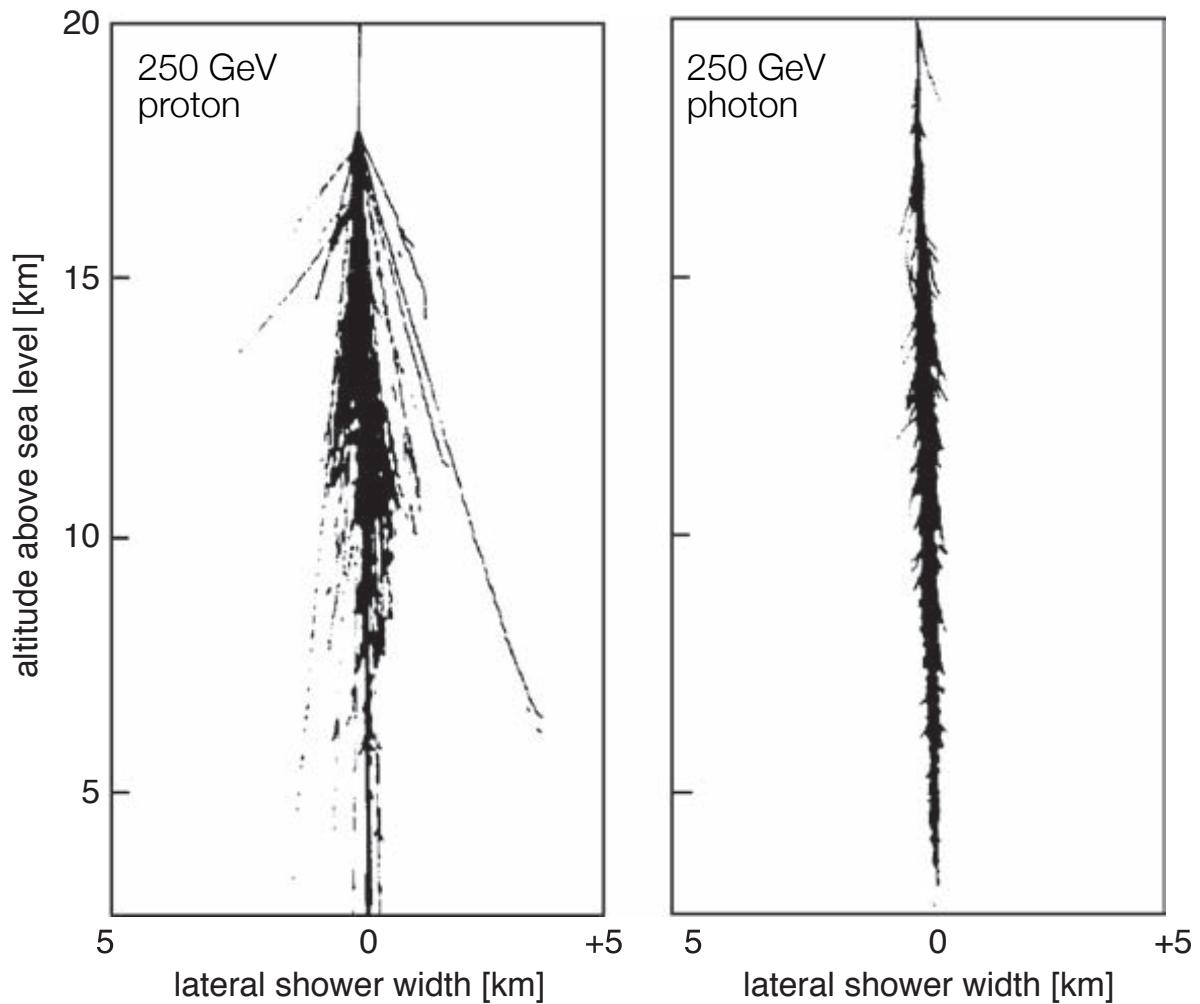
Interaction length characterizes both,  
longitudinal and transverse profile of  
hadronic showers ...

# Hadronic vs. EM showers

## Comparison

hadronic vs. electromagnetic shower ...

[Simulated air showers]



# Hadronic vs. EM showers

Hadronic vs. electromagnetic interaction length:

$$\left. \begin{aligned} X_0 &\sim \frac{A}{Z^2} \\ \lambda_{\text{int}} &\sim A^{1/3} \end{aligned} \right] \rightarrow \frac{\lambda_{\text{int}}}{X_0} \sim A^{4/3}$$

$$\lambda_{\text{int}} \gg X_0$$

[ $\lambda_{\text{int}}/X_0 > 30$  possible; see below]

Typical  
Longitudinal size: 6 ... 9  $\lambda_{\text{int}}$   
[95% containment]

[EM: 15-20  $X_0$ ]

Typical  
Transverse size: one  $\lambda_{\text{int}}$   
[95% containment]

[EM: 2  $R_M$ ; compact]

Hadronic calorimeter need more depth  
than electromagnetic calorimeter ...

Some numerical values for materials typical used in hadron calorimeters

	$\lambda_{\text{int}}$ [cm]	$X_0$ [cm]
Scint	79.4	42.2
LAr	83.7	14.0
Fe	16.8	1.76
Pb	17.1	0.56
U	10.5	0.32
C	38.1	18.8

# Hadronic shower development

Hadronic shower development:  
[estimate similar to e.m. case]

Depth (in units of  $\lambda_{\text{int}}$ ):

$$t = \frac{x}{\lambda_{\text{int}}}$$

Energy in depth  $t$ :

$$E(t) = \frac{E}{\langle n \rangle^t} \quad \& \quad E(t_{\max}) = E_{\text{thr}} \quad [\text{with } E_{\text{thr}} \approx 290 \text{ MeV}]$$

$$E_{\text{thr}} = \frac{E}{\langle n \rangle^{t_{\max}}}$$

Shower maximum:

$$\langle n \rangle^{t_{\max}} = \frac{E}{E_{\text{thr}}}$$

$$t_{\max} = \frac{\ln(E/E_{\text{thr}})}{\ln \langle n \rangle}$$

Number of particles  
lower by factor  $E_{\text{thr}}/E_c$   
compared to e.m. shower ...

Intrinsic resolution:  
worse by factor  $\sqrt{E_{\text{thr}}/E_c}$

But:

Only rough estimate as ...

energy sharing between shower particles  
fluctuates strongly ...

part of the energy is not detectable (neutrinos,  
binding energy); partial compensation possible  
(n-capture & fission)

spatial distribution varies strongly; different  
range of e.g.  $\pi^\pm$  and  $\pi^0$  ...

electromagnetic fraction, i.e. fraction of energy  
deposited by  $\pi^0 \rightarrow \gamma\gamma$  increases with energy ...

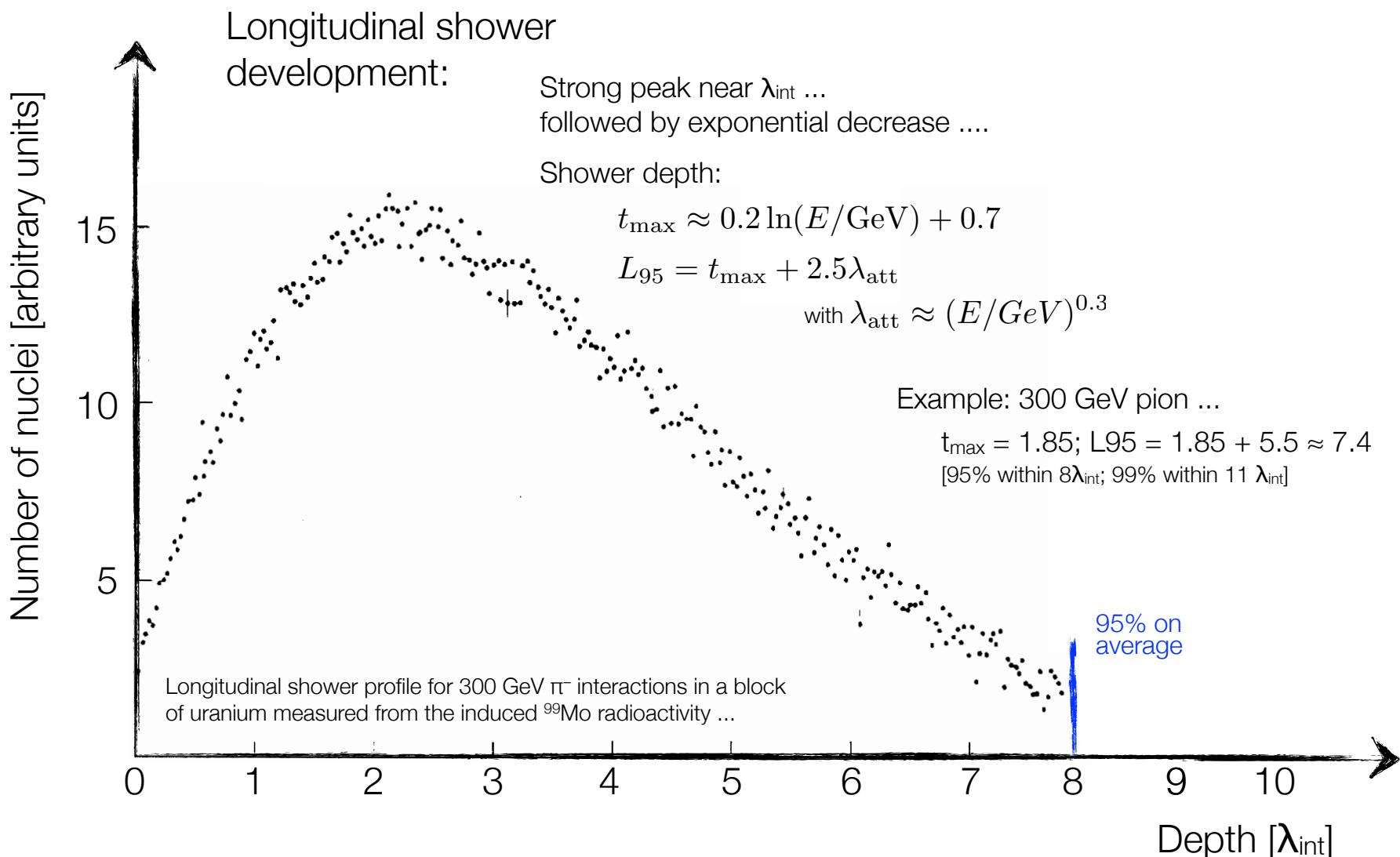
$$f_{\text{em}} \approx f_{\pi^0} \sim \ln E/(1 \text{ GeV})$$

Explanation: charged hadron contribute to electromagnetic  
fraction via  $\pi^- p \rightarrow \pi^0 n$ ; the opposite happens only rarely as  
 $\pi^0$  travel only 0.2  $\mu\text{m}$  before its decay ('one-way street') ...

At energies below 1 GeV hadrons loose their  
energy via ionization only ...

Thus: need Monte Carlo (GEISHA, CALOR, ...)  
to describe shower development correctly ...

# Hadronic shower longitudinal development



# Hadronic shower transverse development

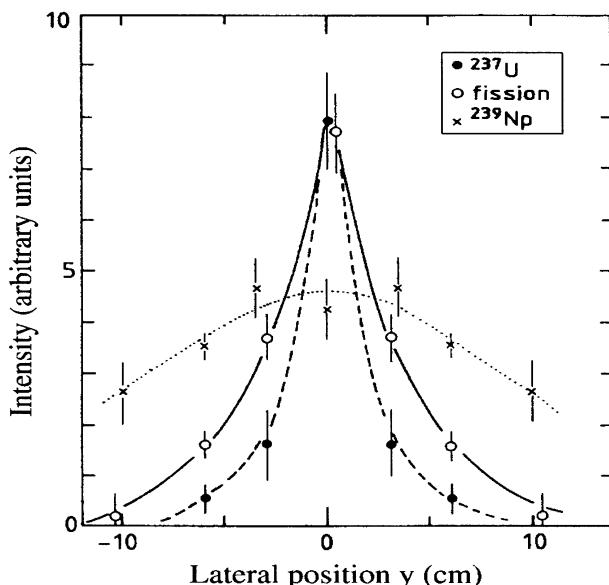
## Transverse shower profile

Typical transverse momenta of secondaries:  $\langle p_t \rangle \simeq 350 \text{ MeV}/c$  ...

Lateral extend at shower maximum:  $R_{95\%} \simeq \lambda_{\text{int}}$  ...

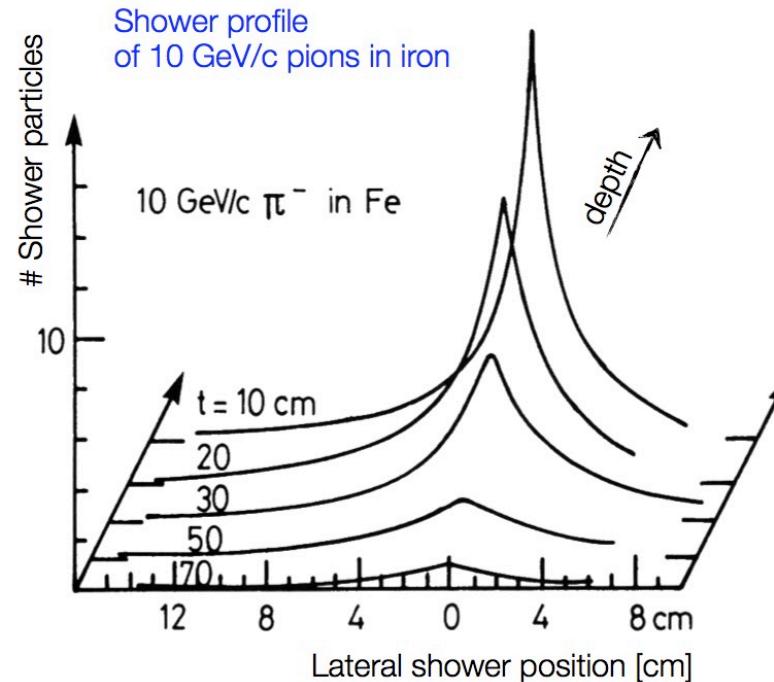
Electromagnetic component leads to relatively well-defined core:  $R \simeq R_M$  ...

Exponential decay after shower maximum ...



Lateral profile for  
300 GeV  $\pi^-$   
[target material  $^{238}\text{U}$ ]  
[measured at depth  $4 \lambda_{\text{int}}$ ]

More  $\pi^0$ 's and  $\gamma$  in core  
Energetic neutrons and charged pions form a wider core  
Thermal neutrons generate broad tail



Measurement from induced radioactivity:

$^{99}\text{Mo}$  (fission): neutron induced ...  
[energetic neutron component]

$^{237}\text{U}$ : mainly produced via  $^{238}\text{U}(\gamma, n)^{237}\text{U}$  ...  
[electromagnetic component]

$^{239}\text{Np}$ : from  $^{239}\text{U}$  decay ...  
[thermal neutrons]

Ordinate indicates decay rate  
of different radioactive nuclides ...