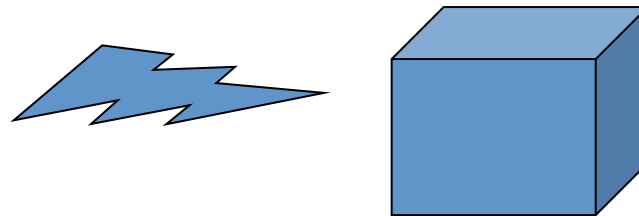


Interactions of Particles/Radiation with Matter



ESIPAP : European School in Instrumentation for Particle and Astroparticle Physics

Non-exhaustive list of « Particles/Radiation » and « Matter »

PARTICLES

- ${}^4_2\text{He}$
- e^\pm
- γ
- $\mu, \gamma, e^\pm, \pi, \nu, p \dots$

RADIATION

- α radiation
- β^\pm radiation
- e.m, X, γ radiation
- cosmic radiation

PARTICLES <--> RADIATION

2 aspects of the same « entity »

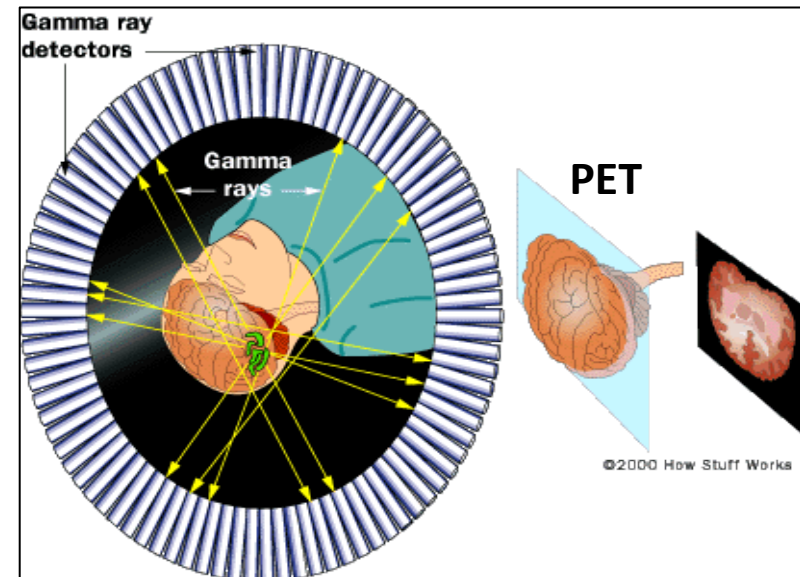
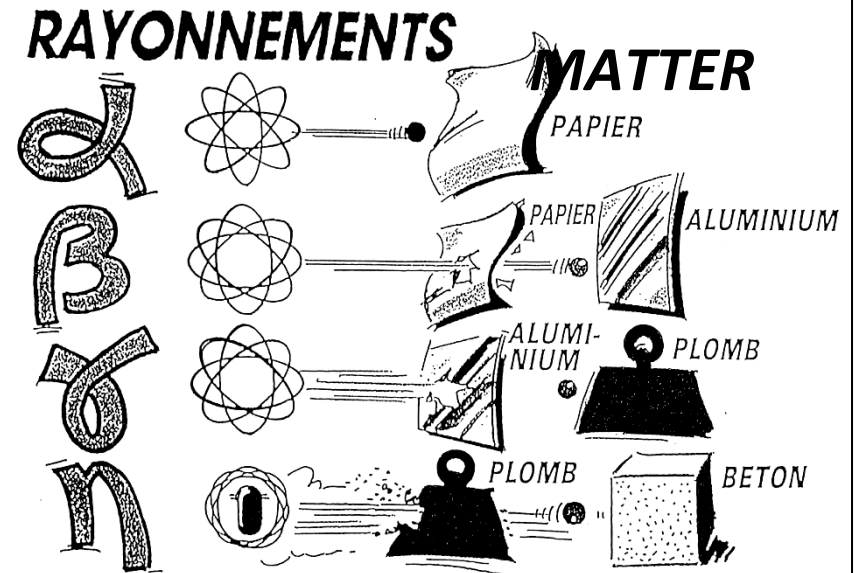
De Broglie relation

$$\lambda = h/p$$

(h = Planck constant)

MATTER

- detectors (research, medical app.,...)
- humain tissu/body (medical app.)
- electronic circuits
- Louvre paintings
- beauty cream, potatoes, ...

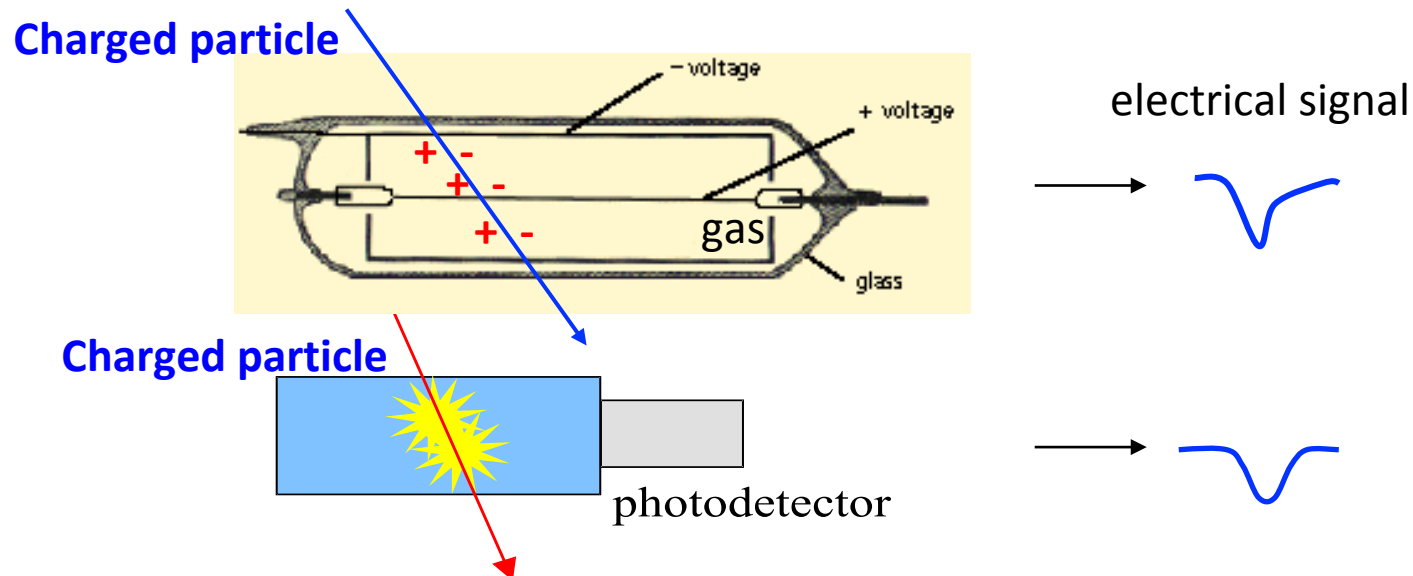


Motivation

- The interaction between **particles & matter** is at the base of several human activities
- Plenty of applications **not only in research** and not **only in Particle & Astroparticle**

Very important for particle detection !

- In order **to detect a particle**, the latter must interact with the material of the detector, and produce 'a (detectable) signal'

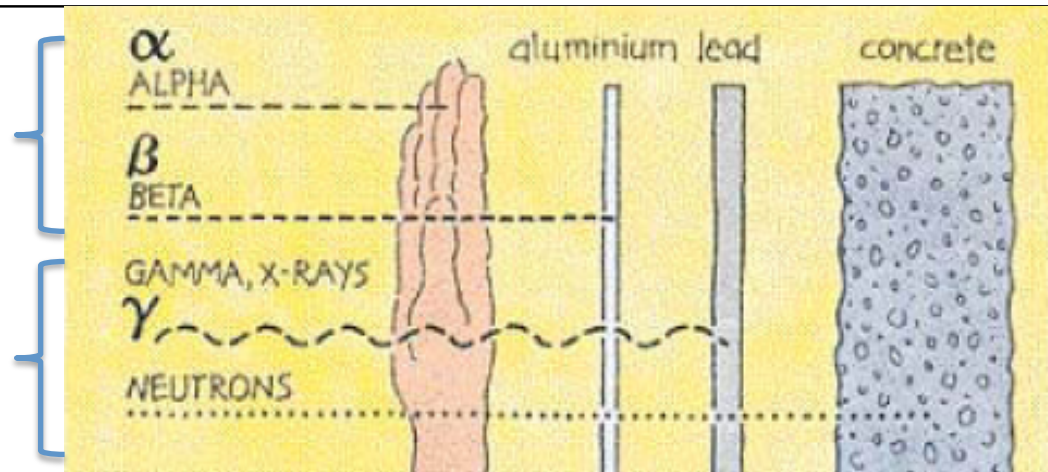


The understanding of **particle detection** requires the knowledge of the
Interactions of particles & matter

Brief outline and bibliography

Two lectures + two tutorials

- **Interaction of charged particles**
 - « heavy » ($m_{\text{Pa}} \gg m_e$)
 - « light » ($m_{\text{Pa}} \sim m_e$)
- **Interaction of neutral particles**
 - Photons
 - Neutral Hadrons: n , π^0 , ...



- **Radiation detection and measurement**, G.F. Knoll, J. Wiley & Sons
- **Experimental Techniques in High Energy Nuclear and Particle Physics**, T. Ferbel, World Scientific
- **Introduction to experimental particle physics**, R. Fernow, Cambridge University Press
- **Techniques for Nuclear and Particle Physics Experiments**, W.R. Leo, Springer-Verlag
- **Detectors for Particle radiation**, K. Kleinknecht, Cambridge University Press
- **Particle detectors**, C. Grupen, Cambridge monographs on particle physics
- **Principles of Radiation Interaction in Matter and Detection**, C. Leroy, P.G. Rancoita, World Scientific
- **Nuclei and particles**, Emilio Segré, W.A. Benjamin
- **High-Energy Particles**, Bruno Rossi, Prentice-Hall



“The classic “

Also: **Particle Data Group**

<http://pdg.lbl.gov/2019/reviews/rpp2019-rev-passage-particles-matter.pdf>

For ‘**professionals**’(*) : **GEANT4 (for GEometry ANd Tracking)**

(Platform for the **simulation** of the **passage of the particles through the matter**

Using Monte Carlo simulation, Open software)

<https://www.sciencedirect.com/science/article/pii/S0168900203013688>

My slides have been inspired by :

Hans Christian Schultz-Coulon’s lectures

Johann Collot @ ESIPAP 2014

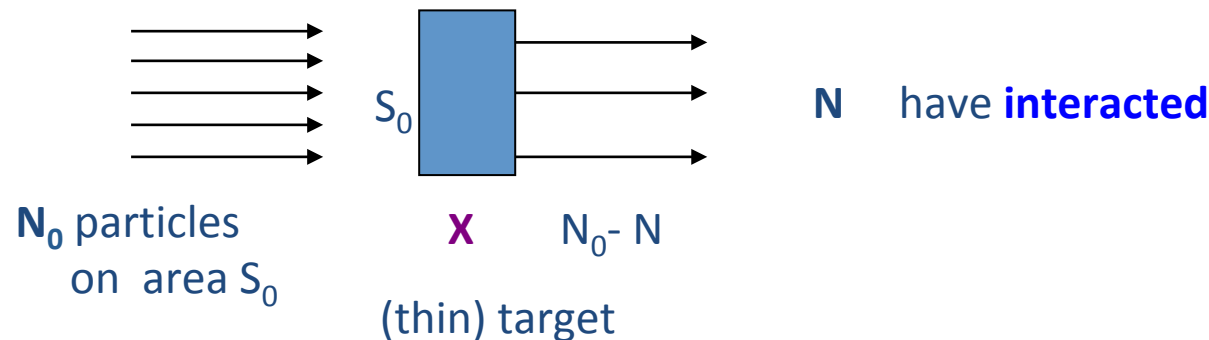
(*) **more exists:**

Fluka

Garfield (simulation gas detectors)

Interaction Cross Section (σ) definition

σ characterises the **probability** of a given **interaction** process



Target parameters:

$n \equiv$ number of target particles

M = target mass

A_{mol} = molar mass

ρ = target density

$N_A = 6.022 \cdot 10^{23} \text{ mol}^{-1}$
(Avogadro number)

$$\sigma \equiv \frac{\text{Number of interactions per number of target particles in unit time}}{\text{Incident flux}}$$

Number of **interactions** per number of **target particles** in unit time = $(1/n) * dN/dt$

Incident flux = $(1/S_0) * dN_0/dt$

$$\sigma \equiv [(1/n) * dN/dt] / [1/S_0 * dN_0/dt] = dN/dN_0 * (S_0 / n)$$

Interaction probability

where $n = (M/A_{\text{mol}}) N_A = (\rho V) (N_A/A_{\text{mol}}) = \rho (S_0 X) (N_A/A_{\text{mol}})$

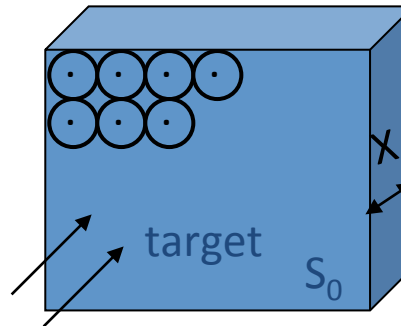
σ doesn't depend from S_0

Cross section (σ)

$$\sigma \equiv (\text{interaction probability}) * (S_0/n)$$

$n \equiv$ number of target particles

$\sigma \equiv$ area of a small disk around a target particle



$[\sigma] = [l]^2 \rightarrow \sigma$ is measured in m^2 or **barn**

$$1 \text{ barn} = 10^{-28} \text{ m}^2 = 10^{-24} \text{ cm}^2$$

$$1 \text{ mbarn} = 10^{-27} \text{ cm}^2$$

Order of magnitude of cross sections :

Neutron of ~ 1 eV on $^{48}_{113}\text{Cd}$ $\sigma = 100 \text{ barn} = 10^{-22} \text{ cm}^2$

'strong interaction'

Neutrino of ~ 1 GeV on p $\sigma = 10^{-38} \text{ cm}^2$

'weak interaction'

*See also
Marco Delmastro
lectures*

Mean free path λ (*)

λ = Average distance traveled between **two consecutive interactions in matter**

Another way of expressing the probability of a given process

$$\lambda \equiv \frac{1}{\sigma n_v}$$

σ total interaction cross-section

n_v number of scattering centers per unit volume

$$n_v = (\rho N_A)/A_{\text{mol}}$$

Order of magnitudes:

Electromagnetic interaction : $\lambda < \sim 1 \mu\text{m}$

Strong interaction : $\lambda > \sim 1 \text{ cm}$

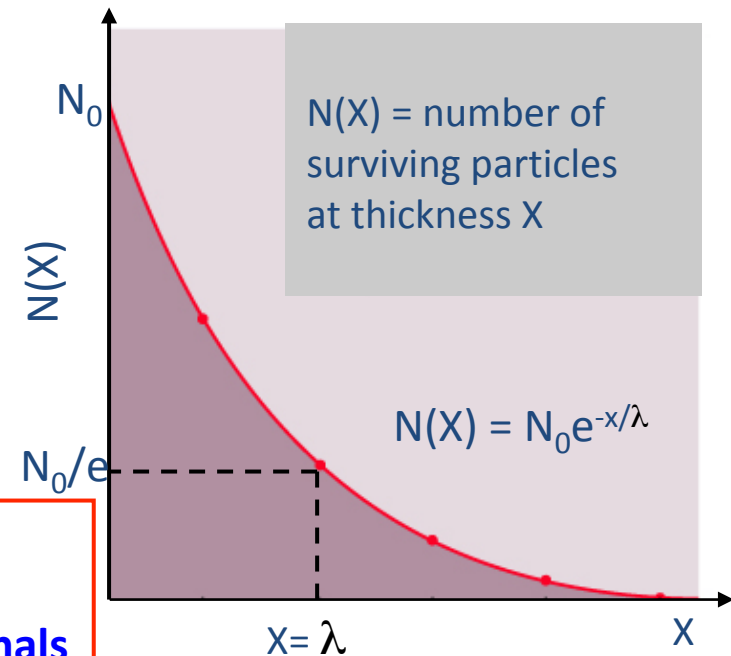
Weak interaction : $\lambda > \sim 10^{15} \text{ m}$

**A practical signal (> 100 interactions or 'hits')
results from the electromagnetic interaction**

Particle detection proceeds in two steps :

1) primary interaction

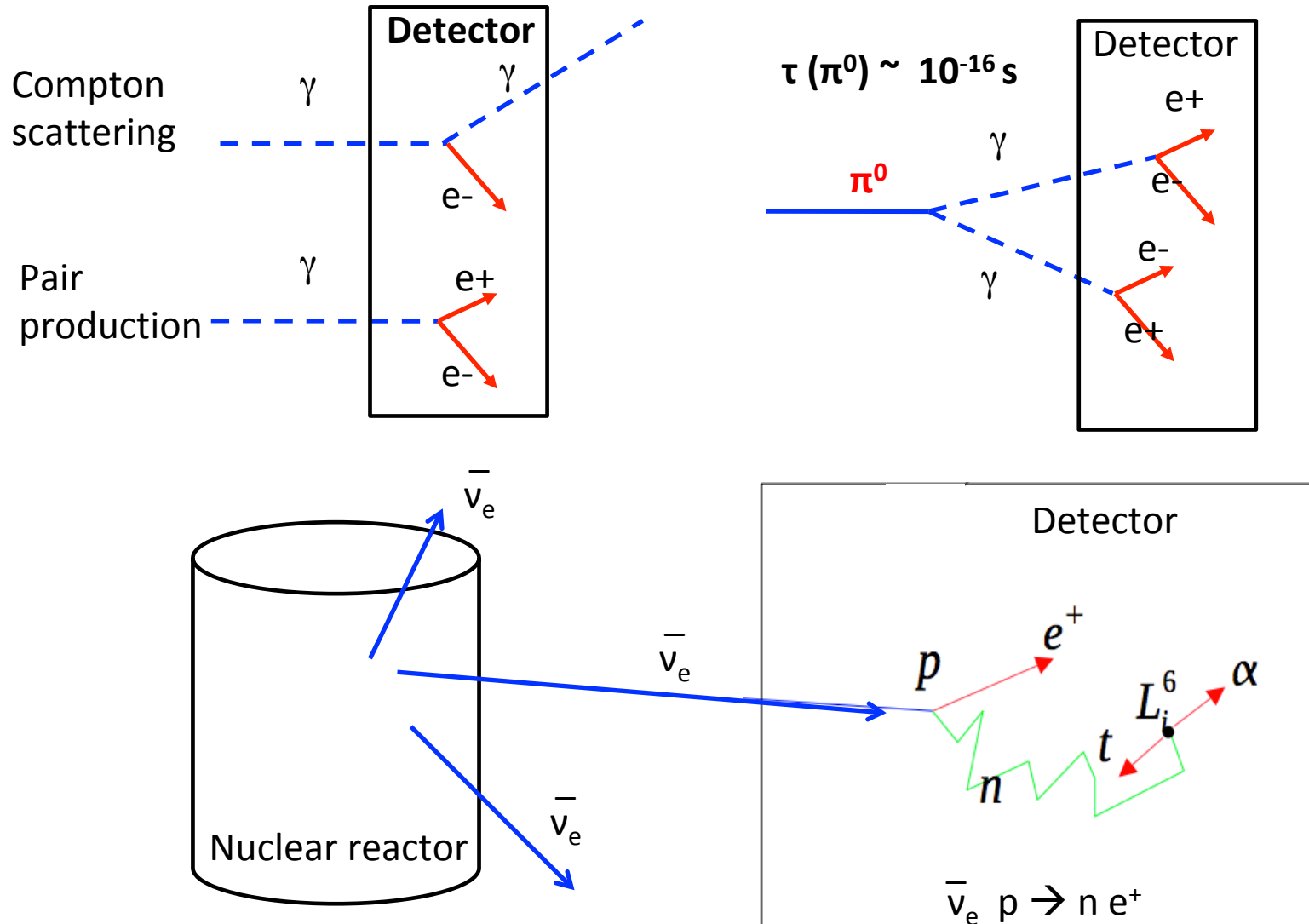
2) charged particle interaction producing the signals



(*) Also: λ = absorption length, interaction length, attenuation length, ...then σ is the cross-section for the corresponding process (see later)

Examples: detection of photons(γ), $\pi^0(2\gamma)$, neutrons(n), neutrinos(ν)

Signals are induced **by e.m. interactions of charged particles** in detectors



Useful relations of relativistic Kinematics and HEP units

- $\vec{p} = m_0 \gamma \vec{v}$ $\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$ $\beta \equiv v/c$ $m_0 \equiv \text{rest mass}$
 $\gamma \equiv \text{Lorentz boost}$
 $m \equiv m_0 \gamma$

- Kinetic energy $E_k = (\gamma-1) m_0 c^2$

- Total energy $E = \sqrt{(pc)^2 + (m_0 c^2)^2}$

- Total energy $E = E_k + m_0 c^2 = m_0 \gamma c^2 = m c^2$ $\gamma = E/(m_0 c^2)$

$E = m c^2$ « equivalence mass & energy »

Units :

$[E] = \text{eV}$ $[m] = \text{eV}/c^2$ $[p] = \text{eV}/c$

« Natural units »

$\hbar = 1$
 $c = 1$

$[c] = \frac{[l]}{[t]}$ $[l] = [t]$

$[E] = [p] = [m] = [v] = [t]^{-1}$

**See Marco Delmastro
lectures**

Outline: main interaction processes

2nd lecture

▪ Charged particle interactions

- 1) **Ionization**: inelastic collision with **electrons** of the atoms
- 2) **Bremsstrahlung**: photon radiation emission by an accelerated charge
- 3) **Multiple Scattering**: elastic collision with **nucleus**
- 4) **Cerenkov & transition radiation effects**: photon emission
- (• 5) **Nuclear interactions (p, π , K)**: processes mediated by strong interactions)

e.m.
interactions

1st lecture

▪ Neutral particle interactions

- Photons :
 - **photoelectric and Compton** effects, **$e^+ e^-$ pair production**
- High energy neutral hadrons with $\tau \sim 10^{-10} \text{ s}$ (n, K^0 , ..) :
 - **nuclear interactions**
- Moderate/low energy neutrons :
 - **scattering** (moderation), **absorption, fission**
- Neutrinos :
 - processes mediated by **weak interactions**

e.m.
interactions

not treated here

After the interaction the particles loose their energy and/or change direction or 'disappear'

Neutral particle interactions

- Photons
- High energy neutral hadrons with $\tau \sim 10^{-10} \text{ s}$ (n , K^0 , ..) :
- Moderate/low energy neutrons

Interactions of photons (γ)

γ : particles with $m_\gamma = 0$, $q_\gamma = 0$, $J^{PC}(\gamma) = 1^{--}$

Since $q_\gamma = 0$, the photons are **indirectly** detected : in their interactions they produce **electrons** and/or **positrons** which subsequently interact (**e.m.**) with matter.

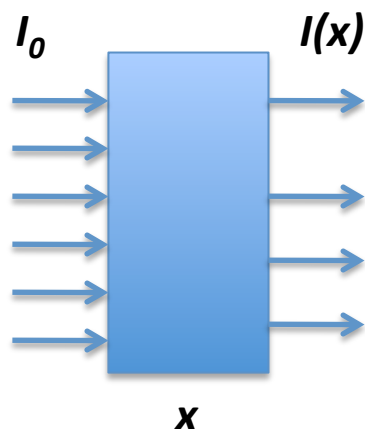
Main processes :

1. Photoelectric effect
2. Compton scattering
3. $e^+ e^-$ pairs production



Photons may be **absorbed** (photoelectric effect or e^+e^- pair creation) or **scattered** (Compton scattering) through large deflection angles.

→ difficult to define a mean range → an attenuation law is introduced :



$$I(x) = I_0 e^{-\mu x}$$

$$\mu = N \sigma = \frac{N_A}{A} \rho \sigma \equiv \frac{1}{\lambda}$$

See also slide 8

μ **absorption coefficient**

N atoms/ m^3

A masse molaire

N_A nombre Avogadro

ρ density

σ **Photon cross section**

λ Mean free path or absorption length

γ Absorption lenght ($\lambda' \equiv 1/\mu'$)

$$I(x) = I_0 e^{-\mu x}$$

$$I(x) = I_0 e^{-\mu' x'}$$

$$x' \equiv x \rho$$

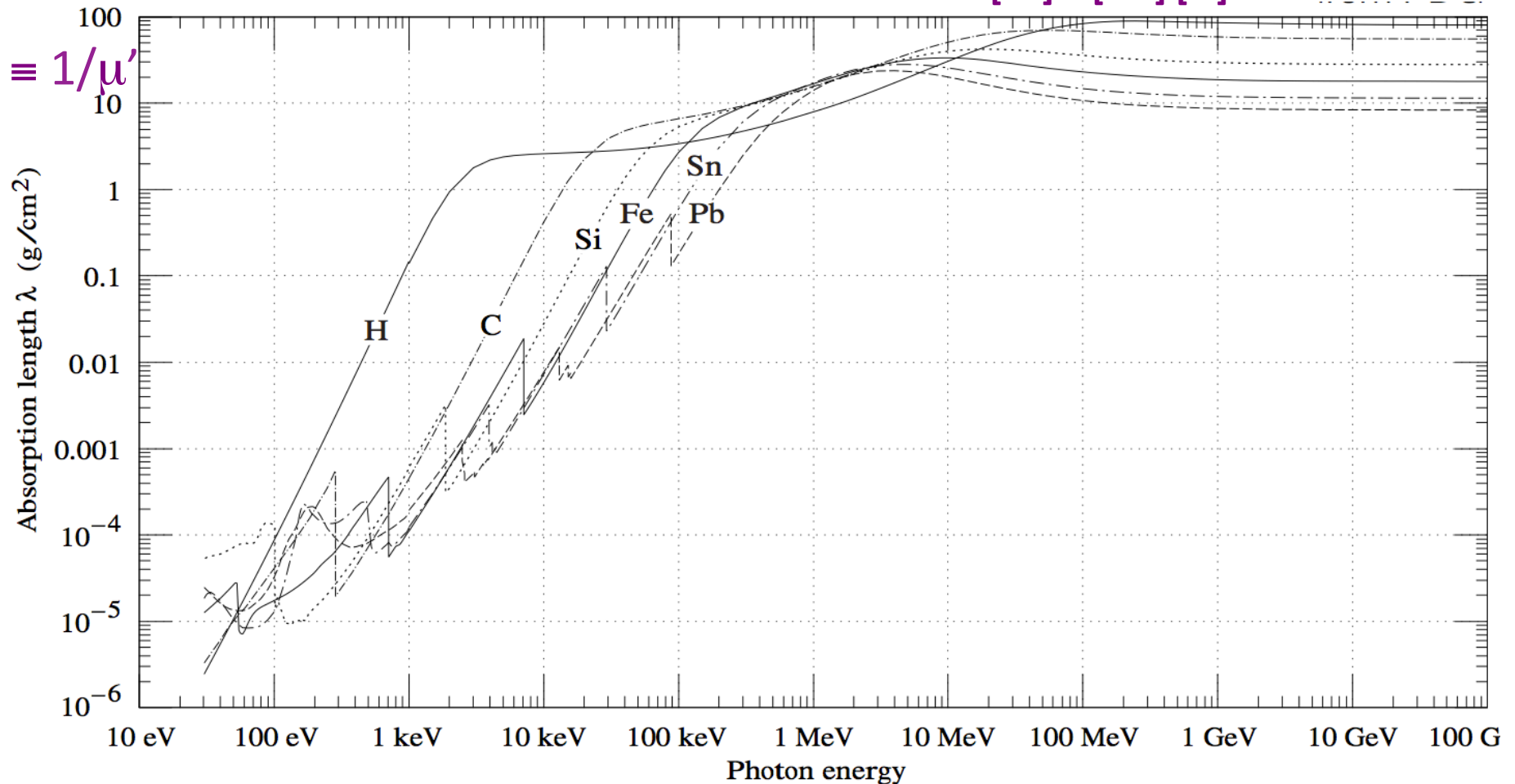
$$\mu' \equiv \mu/\rho$$

$$[x'] = [m] [l]^{-2}$$

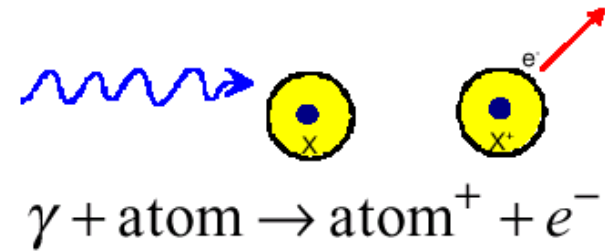
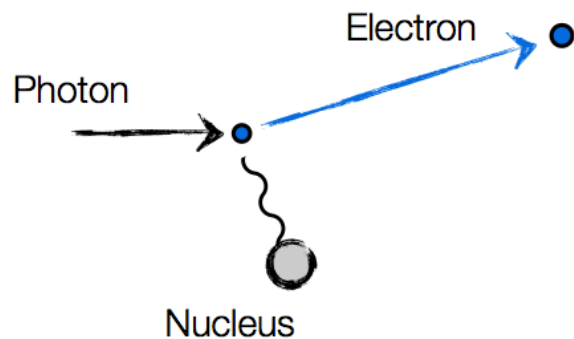
$$[\mu'] = [m]^{-1} [l]^2$$

$$[\lambda] = [m] [l]^{-2}$$

$$\lambda' \equiv 1/\mu'$$



1. Photoelectric effect

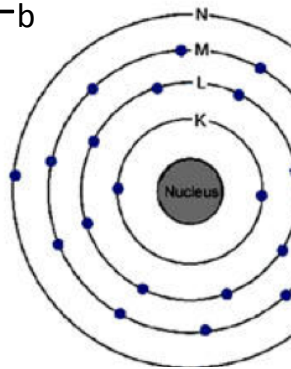


- The energy of the γ is transferred to the electron and the γ disappears

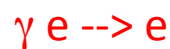
- Energy of the final electron:

$$E_e = E_{\gamma} - E_{\text{electron binding energy}} = h\nu - E_b$$

$$E_b = E_K \text{ or } E_L \text{ or } E_M \text{ etc...}$$

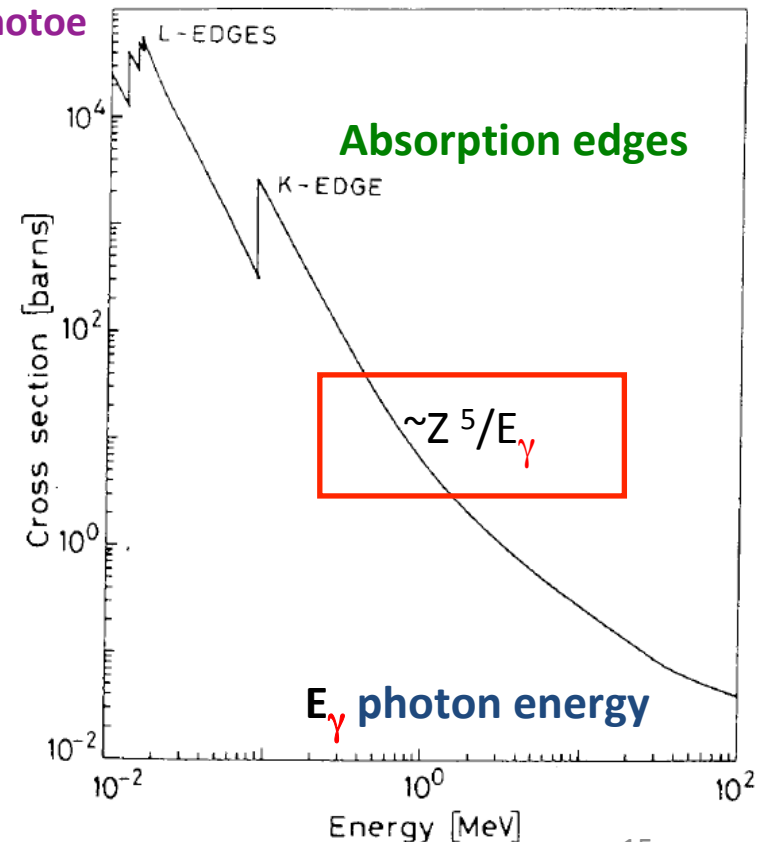


This effect can take place only on **bounded** electrons since the process (on 'free' electrons)



cannot conserve the momentum and energy

σ_{photoe}



1. Photoelectric effect

- At « low » energy ($I_0 \ll E_\gamma \ll m_e c^2$):

$$\sigma_{\text{ph}} = \alpha \pi a_B Z^5 (I_0/E_\gamma)^{7/2}$$

- At « high » energy ($E_\gamma \gg m_e c^2$):

$$\sigma_{\text{ph}} = 2\pi r_e^2 \alpha^4 Z^5 (mc^2)/E_\gamma$$

Example:

$$a_B = 0.53 \cdot 10^{-10} \text{ m}$$

$$I_0 = 13.6 \text{ eV}$$

For $E_\gamma = 100 \text{ KeV}$

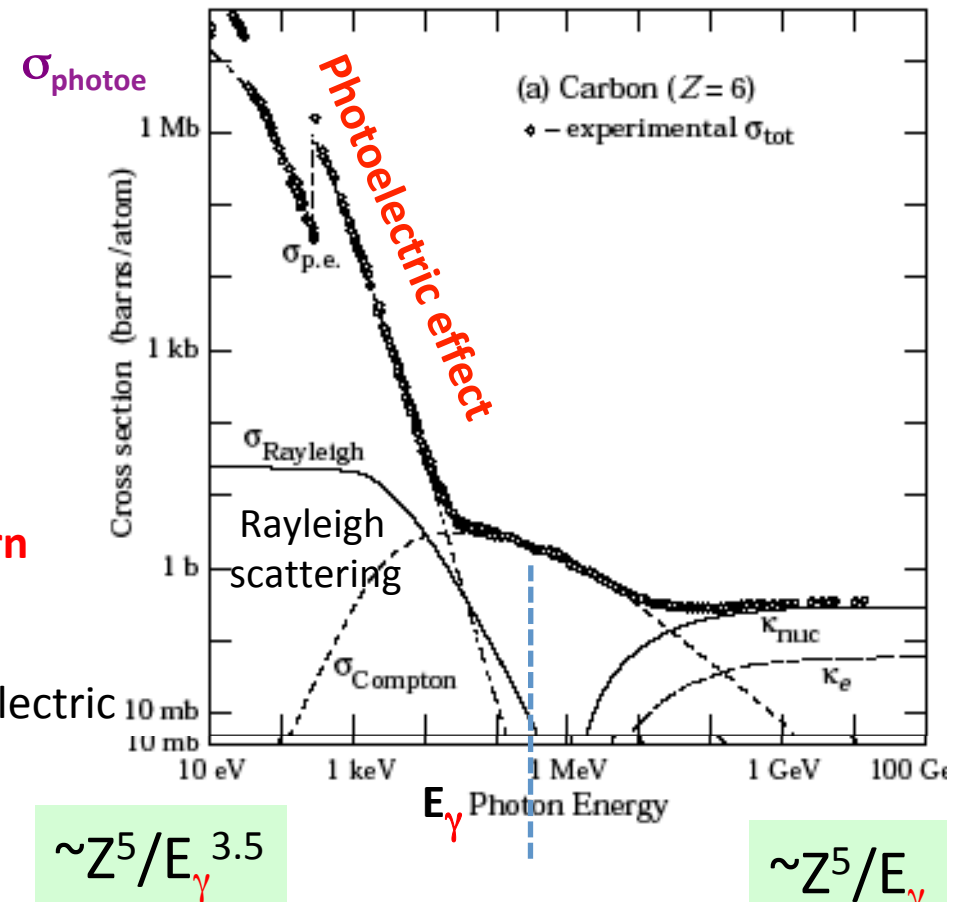
$$\begin{aligned} \sigma(\text{Fe}) &= 29 \text{ barn} \\ \sigma(\text{Pb}) &= 100 \text{ barn} \end{aligned}$$

- At low energy ($E_\gamma < 100 \text{ keV}$), the photoelectric effect dominates the total photon cross section

I_0 = ionisation potential

α fine structure constant

$r_e = \alpha/(m_e c^2)$ = classical electron radius



Atom de-excitation (after photoelectric effect)

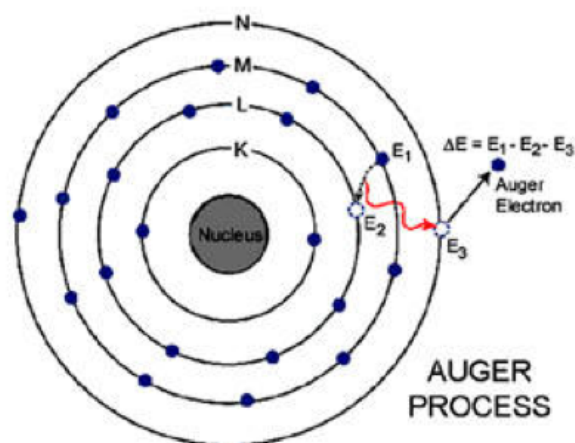
Following the emission of a “photoelectron”, the atom is in an excited state

De-excitation occurs via two effects (time scale: $\sim 10^{-16}$ s)

- Fluorescence: $\text{Atom}^{*+} \rightarrow \text{Atom}^{*+} + \gamma \rightarrow \text{X rays}$
- Auger effect: $\text{Atom}^{*+} \rightarrow \text{Atom}^{*++} + e^- \rightarrow \text{Auger electron}$

Observed
first by
Lisa Meitner

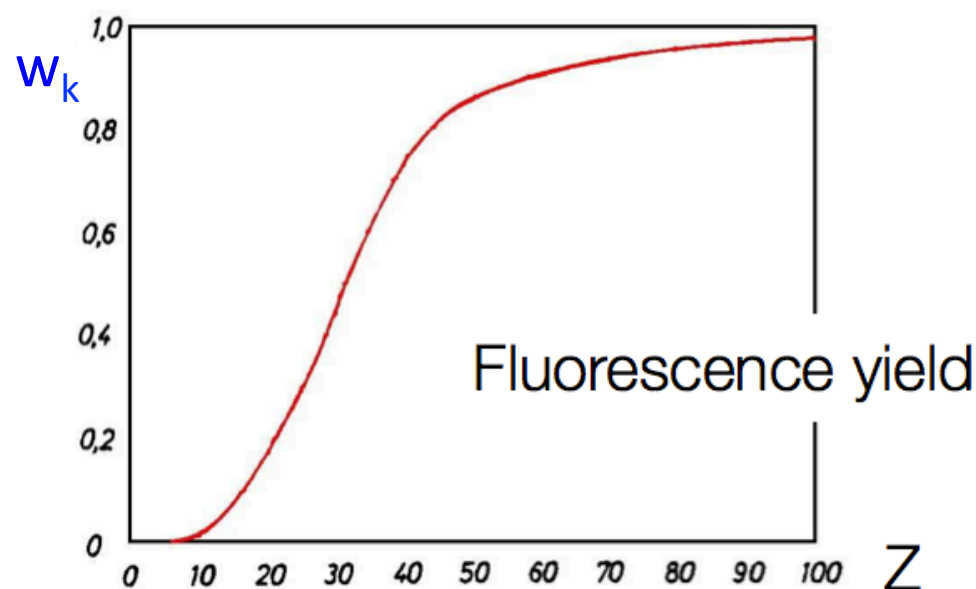
Used for
surface
Spectroscopy
(AES)



Auger electrons deposit energy locally
(small energy $< \sim 10$ KeV)

Fluorescence yield :

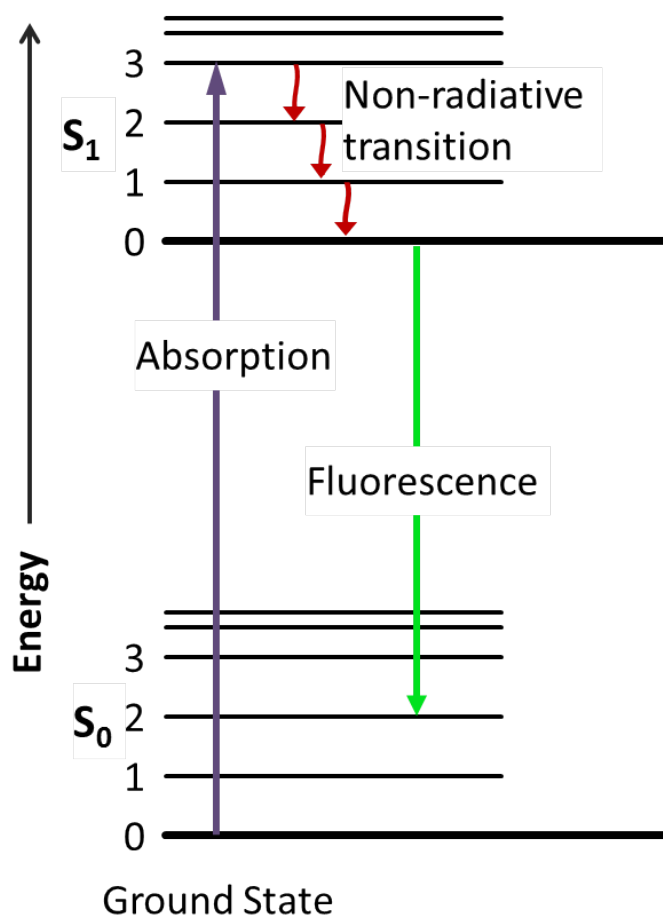
$$W_k = \frac{\text{Prob (fluorescence)}}{\text{Prob (fluorescence) + Prob (Auger)}}$$



General definition of fluorescence

Emission of light (UV to near infrared) by an atom, molecule that has absorbed light or other electromagnetic radiation, within the range of 0.5 to 20 nanoseconds

Energy levels in a molecule :



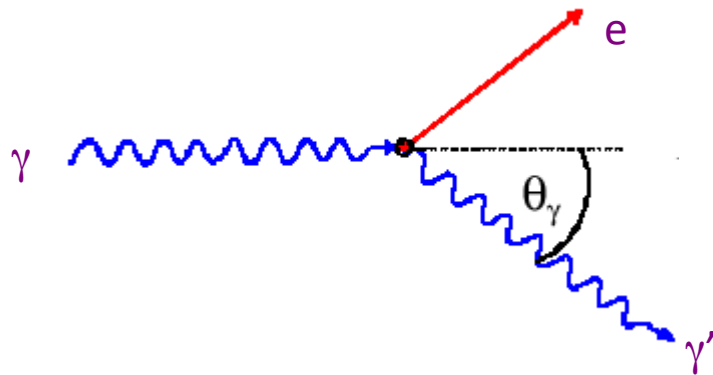
Important for scintillators !

2. Compton scattering

Scattering of γ on « free » electrons

$$\gamma + e \rightarrow \gamma' + e$$

In the matter electrons are bounded. When the γ energy, $E_\gamma \gg$ binding electron energy the electron can be considered as free.



$$E_{\gamma'} = \frac{E_\gamma}{1 + (E_\gamma/m_e c^2) (1 - \cos \theta_\gamma)}$$

Kinetic energy of the outgoing electron E_k^e :

$$E_k^e = E_\gamma - E_{\gamma'} = E_\gamma \frac{(1 - \cos \theta) (E_\gamma/m_e c^2)}{1 + (E_\gamma/m_e c^2) (1 - \cos \theta_\gamma)}$$

γ Forward scattering $\theta_\gamma = 0 \rightarrow E_{\gamma'} = E_{\gamma' \max} = E_\gamma \quad E_e = 0$

Initial photon can give all its energy to final photon

γ Backward scattering $\theta_\gamma = \pi \rightarrow E_{\gamma'} = E_{\gamma' \min} \rightarrow E_k^e$ is max

but not to the $e \rightarrow$
The photon cannot be completely absorbed

Compton Edges in the final e spectrum

- γ Backward scattering $\theta_\gamma = \pi$

$$\gamma + e \rightarrow \gamma' + e$$

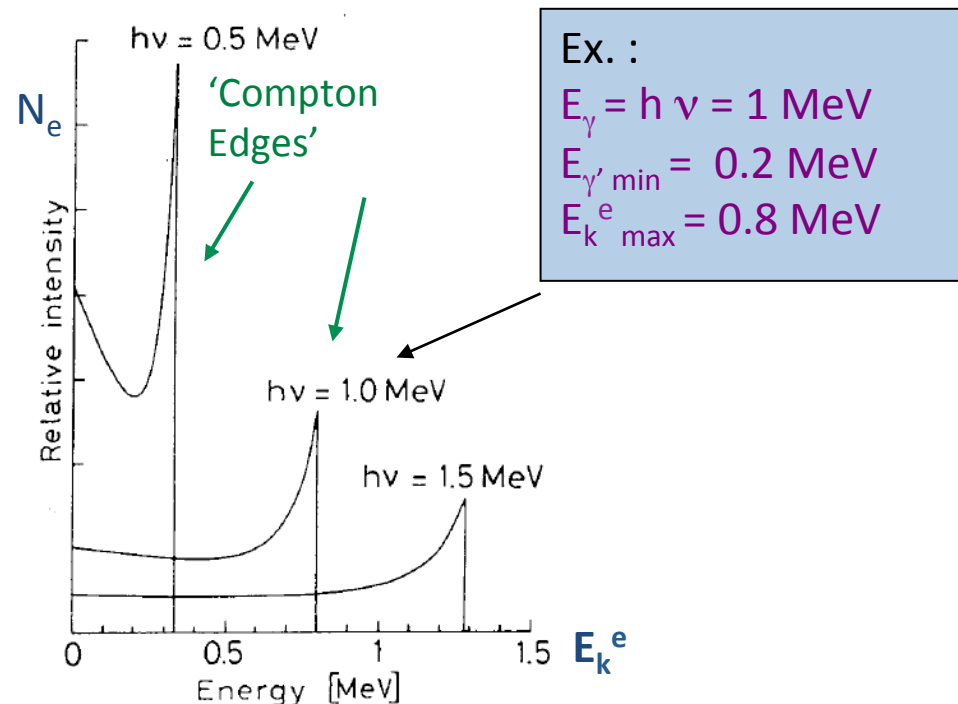
$$E_{\gamma'} \min \rightarrow E_k^e \max$$

$$E_\gamma = E_{\gamma'} + E_k^e$$

$$E_{\gamma'} \min = \frac{E_\gamma}{1 + 2 E_\gamma / m_e c^2}$$

$$E_k^e \max = E_\gamma \frac{2 E_\gamma / m_e c^2}{1 + 2 E_\gamma / m_e c^2}$$

Transfer of complete γ energy to e via Compton scattering is not possible



Important for **single photon detection**: the photon cannot be completely absorbed and the scattered electron misses a small amount of initial energy

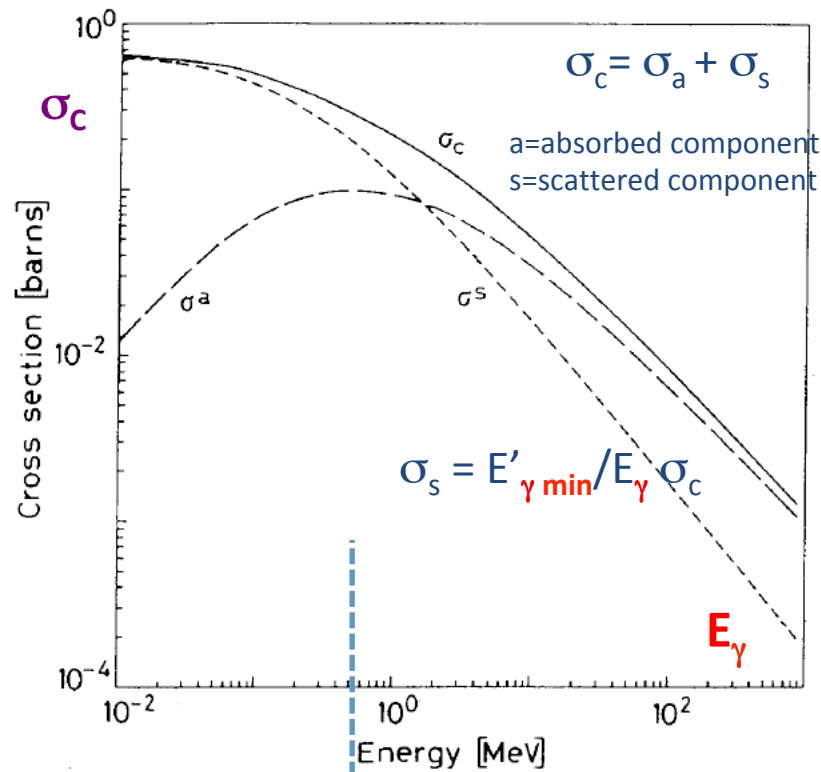
Compton Cross Section

Klein-Nishina Formula (LO QED):

$$\frac{d\sigma_c^e}{d\Omega} = \frac{r_e^2}{2} \frac{1 + \cos^2 \theta_\gamma}{(1 + \epsilon(1 - \cos \theta_\gamma))^2} \left(1 + \frac{\epsilon^2 (1 - \cos \theta_\gamma)^2}{(1 + \cos^2 \theta_\gamma)(1 + \epsilon(1 - \cos \theta_\gamma))} \right) \quad (\text{per electron})$$

$$\sigma_c^e = 2\pi r_e^2 \left(\left(\frac{1+\epsilon}{\epsilon^2} \right) \left\{ \frac{2(1+\epsilon)}{1+2\epsilon} - \frac{1}{\epsilon} \ln(1+2\epsilon) \right\} + \frac{1}{2\epsilon} \ln(1+2\epsilon) - \frac{1+3\epsilon}{(1+2\epsilon)^2} \right) \quad (\text{per electron})$$

$$\epsilon = \frac{E_\gamma}{m_e c^2}$$



@ Small photon energy ($E_\gamma \ll m_e c^2$)

$$\sigma_c = \sigma_{Th} (1 - E_\gamma / (m_e c^2))$$

$$\sigma_{Th} = 8\pi/3 r_e^2 = 0.66 \text{ barn}$$

(σ_{Th}) = Thomson σ

@ Large photon energy ($E_\gamma \gg m_e c^2$)

$$\sigma_c \sim (\ln E_\gamma) / E_\gamma$$

Cross section per atom:

$$\sigma_c^{atom.} = Z \sigma_c^e$$

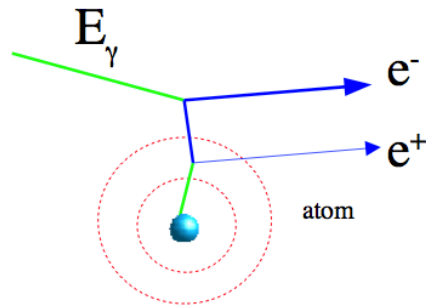
$$r_e = \alpha / (m_e c^2) = \text{classical electron radius}$$

m_e

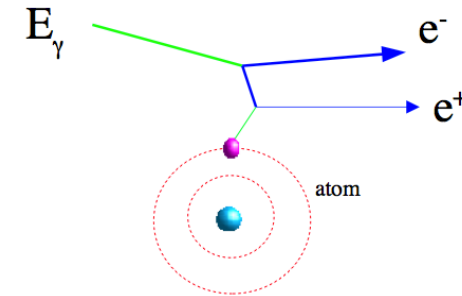
3. Pair production: $\gamma \rightarrow e^+ e^-$

Called also photon conversion

For energy-momentum conservation this process cannot take place in 'vacuum', an interaction with an electromagnetic field is necessary



Pair production in the field of the **nucleus**



Pair production in the field of an **electron**
(smaller probability $\sim 1/Z$)

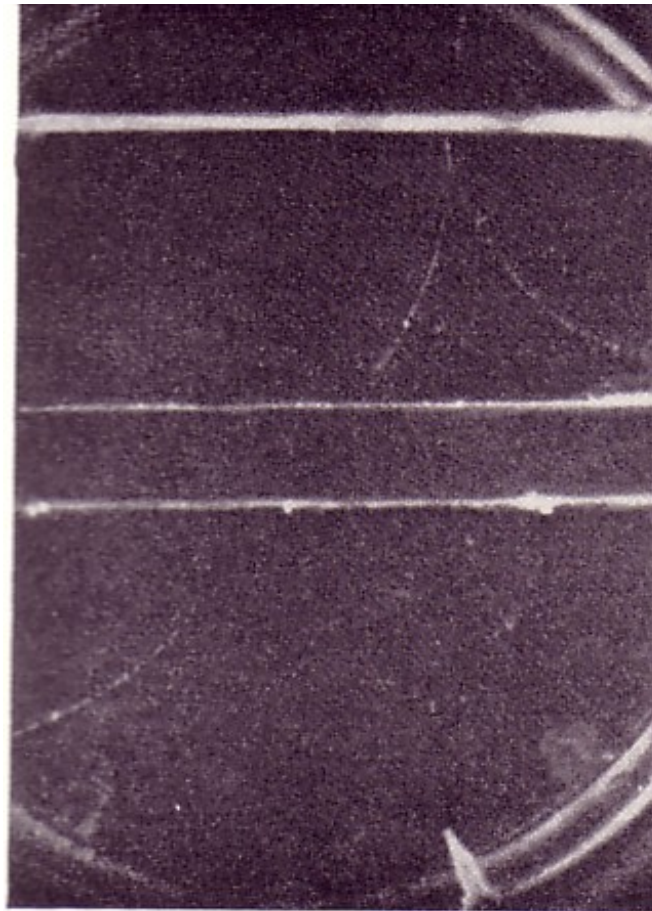
Threshold process : $E_\gamma > 2 m_e c^2 (1 + m_e/m_x)$

$m_x = m_N$ or $m_x = m_e$

Kinetic energy transferred to the "target" (nucleus or electrons)

First experimental observation of a positron

direction of the
high-energy photon



Pb plate

Production of an
electron-positron pair
by a high-energy photon
in a Pb plate

e⁺ e⁻ pair production cross-section

$$\epsilon = \frac{E_\gamma}{m_e}$$

$$1 \ll \epsilon < \frac{1}{\alpha Z^{1/3}}$$

$$\sigma_{pair}^{atom.} = 4\alpha r_e^2 Z^2 \left(\frac{7}{9} \ln(2\epsilon) - \frac{109}{54} \right)$$

$$\epsilon \gg \frac{1}{\alpha Z^{1/3}}$$

$$\sigma_{pair}^{atom.} = 4\alpha r_e^2 Z^2 \left(\frac{7}{9} \ln\left(\frac{183}{Z^{1/3}}\right) - \frac{1}{54} \right)$$

In the high energy regime
($E_\gamma \rightarrow \infty$)

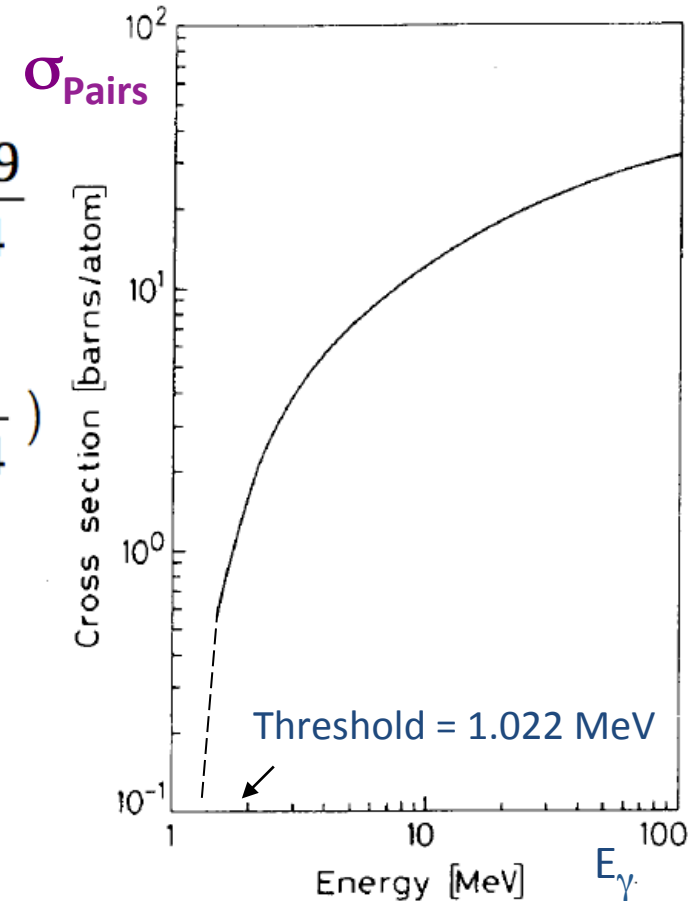
$$\sigma_{pair}^{atom.} \simeq \frac{7}{9} \frac{A}{N_A} \frac{1}{X_0}$$

accurate to within a few percent down to energies as low as 1 GeV, particularly for high-Z materials.

$X_0 =$
radiation length

	ρ [g/cm ³]	X_0 [cm]
H ₂ [fl.]	0.071	865
C	2.27	18.8
Fe	7.87	1.76
Pb	11.35	0.56
Air	$1.2 \cdot 10^{-3}$	$30 \cdot 10^3$

Pair production is
the leading effect at
high energy

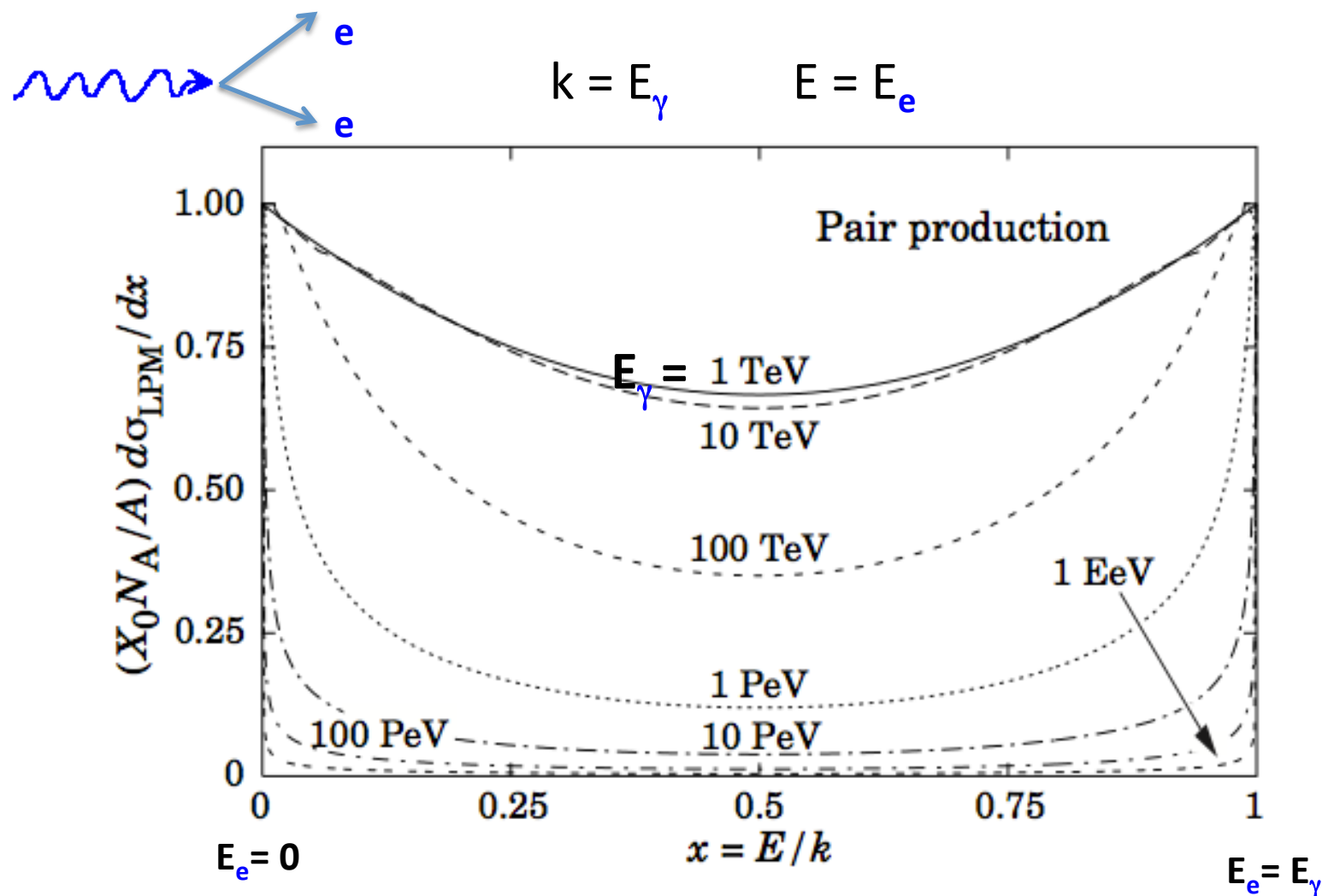


Rises above threshold

reaches saturation for
large E_γ [screening effect]

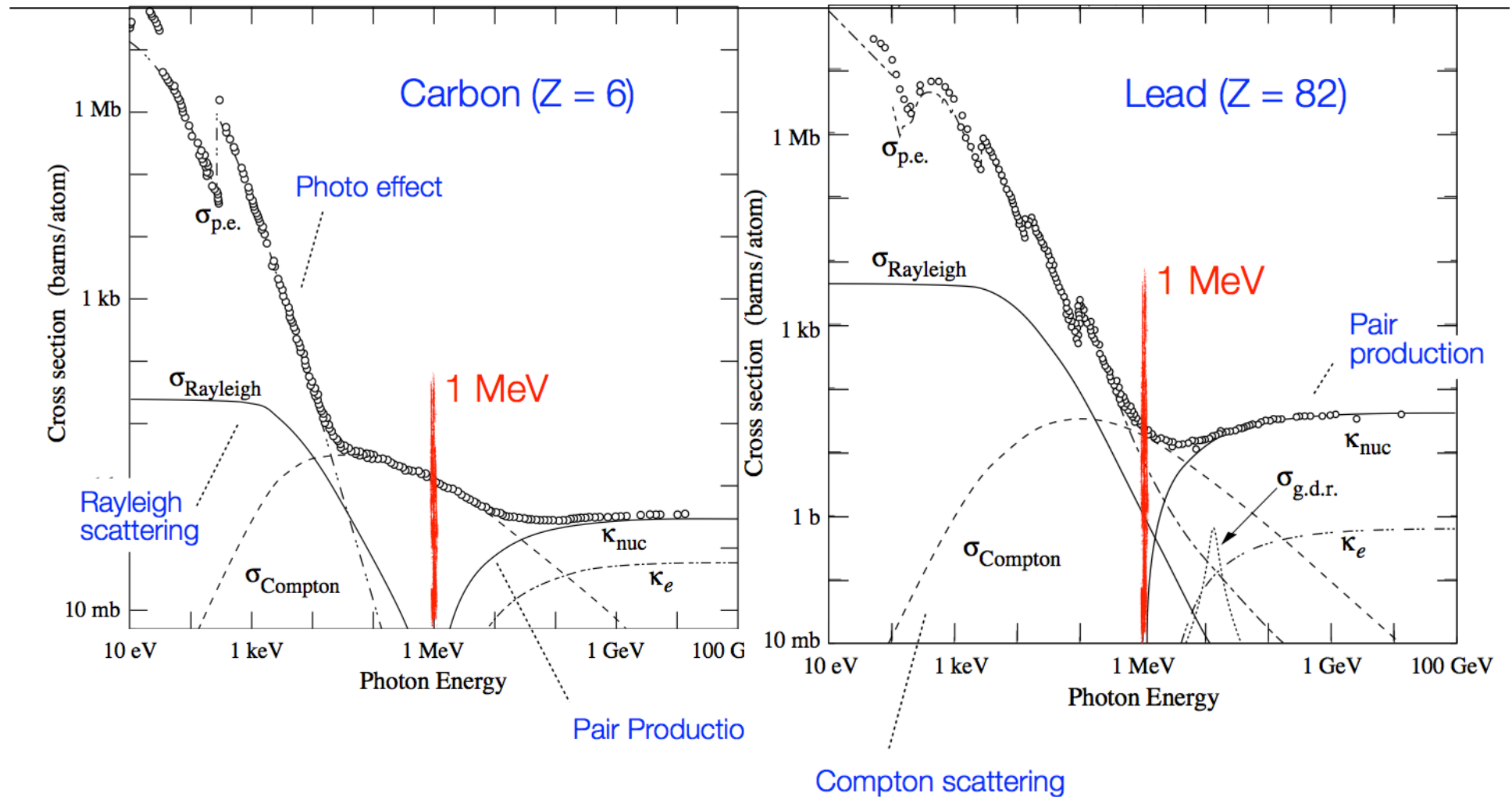
Normalized $e^+ e^-$ pair production cross section

LPM = Landau–Pomeranchuk–Migdal cross section.



fractional electron energy $x = E/k = E_e/E_\gamma$

γ total cross section

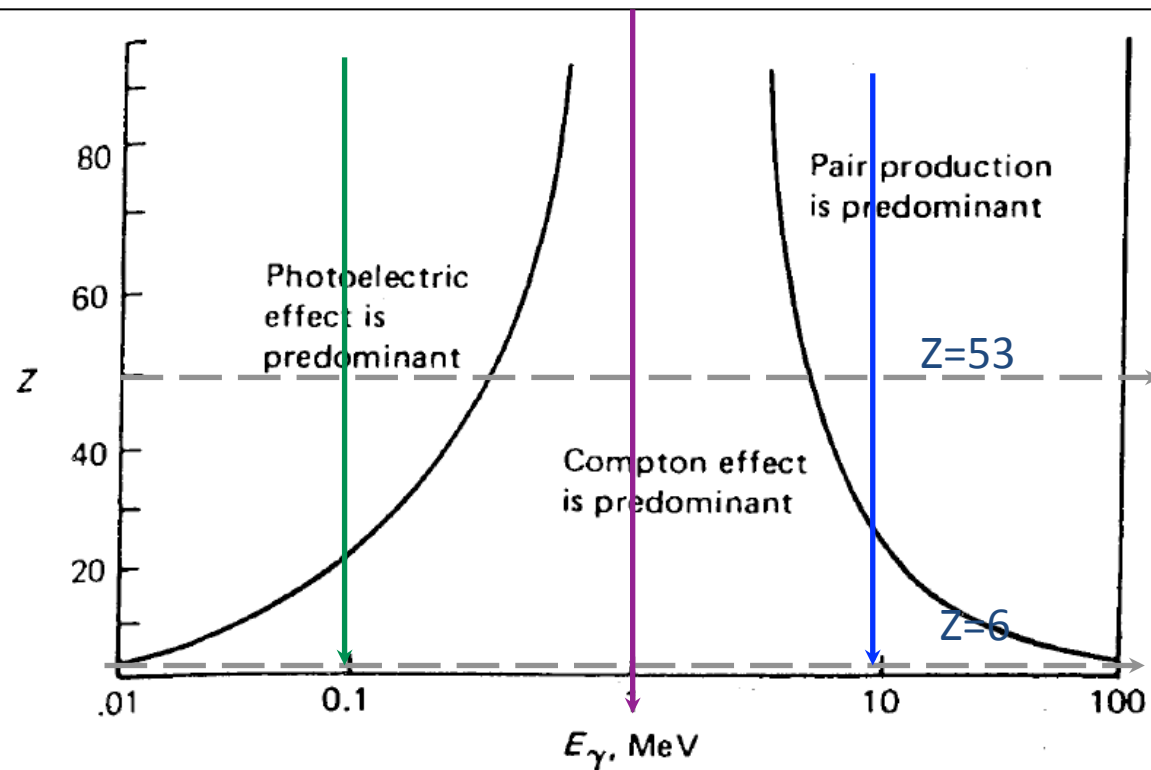


Several other effects take place (not discussed here):

Rayleigh Scattering (scattering on atmosphere particles, blue sky)

Photo Nuclear Interactions (Giant Dipole Resonance, collective excitation of atomic nuclei).

Dependence on Z et on E



$E_\gamma = 0.1$ MeV in C ($Z=6$) Compton effect is dominant
in I ($Z=53$) Photoelectric effect is dominant

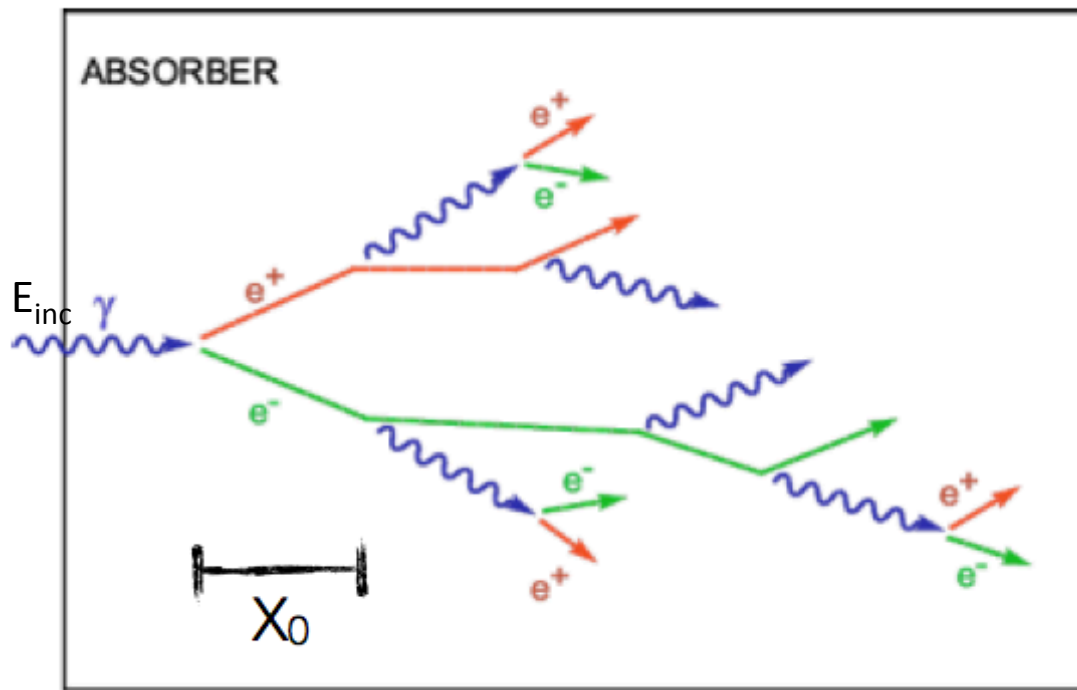
$E_\gamma = 1$ MeV Compton effect is dominant

$E_\gamma = 10$ MeV in C ($Z=6$) Compton effect is dominant
in I ($Z=53$) pair production is dominant

Electromagnetic showers

See M. Delmastro lectures

Dominant processes for photons (and electrons) at very high energies



$$t_{max} = \ln \frac{E_{inc}}{E_c} - \left. \begin{array}{l} 1 \\ 0.5 \end{array} \right\} \begin{array}{l} e^- \\ \text{gamma} \end{array}$$

$$L_{95\%} \approx t_{max} + 0.08 Z + 9.6 [X_0]$$

$L_{95\%}$ = longitudinal shower containment

t_{max} = depth in radiation length units, where the max energy is deposited

E_{in} = incoming photon energy

E_c = critical energy

Also electrons can start e.m. showers

Hadron collisions and interaction lengths

The total cross section for **very high energy hadrons** is expressed as:

$$\sigma_T = \sigma_{\text{elastic}} + \sigma_{\text{inelastic}}$$

The inelastic part of the total cross-section is susceptible to induce a **hadron shower** (increase of particles multiplicity)

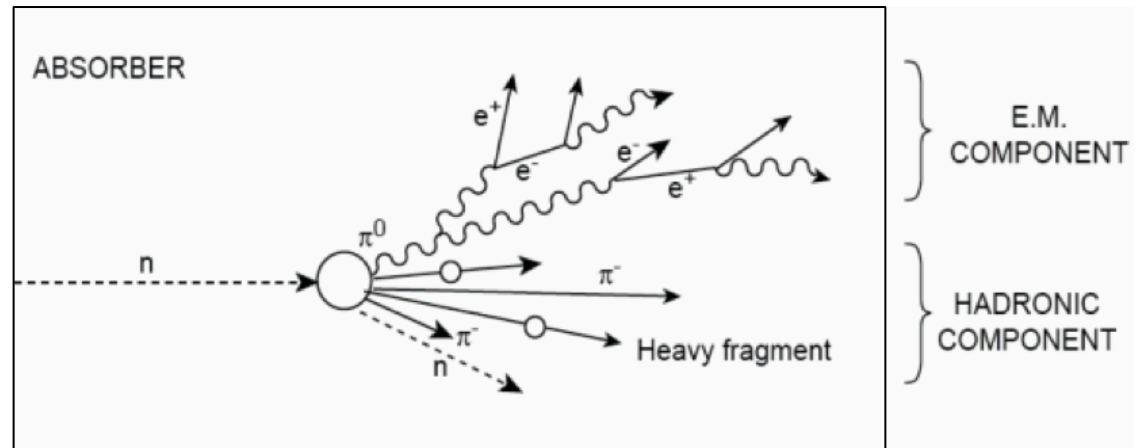
Two mean-lengths are introduced:

- nuclear collision length

$$\lambda_T = \frac{A}{N_A \sigma_T} \text{ g cm}^{-2}$$

- nuclear interaction length

$$\lambda_I = \frac{A}{N_A \sigma_{\text{inelastic}}} \text{ g cm}^{-2}$$



See M. Delmastro slides for more details

95% containment of a hadronic shower is for a material thickness of :

$$L_{95\%}(\text{in units of } \lambda_I) \approx 1 + 1.35 \ln(E(\text{GeV}))$$

→ ~ **10 interaction lengths** are needed to contain a **1 TeV hadronic shower**

In high A materials $\lambda_I > X_0$ This explains why hadron calorimeters are after installed electromagnetic

6. ATOMIC AND NUCLEAR PROPERTIES OF MATERIALS

Table 6.1. Abridged from pdg.lbl.gov/AtomicNuclearProperties by D. E. Groom (2007). Quantities in parentheses are for NTP (20° C and 1 atm), and square brackets indicate quantities evaluated at STP. Boiling points are at 1 atm. Refractive indices n are evaluated at the sodium D line blend (589.2 nm); values $\gg 1$ in brackets are for $(n - 1) \times 10^6$ (gases).

Material	Z	A	$\langle Z/A \rangle$	Nucl.coll. length λ_T {g cm ⁻² }	Nucl.inter. length λ_I {g cm ⁻² }	Rad.len. X_0 {g cm ⁻² }	$dE/dx _{\min}$ { MeV g ⁻¹ cm ² }	Density {g cm ⁻³ } ({gℓ ⁻¹ })	Melting point (K)	Boiling point (K)	Refract. index (@ Na D)
H ₂	1	1.00794(7)	0.99212	42.8	52.0	63.04	(4.103)	0.071(0.084)	13.81	20.28	1.11[132.]
D ₂	1	2.01410177803(8)	0.49650	51.3	71.8	125.97	(2.053)	0.169(0.168)	18.7	23.65	1.11[138.]
He	2	4.002602(2)	0.49967	51.8	71.0	94.32	(1.937)	0.125(0.166)		4.220	1.02[35.0]
Li	3	6.941(2)	0.43221	52.2	71.3	82.78	1.639	0.534	453.6	1615.	
Be	4	9.012182(3)	0.44384	55.3	77.8	65.19	1.595	1.848	1560.	2744.	
C diamond	6	12.0107(8)	0.49955	59.2	85.8	42.70	1.725	3.520			2.42
C graphite	6	12.0107(8)	0.49955	59.2	85.8	42.70	1.742	2.210			
N ₂	7	14.0067(2)	0.49976	61.1	89.7	37.99	(1.825)	0.807(1.165)	63.15	77.29	1.20[298.]
O ₂	8	15.9994(3)	0.50002	61.3	90.2	34.24	(1.801)	1.141(1.332)	54.36	90.20	1.22[271.]
F ₂	9	18.9984032(5)	0.47372	65.0	97.4	32.93	(1.676)	1.507(1.580)	53.53	85.03	[195.]
Ne	10	20.1797(6)	0.49555	65.7	99.0	28.93	(1.724)	1.204(0.839)	24.56	27.07	1.09[67.1]
Al	13	26.9815386(8)	0.48181	69.7	107.2	24.01	1.615	2.699	933.5	2792.	
Si	14	28.0855(3)	0.49848	70.2	108.4	21.82	1.664	2.329	1687.	3538.	3.95
Cl ₂	17	35.453(2)	0.47951	73.8	115.7	19.28	(1.630)	1.574(2.980)	171.6	239.1	[773.]
Ar	18	39.948(1)	0.45059	75.7	119.7	19.55	(1.519)	1.396(1.662)	83.81	87.26	1.23[281.]
Ti	22	47.867(1)	0.45961	78.8	126.2	16.16	1.477	4.540	1941.	3560.	
Fe	26	55.845(2)	0.46557	81.7	132.1	13.84	1.451	7.874	1811.	3134.	
Cu	29	63.546(3)	0.45636	84.2	137.3	12.86	1.403	8.960	1358.	2835.	
Ge	32	72.64(1)	0.44053	86.9	143.0	12.25	1.370	5.323	1211.	3106.	
Sn	50	118.710(7)	0.42119	98.2	166.7	8.82	1.263	7.310	505.1	2875.	
Xe	54	131.293(6)	0.41129	100.8	172.1	8.48	(1.255)	2.953(5.483)	161.4	165.1	1.39[701.]
W	74	183.84(1)	0.40252	110.4	191.9	6.76	1.145	19.300	3695.	5828.	
Pt	78	195.084(9)	0.39983	112.2	195.7	6.54	1.128	21.450	2042.	4098.	
Au	79	196.966569(4)	0.40108	112.5	196.3	6.46	1.134	19.320	1337.	3129.	
Pb	82	207.2(1)	0.39575	114.1	199.6	6.37	1.122	11.350	600.6	2022.	
U	92	[238.02891(3)]	0.38651	118.6	209.0	6.00	1.081	18.950	1408.	4404.	

Material	Z	A	$\langle Z/A \rangle$	Nucl.coll. length λ_T {g cm ⁻² }	Nucl.inter. length λ_I {g cm ⁻² }	Rad.len. X_0 {g cm ⁻² }	$dE/dx _{\min}$ { MeV g ⁻¹ cm ² }	Density {g cm ⁻³ } ({g ℓ ⁻¹ })	Melting point (K)	Boiling point (K)	Refract. index (@ Na D)
Air (dry, 1 atm)			0.49919	61.3	90.1	36.62	(1.815)	(1.205)		78.80	
Shielding concrete			0.50274	65.1	97.5	26.57	1.711	2.300			
Borosilicate glass (Pyrex)			0.49707	64.6	96.5	28.17	1.696	2.230			
Lead glass			0.42101	95.9	158.0	7.87	1.255	6.220			
Standard rock			0.50000	66.8	101.3	26.54	1.688	2.650			
Methane (CH ₄)			0.62334	54.0	73.8	46.47	(2.417)	(0.667)	90.68	111.7	[444.]
Ethane (C ₂ H ₆)			0.59861	55.0	75.9	45.66	(2.304)	(1.263)	90.36	184.5	
Butane (C ₄ H ₁₀)			0.59497	55.5	77.1	45.23	(2.278)	(2.489)	134.9	272.6	
Octane (C ₈ H ₁₈)			0.57778	55.8	77.8	45.00	2.123	0.703	214.4	398.8	
Paraffin (CH ₃ (CH ₂) _n ≈23CH ₃)			0.57275	56.0	78.3	44.85	2.088	0.930			
Nylon (type 6, 6/6)			0.54790	57.5	81.6	41.92	1.973	1.18			
Polycarbonate (Lexan)			0.52697	58.3	83.6	41.50	1.886	1.20			
Polyethylene ([CH ₂ CH ₂] _n)			0.57034	56.1	78.5	44.77	2.079	0.89			
Polyethylene terephthalate (Mylar)			0.52037	58.9	84.9	39.95	1.848	1.40			
Polymethylmethacrylate (acrylic)			0.53937	58.1	82.8	40.55	1.929	1.19			1.49
Polypropylene			0.55998	56.1	78.5	44.77	2.041	0.90			
Polystyrene ([C ₆ H ₅ CHCH ₂] _n)			0.53768	57.5	81.7	43.79	1.936	1.06			1.59
Polytetrafluoroethylene (Teflon)			0.47992	63.5	94.4	34.84	1.671	2.20			
Polyvinyltoluene			0.54141	57.3	81.3	43.90	1.956	1.03			1.58
Aluminum oxide (sapphire)			0.49038	65.5	98.4	27.94	1.647	3.970	2327.	3273.	1.77
Barium fluoride (BaF ₂)			0.42207	90.8	149.0	9.91	1.303	4.893	1641.	2533.	1.47
Carbon dioxide gas (CO ₂)			0.49989	60.7	88.9	36.20	1.819	(1.842)			[449.]
Solid carbon dioxide (dry ice)			0.49989	60.7	88.9	36.20	1.787	1.563	Sublimes at 194.7 K		
Cesium iodide (CsI)			0.41569	100.6	171.5	8.39	1.243	4.510	894.2	1553.	1.79
Lithium fluoride (LiF)			0.46262	61.0	88.7	39.26	1.614	2.635	1121.	1946.	1.39
Lithium hydride (LiH)			0.50321	50.8	68.1	79.62	1.897	0.820	965.		
Lead tungstate (PbWO ₄)			0.41315	100.6	168.3	7.39	1.229	8.300	1403.		2.20
Silicon dioxide (SiO ₂ , fused quartz)			0.49930	65.2	97.8	27.05	1.699	2.200	1986.	3223.	1.46
Sodium chloride (NaCl)			0.55509	71.2	110.1	21.91	1.847	2.170	1075.	1738.	1.54
Sodium iodide (NaI)			0.42697	93.1	154.6	9.49	1.305	3.667	933.2	1577.	1.77
Water (H ₂ O)			0.55509	58.5	83.3	36.08	1.992	1.000(0.756)	273.1	373.1	1.33
Silica aerogel			0.50093	65.0	97.3	27.25	1.740	0.200	(0.03 H ₂ O, 0.97 SiO ₂)		

Neutron interactions

Electric charge of the neutron **n** : $q_n = 0$

➡ The **n** interacts via « **strong interaction** » with nuclei (short range force $\sim 10^{-13}$ cm)

Classification of neutrons:

Cold or ultracold neutrons	$E_n < 0.025$ eV
Thermal or slow neutrons	$E_n \sim 0.025$ eV
Intermediate neutrons	$E_n \sim 0.025$ eV \div 0.1 MeV
Fast neutrons	$E_n \sim 0.1 \div 10$ -20 MeV
High energy neutrons	$E_n > 20$ MeV

Alternative classification:

Slow neutrons (absorbed)	$E_n < \sim 0.5$ MeV	
Fast neutrons	$E_n > \sim 0.5$ MeV	$E = 0.5$ MeV = 'cadmium cutoff'

Main interaction processes of **n**: **scattering (elastic and inelastic), absorption, fission hadron shower production** depending on the neutron energy

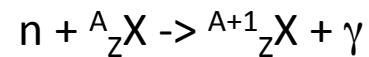
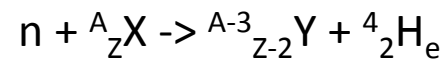
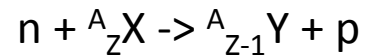
Neutron interactions

Scattering with nuclei : $n + {}^A_ZX \rightarrow {}^A_ZX^{(*)} + n$

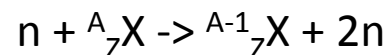
Elastic \rightarrow important for **moderation**

Inelastic

Absorption & Nuclear reactions:



radiative capture of n



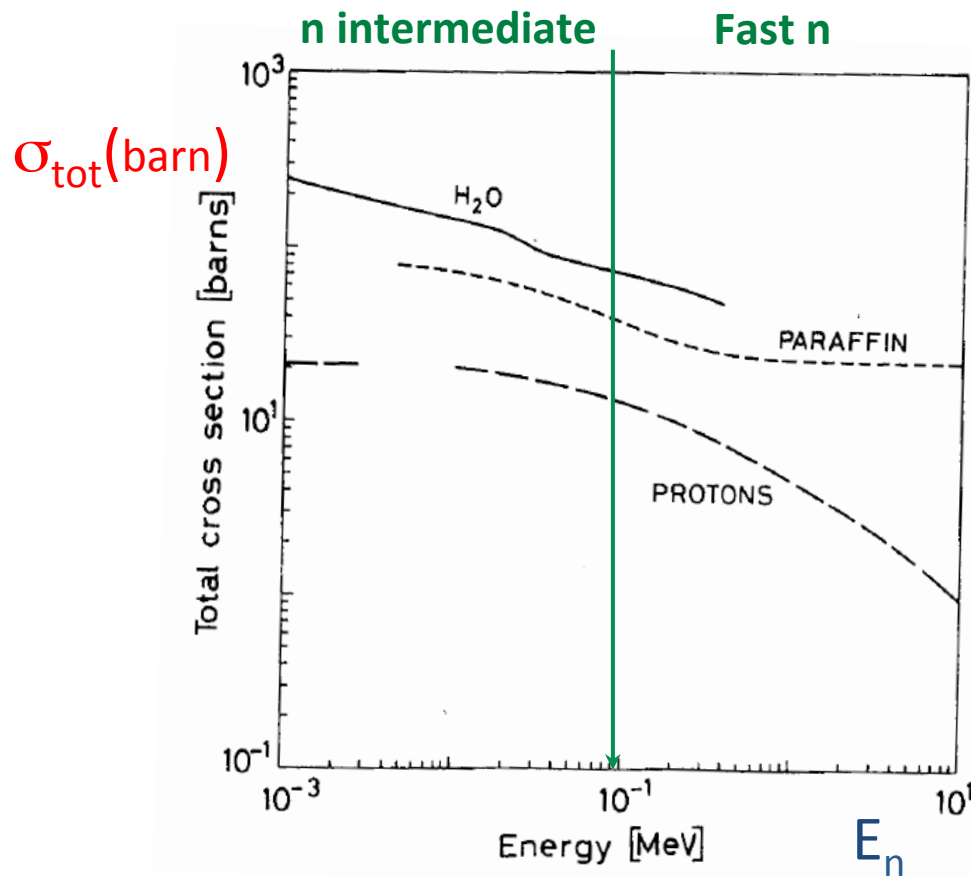
Fission: $n + {}^A_ZX \rightarrow {}^{A1}_{Z2}Y + {}^{A2}_{Z2}Y + n + n + \dots$

Cross section $\approx 1/v_n$ (more probable for low energy) + resonant peaks

Hadron showers $E_n > \sim 100 \text{ MeV}$

Cross section of low energy neutrons (n)

Neutron cross section on H₂O, paraffine and protons

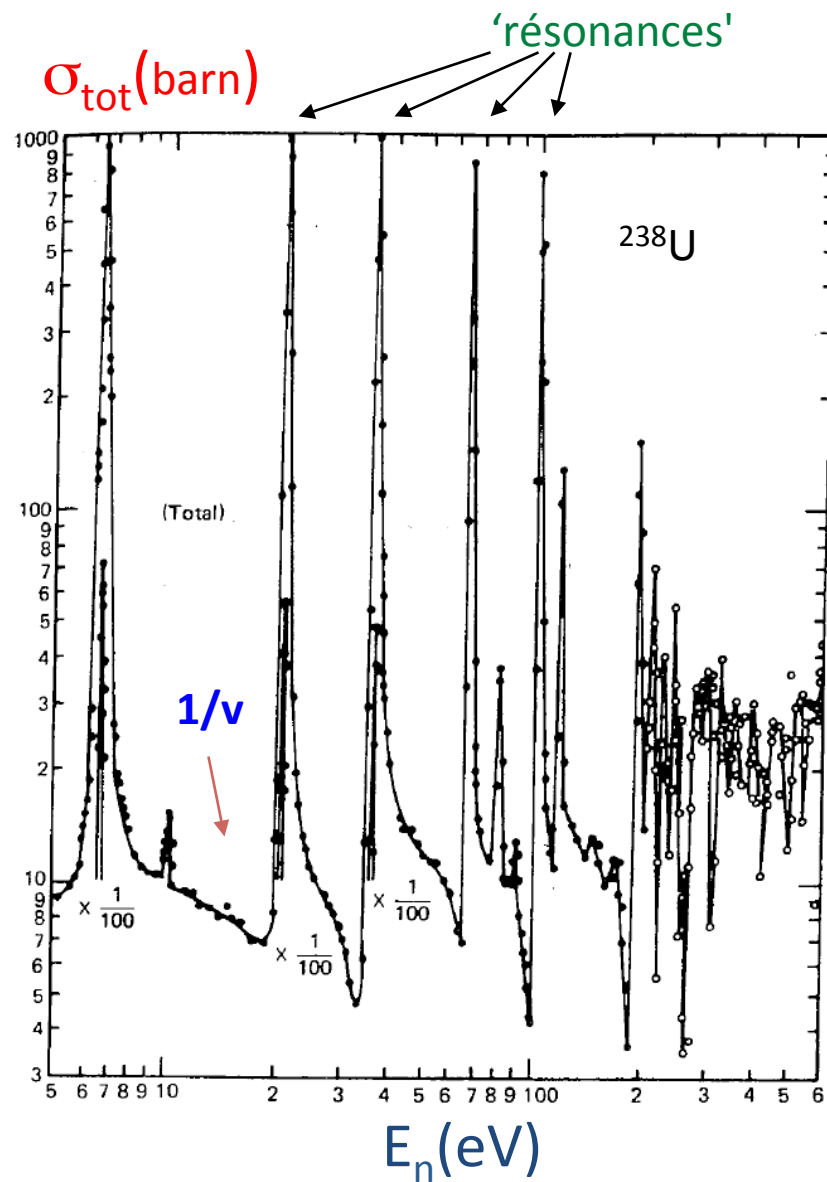


$$1 \text{ barn} = 10^{-24} \text{ cm}^2 = 10^{-28} \text{ m}^2$$

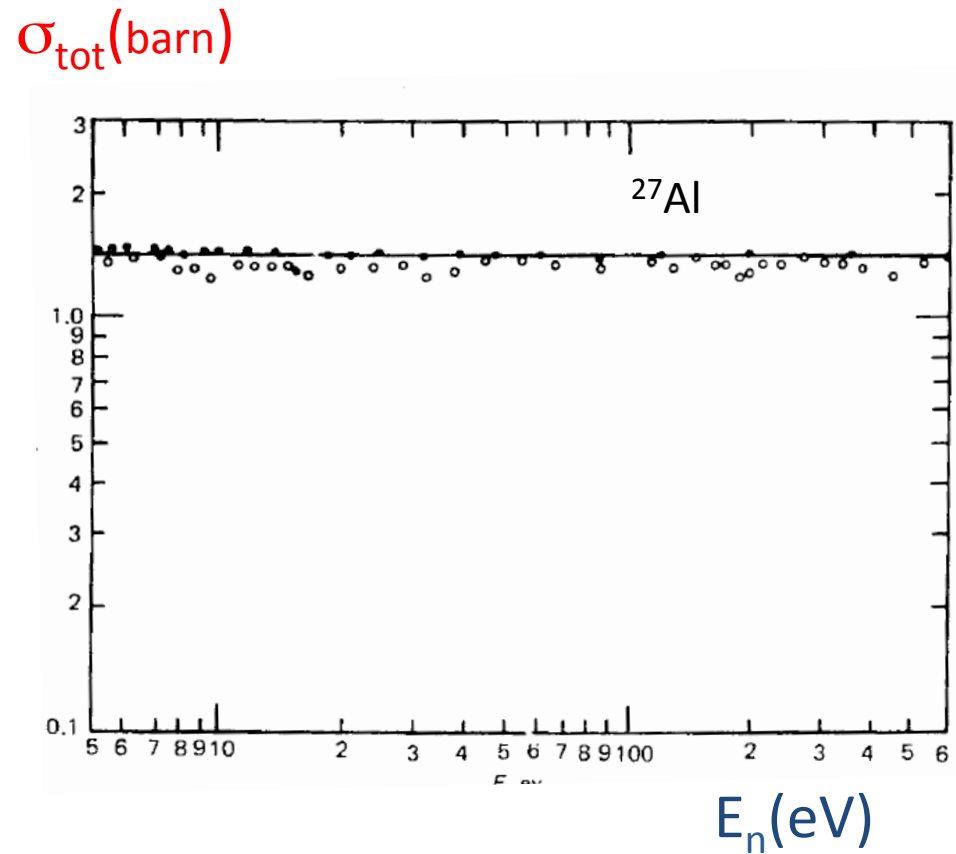
$$\text{Radius of the nucleon } R \approx 10^{-15} \div 10^{-14} \text{ m}$$

$$1 \text{ barn} \approx R^2$$

Low energy neutron (n) cross section



NB : Very different scales on the vertical axis



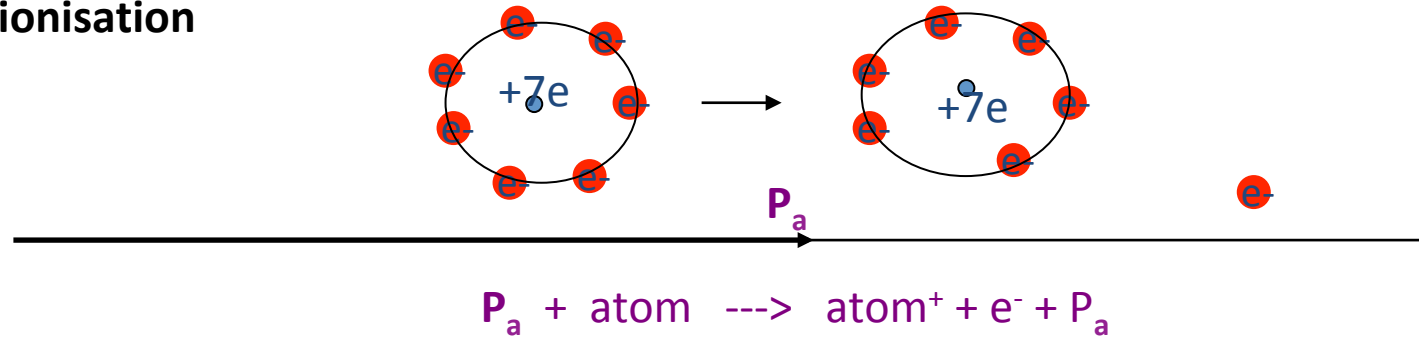
Charged particle interactions

- 1) **Ionization**: inelastic collision with **electrons** of the atoms
- 2) **Bremsstrahlung**: photon radiation emission by an accelerated charge
- 3) **Multiple Scattering**: elastic collision with **nucleus**
- 4) **Cerenkov & transition radiation effects**: photon emission
- (• 5) **Nuclear interactions (p, π, K)**: processes mediated by strong interactions)

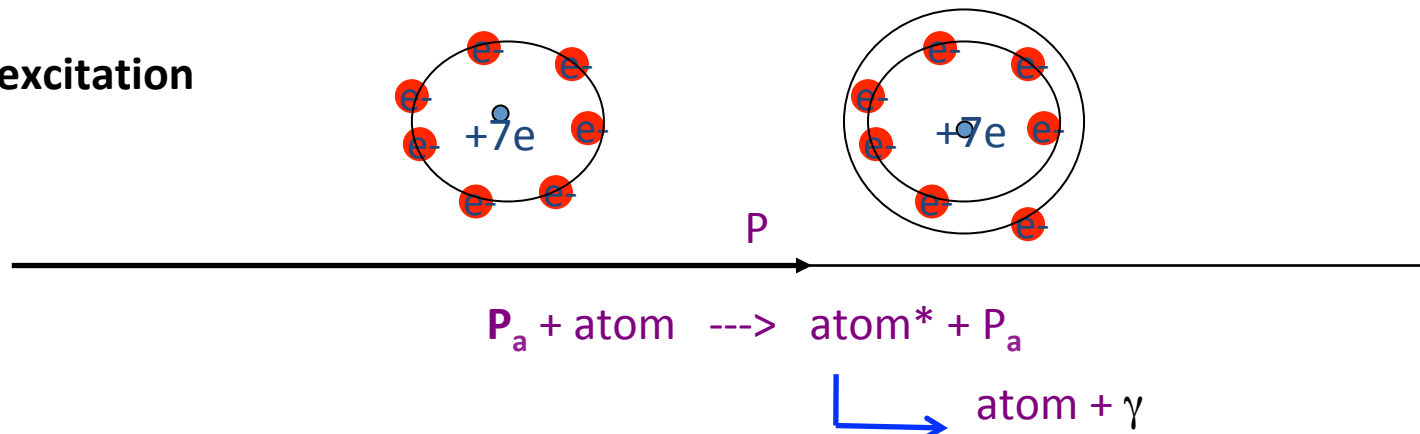
1) Inelastic collision with electrons of the atoms

Main e.m. process for heavy ($M_{P_a} \gg m_e$) charged particles P_a (ex. μ)

- ionisation

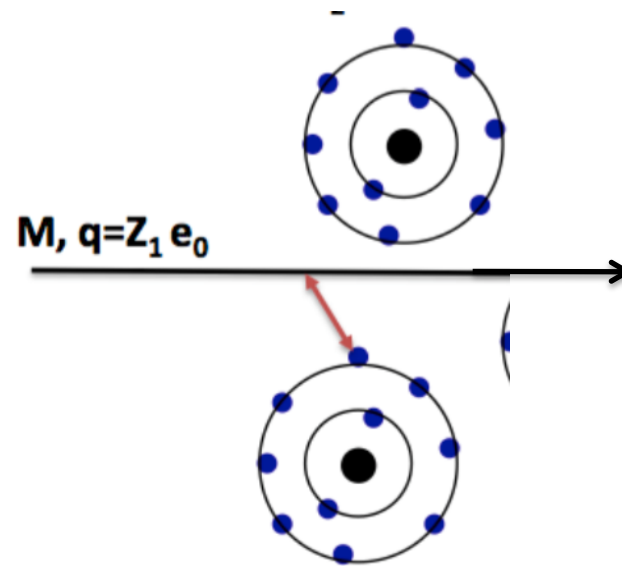


- excitation



- Both processes together (**ionization & excitation**) can also happen
- **Inelastic collisions on nucleus(N)** are much less frequent (since the **energy transfer** depends inversely on the target mass and $m_N \gg m_e$)
- **The particle P_a loses a bit of its energy** (in each of the many collisions), its directions is \sim unchanged.

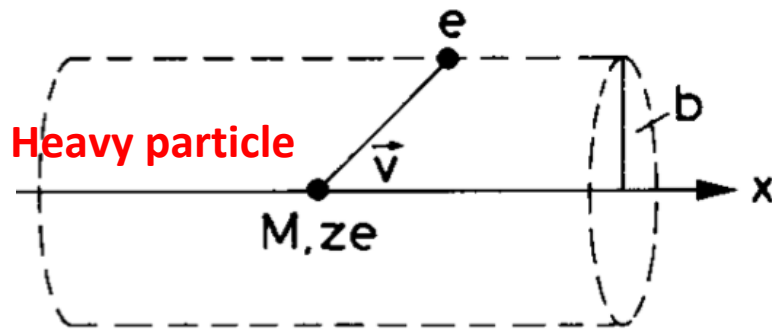
Average energy loss per unit of length ($-dE/dx$) of P_a due to inelastic collisions with electrons of the atom



Analytic formula: **Bethe & Bloch (B&B) formula**

let's derive here a simplified 'semi-relativistic' expression for $-dE/dx$

Simple computation of the average energy loss of particle P_a (derivation of the B&B formula)



Assumptions:

- e considered free and initially at rest
- e moving slightly during interaction
- Heavy particle undeflected ($v \sim \text{const}$)
- Electric force **acting on e** (during $dt = dx/v$):

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$(*) \quad \Delta p_{\perp} = \int F_{\perp} dt = e \int E_{\perp} dt = e \int E_{\perp} dx/v$$

$$E_{\perp} \text{ from Gauss law: } \Phi_S(\vec{E}) = 4\pi ze$$

$$\int E_{\perp} 2\pi b dx = 4\pi ze \quad \int E_{\perp} dx = 2ze/b$$

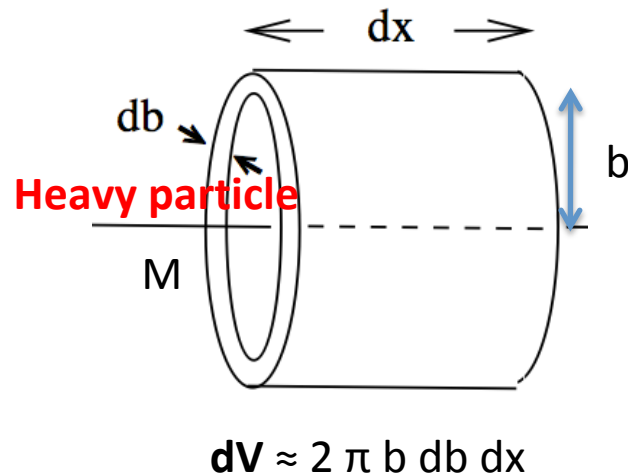
Momentum transferred to **one** electron: $\Delta p = 2ze^2 / (bv)$

Kin. Energy transferred to **one** electron: $\Delta E = \Delta p^2 / (2m_e) = \frac{2z^2 e^4}{m_e v^2 b^2} \quad (\text{non-rel.})$

(*) w.r.t the particle direction

Δp_{\parallel} effects average to 0 (symmetry)

Simple computation of the average energy loss



$$\Delta E = \frac{2 z^2 e^4}{m_e v^2 b^2}$$

- Effect of the interaction of P_a with the electrons in dV (energy loss by P_a):

$$-dE(b) = \Delta E N_c dV = \frac{4\pi z^2 e^4}{m_e v^2} N_c \frac{db}{b} dx$$

$N_c = (\rho N_A Z)/A_{\text{mol}}$ = number of electrons per unit of volume

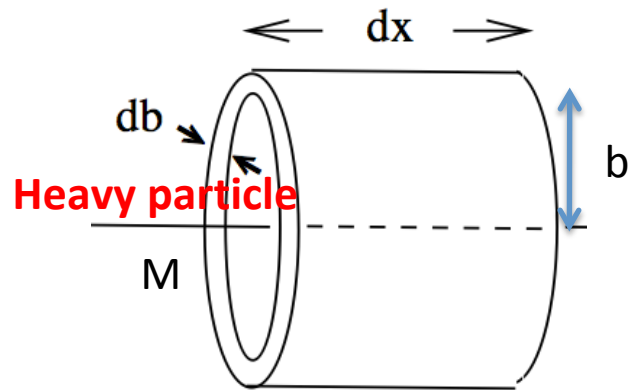
$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_c \ln \frac{b_{\text{max}}}{b_{\text{min}}}$$

De Broglie wavelength of electron

(after an head-on collision $v_e \approx$ particle velocity)

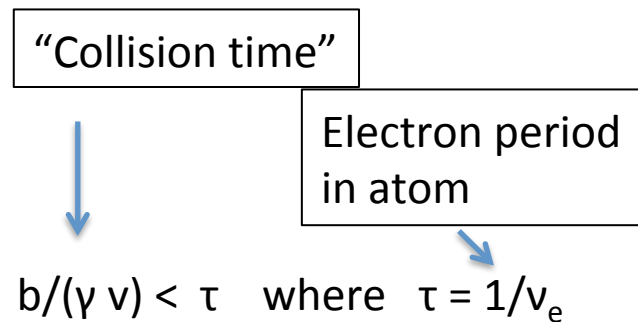
b_{min} from De Broglie wavelength $b_{\text{min}} = \lambda_e = h/p_{\text{emax}} \sim h/(m_e \gamma v)$

Simple computation of the average energy loss



$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_c \ln \frac{b_{\max}}{b_{\min}}$$

b_{\max} from “adiabatic invariance” :
the perturbation should occur in a time short compared to the revolution period τ of the bound electron



Particle velocity

Orbital frequency

$$b_{\max} = (\gamma v)/v_e$$

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_c \ln \frac{m_e c^2 \beta^2 \gamma^2}{h v_e}$$

$$I = h v_e$$

Close to the Bethe&Bloch formula (within a factor ~ 2)

Average energy loss by a charged particle ($m_{Pa} \gg m_e$) in matter

Incident charged

'heavy' particle P_a of energy E, M



matter (e.x. gaz of a detector)

Bethe-Bloch formula (B & B)

$$-\frac{dE}{dx} = K \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} - 2 \beta^2 - \delta - 2 \frac{C}{Z} \right]$$

r_e = classic radius of electron = $\alpha/(m_e c^2) = 2.8$ fm

m_e = electron mass = 511 KeV

z = charge of incident particle in unit of e

β = particle speed in unit of c

$\gamma = 1/\sqrt{1-\beta^2}$

T_{max} = maximum Kin energy transferred in a collision)

ρ = density of the matter

Z, A = atomic number, atomic weight of the matter

I = effective excitation potential of the matter

Difficult to compute --> obtained from dE/dx

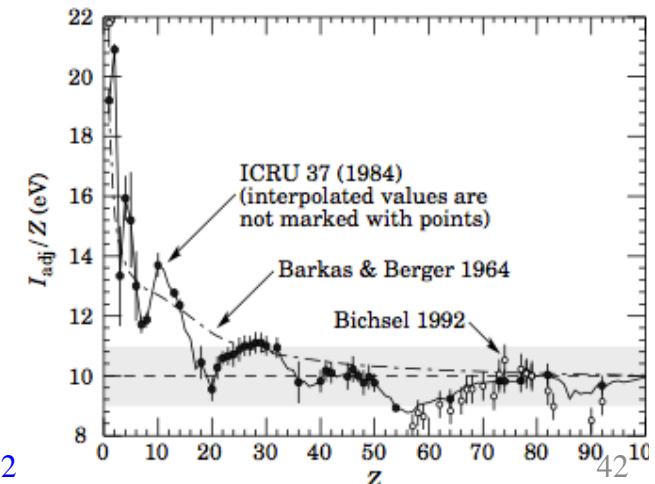
$$I \text{ (eV)} = (12 + 7/Z) Z \quad (Z \leq 12)$$

$$I \text{ (eV)} = (9.76 + 58.8 Z^{-1.19}) Z \quad (Z \geq 12)$$

$$2K = 4 \pi N_A r_e^2 m_e c^2 = 0.307 \text{ MeV g}^{-1} \text{ cm}^2$$

$$T_{max} = E_e^{max} - m_e = \frac{2m_e \beta^2 \gamma^2}{(E_{CM}/M)^2}$$

$$T_{max} \sim 2 m_e c^2 \beta^2 \gamma^2 \text{ for } \gamma \ll m_{Pa}/(2 m_e)$$



Shell (C) and Density(δ) effect corrections

C = Relevant at low energy. Small correction. The particle velocity \sim orbital velocity of e
 \rightarrow the assumption that atomic electrons initially are at rest breaks.
 Takes into account binding energy. The energy loss is reduced.
 The capture process of the particle is possible

δ = “Density effect”. Relevant at high energy.

The electric field of the particle polarise the atoms of the matter
 \rightarrow The energy loss is reduced since shielding of electrical field far from the particle path \rightarrow **moderation of the relativistic rise**
 It depends on the particle speed and on the matter density

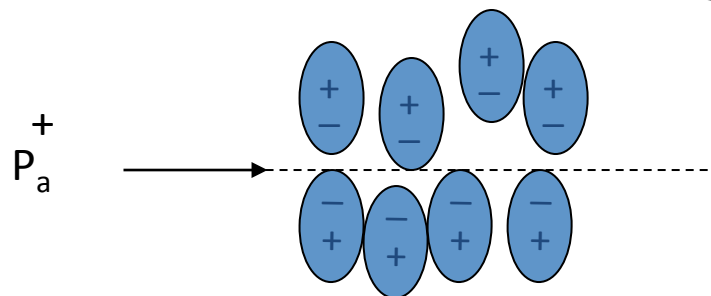
Density effect leads to “saturation” at high energy

For high energy: $\delta/2 \rightarrow \ln(\hbar\omega_p/I) + \ln \beta\gamma - 1/2$

$\hbar\omega_p \triangleq$ “Plasma energy” =

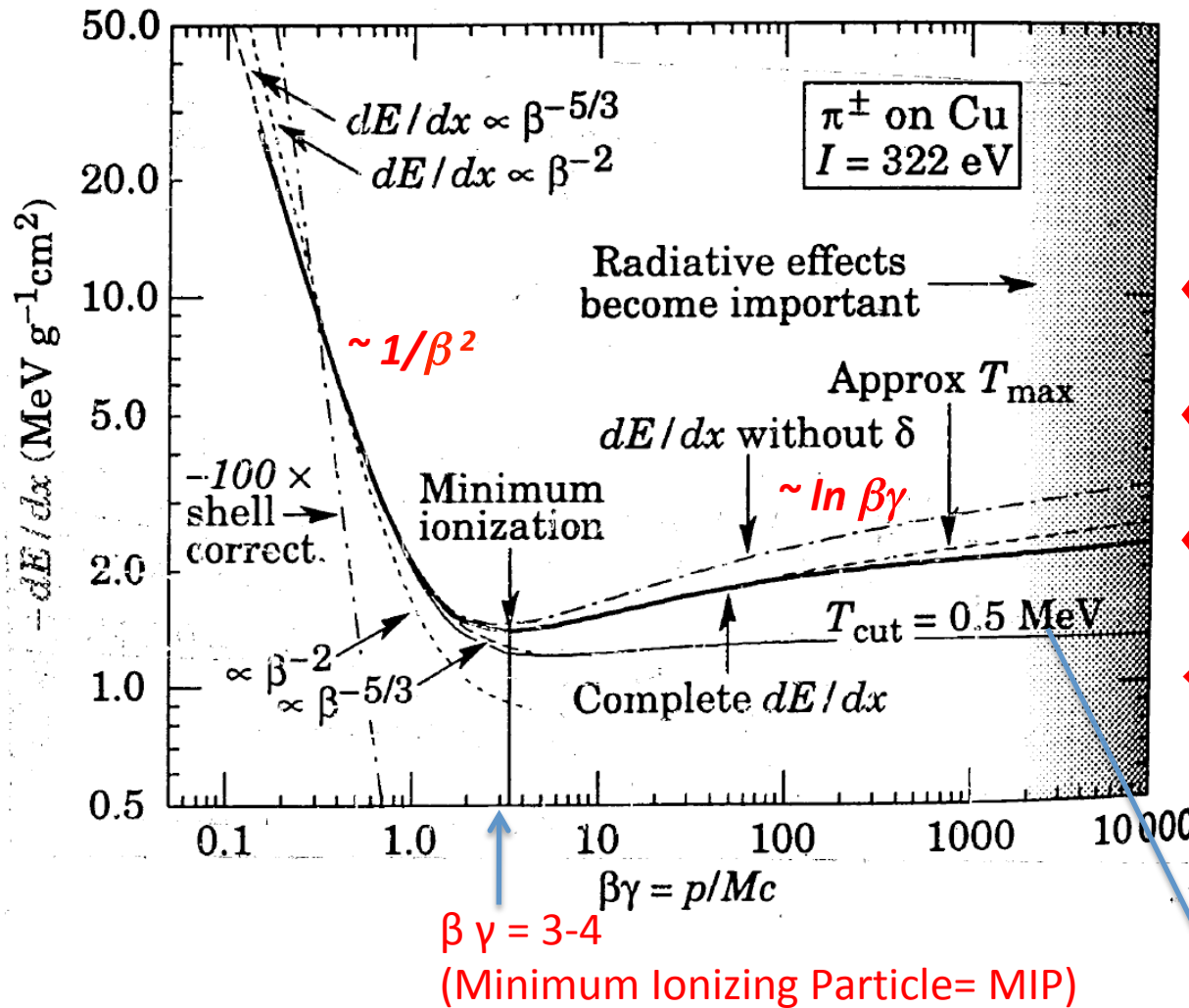
$$\sqrt{\rho \langle Z/A \rangle} \times 28.816 \text{ eV}$$

(ρ in g cm^{-3})



Polarisation effect (correction δ)
 \rightarrow reduction of the energy loss

Stopping power or mean specific energy loss = dE/dx ($M_{pa} \gg m_e$)



Few remarks:

- ◆ $dE/dx = f(\beta)$
- ◆ dE/dx doesn't depend on M_{pa}
- ◆ $dE/dx \propto z^2$ (particle charge)
- ◆ On vertical l' axis
 $-dE/(\rho dx)$ (MeV cm²)/g

Why?

T_{cut}
'restricted energy loss'
discussed later

See Marco Delmastro lectures for explanation of $1/\beta^2$ and $\ln \beta\gamma$

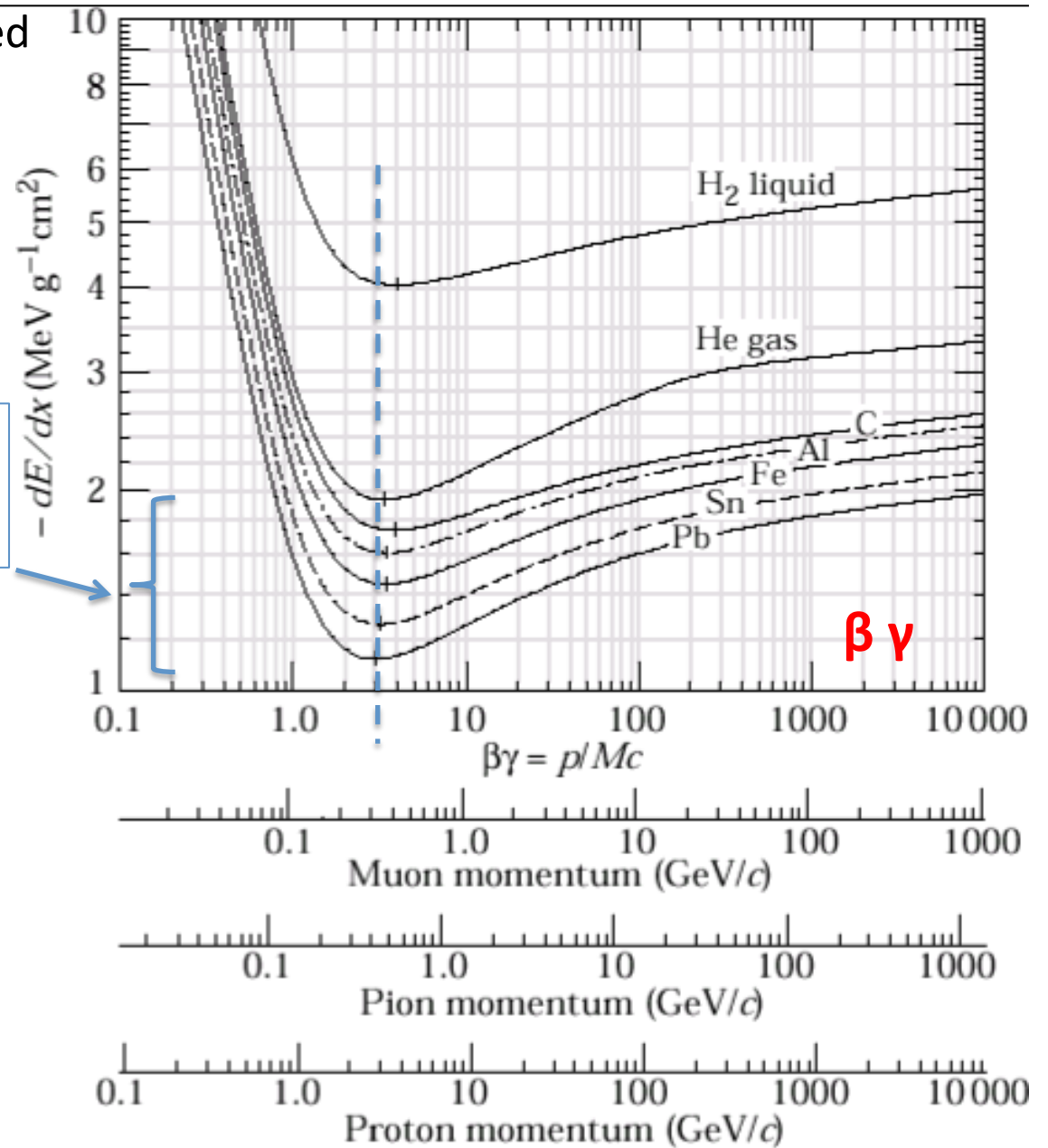
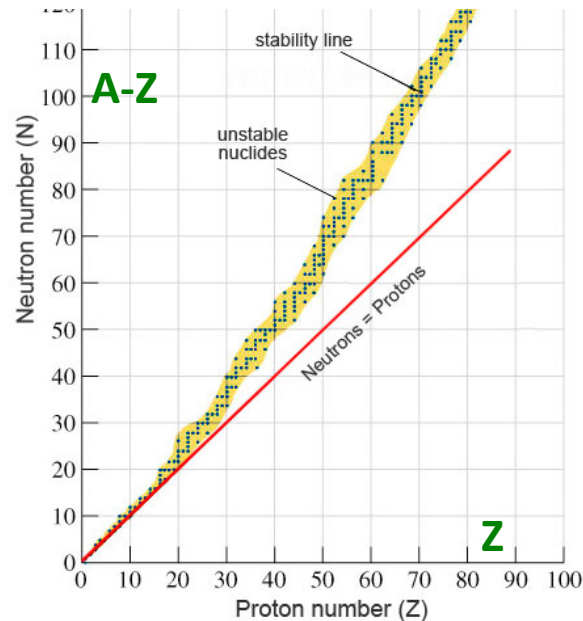
Stopping power

* If material thickness is measured in ρdx (g/cm²)

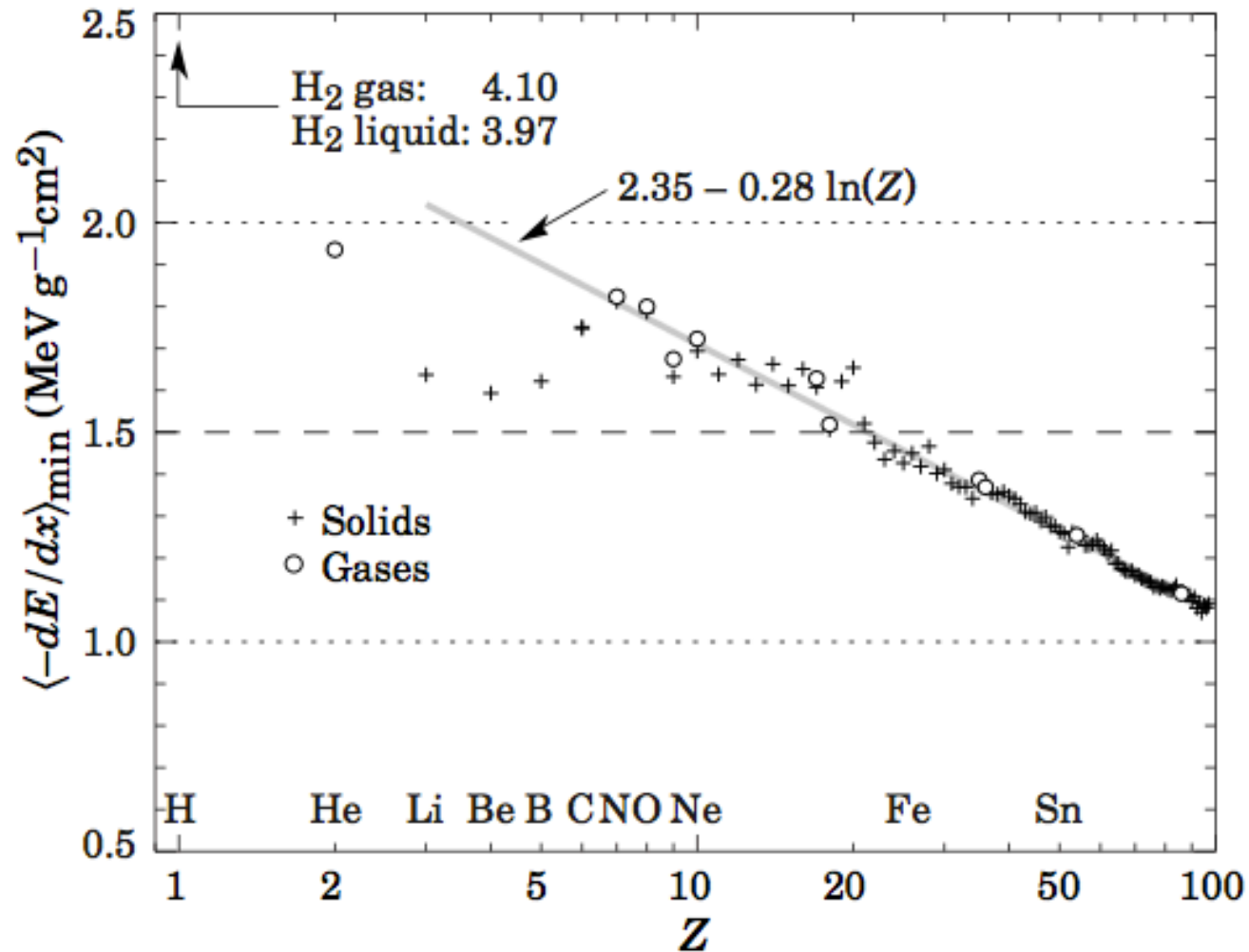
→ on vertical l'axis
 $-dE/(\rho dx)$ (MeV cm²)/g

→ the dependence on the material is reduced
 ($Z/A \sim 0.5$)

1-2 MeV g⁻¹cm²
 « minimum of ionisation »



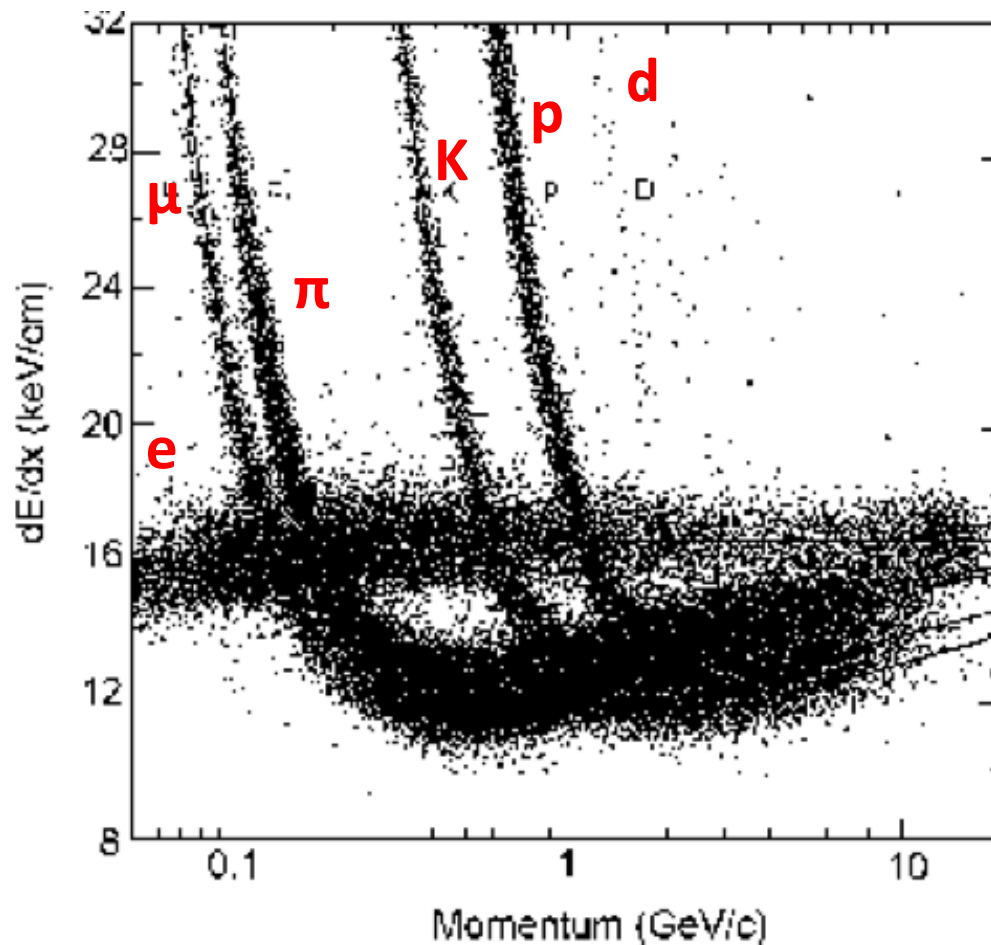
Stopping power at the *minimum of ionization* in greater detail



Use of dE/dx for particle identification

$$\bullet \vec{p} = m \gamma c \vec{\beta} \quad \gamma \equiv \frac{1}{\sqrt{1 - \beta^2}}$$

Measuring **independently** p and $\gamma \beta$ one can extract $m \rightarrow$ particle identification



Plot from
PEP4-9
Time Projection Chamber (TPC)
@SLAC (late '70)

Knock-on electrons or delta(δ) rays or secondary electrons

High energy transfers generates **secondary electrons (from delta rays)**

Distribution (prob.) of δ with **kinetic energies** $T \gg I$:

$$\frac{d^2 N}{dT dx} = \frac{1}{2} K z^2 \frac{Z}{A} \frac{1}{\beta^2} \frac{F(T)}{T^2} \cdot \text{MeV}^{-1} \text{cm}^2 \text{g}^{-1}$$

$$K = 0.307$$

$F(T)$ = **Spin dependent factor**

β , m_{pa} = speed and mass of **primary** particle

x = “**mass thickness**” ($\rho \cdot t$)

Spin 0 $F(T) = F_0(T) = \left(1 - \beta^2 \frac{T}{T_{max}}\right)$

Spin 1/2 $F(T) = F_{1/2}(T) = F_0(T) + \frac{1}{2} \left(\frac{T}{E}\right)^2$

Spin 1 $F(T) = F_1(T) = F_0(T) \left(1 + \frac{1}{3} \frac{T m_e}{m_{pa}^2}\right) + \frac{1}{3} \left(\frac{T}{E}\right)^2 \left(1 + \frac{1}{2} \frac{T m_e}{m_{pa}^2}\right)$

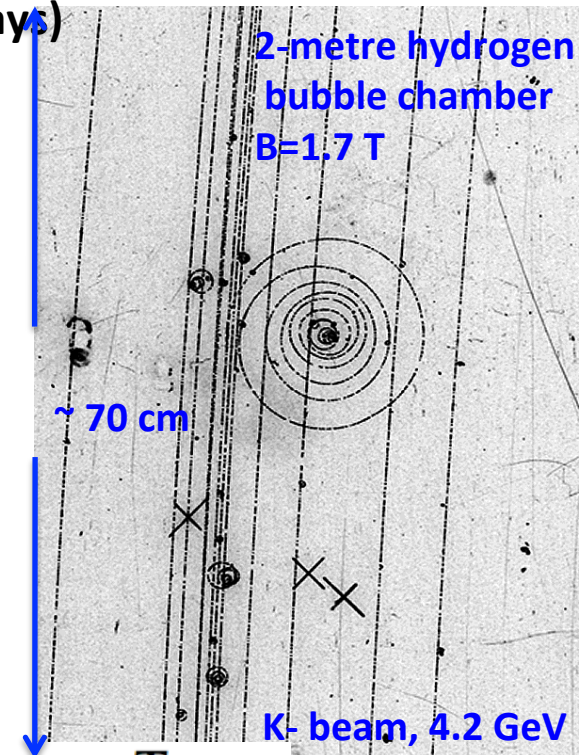
For $T \ll T_{max}$ & $T \ll m_{pa}^2 / m_e$ & $F(T) = 1$:

approximate probability to generate a δ with $T > T_s$

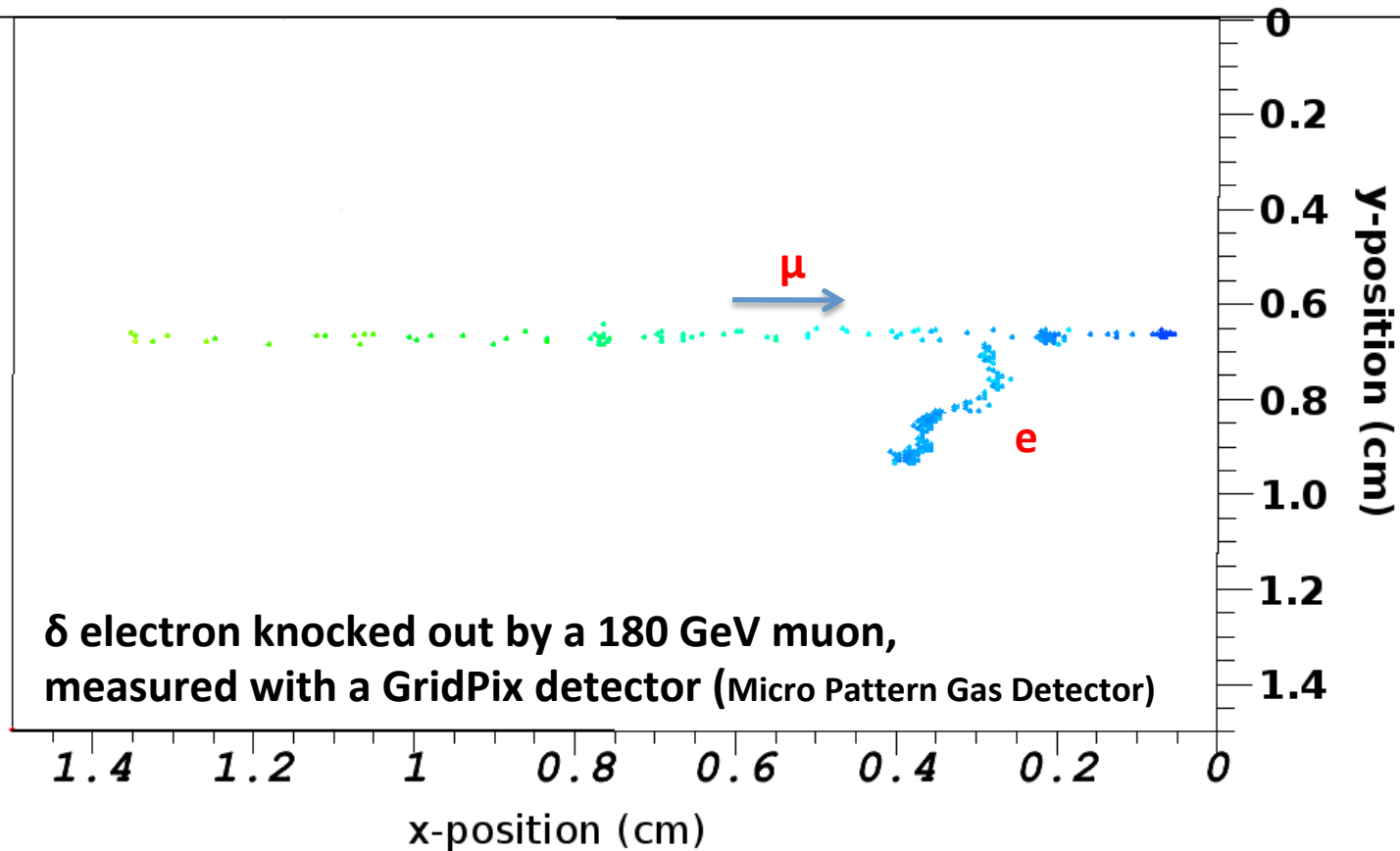
in a thin absorber of thickness x :

$$w(T_s, E, x) \simeq 0.3071 x \frac{z^2 Z}{A(g) \beta^2} \frac{1}{T_s}$$

Lucia Di Ciaccio - ESIPAP IPM - January 2022



Delta(δ) rays in Micro Pattern Gas Detector



δ rays produce ionization. This is called secondary to distinguish from the primary (impinging particle)

For a $\beta \approx 1$ particle, on average **one** collision with **$T > 10$ keV** along a path length of **90 cm** of **Ar** gas

δ rays are \sim rare, why to care?

Restricted energy loss

- **δ rays** may escape the detector if it is too thin
 - The average energy deposits are very often much smaller than predicted by **Bethe & Bloch**

If the energy transferred is restricted to $T \leq T_{\text{cut}} \leq T_{\text{max}} \rightarrow$ “restricted energy loss”

$$-\left. \frac{dE}{dx} \right|_{T < T_{\text{cut}}} = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{\text{cut}}}{I^2} - \frac{\beta^2}{2} \left(1 + \frac{T_{\text{cut}}}{T_{\text{max}}} \right) - \frac{\delta}{2} \right]$$

The difference between the **restricted energy loss** formula and the **B & B** is given by the contribution of the (escaping) **δ rays**

At very high energies ($\beta \gamma > 10^{S1}$, **$S1 \sim 2-5$**) the stopping power reaches a constant called “**Fermi plateau**”:

$$-\left(\frac{dE}{dx} \right) \left[\frac{\text{MeV}}{\text{g/cm}^2} \right] = 0.3071 \frac{z^2 Z}{2.A(g)} \ln \left(\frac{2m_e T_{\text{cut}}}{(h\nu_p)^2} \right)$$

$$h\nu_p =$$

$$\hbar\omega_p \text{ “Plasma energy”} =$$

$$\sqrt{\rho \langle Z/A \rangle} \times 28.816 \text{ eV} \\ (\rho \text{ in g cm}^{-3})$$

**$S1$, $h\nu_p$ = “density effect” parameters
depending on the material**

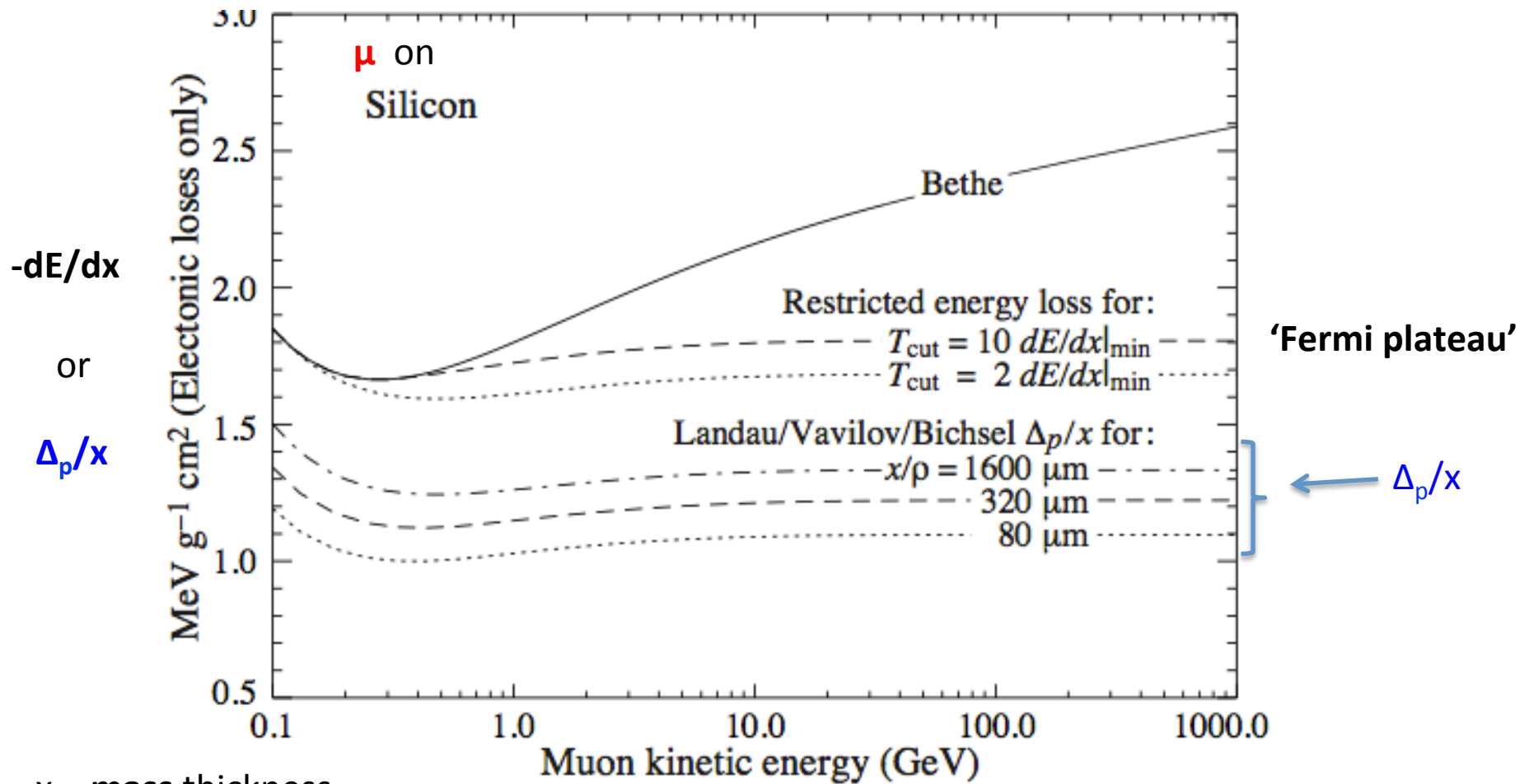
Density effect parameters

Table 2.1 Values of Z , Z/A , I , ρ in units of g/cm^3 , $h\nu_p$ and density-effect parameters S_0 , S_1 , a , md , and δ_0 for elemental substances.

El.	Z	Z/A	I eV	ρ	$h\nu_p$ eV	S_0	S_1	a	md	δ_0
He	2	0.500	41.8	$1.66 \cdot 10^{-4}$	0.26	2.202	3.612	0.134	5.835	0.00
Li	3	0.432	40.0	0.53	13.84	0.130	1.640	0.951	2.500	0.14
O	8	0.500	95.0	$1.33 \cdot 10^{-3}$	0.74	1.754	4.321	0.118	3.291	0.00
Ne	10	0.496	137.0	$8.36 \cdot 10^{-4}$	0.59	2.074	4.642	0.081	3.577	0.00
Al	13	0.482	166.0	2.70	32.86	0.171	3.013	0.080	3.635	0.12
Si	14	0.498	173.0	2.33	31.06	0.201	2.872	0.149	3.255	0.14
Ar	18	0.451	188.0	$1.66 \cdot 10^{-3}$	0.79	1.764	4.486	0.197	2.962	0.00
Fe	26	0.466	286.0	7.87	55.17	-0.001	3.153	0.147	2.963	0.12
Cu	29	0.456	322.0	8.96	58.27	-0.025	3.279	0.143	2.904	0.08
Ge	32	0.441	350.0	5.32	44.14	0.338	3.610	0.072	3.331	0.14
Kr	36	0.430	352.0	$3.48 \cdot 10^{-3}$	1.11	1.716	5.075	0.074	3.405	0.00
Ag	47	0.436	470.0	10.50	61.64	0.066	3.107	0.246	2.690	0.14
Xe	54	0.411	482.0	$5.49 \cdot 10^{-3}$	1.37	1.563	4.737	0.233	2.741	0.0
Ta	73	0.403	718.0	16.65	74.69	0.212	3.481	0.178	2.762	0.14
W	74	0.403	727.0	19.30	80.32	0.217	3.496	0.155	2.845	0.14
Au	79	0.401	790.0	19.32	80.22	0.202	3.698	0.098	3.110	0.14
Pb	82	0.396	823.0	11.35	61.07	0.378	3.807	0.094	3.161	0.14
U	92	0.387	890.0	18.95	77.99	0.226	3.372	0.197	2.817	0.14

Data are from [Sternheimer, Berger and Seltzer (1984)]

Restricted energy loss



Another important parameter is :

Δ_p = most probable energy loss (explained later)

-dE/dx Fluctuations → Energy straggling

- **Bethe-Bloch** formula describes **mean energy loss** per unit of length.

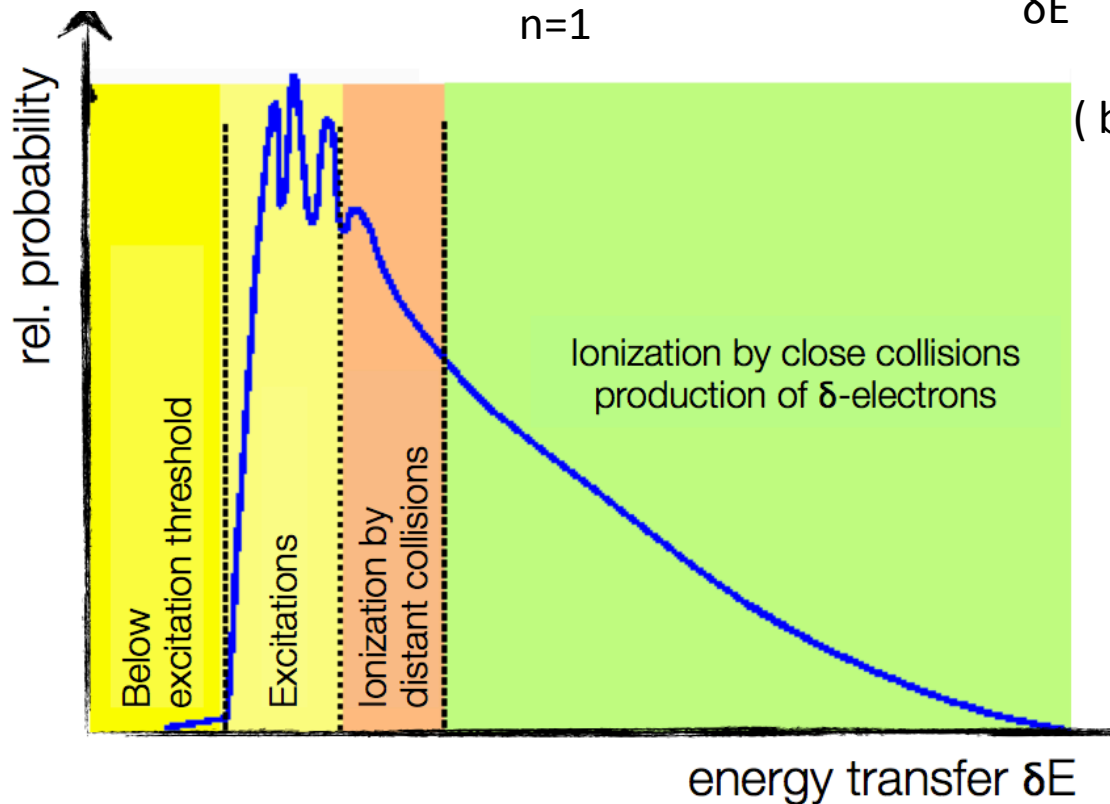
The actual energy loss ΔE in a material of thickness x is:

$$\Delta E = \sum_{n=1}^N \delta E_n(\beta)$$

- N number of collisions
- δE energy loss in a **single one collision**

δE stochastic fluctuations
→ **energy straggling**

(besides it depends on β of the particle)



Complex subject first studied by L. **Landau** and then by P.V. **Vavilov**

No general exact solutions, few approximate formulas help to estimate it. Introduce :

Significance parameter : K

$$K = \epsilon / T_{\max}$$

mean energy loss = $153.4 \frac{z^2}{\beta^2} \frac{Z}{A} \rho \delta x$ keV,

in thickness ρdx

NB: ΔE depends on thickness x

ΔE (energy loss) distribution

➤ Thin absorbers (K << 1):

- **Landau distribution.** Not analytic, useful approximation :

$$L(\lambda) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(\lambda + e^{-\lambda})\right) \quad \lambda = \frac{\Delta E - \Delta E_{MP}}{\epsilon}$$

ΔE_{MP} = **M**ost **P**robable value
 € See previous slide

$$\Delta E_{MP} = \Delta E_{Bethe} + \epsilon \left(\beta^2 + \ln\left(\frac{\epsilon}{T_{max}}\right) + 0.194 \right) MeV$$

- **Improved (I)** generalized energy loss distribution : convolution of a Landau with a Gaussian (takes better into account **distant collisions**)

$$f(\Delta E, x)_I = \frac{1}{\sqrt{2\pi}\sigma_I^2} \int_{-\infty}^{+\infty} L(\Delta E - \Delta E', x) \exp\left(-\frac{\Delta E'^2}{2\sigma_I^2}\right) d(\Delta E')$$

$$\sigma_I \sim (1/\beta^2) \ln \beta$$

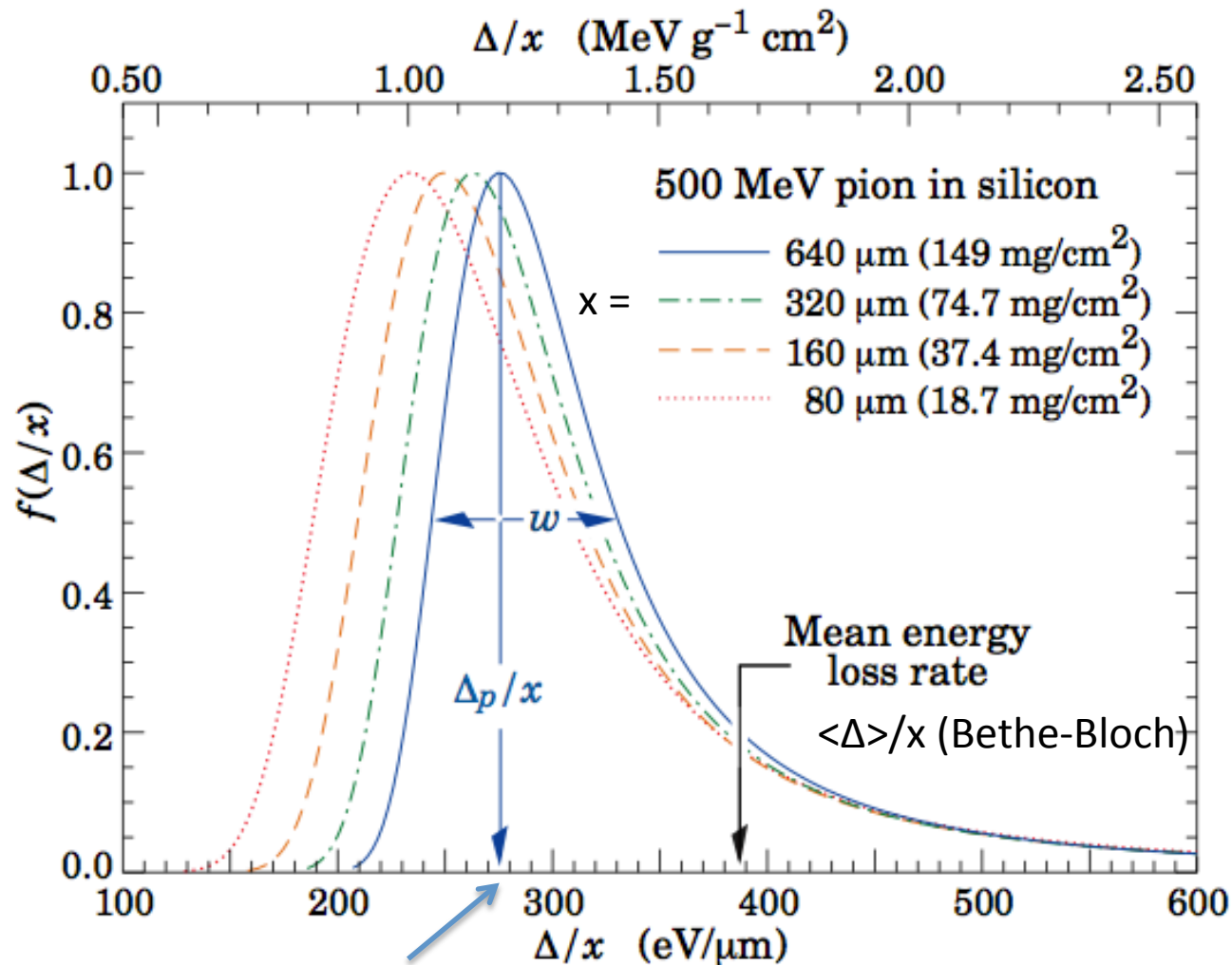
➤ Thick absorbers (K >> 1):

The distribution tends to a
Gaussian

$$f(\Delta E, x) \simeq \frac{1}{\sqrt{2\pi T_{max} \epsilon \left(1 - \frac{\beta^2}{2}\right)}} \exp\left(-\frac{(\Delta E - \Delta E_{Bethe})^2}{2 T_{max} \epsilon \left(1 - \frac{\beta^2}{2}\right)}\right)$$

x = thickness

Energy loss (Δ) distribution (straggling function)



Δ = energy loss
 x = thickness

Δ_p = most probable value

Important to describe the energy loss by a single particle

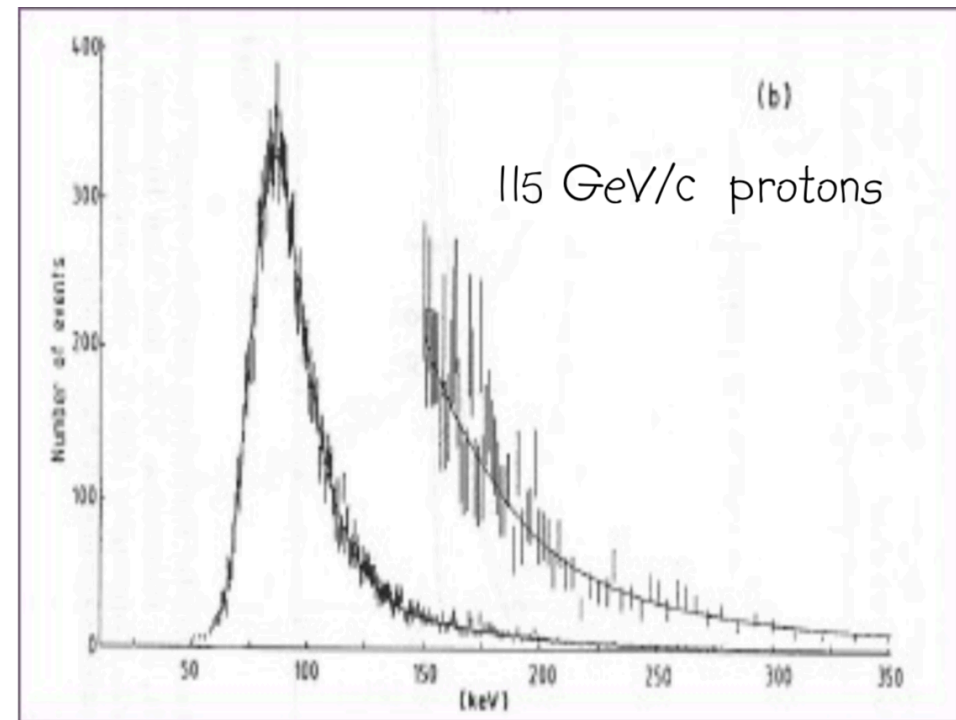
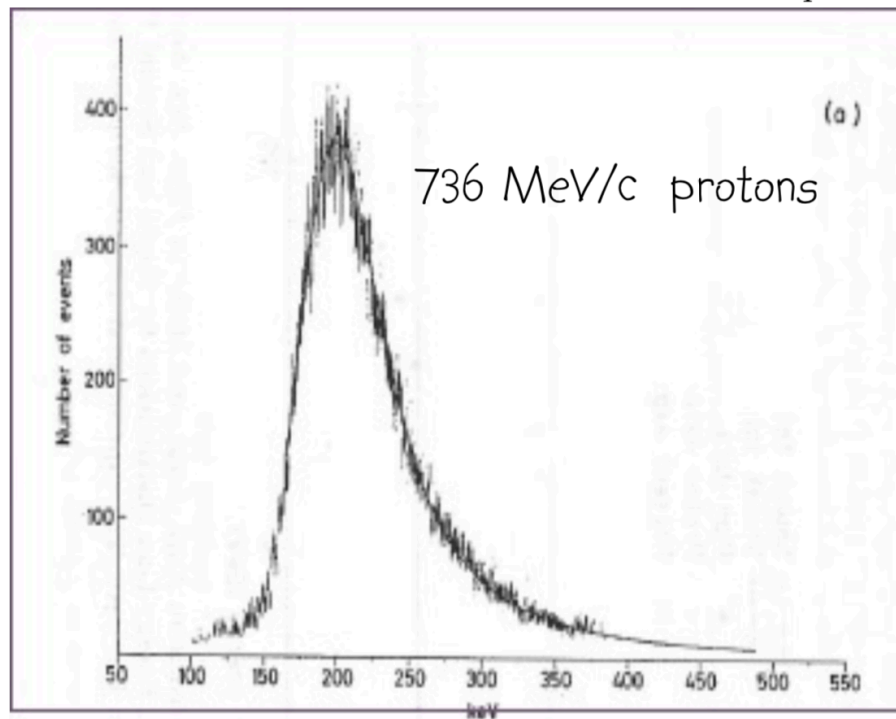


Fig. 2.10 Curves (a) and (b) (adapted and republished with permission from Hancock, S., James, F., Movchet, J., Rancoita, P.G. and Van Rossum, L., *Phys. Rev. A* **28**, 615 (1983); Copyright (1983) by the American Physical Society) show the energy loss spectra at 0.736 and 115 GeV/c of incoming particle momentum. Continuous curves are the complete fit to experimental data, i.e., the Landau straggling function folded over the Gaussian distribution taking into account distant collisions.

Stopping power of a compound medium

- For a compound of f elements:

$$-\frac{dE}{\rho dx} = \sum_1^f w_i \frac{dE}{\rho_i dx}$$

ρ_i = density of element i

$\frac{dE}{\rho_i dx}$ = stopping power of element i

w_i = mass fraction of element i

$$w_i = (N_i A_i) / A_m$$

N_i = number of atoms of element i

A_i = atomic weight of element i

A_m = molar mass of compound

$$A_m = \sum N_i A_i$$

- It is also possible to use effective quantities (empirical):

$$Z_{\text{eff}} = \sum N_i Z_i$$

$$A_{\text{eff}} = \sum N_i A_i$$

$$\ln I_{\text{eff}} = (\sum N_i Z_i \ln I_i) / Z_{\text{eff}}$$

$$\delta_{\text{eff}} = (\sum N_i Z_i \delta_i) / Z_{\text{eff}}$$

$$C_{\text{eff}} = \sum N_i C_i$$

Particle Range in matter : R

- Charged particles ionize, loose energy until their energy E_k is (almost) zero. The distance to this point is called the **range of the particle**: R

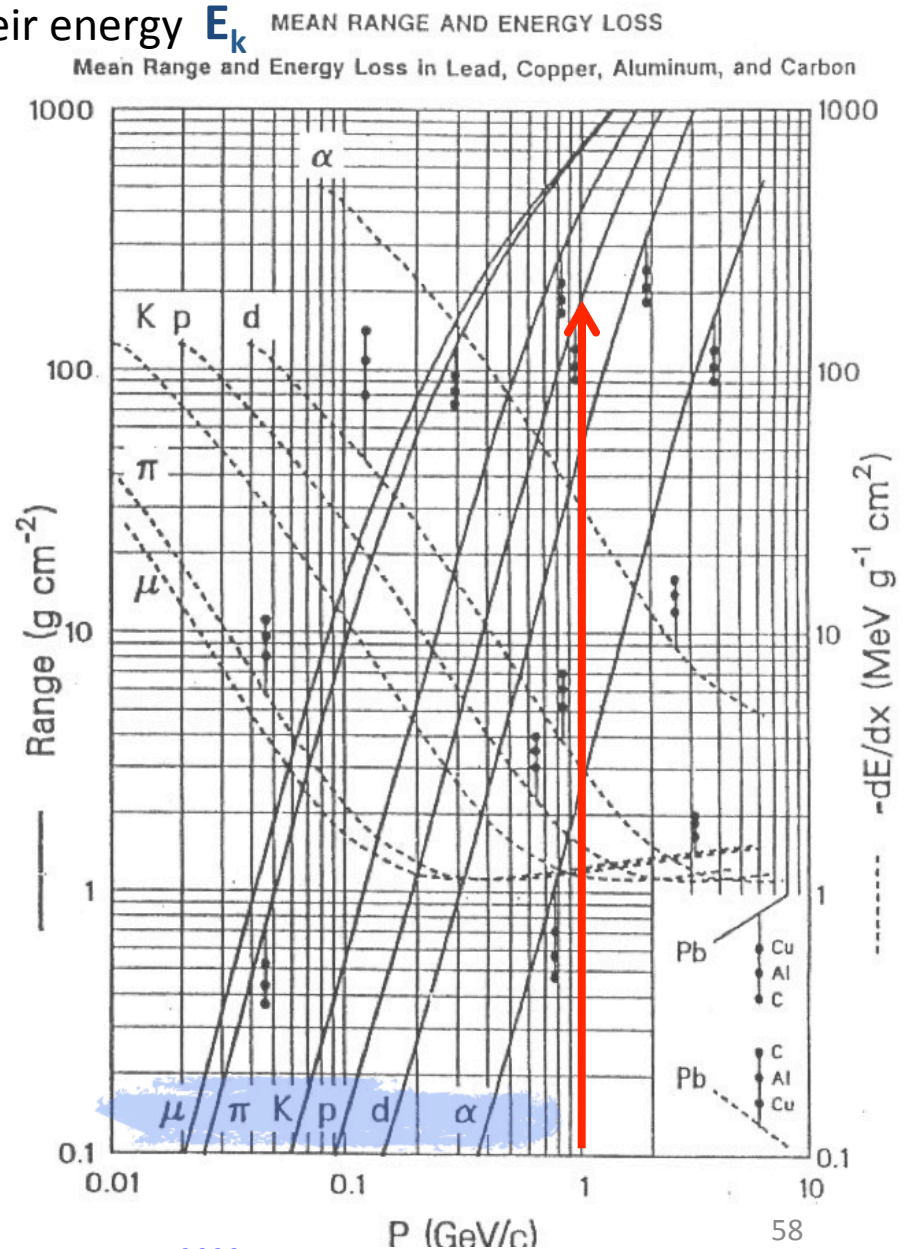
$$R(E_k) = \int_{E_k}^{\sim 0} \left(\frac{dx}{dE} \right) dE = \int_{E_k}^{\sim 0} \left(\frac{dE}{dx} \right)^{-1} dE$$

- This expression **ignores** the Coulomb **scattering** (producing a zig-zag trajectory of P_a)
- A **mean range** $\langle R \rangle$ is defined as the distance at which **half of the initial particles** have been stopped.
If $E_k > 1 \text{ MeV}$, $R \approx \langle R \rangle$

Proton with $p = 1 \text{ GeV}$ on target

lead ($\rho = 11.34 \text{ g/cm}^3$)

$$R = 200/11.34 \sim 20 \text{ cm}$$



Particle Range in matter : R

- R may be used to evaluate the particle energy

$$R \propto E_k^b \quad b \sim 1.75 \quad \text{for } E_k < \text{minimum ionisation}$$

- Scaling laws

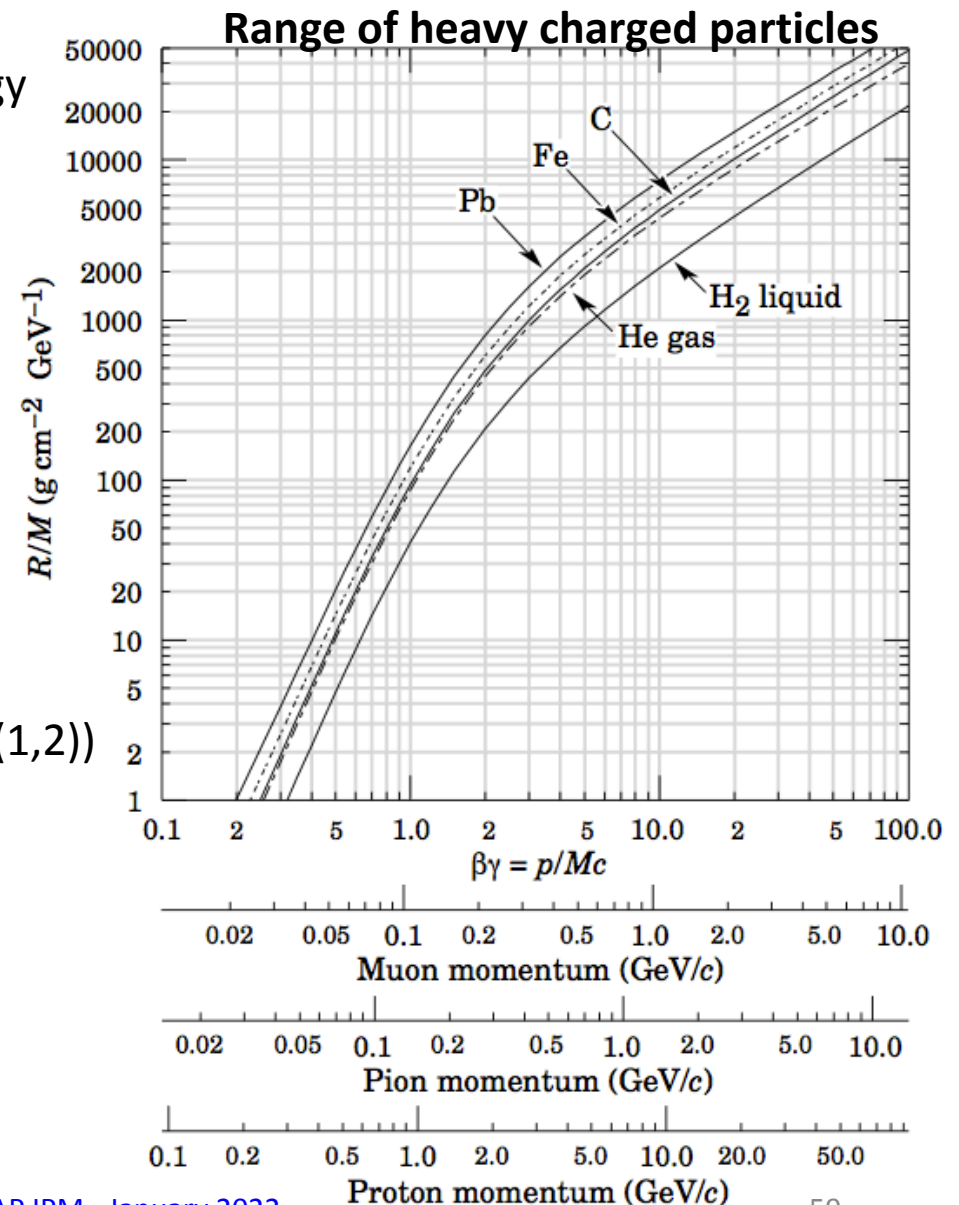
- Particle 1 and 2, in same material

$$R_2(E_{k2}) \propto \frac{M_2 z_1^2}{M_1 z_2^2} R_1(E_{k1} * M_1/M_2)$$

- Same particle in two different materials(1,2))

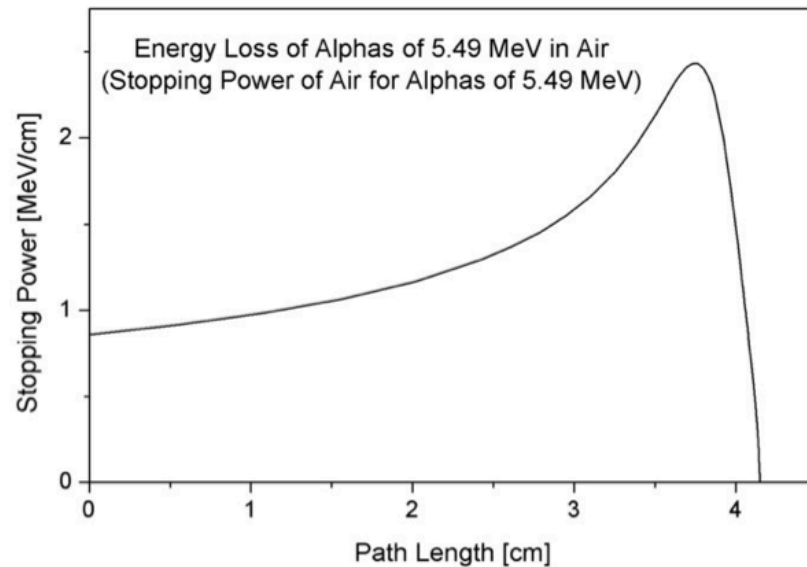
$$\frac{R_1}{R_2} = \frac{\rho_2}{\rho_1} \frac{\sqrt{A_1}}{\sqrt{A_2}} \quad R(\text{cm})$$

- Compounds: $R_{\text{compound}} = A_m / \sum (N_i A_i / R_i)$



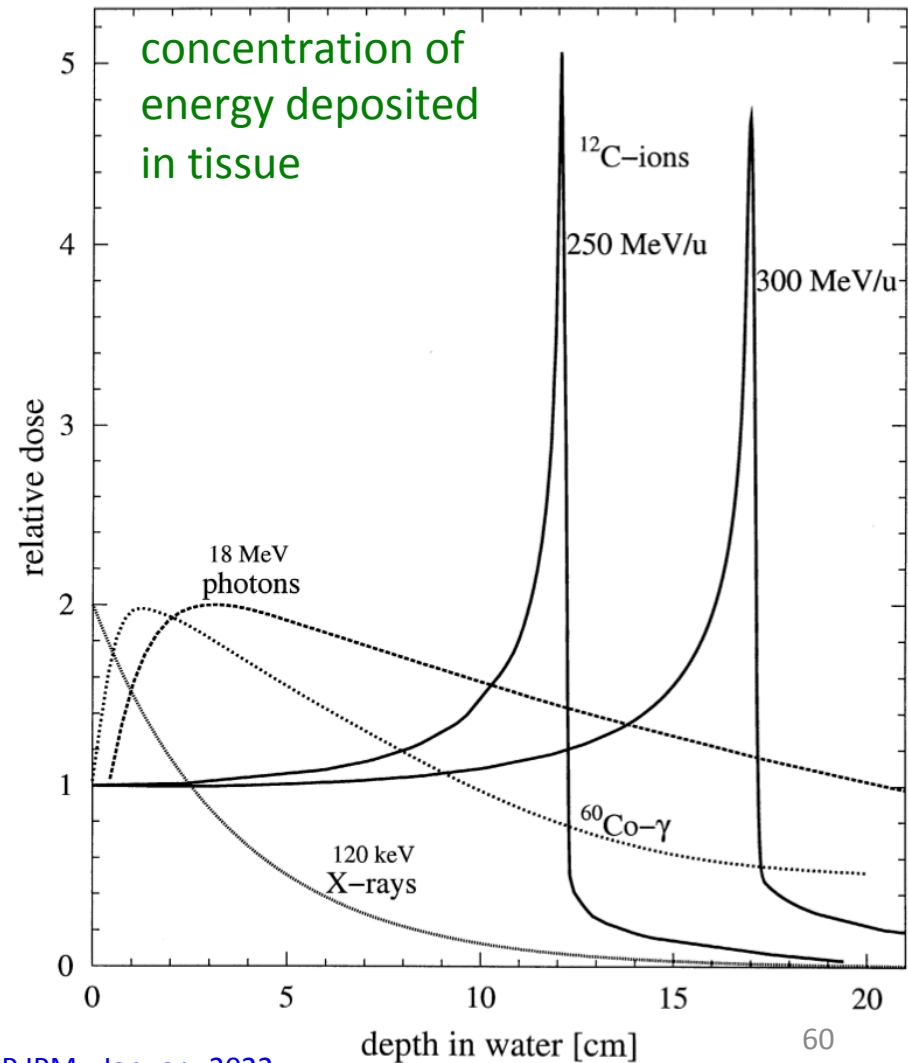
Mean Particle Range

- If the medium is thick enough, a particle will progressively decelerate while increasing its stopping power ($\beta^{-5/3}$) until it reaches a maximum (called the **Bragg peak**).

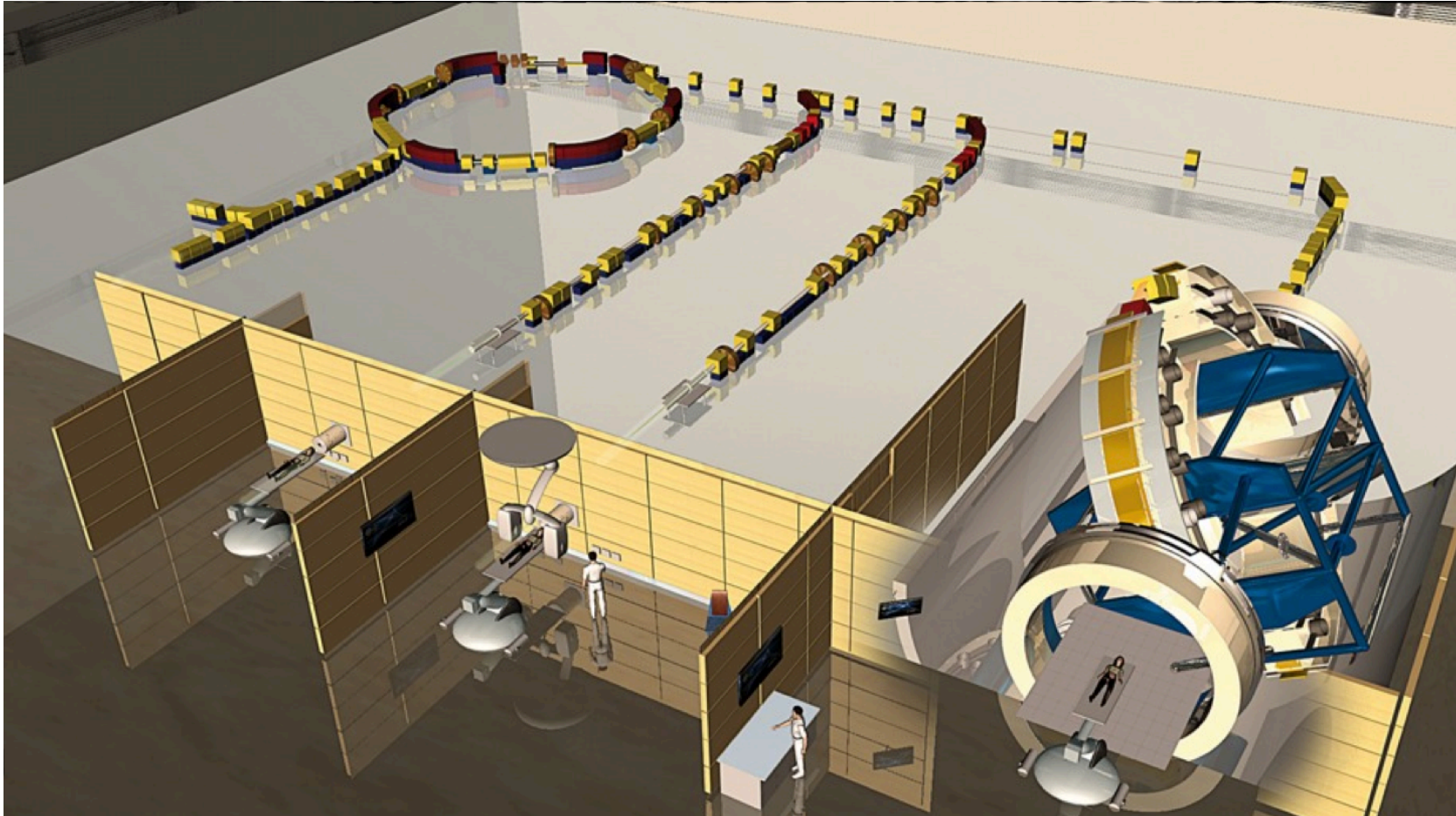


- Possibility to precisely deposit dose at well defined depth dependent on E_{beam} (Remember also dependence on z^2)

**Applications:
Tumor therapy**



Heidelberg Ion-Beam Therapy Center (HIT)



~ 50 centers around the world

Stopping power of e^\pm by ionization and excitation in matter

For e^\pm the **Bethe-Bloch formula** must be **modified** since:

- 1) the change in direction of the particle was neglected; for e^\pm this approximation is not valid (scattering on particle with same mass)
- 2) Pauli Principle : the incoming and outgoing particles are the identical particles

$$-\frac{dE}{dx} = 2 \pi N_A r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \frac{\tau^2 (\tau+2)}{2(1/m_e c^2)^2} + F(\tau) - \delta - 2 \frac{C}{Z} \right]$$

For electrons: $F(\tau) = 1 - \beta^2 + \frac{(\tau^2/8) - (2\tau+1) \ln 2}{(\tau+1)^2}$ $\tau = \frac{1}{\sqrt{1-\beta^2}} - 1 = E_k/(mc^2)$

For positrons : $F(\tau) = 2 \ln 2 - \frac{\beta^2}{12} \left(23 + \frac{14}{\tau+2} + \frac{10}{(\tau+2)^2} + \frac{4}{(\tau+2)^3} \right)$

e^\pm lose more energy wrt heavier particles since they interact with particles of the same mass

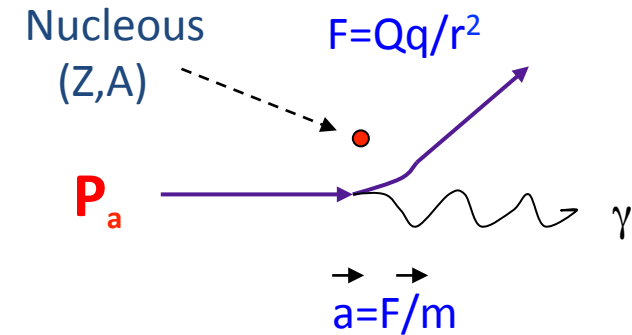
▪ When a positron comes to a rest it annihilates : $e^+ + e^- \rightarrow \gamma \gamma$ of 511 keV each

▪ A positron may also undergo an annihilation in flight:
with a cross section :

$$\sigma(Z, E) = \frac{Z \pi r_e^2}{\gamma+1} \left[\frac{\gamma^2+4\gamma+1}{\gamma^2-1} \ln(\gamma + \sqrt{\gamma^2-1}) - \frac{\gamma+3}{\sqrt{\gamma^2-1}} \right]$$

2. Bremsstrahlung. Mean radiative energy loss.

- An accelerated (or decelerated) charged particle (P_a) emits electromagnetic radiation (γ)
- **Very fundamental process !**
- Here the process takes place in **the Coulomb field of the nucleus**. The amount of **screening** from the atomic electrons plays an important role
- Relevant in particular for e^\pm due to their small mass



$N = \text{atoms/cm}^3$ ($N = \rho N_A/A$)
 $Z = \text{atomic number}$

$E_0 = \text{Initial energy of particle } P_a$

$\nu_0 = E_0/h$

$h\nu = \text{energy of emitted } \gamma$

$\frac{d\sigma}{d\nu} = \text{Differential cross section of the bremsstrahlung process}$

If $P_a = \text{electron}$:

If $E_0 \gg m_e c^2$ et $E_0 \ll 137 m_e c^2 / Z^{1/3}$ $\phi(Z^2) = 4\alpha Z^2 r_e^2 \ln(2E_0/m_e c^2 - 1/3 - f(Z))$ $\alpha = 1/137$

If $E_0 \gg 137 m_e c^2 / Z^{1/3}$ $\phi(Z^2) = 4\alpha Z^2 r_e^2 \ln(183 Z^{-1/3} - 1/18 - f(Z))$
 $r_e = \alpha/(m_e c^2)$

See W.R. Leo

$f(Z) = \text{Coulomb correction}$

2. Bremsstrahlung – Energy Spectrum

LPM = Landau–Pomeranchuk–Migdal cross section.

Normalized bremsstrahlung cross section vs $y (= k / E_0)$ where $k = E_\gamma$
 $\rightarrow y = \text{fraction}$ of the electron energy (E_0) transferred to the radiated γ

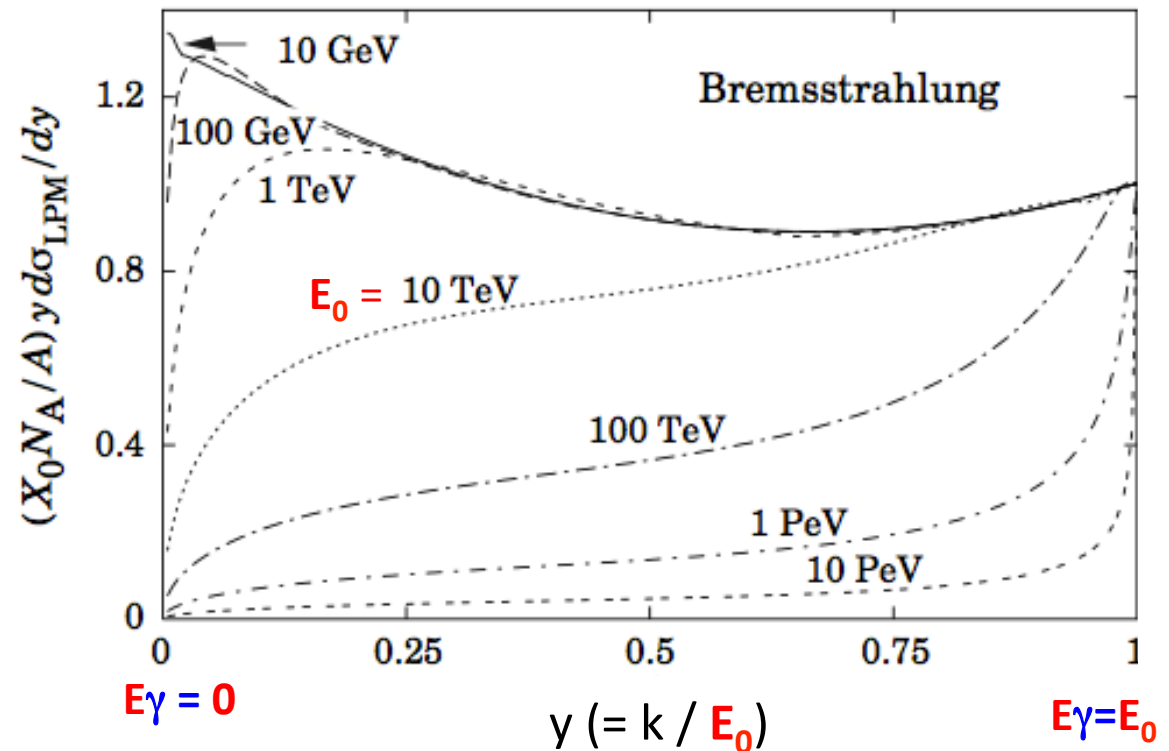
$$K d\sigma/dk = \nu d\sigma/d\nu \quad (\text{for given } E_0)$$

For high energy E_0 (small y):

$$\frac{d\sigma}{dk} = \left(\frac{1}{k} \right) \frac{A}{X_0 N_A} \left(\frac{4}{3} - \frac{4}{3}y + y^2 \right)$$

Formula accurate except for
 $y=1$ and $y=0$

see PDG for further details



LPM = Landau–Pomeranchuk–Migdal cross section.

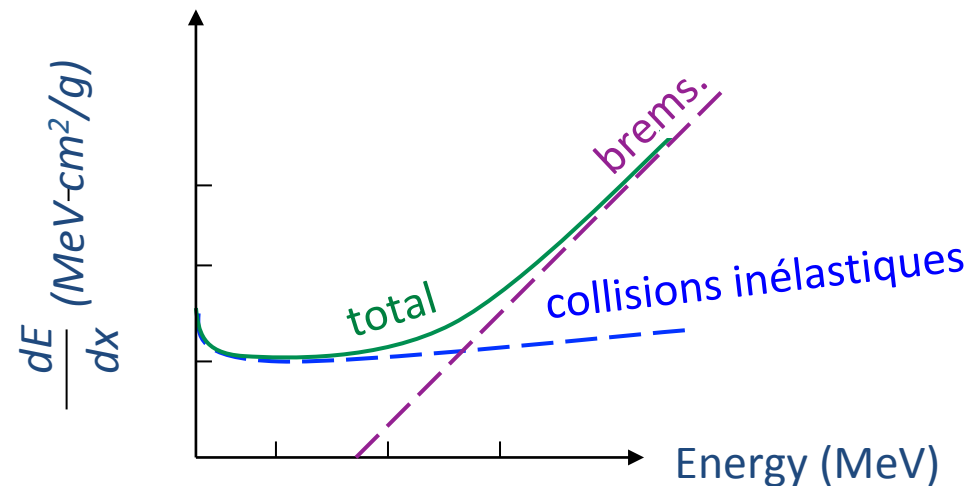
Bremsstrahlung. Mean radiative energy loss

For a particle of charge **z** and mass **m**:

$$\frac{dE}{dx}_{\text{brem}}(z, m) = \left(\frac{m_e}{m}\right)^2 z^2 \frac{dE}{dx}_{\text{brem}}(e^-)$$

- Relevant in particular for **e^\pm** due to their small mass
- Shown so far is the mean energy loss due interaction **in the field of the nucleus**
- Contribution also from radiation which arises in the fields of the **atomic electrons**.
- Cross section are given by the above formula but replacing **Z^2** with **Z** .
- The overall contribution can be approximated by replacing **Z^2** by **$Z(Z+1)$** in all the above formulas

Comparison -dE/dx Bremsstrahlung vs ionisation/excitation



- The **average energy loss** due to **ionisation/excitation** increases with the **log of the energy** and linearly with **Z** :
- The **average energy loss** due to **brem** increases **linearly** with **energy** and **linearly** with **E** and **Z²** :

$$\left. \frac{dE}{dx} \right)_{\text{ion./excit.}} \propto Z/A, \quad 1/\beta^2 \ln E$$

$$\left. \frac{dE}{dx} \right)_{\text{brem}} \propto Z^2/A, \quad E, \quad 1/m^2$$

Energy loss due to **brem** is a discrete process: results from the emission of $\sim 1 \gamma$ ou 2γ
 --> fluctuations

Critical energy (E_c)

- The relevance of **bremstrahlung** wrt **ionisation** depends on the **critical energy (E_c)** of the particle P_a in the material
- The **critical energy (E_c)** is the energy at which the **ionization stopping power** is **equal** to the mean **radiative energy loss**.

$$\begin{array}{ccc}
 @ E = E_c & @ E > E_c & @ E < E_c \\
 \left(\frac{dE}{dx} \right)_{\text{brem.}} = \left(\frac{dE}{dx} \right)_{\text{ion}} & \left(\frac{dE}{dx} \right)_{\text{brem.}} > \left(\frac{dE}{dx} \right)_{\text{ion}} & \left(\frac{dE}{dx} \right)_{\text{brem.}} < \left(\frac{dE}{dx} \right)_{\text{ion}}
 \end{array}$$

For e^\pm in :

Pb	$E_c = 7.4 \text{ MeV}$
Cu	$E_c = 24.8 \text{ MeV}$
Fe	$E_c = 27.4 \text{ MeV}$
Al	$E_c = 51 \text{ MeV}$

For liquid and solids: $E_c \sim 610 \text{ MeV}/(Z+1.24)$

For gas $E_c \sim 710 \text{ MeV}/(Z + 0.92)$

For other particles E_c would scale according to the square of their masses with respect to the electron mass.

Radiation lenght X_0

For $E \gg E_c$ $\left(\frac{dE}{dx} \right)_{\text{tot}} \approx \left(\frac{dE}{dx} \right)_{\text{brem.}}$ $N = \text{atoms/cm}^3$

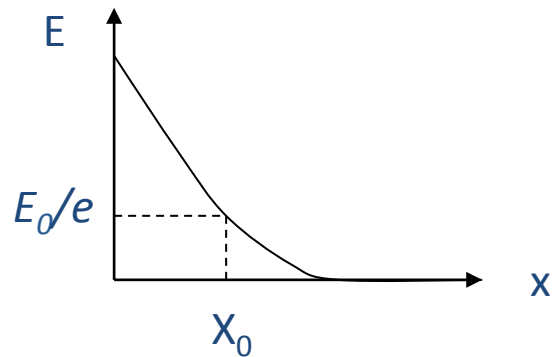
$$- \frac{dE}{dx} = N E_0 \phi$$

$$\Rightarrow - \frac{dE}{E} = N \phi dx$$



$$E = E_0 e^{-x/X_0}$$

$$X_0 \equiv 1/(N \phi)$$



$X_0 \equiv 1/(N \phi) \equiv$ radiation lenght \equiv
distance after which an high energy electron has
lost $1/e$ of his energy by radiation

Mean radiated energy of an electron over
a path x in the medium:

$$E_{\text{brem.}}(e^-) = E (1 - e^{-x/X_0})$$

Radiation lenght X_0

$$X_0 \left\{ \begin{array}{l} \text{Pb} = 0.56 \text{ cm} \\ \text{Fe} = 1.76 \text{ cm} \\ \text{Air} = 30050 \text{ cm} \end{array} \right.$$

$$X'_0 \equiv X_0 \rho \qquad X'_0 = \frac{716.4 \text{ g cm}^{-2} A}{Z(Z+1) \ln(287/\sqrt{Z})}$$

Expressing the mean radiated energy in unit of X'_0

→ The probability of the process becomes less dependent on the material

Pour un composé de N éléments :

$$\frac{1}{X_0} = \sum_i w_i \frac{1}{X_{0i}}$$

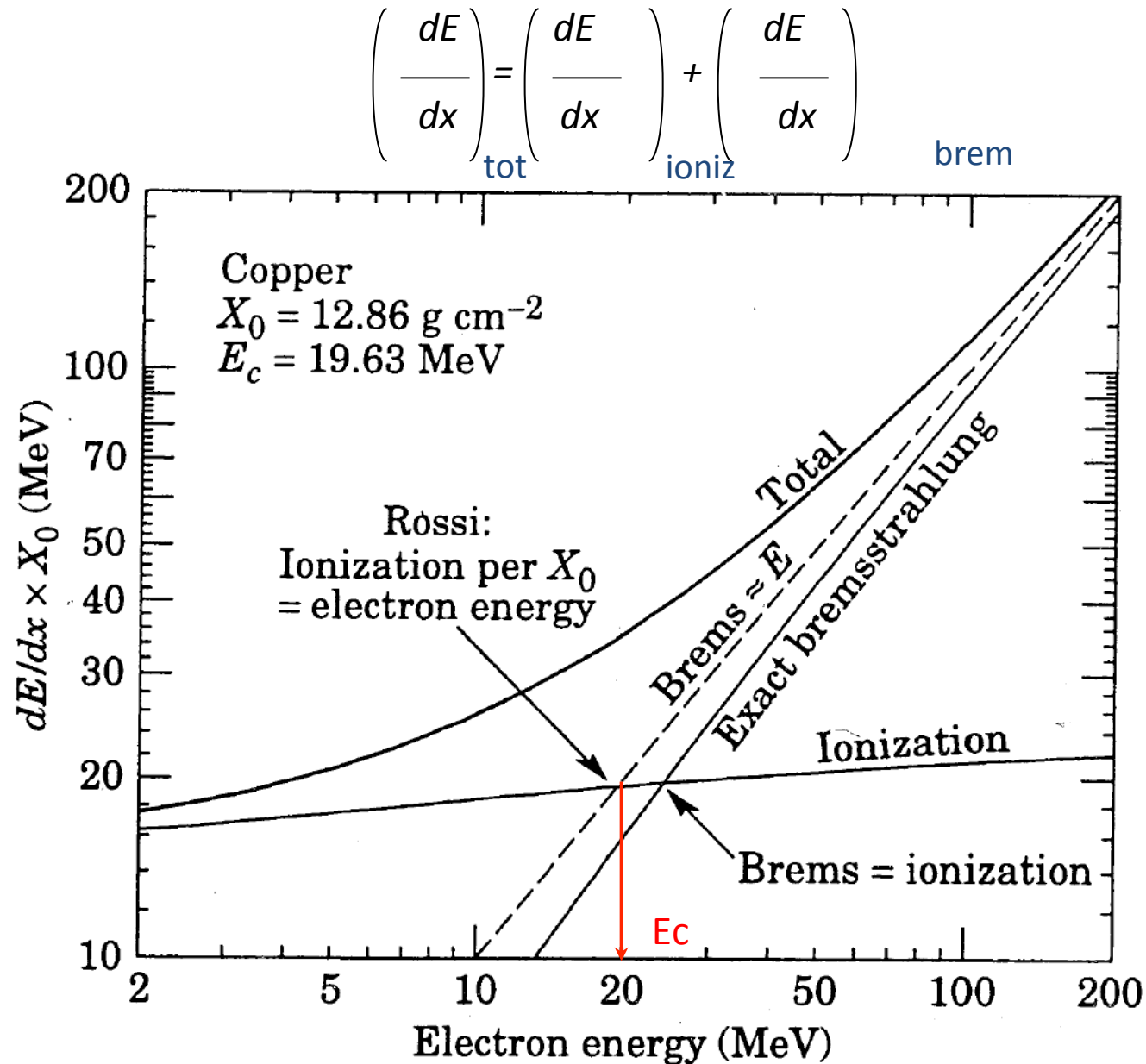
w_i = fraction in mass of element i

X_{0i} = radiation lenght of element i

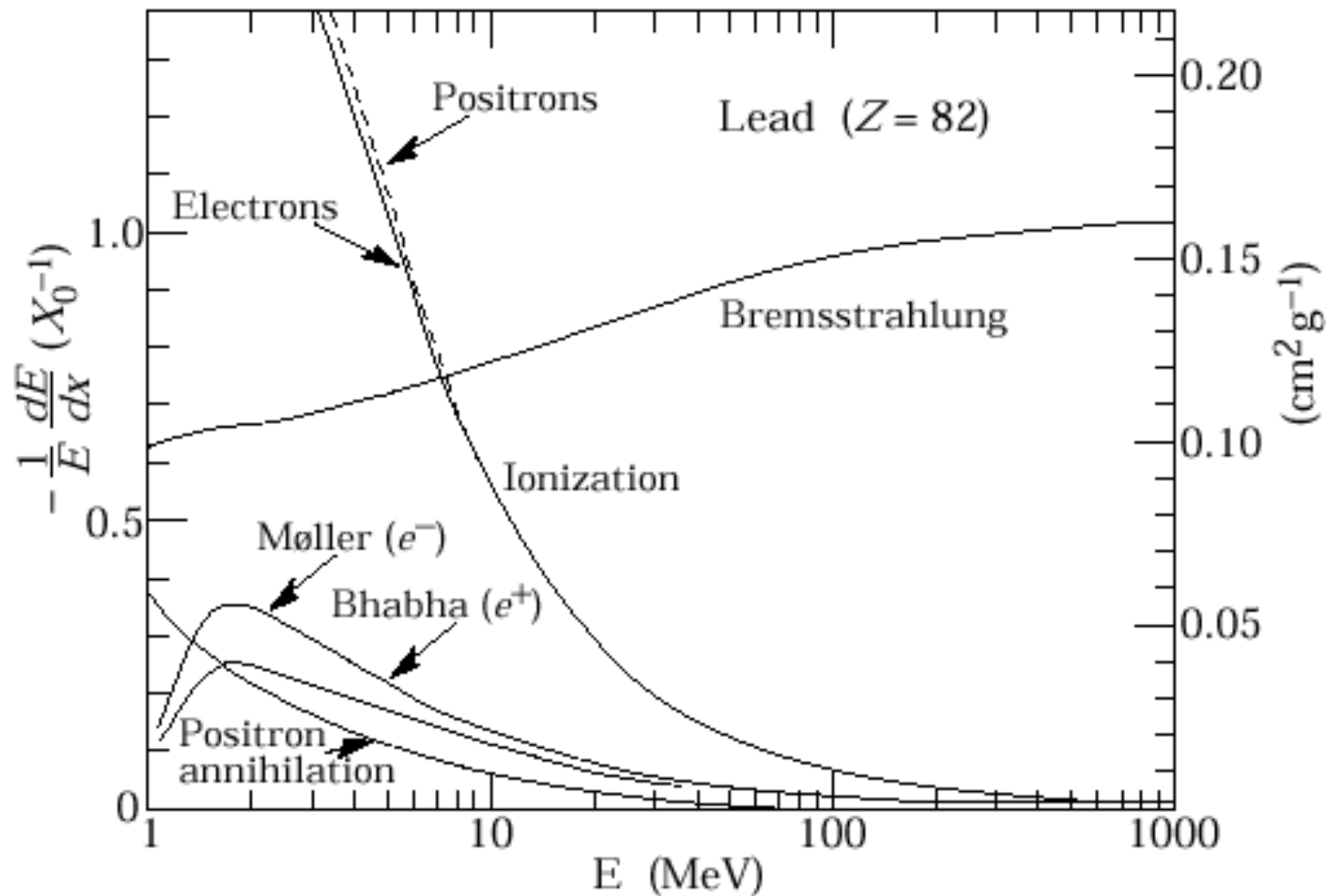
For electrons

<i>medium</i>	<i>Z</i>	<i>A</i>	<i>X₀ (g/cm²)</i>	<i>X₀ (cm)</i>	<i>E_c (MeV)</i>
hydrogen	1	1.01	63	700000	350
helium	2	4	94	530000	250
lithium	3	6.94	83	156	180
carbon	6	12.01	43	18.8	90
nitrogen	7	14.01	38	30500	85
oxygen	8	16	34	24000	75
aluminium	13	26.98	24	8.9	40
silicon	14	28.09	22	9.4	39
iron	26	55.85	13.9	1.76	20.7
copper	29	63.55	12.9	1.43	18.8
silver	47	109.9	9.3	0.89	11.9
tungsten	74	183.9	6.8	0.35	8
lead	82	207.2	6.4	0.56	7.4
air	7.3	14.4	37	30000	84
silica (SiO ₂)	11.2	21.7	27	12	57
water	7.5	14.2	36	36	83

Electron interactions in copper : higher energies



Interactions of electrons in lead: a more complete picture

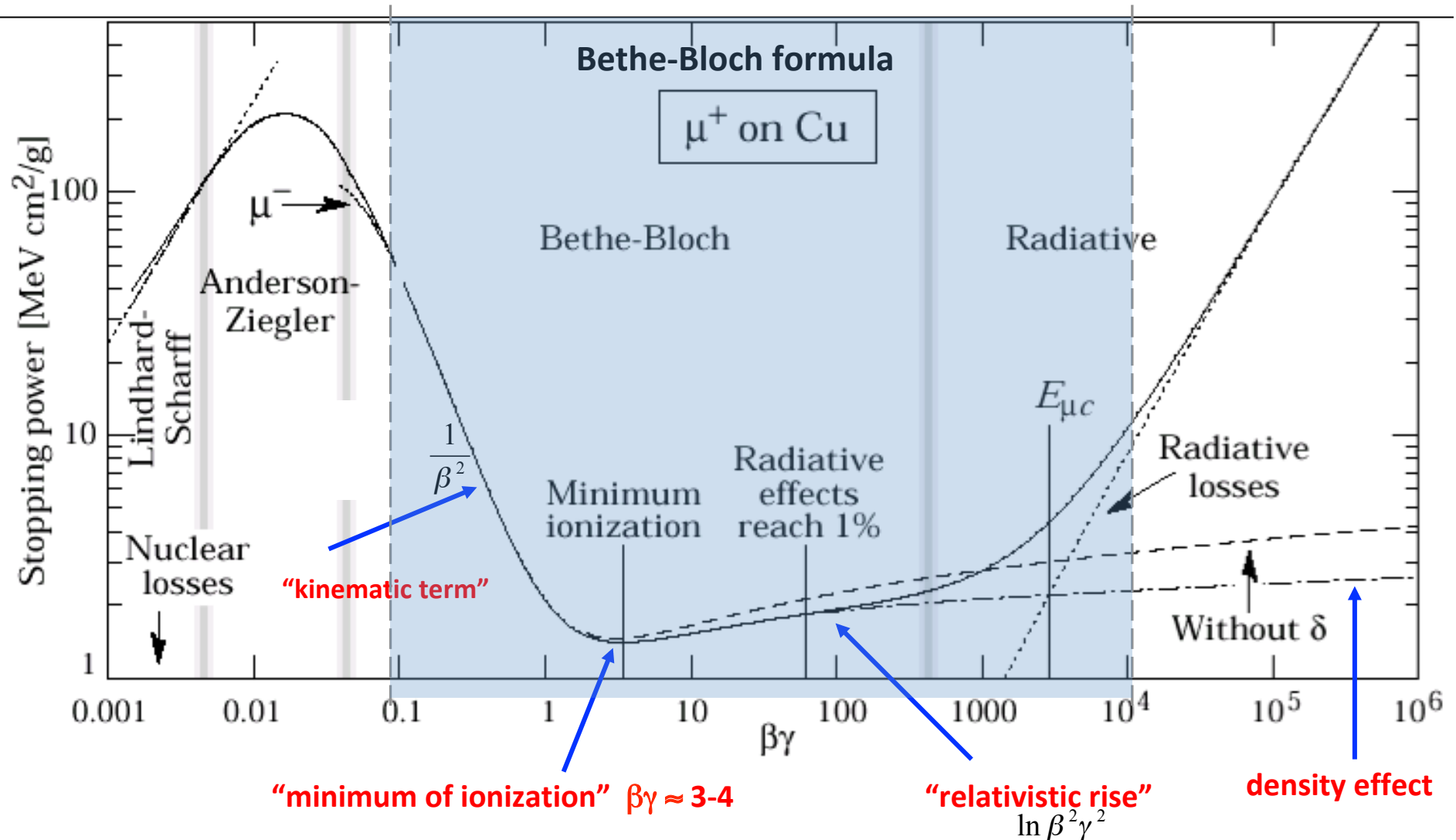


Moller scattering $e^- e^- \rightarrow e^- e^-$

Bhabha scattering $e^+ e^- \rightarrow e^+ e^-$

Positron annihilation $e^+ e^- \rightarrow 2\gamma$

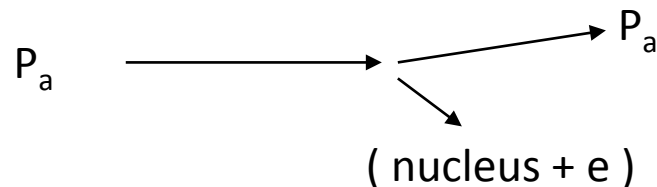
Total energy lost by a muon (μ) per unit length in copper



At very low energy the **Bethe-Bloch** formula is not valid since the speed of the interacting particle is \sim speed of electrons in the atoms. For $\beta\gamma < 0.05$ there are only phenomenological fitting formulae

3. Elastic scattering with nuclei

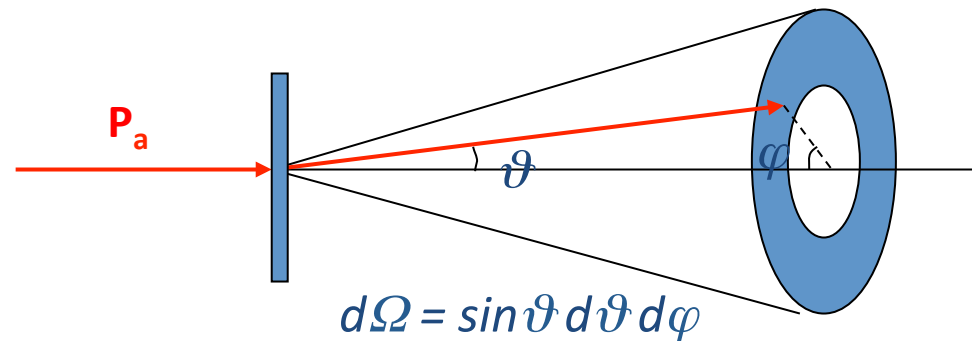
A **charged particle** P_a traversing a medium is deflected many times (mainly) by small-angles essentially due to **Coulomb scattering** in the electromagnetic field of **the nuclei**.



The **energy loss** (or transferred to the nuclei) is small ($m_{\text{nucleus}} \gg m_{P_a}$) therefore **neglected**, The change of direction is important.

- A **single collision** is described by the Rutherford formula (ignores spin and screening effects)

$$\frac{d\sigma}{d\Omega} = 4zZr_e^2 \left(\frac{m_e c}{\beta p} \right)^2 \frac{1}{\sin^4 \theta/2}$$



- **Multiple scattering:** $N_{\text{collisions}} > 20$

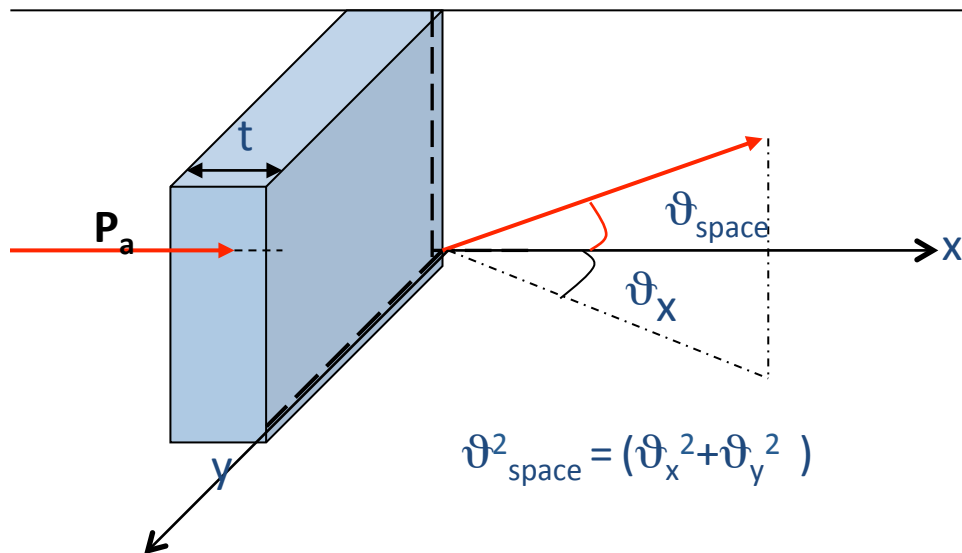
The particle follows a zig-zag trajectory

Deflection angles are described by the **Molière theory**



H. A. Bethe" "Molière's Theory of Multiple Scattering" Phys. Rev. **89**, 1256 – Published March 1953

3. Multiple scattering through small angles ($< \sim 10^\circ$)



For small scattering angles, the distribution of $\vartheta_x \approx$ Gaussian

$$\text{prob}(\vartheta_x) d\vartheta_x = \frac{1}{\sqrt{2\pi} \sigma_0} \exp(-\vartheta_x^2 / (2 \sigma_0^2)) d\vartheta_x$$

(similar for ϑ_y and $\vartheta_{\text{space}}^2 = \vartheta_x^2 + \vartheta_y^2$)

Where:

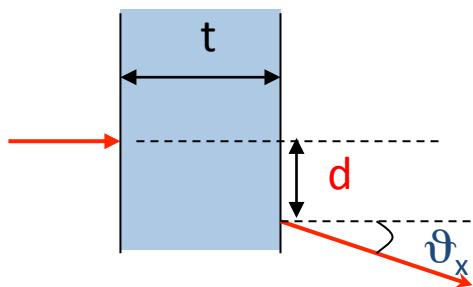
$$\sigma_0 = \frac{13.6 \text{ MeV}}{\beta p} |z| \sqrt{\frac{t}{X_0} \left(1 + 0.038 \ln \frac{t}{X_0} \right)}$$

t = medium thickness

ρ = matter density

X_0 = radiation length

β, p = speed/c and momentum of the incident particle



Particles emerging from the the medium are also laterally shifted :

$$d^{\text{rms}} = \frac{1}{\sqrt{3}} t \sigma_0$$

Momentum resolution

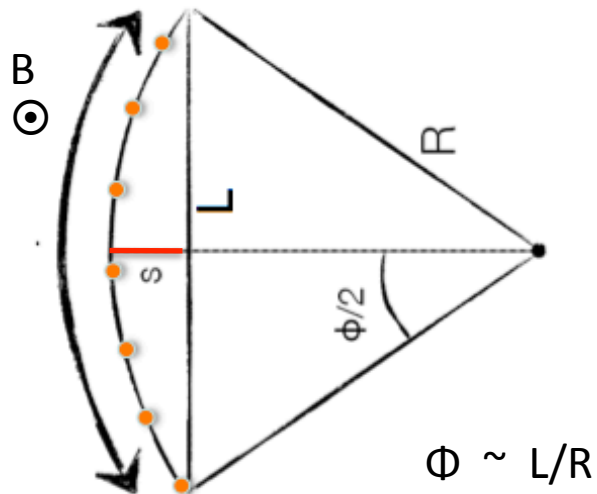
Multiple scattering impacts the measurement of the momentum

Assume $\mathbf{B} \perp \mathbf{v}$ particle:

$$Mv^2/R = q |\vec{v} \wedge \vec{B}|$$

$$\mathbf{p} = B \mathbf{R} \quad (q = 1)$$

The momentum is measured from R , which is obtained from \mathbf{L} and \mathbf{s}



$$s = R - R \cos \frac{\phi}{2} \approx R \frac{\phi^2}{8}$$

$$s = R \frac{L^2}{8R^2} = \frac{L^2}{8R}$$

$$R = \frac{L^2}{8s}$$

The precision on the momentum will depend on the precision on the track reconstruction and also on the **multiple scattering that the particle undergoes**

Momentum resolution

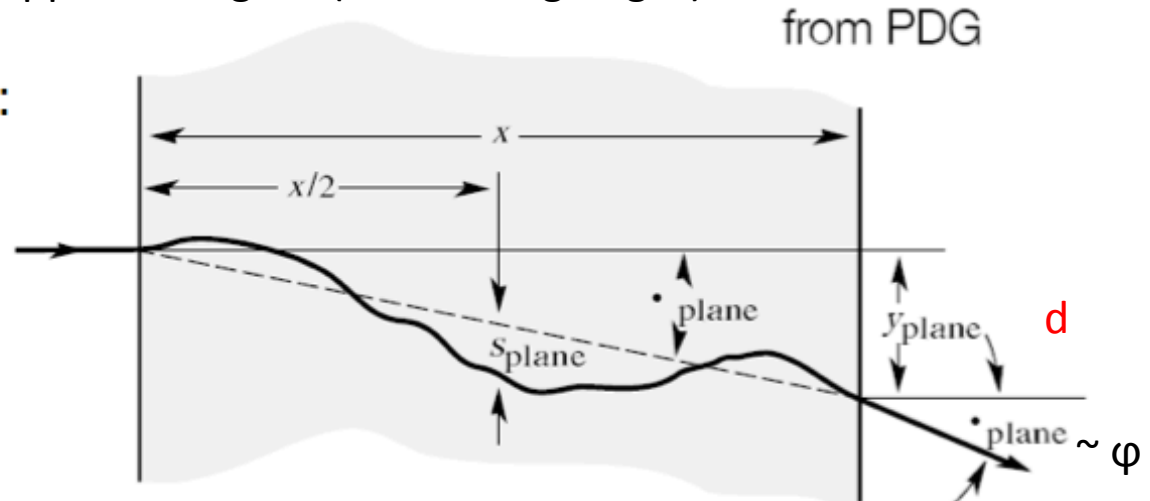
$$p = q B R \sim B L^2$$

Multiple scattering introduces an apparent sagitta (0.5% in Argon gas)

Multiple scattering contribution:

$z=1$

$$\sigma_\phi \approx \frac{14 \text{ MeV}/c}{p} \sqrt{\frac{L}{X_0}}$$



$$p = q B R \quad R = \frac{L}{\phi}$$

At small momenta this limits resolution of momentum measurement ...

$$\frac{\sigma_\phi}{\phi} = \frac{14 \text{ MeV}/c}{p} \sqrt{\frac{L}{X_0}} \cdot \frac{R}{L} = \frac{14 \text{ MeV}/c}{p} \sqrt{\frac{1}{L X_0}} \cdot \frac{p}{e B} \sim \frac{1}{\sqrt{L X_0 B}}$$

momentum independent

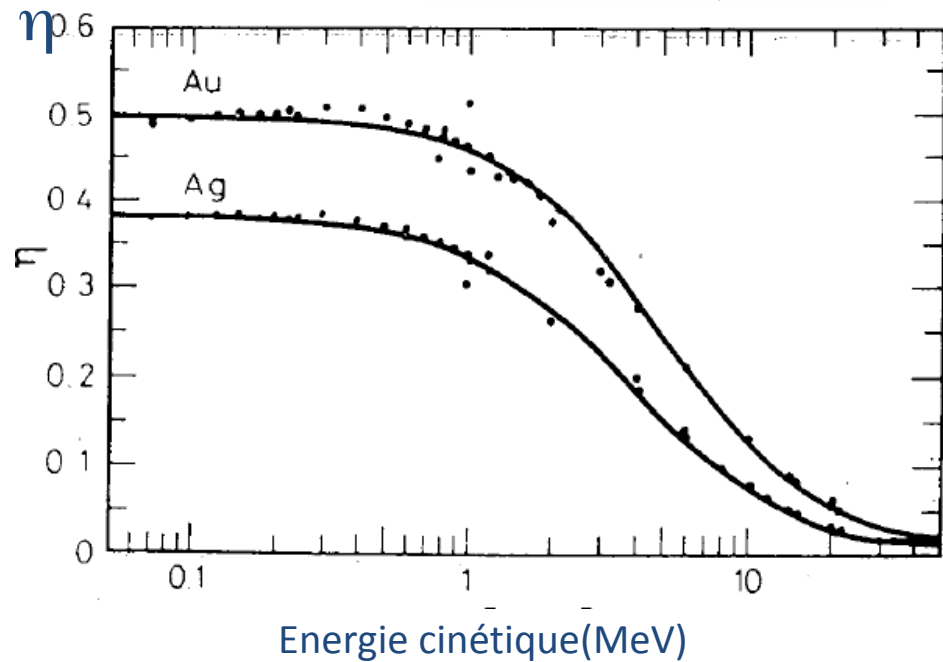
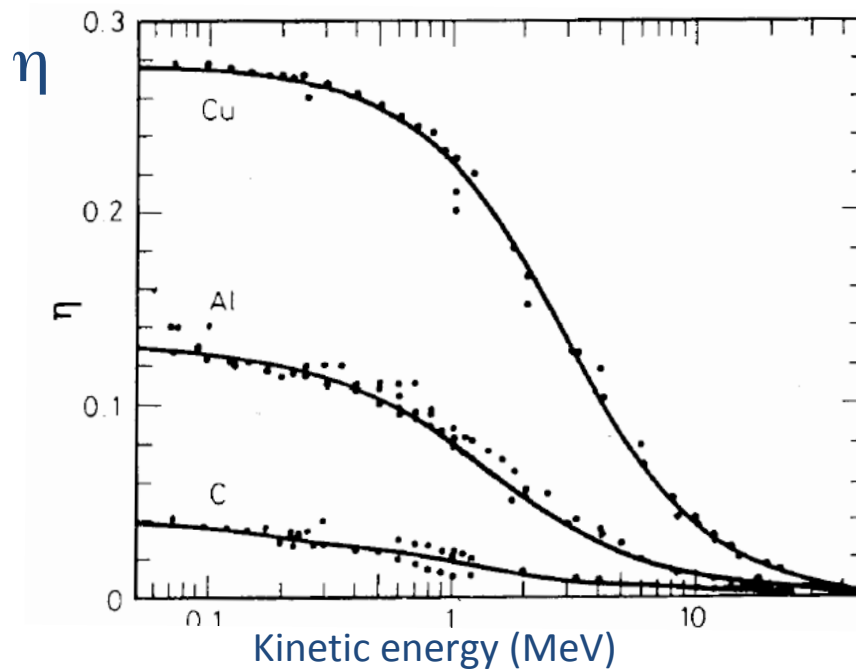
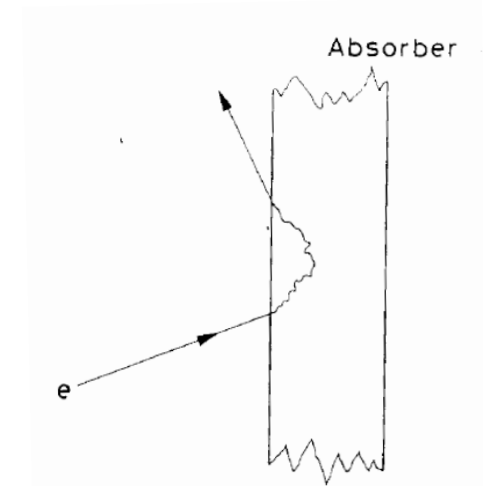
$$\left(\frac{\sigma_p}{p}\right)^2 = \text{const} \cdot \left(\frac{p}{B L^2}\right)^2 + \text{const} \cdot \left(\frac{1}{B \sqrt{L X_0}}\right)^2$$

Due to multiple scattering

3. Back-scattering of electrons

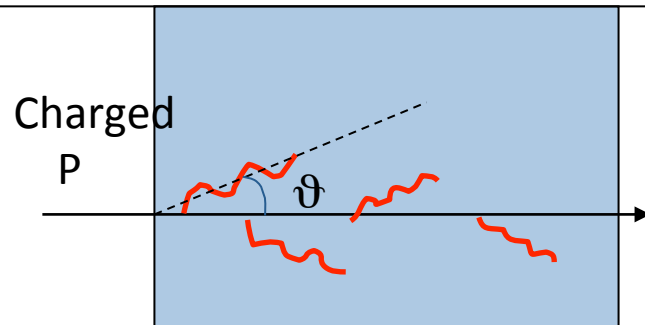
- * increases with Z of the material
- * is relevant for low energy electrons

$$\eta = \frac{\text{Number of backscattered electrons}}{\text{Number of incident electrons}}$$



Effect to take into account when building a detector for low energy electrons ($< \sim 10$ MeV)

4. Cherenkov light emission

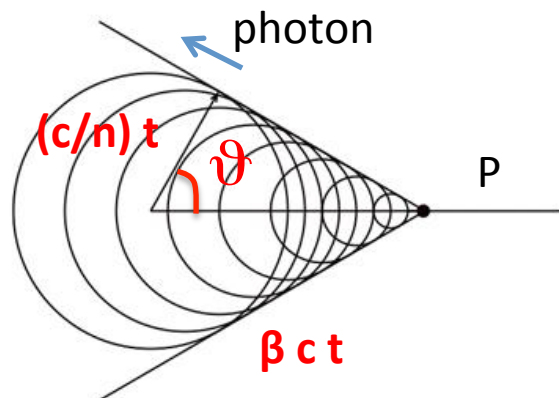


Radiation emitted when a charged particle crosses a medium at a speed $>$ than the **phase velocity of light** in the medium

$$v_{\text{particle}} > c/n$$

n = refracting index

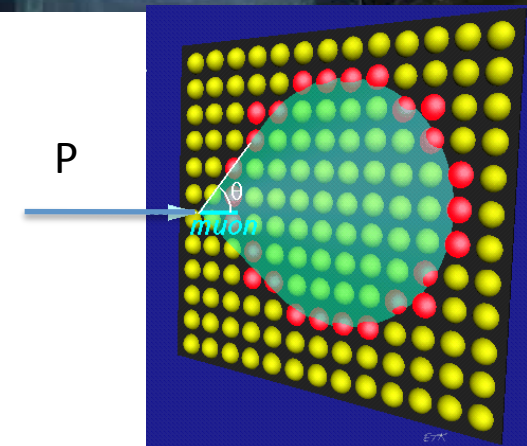
- The medium is **electrically polarized** by the particle's electric field (oscillating dipoles)
- When the particle travels fast this effect is left in the **wake** of the particle.
- The emitted energy radiates as a **coherent shockwave**



$$\cos \vartheta = \frac{1}{\beta n}$$



Cherenkov emission (e) TRIGA reactor



4. Cerenkov light emission

- Number of photons N emitted per unit path length and unit of wave length

$$\frac{dN}{dx d\lambda} = 2\pi\alpha \frac{1}{\lambda^2} \left(1 - \frac{1}{\beta^2 n^2(\lambda)}\right) z^2$$

- Number of photons per unit path length is:

$$\frac{dN}{dx} = 2\pi\alpha z^2 \int_{\beta n > 1} \left(1 - \frac{1}{\beta^2 n^2}\right) \frac{d\lambda}{\lambda^2}$$

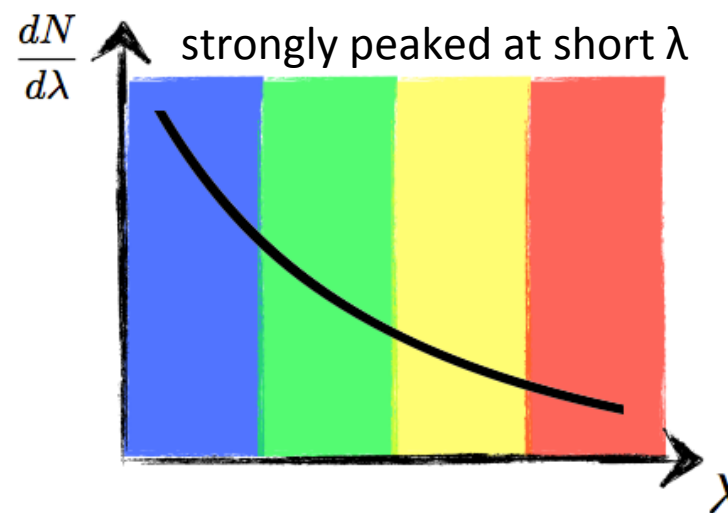
Assuming $n \sim \text{const}$ over the wavelength region detected

$$\frac{dN}{dx} = 2\pi\alpha \sin^2 \theta \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) z^2$$

in λ range 350-500 nm (photomultiplier sensitivity range),

$$\frac{dN}{dx} = 390 \sin^2 \theta \text{ photons/cm}$$

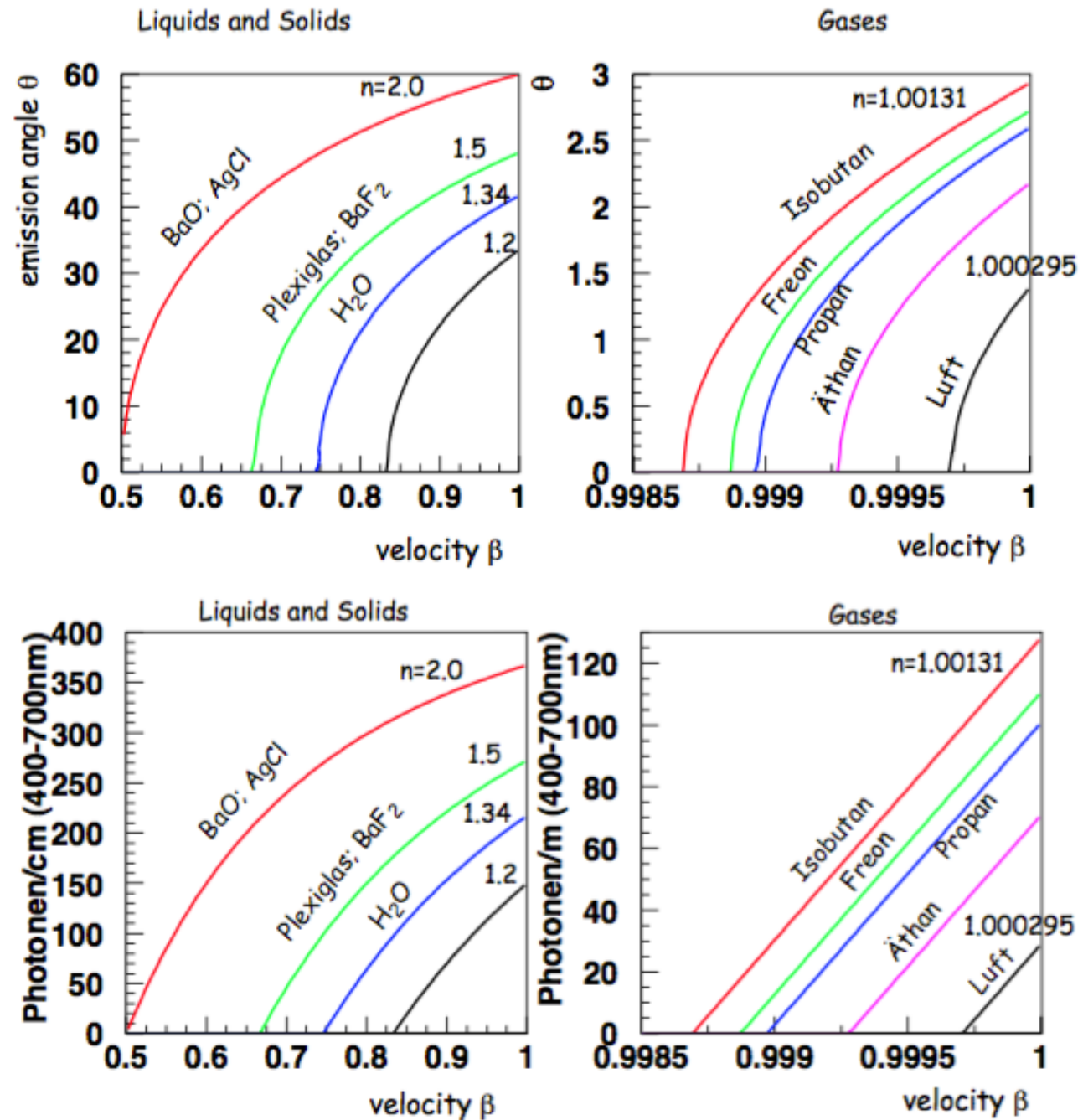
dE/dx due to Cherenkov radiation is small compared to ionization loss ($< 1\%$) and much weaker than scintillating output. It can be neglected in energy loss of a particle, but is Important for particle detection



$n = \text{refracting index}$

Cerenkov angle

$$\cos \vartheta = \frac{1}{\beta n}$$



Photon yield

4.Cerenkov light emission

$$\beta > 1/n$$

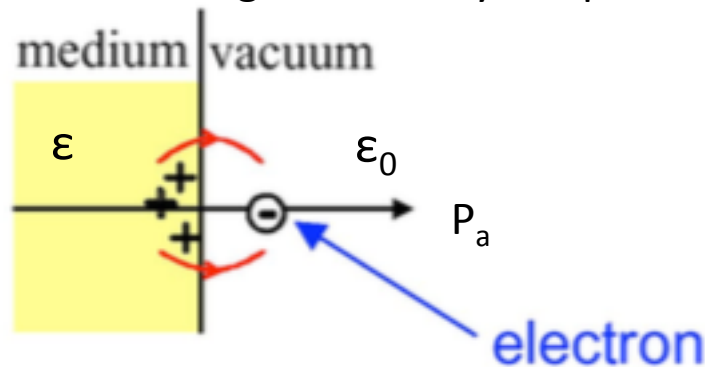
Parameters of Typical Radiator

Medium	n	β_{thr}	$\theta_{\text{max}} [\beta=1]$	$N_{\text{ph}} [\text{eV}^{-1} \text{ cm}^{-1}]$
Air	1.000283	0.9997	1.36	0.208
Isobutan	1.00127	0.9987	2.89	0.941
Water	1.33	0.752	41.2	160.8
Quartz	1.46	0.685	46.7	196.4

4. Transition radiation

- When a relativistic charged particle crosses a boundary between media of different **dielectric properties** radiation is emitted mostly in the **X-ray domain (5-15 KeV)**

The electric field generated by the particle is different on the two sides



<https://arxiv.org/pdf/1111.4188v1.pdf>

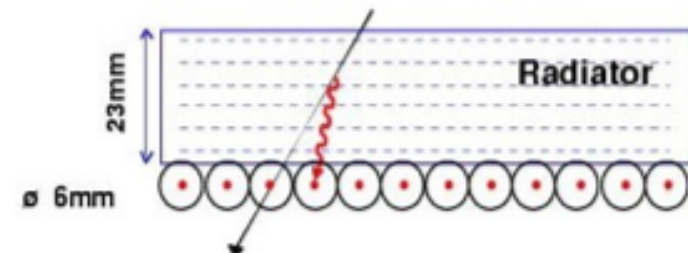
(ω_p = plasma energy of medium)

- The radiation is emitted in a cone at an angle **$\cos \theta = 1/\gamma$**
- Number of photons:
$$N_\gamma(\hbar\omega > \hbar\omega_0) = \frac{\alpha z^2}{\pi} \left[\left(\ln \frac{\gamma \hbar\omega_p}{\hbar\omega_0} - 1 \right)^2 + \frac{\pi^2}{12} \right]$$
- The probability of radiation per transition surface is **low $\sim 1/2 \alpha$** (fine structure constant)

TRD Module

TR in AMS detector:

- polypropylene/polyethylene fibers
- Xe/CO₂ straw tubes



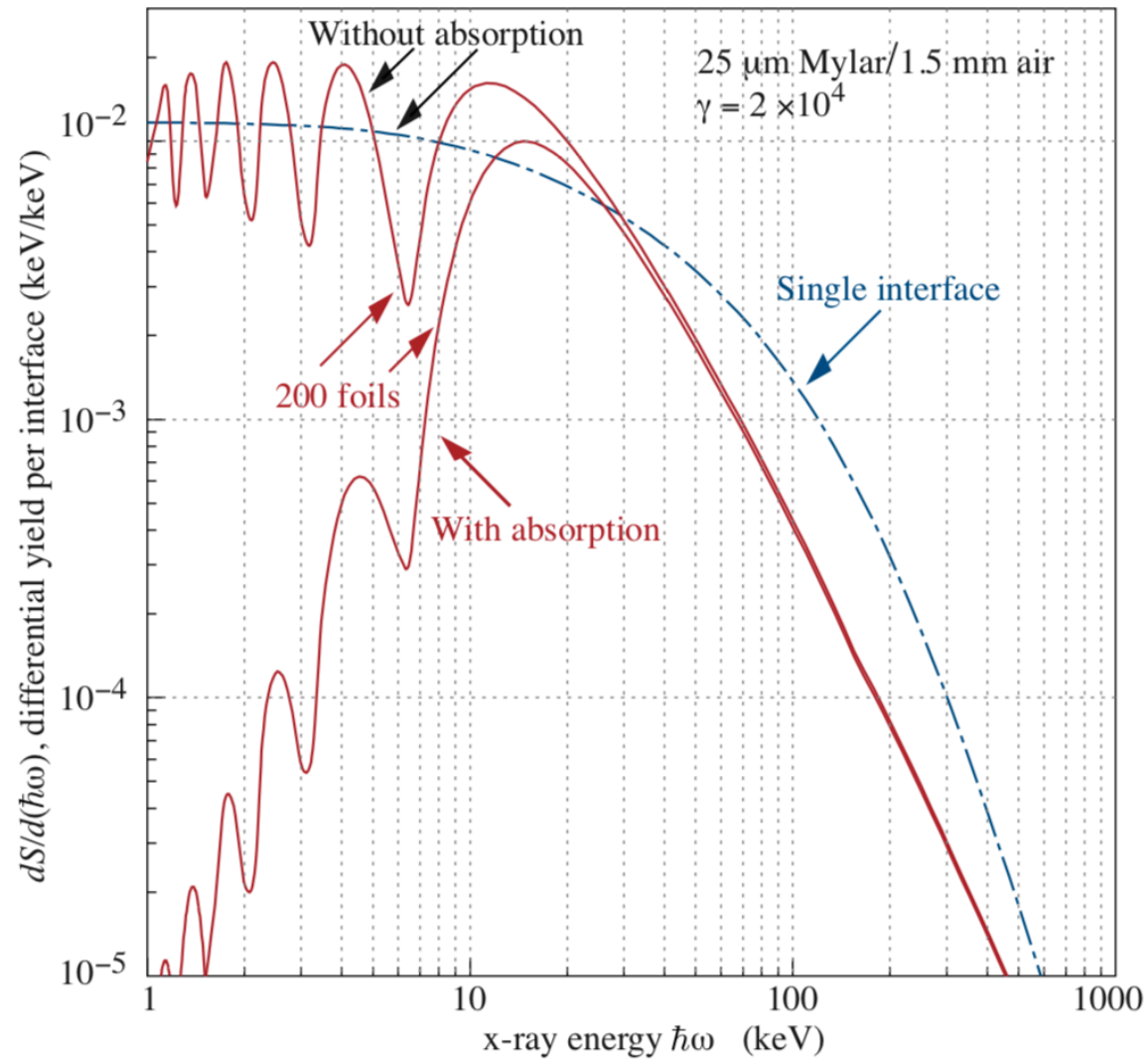


Figure 33.27: X-ray photon energy spectra for a radiator consisting of 200 $25\ \mu\text{m}$ thick foils of Mylar with 1.5 mm spacing in air (solid lines) and for a single surface (dashed line). Curves are shown with and without absorption. Adapted from Ref. 88.

4. Transition radiation

- The energy of radiated photons increases as a function of γ of particle

Energy radiated when a particle z crosses the boundary between vacuum et medium (ω_p = plasma energy)

$$I = \alpha z^2 \gamma \hbar \omega_p / 3$$

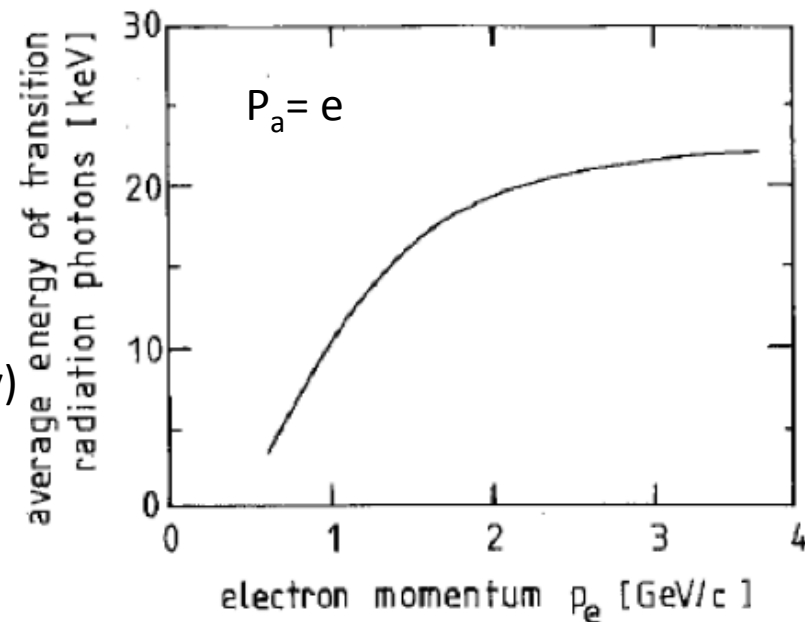
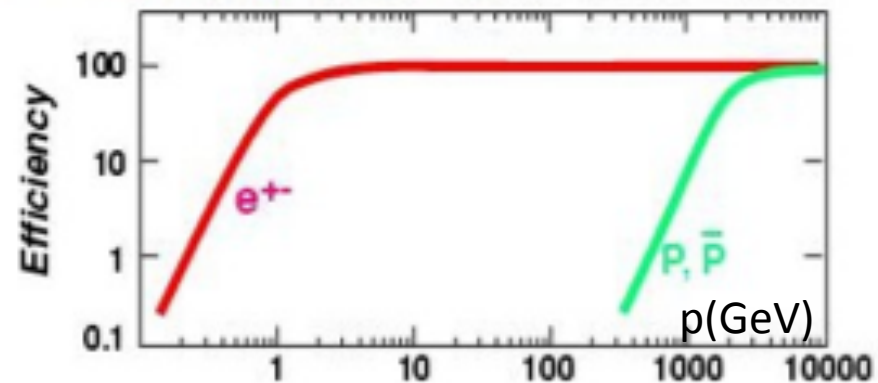


Fig. 6.21. Typical dependence of the average energy of transition radiation photons on the electron momentum for standard radiator arrangements [450].

e^\pm / hadron rejection $> 10^3$



Useful for particle identification

