Towards the precise description of Composite Higgs models at colliders

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Outline

Introduction: Composite Higgs (CH) models

- Technicolor: 4D confining gauge theory G_{HC} with fermionic matter $\to$ dynamical EW symmetry breaking (hierarchy problem)
  (Weinberg 76, Susskind 79)

$$\langle\psi\psi\rangle \sim f^3 \to f = v$$

- CH: Vacuum misalignment (Higgs is a pNGB) (Little-hierarchy problem and doublet nature of Higgs)
  (Georgi, Kaplan 84', Agashe, Contino, Pomarol 05)

$$v = f \sin\theta$$

Model example (Gripaios, Pomarol, Riva, Serra 0902.1483):

$$\langle\psi^I_{\alpha,c}\psi^J_{\beta,c'}\epsilon^{\alpha\beta}\epsilon^{cc'}\rangle \sim f^3 E^{IJ}_\psi, \quad E_\psi = \cos\theta E^-_\psi + \sin\theta E^{\mathrm{B}}_\psi$$

	Sp(4)	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	SU(4)	SU(6)
$\binom{\psi_1}{\psi_2}$	$\square$	**1**	**2**	0	**4**	**1**
$\psi_{3,4}$	$\square$	**1**	**1**	$\pm 1/2$		**1**

Partial Compositeness

- CH: flavour scale $\Lambda_F > 10^4$ TeV $\gg \Lambda_{TC}$ generates low energy 4-fermion interactions (Q =SM fermions, ψ = hyper-fermions)

$$\alpha\frac{\bar\psi\psi\bar\psi\psi}{\Lambda_F^2} + \boxed{\beta\frac{\bar\psi\psi\bar Q Q}{\Lambda_F^2}}_{\text{ETC}} + \boxed{\kappa\frac{\psi\psi\psi Q}{\Lambda_F^2} + h.c.}_{\text{PC}} + \boxed{\gamma\frac{\bar Q Q \bar Q Q}{\Lambda_F^2}}_{\text{FCNC}}$$

  - Extended Technicolor (ETC) (Dimopoulos, Susskind 79, Eichten, Lane 80)
  - Partial Compositeness (PC) (Kaplan 91)
  - Yukawa is NOT the only relevant operator. Enhanced w.r.t. 4-fermion FCNC operators? $\rightarrow$ Walking Technicolor and large anomalous dimension (Holdom 81)

Model example Barnard, Gherghetta, Ray 1311.6562, Ferretti, Karateev 1312.5330

	Sp(4)	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	SU(4)	SU(6)
$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$	$\square$	**1**	**2**	0	**4**	**1**
$\psi_{3,4}$	$\square$	**1**	**1**	$\pm 1/2$		**1**
$\chi_{1,2,3}$	$\boxminus$	**3**	**1**	$2/3$	**1**	**6**
$\chi_{4,5,6}$	$\boxminus$	$\bar{\mathbf{3}}$	**1**	$-2/3$	**1**	

Condensation and composite states

	spin	SU(4)×SU(6)	Sp(4)×SO(6)	names
QQ	0	$(\mathbf{6},\mathbf{1})$	$(\mathbf{1},\mathbf{1})$	σ
			$(\mathbf{5},\mathbf{1})$	π
$\chi\chi$	0	$(\mathbf{1},\mathbf{21})$	$(\mathbf{1},\mathbf{1})$	σ_c
			$(\mathbf{1},\mathbf{20})$	π_c
χQQ	1/2	$(\mathbf{6},\mathbf{6})$	$(\mathbf{1},\mathbf{6})$	ψ_1^1
			$(\mathbf{5},\mathbf{6})$	ψ_1^5
$\chi\bar{Q}\bar{Q}$	1/2	$(\mathbf{6},\mathbf{6})$	$(\mathbf{1},\mathbf{6})$	ψ_2^1
			$(\mathbf{5},\mathbf{6})$	ψ_2^5
$Q\bar{\chi}\bar{Q}$	1/2	$(\mathbf{1},\bar{\mathbf{6}})$	$(\mathbf{1},\mathbf{6})$	ψ_3
$Q\bar{\chi}\bar{Q}$	1/2	$(\mathbf{15},\bar{\mathbf{6}})$	$(\mathbf{5},\mathbf{6})$	ψ_4^5
			$(\mathbf{10},\mathbf{6})$	ψ_4^{10}
$\bar{Q}\sigma^\mu Q$	1	$(\mathbf{15},\mathbf{1})$	$(\mathbf{5},\mathbf{1})$	a
			$(\mathbf{10},\mathbf{1})$	ρ
$\bar{\chi}\sigma^\mu\chi$	1	$(\mathbf{1},\mathbf{35})$	$(\mathbf{1},\mathbf{20})$	a_c
			$(\mathbf{1},\mathbf{15})$	ρ_c

Below condensation scale hypercolor-neutral composite states are formed

- Scalars: pNGBs π (and QCD charged π_c), σ excitation
- Vectors ρ^μ, a^μ (ρ_c^μ, a_c^μ)
- Fermions (including top partners)
- $U(1)$ ALP (anomalous and non-anomalous)
- Extended content: conformal dynamics, dark matter candidates (e.g. 1808.07515), ...

Goal

A common framework for the collider simulation of CH models.

- Construction of the Lagrangian within the CCWZ formalism
  Callan, Coleman, Wess, Zumino 69
- Power counting ($\sim$ EWχL): p^2, $g^2 f^2$, $m_\psi f$, yf^2
- Precision: Incorporation of operators at NLO in the chiral expansion. Only pNGBs, EW bosons, SM fermions.
- Exploration: the production and decays of new states, pNGBs, and heavy resonances.
- Respecting the approximate symmetries of the dynamical breaking, including relations in the interactions between pNGB, gauge bosons, SM fermion and heavy composite states.
- Implementation in FeynRules/UFO (in progress)
  Flexible tool to simulate collider events e.g. in MadGraph, using all its features like polarization selection, possibility to include radiative corrections, parton shower etc.

pNGBs, EW bosons and SM fermions

$$\mathcal{L}_{LO} = \frac{f^2}{N^2}\langle x^\mu x_\mu + \chi_+\rangle - \frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$
$$+ C_g f^4 g^2 \langle \Gamma^{g,i}\Sigma_0\Gamma^{g,iT}\Sigma_0\rangle + C_g' f^4 g'^2\langle\Gamma^{g'}\Sigma_0\Gamma^{g'T}\Sigma_0\rangle.$$

- pNGBs of the spontaneous G/H breaking are the lightest states
- pNGBs π from $\langle\psi\psi\rangle$ condensate gets masses from electroweak loops, hyper-fermion masses and top interactions
  $\rightarrow m_\pi^2 \sim \mathcal{O}(g^2 f^2, m_\psi f, y f^2)$
- pNGBs π_c from $\langle\chi\chi\rangle$ condensate gets masses from gluon loops and hyper-fermion masses, $\rightarrow m_{\pi_c}^2 \sim \mathcal{O}(g_s^2 f^2, m_\chi f)$

Electro-weak coset	$SU(2)_L \times U(1)_Y$
$SU(5)/SO(5)$	$\mathbf{3}_{\pm1} + \mathbf{3}_0 + \mathbf{2}_{\pm1/2} + \mathbf{1}_0$
$SU(4)/Sp(4)$	$\mathbf{2}_{\pm1/2} + \mathbf{1}_0$
$SU(4)\times SU(4)'/SU(4)_D$	$\mathbf{3}_0 + \mathbf{2}_{\pm1/2} + \mathbf{2'}_{\pm1/2} + \mathbf{1}_{\pm1} + \mathbf{1}_0 + \mathbf{1'}_0$
Color coset	$SU(3)_c \times U(1)_Y$
$SU(6)/SO(6)$	$\mathbf{8}_0 + \mathbf{6}_{(-2/3 \text{ or } 4/3)} + \bar{\mathbf{6}}_{(2/3 \text{ or } -4/3)}$
$SU(6)/Sp(6)$	$\mathbf{8}_0 + \mathbf{3}_{2/3} + \bar{\mathbf{3}}_{-2/3}$
$SU(3)\times SU(3)'/SU(3)_D$	$\mathbf{8}_0$

- Wess-Zumino-Witten terms Wess, Zumino 71, Witten 83 - Topological terms universal and model independent, besides being phenomenologically relevant for the description of bosonic decays of pNGBs
- Linear term in the pNGB:

$$\mathcal{L}_{WZW} = \frac{\dim(\psi)}{48\pi^2 f}\langle 2F_{\mu\nu}\widetilde{F}^{\mu\nu}(\Omega\Pi\Omega^\dagger + \Omega^\dagger\Pi\Omega) + \Omega^\dagger F_{\mu\nu}\Omega\Pi\Omega\widetilde{F}^{\mu\nu}\Omega^\dagger + \Omega F_{\mu\nu}\Omega^\dagger\Pi\Omega^\dagger\widetilde{F}^{\mu\nu}\Omega\rangle\,.$$

- SM fermions as spurions of G - more model dependent

$$\Psi = \begin{cases}\xi^\dagger\Xi_{A/S}\xi^*\Sigma_0 \\ \xi^\dagger\Xi_{Adj}\xi\end{cases} \rightarrow h\psi h^\dagger.$$

$$\mathcal{L}_\Psi = \bar{Q}i\not{D}Q + \bar{L}i\not{D}L + \bar{u}i\not{D}u + \bar{d}i\not{D}d + \bar{e}i\not{D}e - \frac{f}{4\pi}\langle Y_u\bar{\psi}_Q\psi_u + Y_d\bar{\psi}_Q\psi_d + Y_e\bar{\psi}_L\psi_e + Y_\nu\bar{\psi}_L\psi_\nu + \text{h.c.}\rangle + \text{ double-trace terms} - V_\psi$$

Next-to-leading order Lagrangian (pNGBs and EW bosons)

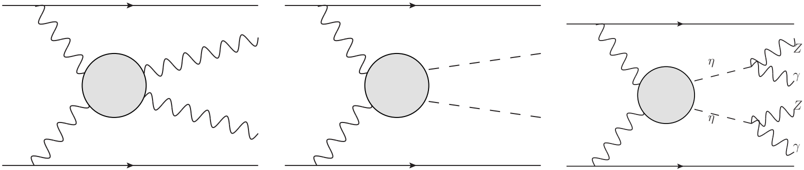
- The Leutwyler-Gasser terms Gasser, Leutwyler 83, 84 for a generic coset Bijnens, Lu 0910.5424

$$\mathcal{L}_{p^4} = \frac{1}{16\pi^2}\{L_0\langle x^\mu x^\nu x_\mu x_\nu\rangle + L_1\langle x^\mu x_\mu\rangle\langle x^\nu x_\nu\rangle + L_2\langle x^\mu x^\nu\rangle\langle x_\mu x_\nu\rangle + L_3\langle x^\mu x_\mu x^\nu x_\nu\rangle$$
$$+ L_4\langle x^\mu x_\mu\rangle\langle\chi_+\rangle + L_5\langle x^\mu x_\mu \chi_+\rangle + L_6\langle\chi_+\rangle^2 + L_7\langle\chi_-\rangle^2 + \frac{1}{2}L_8\langle\chi_+^2 + \chi_-^2\rangle$$
$$- \mathrm{i}L_9\langle\tilde{f}_{\mu\nu}x^\mu x^\nu\rangle + \frac{1}{4}L_{10}\langle\tilde{f}_{\mu\nu}^2 - \hat{f}_{\mu\nu}^2\rangle\}$$

- Similar to propagating photons Urech 94, with EW bosons

$$\mathcal{L}_{g^2p^2} = \frac{g^2f^2}{16\pi^2}\{K_1\langle x_\mu x^\mu\rangle\langle\Gamma^2\rangle + K_2\langle x_\mu x^\mu\rangle\langle\Gamma^{g,i}\Sigma_0\Gamma^{g,iT}\Sigma_0\rangle + K_3\langle x_\mu\Gamma\rangle\langle x^\mu\Gamma\rangle$$
$$+ K_5\langle x_\mu x^\mu\Gamma^2\rangle + K_6(\langle x_\mu x^\mu\Gamma^{g,i}\Sigma_0\Gamma^{g,iT}\Sigma_0\rangle + \text{h.c.}) + K_7\langle\chi_+\rangle\langle\Gamma^2\rangle$$
$$+ K_8\langle\chi_+\rangle\langle\Gamma^{g,i}\Sigma_0\Gamma^{g,iT}\Sigma_0\rangle + K_9\langle\chi_+\Gamma^2\rangle + K_{10}(\langle\chi_+\Gamma^{g,i}\Sigma_0\Gamma^{g,iT}\Sigma_0\rangle + \text{h.c.})$$
$$+ K_{11}(\langle\chi_-\Gamma^{g,i}\Sigma_0\Gamma^{g,iT}\Sigma_0\rangle + \text{h.c.}) + K_{12}(\langle x^\mu D_\mu\Gamma^{g,i}\Sigma_0\Gamma^{g,iT}\Sigma_0\rangle + \text{h.c.})$$
$$+ K_{13}D_\mu\Gamma^{g,i}\Sigma_0(D^\mu\Gamma^{g,i})^T\Sigma_0 + K_{14}D_\mu\Gamma^{g,i}D^\mu\Gamma^{g,iT}\}$$

Towards Predictions: (p)GBS at Colliders (preliminary)



- GBS are embedded in more complicated processes at colliders.
- Longitudinal weak bosons are manifestations of the GBs (equivalence theorem)
- Polarized scattering with `MadGraph_aMC@NLO` DBF, Mattelaer, Ruiz, Shil 1912.01725

$$q_1 q_2 \;\to\; q_1' q_2' W^+_\lambda W^-_{\lambda'}, \quad \lambda = 0, T$$

- Di-Higgs via VBF

$$q_1 q_2 \;\to\; q_1' q_2' hh$$

- Di-pNGBs via VBF:

$$q_1 q_2 \;\to\; q_1' q_2' \eta\eta, \quad q_1 q_2 \;\to\; q_1' q_2' \pi^0 \pi^0, \;\cdots$$

Minimal CH $SO(5)/SO(4)$ at $\mathcal{O}(p^4)$ and tree-level

- Peculiar feature $SO(4) \sim SU(2)_L \times SU(2)_R$. Objects can be split in L-R.

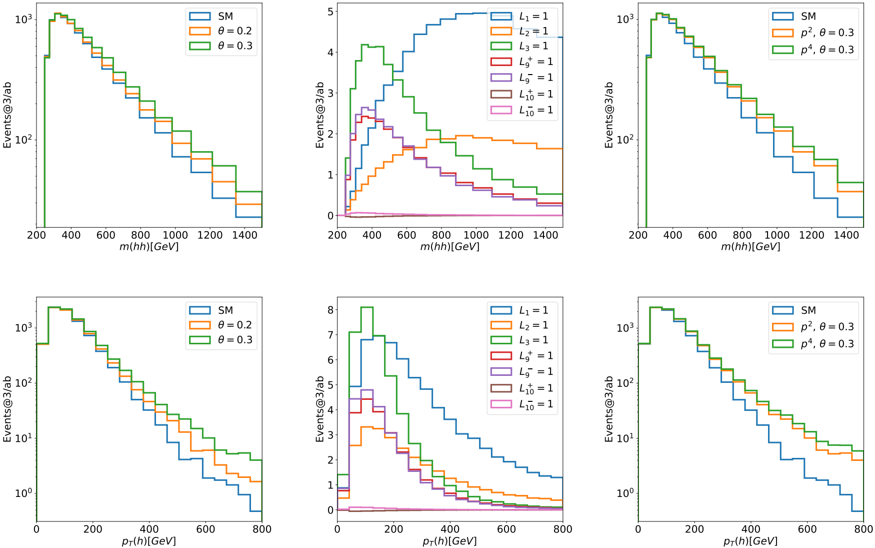
$$\mathcal{L}_4^{MCH} = \frac{1}{16\pi^2}\Big\{L_1\langle x^\mu x_\mu\rangle\langle x^\nu x_\nu\rangle + L_2\langle x^\mu x^\nu\rangle\langle x_\mu x_\nu\rangle + L_3^-\langle s_L^{\mu\nu} s_{\mu\nu,L} - s_R^{\mu\nu} s_{\mu\nu,R}\rangle$$
$$- \mathrm{i}L_9^+\langle(f_L^{\mu\nu} + f_R^{\mu\nu})x_\mu x_\nu\rangle - \mathrm{i}L_9^-\langle(f_L^{\mu\nu} - f_R^{\mu\nu})x_\mu x_\nu\rangle$$
$$+ \frac{1}{4}L_{10}^+\langle\hat{f}_{\mu\nu}\hat{f}^{\mu\nu}\rangle + \frac{1}{4}L_{10}^-\langle f_L^{\mu\nu} f_{\mu\nu,L} - f_R^{\mu\nu} f_{\mu\nu,R}\rangle\Big\}$$

- The higher dimension operators induce corrections to the kinetic terms, as

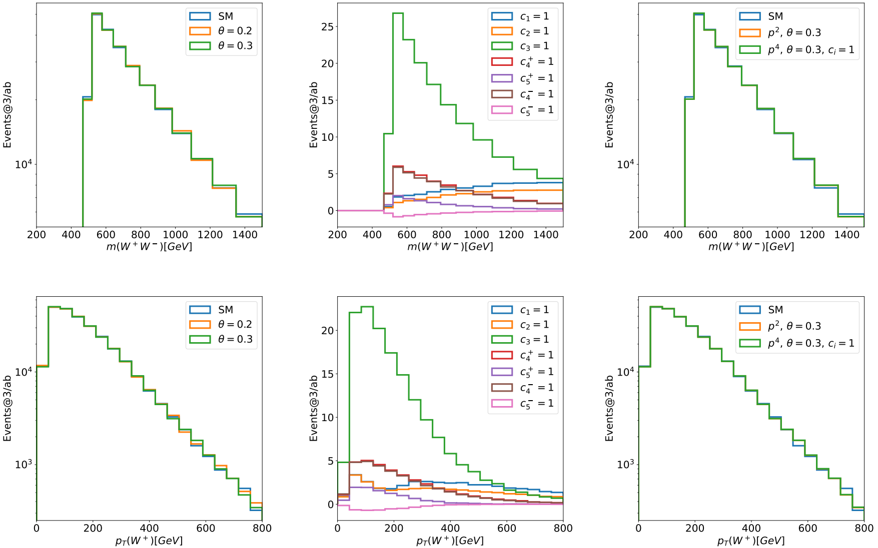
$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu}[1 + X_B] - \frac{1}{4}W^i_{\mu\nu}W^{i\mu\nu}[1 + X_W] - \frac{1}{4}W^3_{\mu\nu}B^{\mu\nu}X_{BW}$$

- The implementation in FEYNRULES requires the fields to have canonically normalized kinetic terms and diagonal masses.

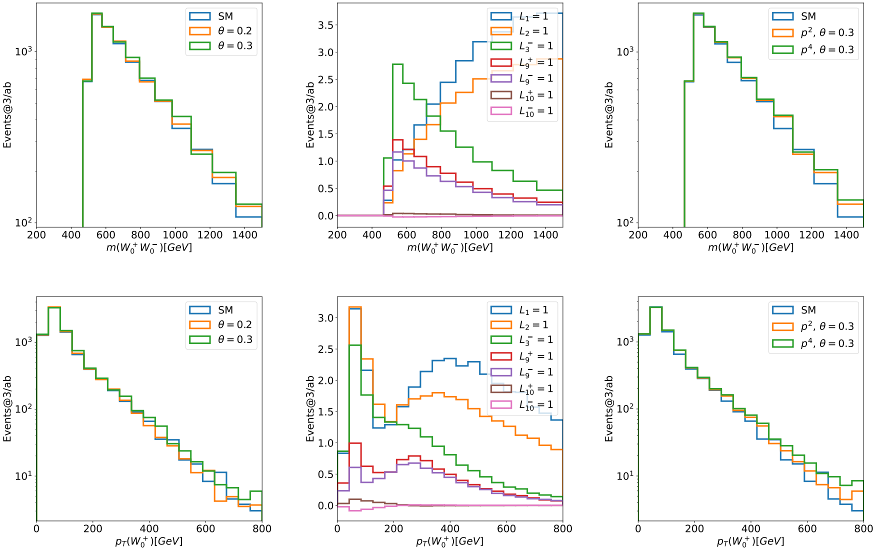
Di-Higgs via VBF $pp \to jjhh$



VBS $pp \to jjW^+W^-$

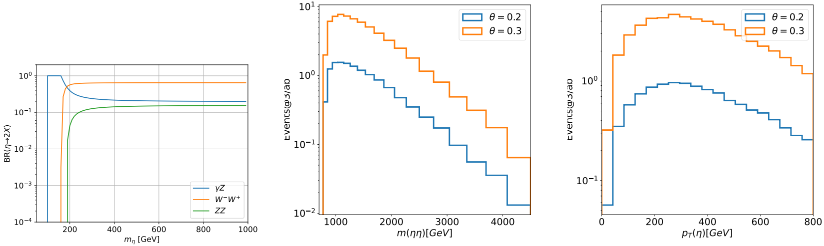


Longitudinally polarized VBS $pp \to jjW_0^+ W_0^-$



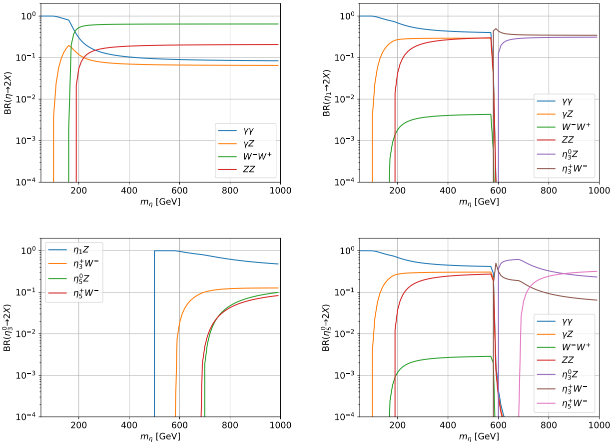
pNGB pair production in non-minimal models at $\mathcal{O}(p^2)$

- The $SU(4)/Sp(4)$ model contains 5 (p)NGBs. The Higgs bi-doublet **(2,2)** and a singlet η **(1,1)** under custodial $SU(2)_L \times SU(2)_R$ group.
- BR considering only 2-body bosonic decays at tree level

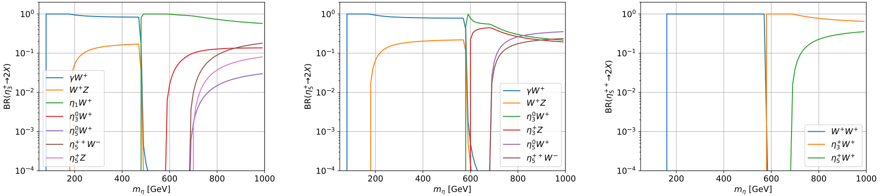


- If η is light enough VBF and η^2-strahlung can compete with production via Higgs and top loops e.g. DBF, Ferretti, Huang, Shu 2005.13578

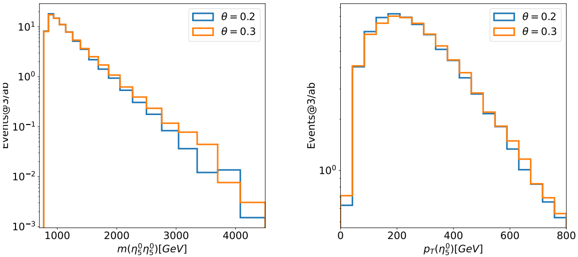
- The $SU(5)/SO(5)$ model contains $14 \subset SO(5)$ (p)NGBs
- 14 of $SO(5) \to (2,2) + (3,3) + (1,1)$ of $SU(2)_L \times SU(2)_R$
- $(3,3) \to 5 + 3 + 1$ $(\eta_5,\ \eta_3,\ \eta_1)$ $(1,1) \to 1$ (η) of $SU(2)_V$.



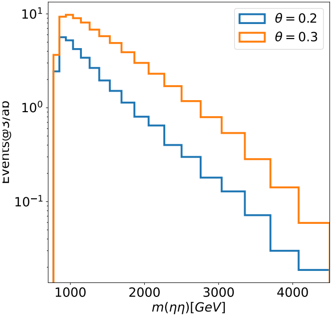
$\theta = 0.3,\ m_{\eta_5^{\pm\pm,\pm,0}} = 600\,\text{GeV},\ m_{\eta_3^{\pm,0}} = 500\,\text{GeV},\ m_{\eta_1} = 400\,\text{GeV},\ m_\eta = 300\,\text{GeV}.$

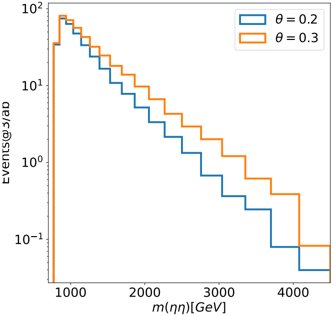


$$pp \to jj\eta_5^0\eta_5^0$$



$$pp \to jj\eta_3^+\eta_3^+ \qquad pp \to jj\eta_5^{++}\eta_5^{--}$$





States can also be produced via Drell-Yan processes, that can also be simulated in the framework.

Heavy Composite Resonances

Scalar excitation σ

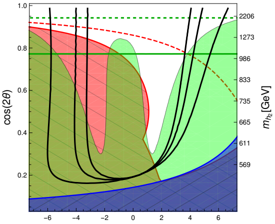
- Incorporated via expansion parameters

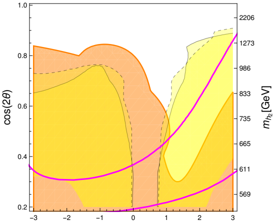
$$\mathcal{L}_\sigma = \frac{1}{2}\kappa_0(\sigma/f)f^2\langle x_\mu x^\mu\rangle + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}M_\sigma^2\sigma^2 + \kappa_i(\sigma/f)\mathcal{O}_i$$

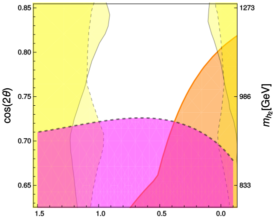
$$\kappa_i(\sigma/f) = 1 + \kappa'\sigma/f + \cdots$$

- Indication of light 0^+ scalar in near conformal dynamics e.g. Hasenfratz, Rebbi, Witzel 16, Elander, Piai, 17
- An example: A lightish 0^+ DBF, Cacciapaglia, Deandrea 1809.09146

EWPO+Higgs $\qquad$ $\sigma \to ZZ$ (gg+VBF) $\qquad$ $\sigma \to ZZ$, tt







Spin-1 ρ^μ, a^μ

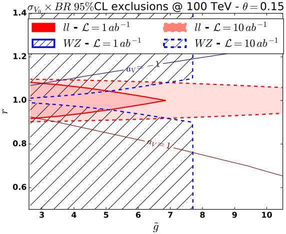
- Incorporated via the Local Hidden Symmetry construction $\rightarrow$ LO parameters: $\tilde{g}$, r, M_V, M_A

$$
\begin{aligned}
\mathcal{L}_v &= -\frac{1}{2\widetilde{g}^2}\langle \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\rangle + \frac{1}{2}f_0^2\,\langle x_{0\mu}x_0^\mu\rangle + \frac{1}{2}f_1^2\,\langle x_{1\mu}x_1^\mu\rangle \\
&+ r\,f_1^2\,\langle x_{0\mu}Kx_1^\mu K^\dagger\rangle + \frac{1}{2}f_K^2\,\langle D^\mu K\;D_\mu K^\dagger\rangle\,.
\end{aligned}
$$

$$
\mathcal{F}_\mu = \mathcal{V}_\mu + \mathcal{A}_\mu = \sum_{a=1}^{10}\mathcal{V}_\mu^a V_a + \sum_{a=1}^{5}\mathcal{A}_\mu^a Y_a.
$$

- An example: $SU(4)/Sp(4)$ implementation (DBF, Cacciapaglia, Cai, Deandrea, Frandsen 16, DBF, Ferrarese 1705.02787)

		SU(2)$_V$	SU(2)$_L\times$ SU(2)$_R$	TC	CH
$\mathcal{V}$	$v_\mu^{0,\pm}$	3	(3,1)$\oplus$(1,3)	$\vec{\rho}_\mu$	$\vec{\rho}_\mu$
	$s_\mu^{0,\pm}$	3			$\vec{a}_\mu$
	$\tilde{s}_\mu^{0,\pm}$	3	(2,2)		
	$\tilde{v}_\mu^{0}$	1			
$\mathcal{A}$	$a_\mu^{0,\pm}$	3	(2,2)	$\vec{a}_\mu$	
	x_μ^{0}	1			
	$\tilde{x}_\mu^{0}$	1	(1,1)		



Conclusion

- CH + PC continues to be a promising alternative to the SM.
- The developed tools will allow an unified framework for the study of CH models at colliders
  - with the non-linear structure of the symmetry breaking
- Exploratory:
  - pNGBs in non-minimal models
  - Resonances: vectors (added via local hidden symmetry), scalar excitation, top partners (see Avik Banerjee's talk)
  - Appropriate for the study of pNGB pair production and other BSM signatures.
- Precision: incorporation of NLO terms,
  - Renormalization program for pNGB/EW boson loops
  - Suited for a robust test of (Generalized) Universal Relations Liu, Low, Yin 1809.09126
  - the phenomenological study of VBS and di-Higgs production (relations including quartic interactions)

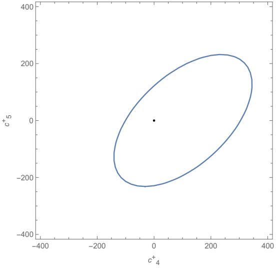
Backup

Anomalous Trilinear gauge coupling

The coefficients L_3^-, $L_9^\pm$ and L_{10}^+ contribute to aTGC

$$\delta g_{1,z} = \left(\frac{m_Z^2}{(4\pi f)^2}\right)(-2L_3^- + L_9^-)\cos\theta + L_9^+,$$
$$\delta\kappa_\gamma = \left(\frac{m_W^2}{(4\pi f)^2}\right)(-4(L_9^+ - 2L_{10}^+))$$

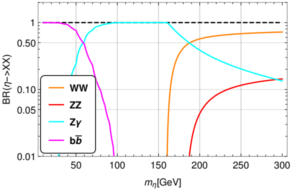
Fit Falkowski, Gonzalez-Alonso, Greljo, Marzocca, 1508.00581 not very strong bounds (expected $L \sim \mathcal{O}(1)$)

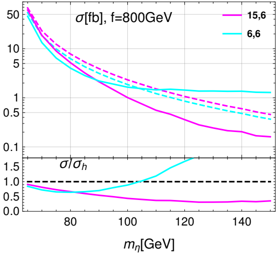


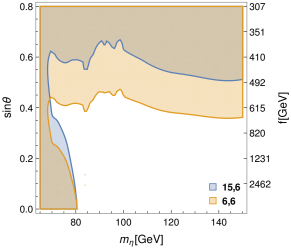
Anomalous Quartic couplings

$$
\begin{aligned}
\mathcal{L}_{QGC} = & -g^2(\mathcal{O}_{WW,1} - \mathcal{O}_{WW_2}) \left[1 + \frac{m_W^2}{16\pi^2 f^2}\left(-8\cos\theta L_3^- + L_4^+ + \cos\theta L_4^-\right)\right] \\
& + \frac{m_W^4}{\pi^2 f^4}\left[2L_1 \mathcal{O}_{WW,1} + L_2(\mathcal{O}_{WW,1} + \mathcal{O}_{WW,2})\right] \\
& + g^2 c_W^2 (\mathcal{O}_{WZ,1} - \mathcal{O}_{WZ_2}) \left[1 + \frac{m_Z^2}{16\pi^2 f^2}\left(8\cos\theta L_3^- - (1 - 4s_W^4)L_4^+ + \cos\theta L_4^- + s_W^4 L_5^+\right)\right] \\
& + \frac{m_W^2 m_Z^2}{\pi^2 f^4}\left(2L_1 \mathcal{O}_{WZ,1} + L_2 \mathcal{O}_{WZ,2}\right) \\
& - e^2 \frac{c_W}{s_W}(\mathcal{O}_{AZ,1} - \mathcal{O}_{AZ_2}) \left[1 + \frac{m_Z^2}{16\pi^2 f^2}\left(-16\cos\theta L_3^- + 2(4s_W^4 - 4s_W^2 - 1)L_4^+ - \cos\theta L_4^- + s_W^4 L_5^+\right)\right] \\
& + \frac{m_W^2 m_Z^2}{\pi^2 f^4}\left(2L_1 \mathcal{O}_{WZ,1} + L_2 \mathcal{O}_{WZ,2}\right) \\
& + \frac{m_Z^4}{\pi^2 f^4} 2\left(L_1 \mathcal{O}_{ZZ,1} + L_2 \mathcal{O}_{ZZ,2}\right) \\
& + e^2(\mathcal{O}_{WA,1} - \mathcal{O}_{WA_2}) \left[1 + \frac{e^2 v^2}{16\pi^2 f^2}\left(2L_4^+ - L_5^+\right)\right]
\end{aligned}
$$

η of SU(4)/Sp(4) 2005.13578







CCWZ

- Vacuum $\Sigma_0 = \cos\theta\Sigma_B + \sin\theta\Sigma_H$.
- Minimization $\cos\theta_{min} = \frac{2C_m}{y_t' C_t}$, for $y_t' C_t > 2|C_m|$.
- Generators

$$
\begin{aligned}
V^a \cdot \Sigma_0 + \Sigma_0 \cdot V^{a\,T} &= 0\,, & S^a \cdot \Sigma_B + \Sigma_B \cdot S^{a\,T} &= 0\,,\\
Y^a \cdot \Sigma_0 - \Sigma_0 \cdot Y^{a\,T} &= 0\,. & X^a \cdot \Sigma_B - \Sigma_B \cdot X^{a\,T} &= 0\,,
\end{aligned}
$$

$$U = \exp\left[\frac{i\sqrt{2}}{f}\sum_{a=1}^{5}\pi^a Y^a\right],$$

$$
\begin{aligned}
\omega_\mu &= U^\dagger D_\mu U,\\
D_\mu &= \partial_\mu - igW^i_\mu S^i - ig' B_\mu S^6,\\
x_\mu &= 2\mathrm{Tr}\left[Y_a\omega_\mu\right] Y^a\ ,\\
s_\mu &= 2\mathrm{Tr}\left[V_a\omega_\mu\right] V^a\ .
\end{aligned}
$$

Hidden Local Symmetry (HLS)

- Enhance the symmetry group $\mathrm{SU}(4)_0 \times \mathrm{SU}(4)_1$, and embed the SM gauge bosons in $\mathrm{SU}(4)_0$ and the heavy resonances in $\mathrm{SU}(4)_1$. $\mathrm{SU}(4)_i \to \mathrm{Sp}(4)_i$. $\mathrm{Sp}(4)_0 \times \mathrm{Sp}(4)_1 \to \mathrm{Sp}(4)$ by a sigma field K

$$U_0 = \exp\left[\frac{i\sqrt{2}}{f_0}\sum_{a=1}^{5}(\pi_0^a Y^a)\right], \quad U_1 = \exp\left[\frac{i\sqrt{2}}{f_1}\sum_{a=1}^{5}(\pi_1^a Y^a)\right]. \tag{1}$$

$$\begin{aligned} D_\mu U_0 &= (\partial_\mu - igW_\mu^i S^i - ig' B_\mu S^6) U_0 \,, \\ D_\mu U_1 &= (\partial_\mu - i\widetilde{g}\mathcal{V}_\mu^a V^a - i\widetilde{g}\mathcal{A}_\mu^b Y^b) U_1 \,. \end{aligned} \tag{2}$$

$$\begin{aligned} K &= \exp\left[ik^a V^a / f_K\right] \,, \\ D_\mu K &= \partial_\mu K - i v_{0\mu} K + i K v_{1\mu} \end{aligned} \tag{3}$$

$\boldsymbol{\mathcal{F}}_\mu = \boldsymbol{\mathcal{V}}_\mu + \boldsymbol{\mathcal{A}}_\mu = \sum_{a=1}^{d_H} \mathcal{V}_\mu^a V_a + \sum_{a=1}^{d_G - d_H} \mathcal{A}_\mu^a Y_a,$

$$\begin{aligned} \mathcal{L}_v &= -\frac{1}{2\widetilde{g}^2}\,\langle \boldsymbol{\mathcal{F}}_{\mu\nu}\boldsymbol{\mathcal{F}}^{\mu\nu}\rangle + \frac{1}{2}f_0^2\,\langle x_{0\mu}x_0^\mu\rangle + \frac{1}{2}f_1^2\,\langle x_{1\mu}x_1^\mu\rangle \\ &+ r\, f_1^2\,\langle x_{0\mu}K x_1^\mu K^\dagger\rangle + \frac{1}{2}f_K^2\,\langle D^\mu K\; D_\mu K^\dagger\rangle \,. \end{aligned}$$

GBS amplitudes

- $\pi\pi \to \pi\pi$ scattering amplitudes expanded in partial wave amplitudes

$$\mathcal{A}(s,t) = 32\pi \sum_{J=0}^{\infty} a_J(s)(2J+1)P_J(\cos\theta)$$

- In order to force elasticity (at least below new heavy states appear), decompose amplitude in conserved quantum number
- **Template: SU(4)/Sp(4), FMCHM**, decompose in multiplets of Sp(4) (very good symmetry at high energy)

$$5 \otimes 5 = 1 \oplus 10 \oplus 14 \equiv A \oplus B \oplus C$$

PC models

Ferretti, Karateev, 13, Cacciapaglia, Ferretti, Flacke, Serôdio 17, 19

Coset	HC	ψ	χ	$-q_\chi/q_\psi$	Baryon	Name	Lattice
$\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{SO(6)}$	SO(7)	$5 \times \mathbf{F}$	$6 \times \mathbf{Sp}$	5/6	$\psi\chi\chi$	M1	
	SO(9)	$5 \times \mathbf{F}$	$6 \times \mathbf{Sp}$	5/12	$\psi\chi\chi$	M2	
	SO(7)	$5 \times \mathbf{Sp}$	$6 \times \mathrm{F}$	5/6	$\psi\psi\chi$	M3	
	SO(9)	$5 \times \mathbf{Sp}$	$6 \times \mathrm{F}$	5/3	$\psi\psi\chi$	M4	
$\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{Sp(6)}$	Sp(4)	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	5/3	$\psi\chi\chi$	M5	$\surd$
$\frac{SU(5)}{SO(5)} \times \frac{SU(3)^2}{SU(3)}$	SU(4)	$5 \times \mathbf{A}_2$	$3 \times (\mathbf{F}, \overline{\mathbf{F}})$	5/3	$\psi\chi\chi$	M6	$\surd$
	SO(10)	$5 \times \mathbf{F}$	$3 \times (\mathbf{Sp}, \overline{\mathbf{Sp}})$	5/12	$\psi\chi\chi$	M7	
$\frac{SU(4)}{Sp(4)} \times \frac{SU(6)}{SO(6)}$	Sp(4)	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	1/3	$\psi\psi\chi$	M8	$\surd$
	SO(11)	$4 \times \mathbf{Sp}$	$6 \times \mathbf{F}$	8/3	$\psi\psi\chi$	M9	
$\frac{SU(4)^2}{SU(4)} \times \frac{SU(6)}{SO(6)}$	SO(10)	$4 \times (\mathbf{Sp}, \overline{\mathbf{Sp}})$	$6 \times \mathbf{F}$	8/3	$\psi\psi\chi$	M10	
	SU(4)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$6 \times \mathbf{A}_2$	2/3	$\psi\psi\chi$	M11	$\surd$
$\frac{SU(4)^2}{SU(4)} \times \frac{SU(3)^2}{SU(3)}$	SU(5)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \overline{\mathbf{A}_2})$	4/9	$\psi\psi\chi$	M12	

Linearization and Universal relations Liu, Low, Yin 1809.09126

- "Linearization" to SMEFT in the SILH basis (Giudice, Grojean, Pomarol, Rattazzi, 07) with extra dim-8 subset

$$\mathcal{L} = \sum_{i=H,T,y,6} \frac{c_i}{f^2}\mathcal{O}_i + \sum_{i=W,B,HW,HB} \frac{c_i}{m_\rho^2}\mathcal{O}_i + \frac{c_i^8}{f^2 m_\rho^2}(H^\dagger H)\mathcal{O}_i$$

- Universal relations from couplings due to the non-linear nature of the symmetry realization can only be respect by including dim-6 and dim-8 operators of the linear framework, e.g. the couplings $Z_{\mu\nu}A^{\mu\nu}[C_4^h h/v + C_4^{2h}(h/v)^2]$ respect

$$\frac{C_4^{2h}}{C_4^h} = \frac{1}{2}\cos\theta \approx \frac{1}{2}\left(1 + \frac{c_{HW}^8 - c_{HB}^8}{c_{HW} - c_{HB}}(v/f)^2\right)$$

- Moreover, leading operators in strong VBS $\mathcal{O}_{1,2}$ appear only at dim-8 in the SMEFT.

$f^2\mathcal{O}_3$	$f^2\mathcal{O}_4^+$	$f^2\mathcal{O}_4^-$
$-4(\mathcal{O}_W - \mathcal{O}_B)$	$2(\mathcal{O}_{HW} + \mathcal{O}_{HB})$	$2(\mathcal{O}_{HW} - \mathcal{O}_{HB})$
$f^2\mathcal{O}_5^+$	$f^2\mathcal{O}_5^-$	
$4\left[\mathcal{O}_W + \mathcal{O}_B - (\mathcal{O}_{HW} + \mathcal{O}_{HB})\right]$	$-4\left[\mathcal{O}_W - \mathcal{O}_B - (\mathcal{O}_{HW} - \mathcal{O}_{HB})\right]$	

$c_H = 1, \quad c_y = -1/3, \quad c_6 = -4/3$

$$\mathcal{O}_H = \frac{1}{2}\partial_\mu(H^\dagger H)\partial^\mu(H^\dagger H), \quad \mathcal{O}_T = \frac{1}{2}(H^\dagger \overleftrightarrow{D}^\mu H)(H^\dagger \overleftrightarrow{D}_\mu H)$$

$$\mathcal{O}_6 = \lambda(H^\dagger H)^3, \quad \mathcal{O}_y = y_f H^\dagger H \bar{f}_L H f_R$$

$$\mathcal{O}_W = \frac{ig}{2}\left(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H\right) D^\nu W^a_{\mu\nu}, \qquad \mathcal{O}_B = \frac{ig'}{2}\left(H^\dagger \overleftrightarrow{D}^\mu H\right) \partial^\nu B_{\mu\nu}$$

$$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu\nu}, \qquad \mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu} \; .$$

$$c_H = 1, \quad c_y = -1/3, \quad c_6 = -4/3$$

$f^2\mathcal{O}_3$	$f^2\mathcal{O}_4^+$	$f^2\mathcal{O}_4^-$
$-4(\mathcal{O}_W - \mathcal{O}_B)$	$2(\mathcal{O}_{HW} + \mathcal{O}_{HB})$	$2(\mathcal{O}_{HW} - \mathcal{O}_{HB})$
$f^2\mathcal{O}_5^+$	$f^2\mathcal{O}_5^-$	
$4\,[\mathcal{O}_W + \mathcal{O}_B - (\mathcal{O}_{HW} + \mathcal{O}_{HB})]$	$-4\,[\mathcal{O}_W - \mathcal{O}_B - (\mathcal{O}_{HW} - \mathcal{O}_{HB})]$	

Example of couplings table from Liu, Low, Yin 1809.09126

$\mathcal{I}_i^h$	C_i^h (NL)	C_i^h (D6)
(1) $\frac{h}{v} Z_\mu \mathcal{D}^{\mu\nu} Z_\nu$	$\frac{4c_{2w}}{c_w^2}\left(-2c_3 + c_4^-\right) + \frac{4}{c_w^2} c_4^+ \cos\theta$	$2(c_W + c_{HW}) + 2t_w^2(c_B + c_{HB})$
(2) $\frac{h}{v} Z_{\mu\nu} Z^{\mu\nu}$	$-\frac{2c_{2w}}{c_w^2}\left(c_4^- + 2c_5^-\right) - \frac{2}{c_w^2}\left(c_4^+ - 2c_5^+\right)\cos\theta$	$-(c_{HW} + t_w^2 c_{HB})$
(3) $\frac{h}{v} Z_\mu \mathcal{D}^{\mu\nu} A_\nu$	$8\left(-2c_3 + c_4^-\right) t_w$	$2t_w(c_W + c_{HW}) - 2t_w(c_B + c_{HB})$
(4) $\frac{h}{v} Z_{\mu\nu} A^{\mu\nu}$	$-4\left(c_4^- + 2c_5^-\right) t_w$	$-t_w(c_{HW} - c_{HB})$
(5) $\frac{h}{v} W_\mu^+ \mathcal{D}^{\mu\nu} W_\nu^- + h.c.$	$4(-2c_3 + c_4^-) + 4c_4^+ \cos\theta$	$2(c_W + c_{HW})$
(6) $\frac{h}{v} W_{\mu\nu}^+ W^{-\mu\nu}$	$-4(c_4^- + 2c_5^-) - 4\left(c_4^+ - 2c_5^+\right)\cos\theta$	$-2c_{HW}$

$$\mathcal{D}^{\mu\nu} = \partial^\mu \partial^\nu - g^{\mu\nu}\partial^2$$

$\mathcal{I}_i^{2h}$	C_i^{2h} (NL)	C_i^{2h} (D6)
(1) $\frac{h^2}{v^2} Z_\mu \mathcal{D}^{\mu\nu} Z_\nu$	$\frac{2c_{2w}}{c_w^2}\left(-2c_3 + c_4^-\right)\cos\theta$ $+\frac{2}{c_w^2} c_4^+ \cos 2\theta$	$\frac{1}{2} C_1^h$
(2) $\frac{h^2}{v^2} Z_{\mu\nu} Z^{\mu\nu}$	$-\frac{c_{2w}}{c_w^2}\left(c_4^- + 2c_5^-\right)\cos\theta$ $-\frac{1}{c_w^2}\left(c_4^+ - 2c_5^+\right)\cos 2\theta$	$\frac{1}{2} C_2^h$
(3) $\frac{h^2}{v^2} Z_\mu \mathcal{D}^{\mu\nu} A_\nu$	$4t_w\left(-2c_3 + c_4^-\right)\cos\theta$	$\frac{1}{2} C_3^h$
(4) $\frac{h^2}{v^2} Z_{\mu\nu} A^{\mu\nu}$	$-2t_w\left(c_4^- + 2c_5^-\right)\cos\theta$	$\frac{1}{2} C_4^h$
(5) $\frac{h^2}{v^2} W_\mu^+ \mathcal{D}^{\mu\nu} W_\nu^- + h.c.$	$2(-2c_3 + c_4^-)\cos\theta$ $+2c_4^+\cos 2\theta$	$\frac{1}{2} C_5^h$
(6) $\frac{h^2}{v^2} W_{\mu\nu}^+ W^{-\mu\nu}$	$-2\left(c_4^- + 2c_5^-\right)\cos\theta$ $-2\left(c_4^+ - 2c_5^+\right)\cos 2\theta$	$\frac{1}{2} C_6^h$
(7) $\frac{(\partial_\nu h)^2}{v^2} Z_\mu Z^\mu$	$\frac{8}{c_w^2} c_1 \sin^2\theta$	$\times$
(8) $\frac{\partial_\mu h \partial_\nu h}{v^2} Z^\mu Z^\nu$	$\frac{8}{c_w^2} c_2 \sin^2\theta$	$\times$
(9) $\frac{(\partial_\nu h)^2}{v^2} W_\mu^+ W^{-\mu}$	$16 c_1 \sin^2\theta$	$\times$
(10) $\frac{\partial^\mu h \partial^\nu h}{v^2} W_\mu^+ W_\nu^-$	$16 c_2 \sin^2\theta$	$\times$