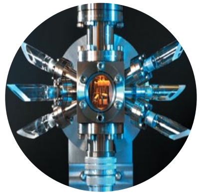
Space-based Atomic Clocks for Geodesy

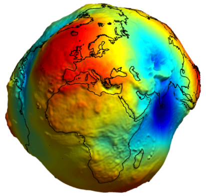
Hu Wu, Jürgen Müller

Institute of Geodesy (IfE), Leibniz University Hannover, Germany



Community Workshop on Cold Atoms in Space

September 23-24, 2021 (online)

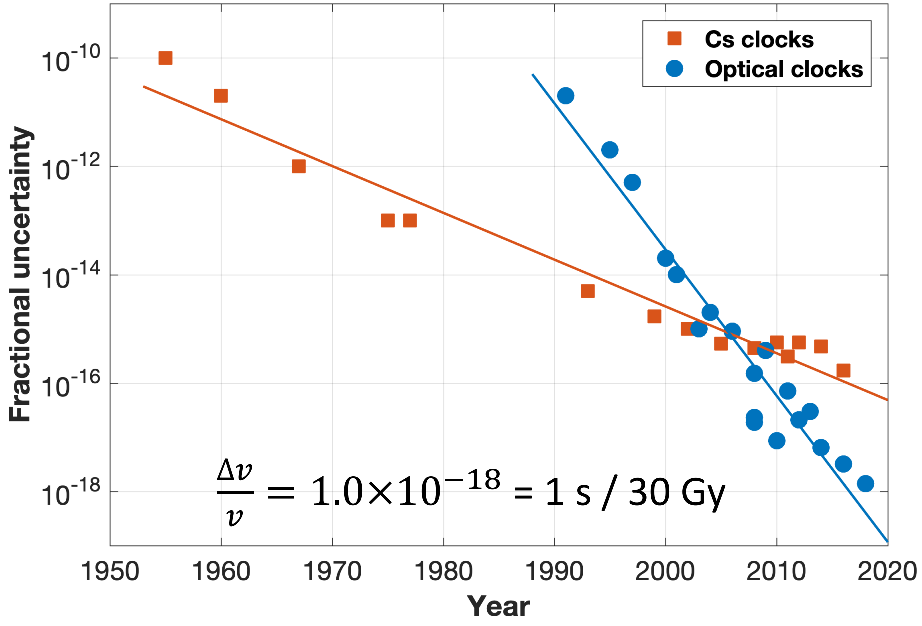


Relativistic Geodesy with Clock Networks

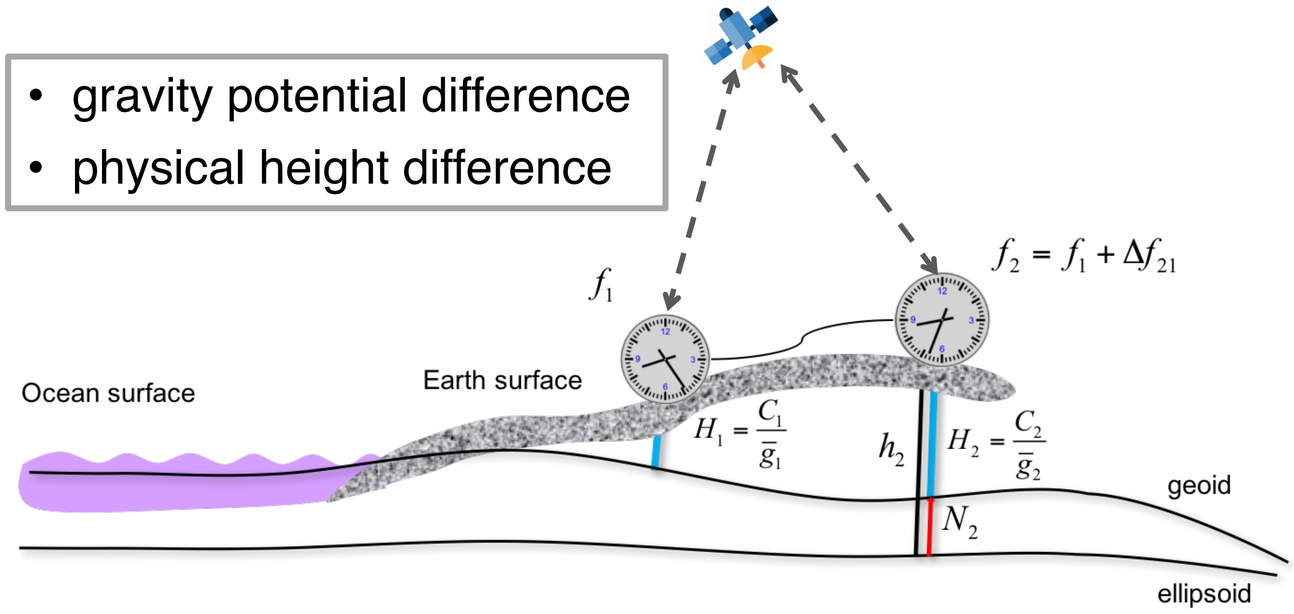
$$\frac{f_2 - f_1}{f_1} = \frac{W_2 - W_1}{c^2} = \frac{-C_2 + C_1}{c^2}$$

Arne Bjerhammar (1985, 1986)

$$\frac{\Delta f}{f}\ (1.0{\times}10^{-18}) \sim \Delta W\ (0.1\ \mathrm{m^2/s^2}) \sim \Delta h\ (1.0\ \mathrm{cm})$$



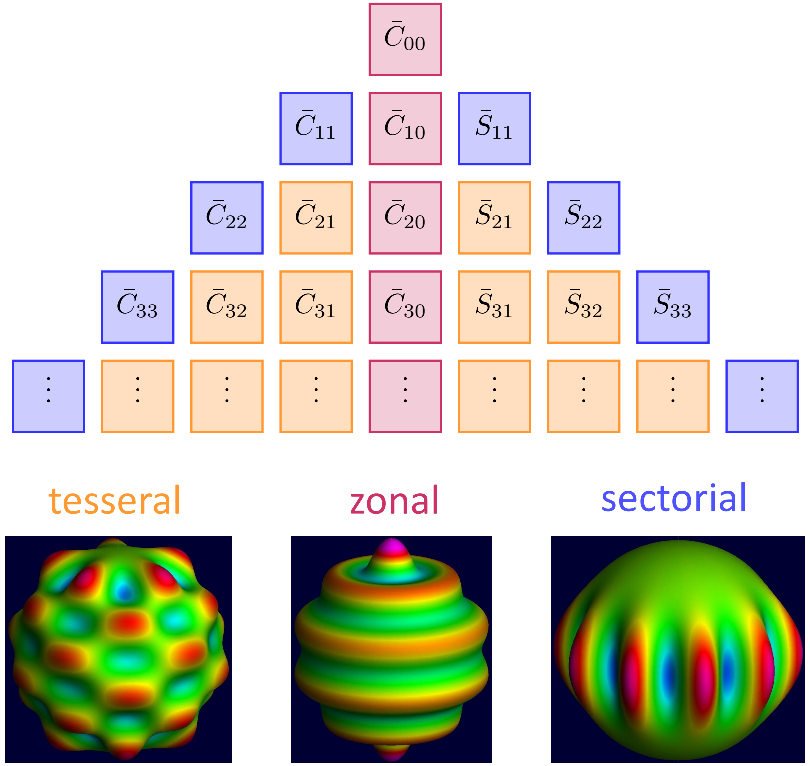
- gravity potential difference
- physical height difference



Our Earth's Gravity Field

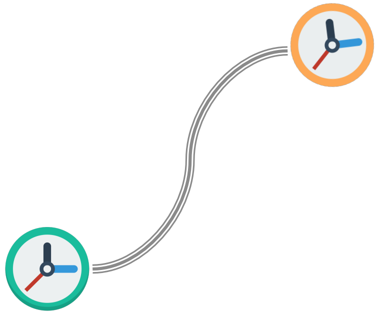
Mathematical expression of the global gravity field:

$$V = \frac{GM}{r} + \frac{GM}{R}\sum_{n=2}^{N}\left(\frac{R}{r}\right)^{n+1}\sum_{m=0}^{n}\left[\bar{C}_{nm}\cos(m\lambda) + \bar{S}_{nm}\sin(m\lambda)\right]\bar{P}_{nm}(\cos\theta)$$

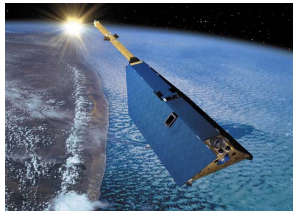


Earth's Gravity field observations:

- gravity potential
- gravity accelerations (**gravimetry**)
- gravity gradients (**gradiometry**)

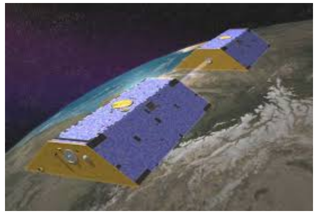


CHAMP



2000 – 2010

GRACE

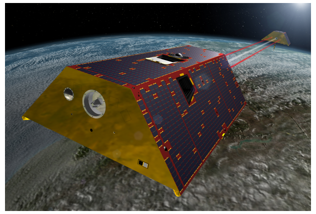


2002 – 2017

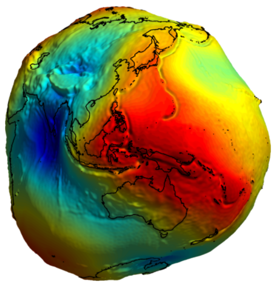
GOCE

2009 – 2013

GRACE-FO



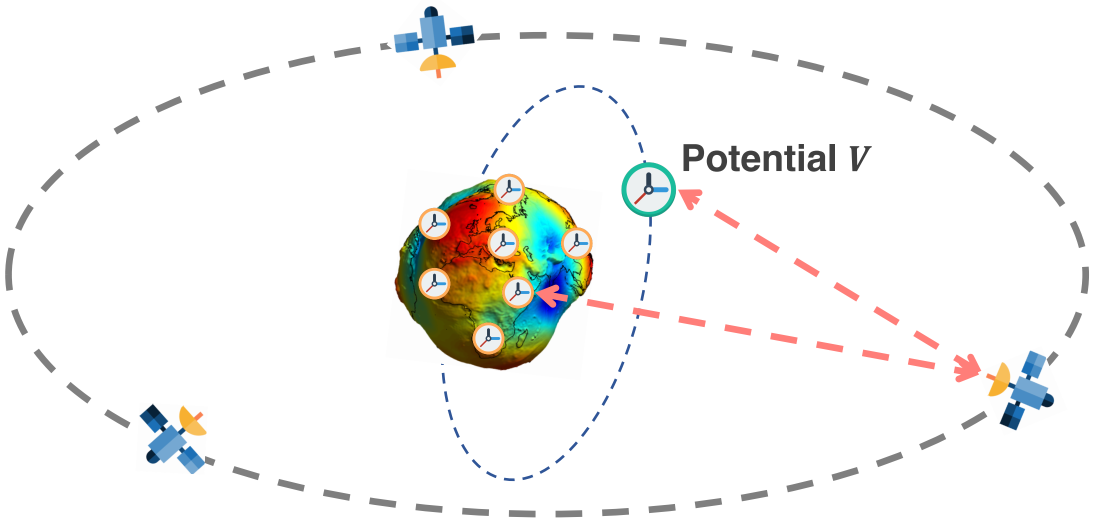
2018 – present



Space-Borne Clocks for Gravity Field Determination

Two possible cases by using space-based clocks for Earth’s gravity field observations

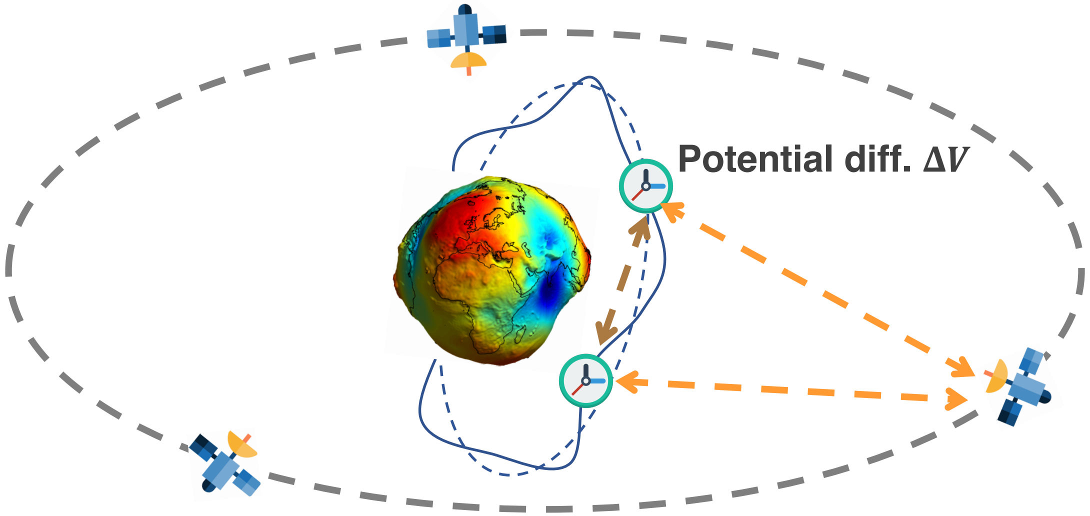
Case 1: Space-to-ground clock comparison



Reference clocks on ground   Target clock on satellite

Stringent requirement of a well-distributed reference clocks over the world

Case 2: Space-to-space clock comparison

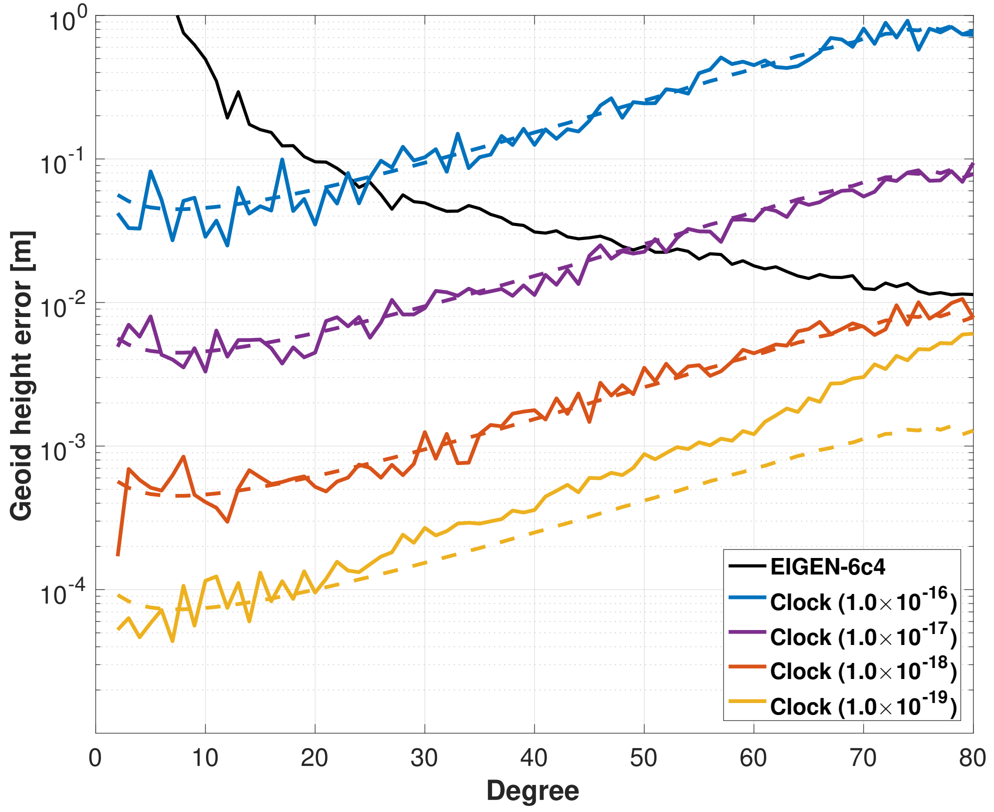


A pair of clocks on co-orbiting satellites

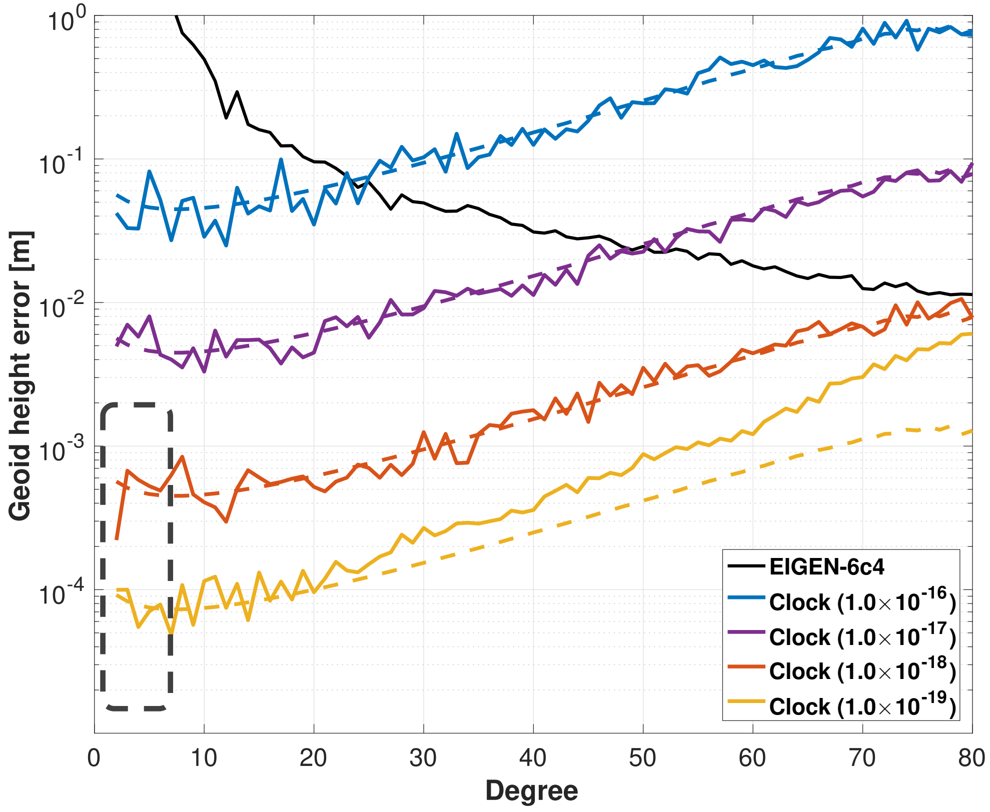
This case is considered for the simulation study.

Gravity Field Results

With clock noise only



With clock and AOD noise



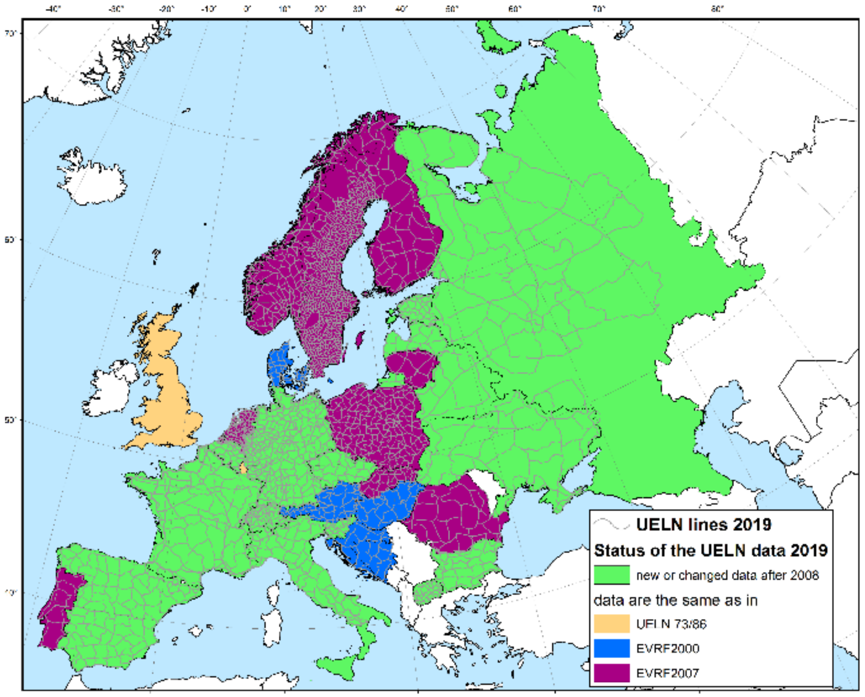
One-month GRACE orbit @ ~475 km

- Contribute mostly to very-low degree coefficients
- More robust to the AOD noise

Height Reference Frame / Levelling Network

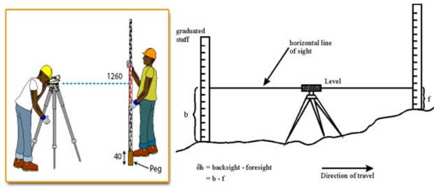
To realize an **International Height Reference System** with the level of **1 cm** is one major task of the International Association of Geodesy (IAG).

United European Levelling Network 2019

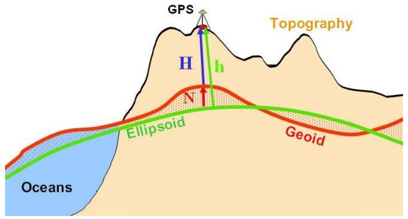


Methods to realize height systems:

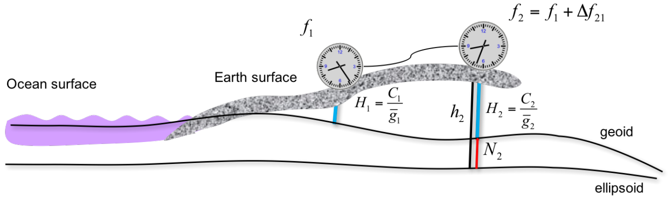
- **Geodetic levelling:**
  - Time, cost and labor consuming;
  - Expensive for maintenance



- **GNSS/geoid modelling:**
  - GNSS error in vertical direction;
  - Challenges in geoid modelling

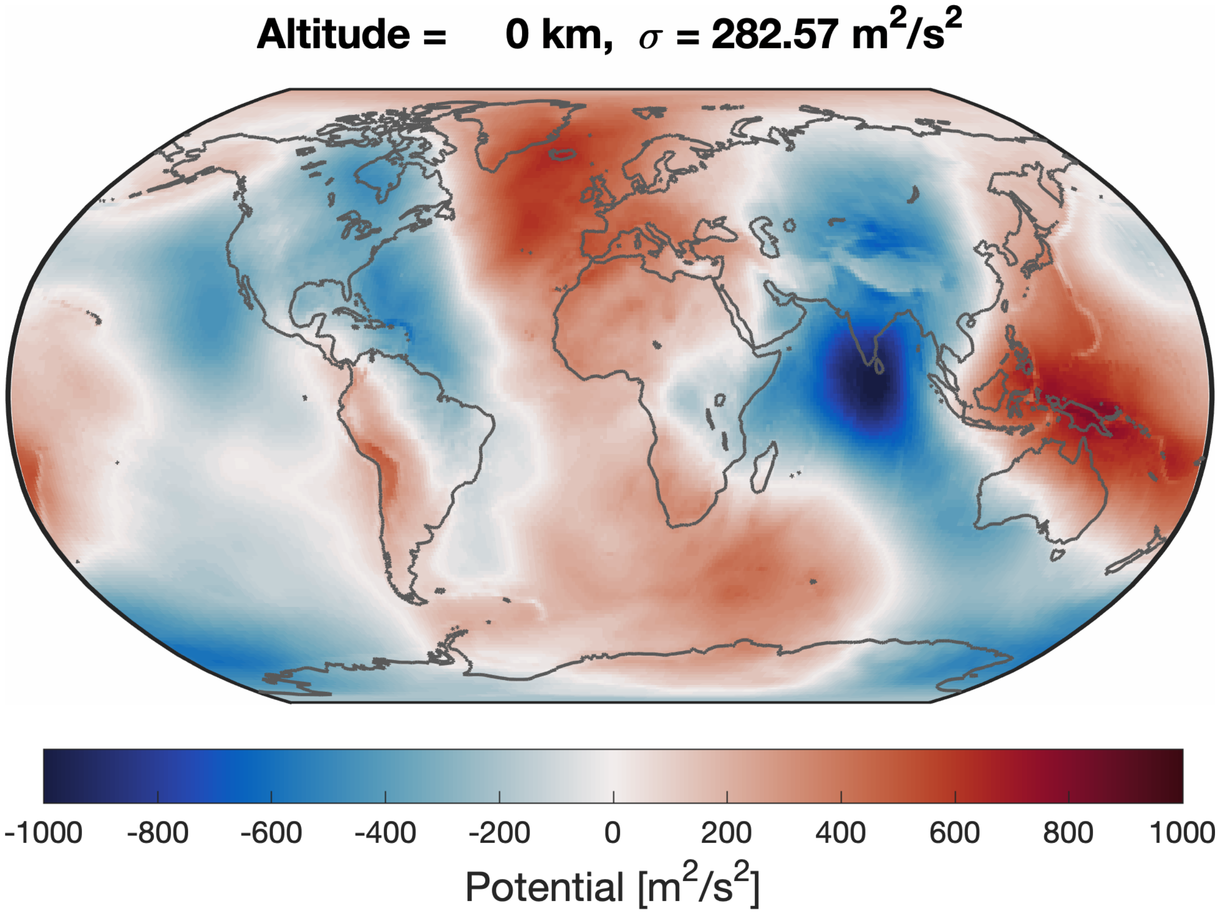


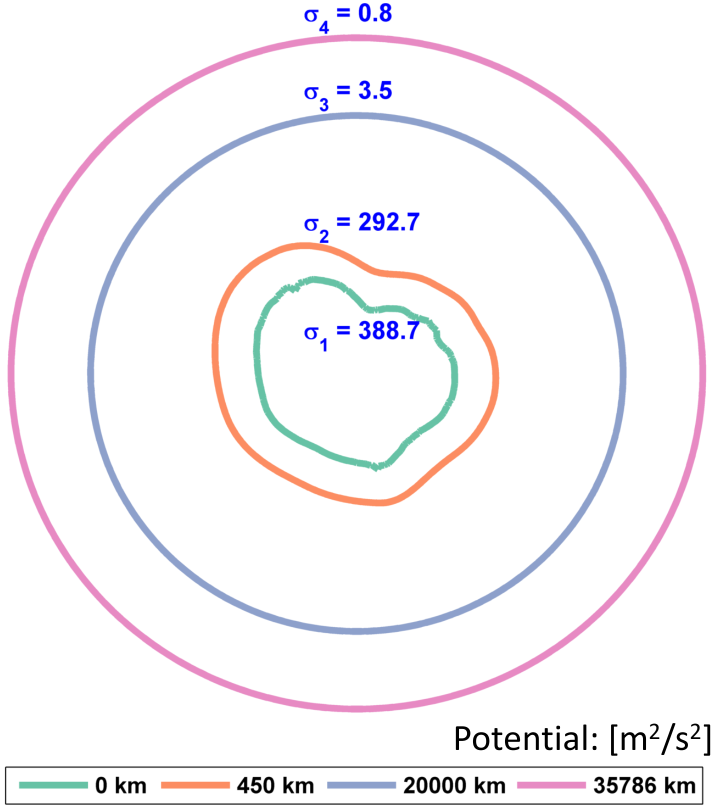
- **Chronometric levelling:**
  - Precisely determine height difference between distant points in days



Space Geodetic Reference Frame

Variations of gravity potential at different altitudes

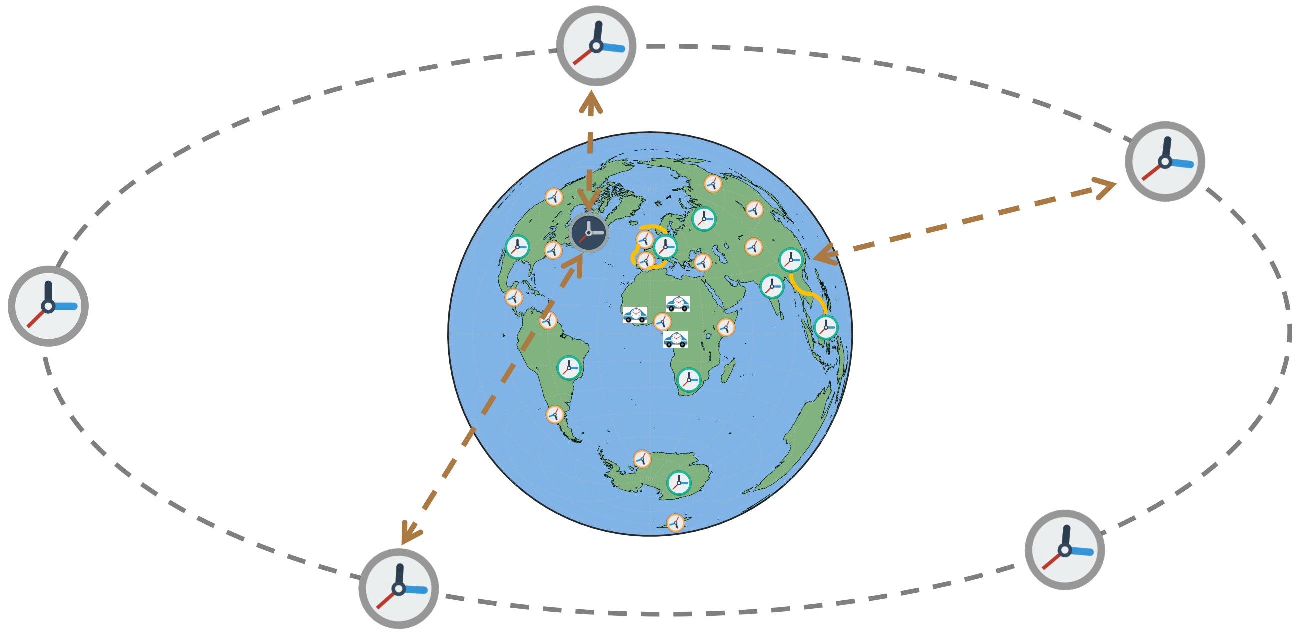




High orbits, e.g., the geostationary orbit, are good choices for reference clocks in space.

Space Geodetic Reference Frame

- spaced clock W_S
- datum clock W_0
- core clock W_P
- national clock W_P
- transportable clock



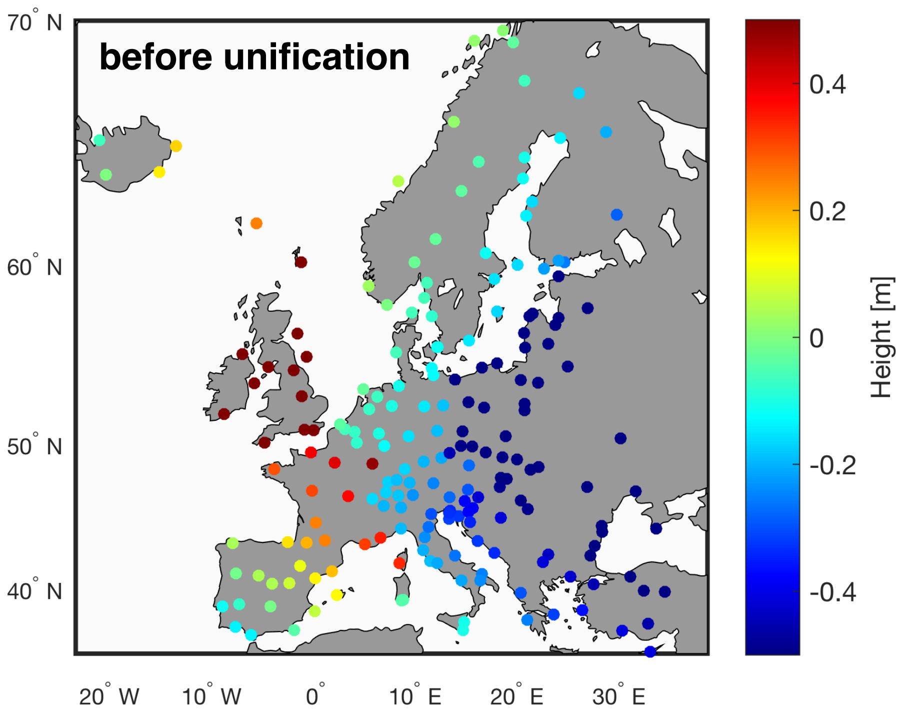
- a few clocks realize a reference equipotential surface in space
- a hybrid clock network serves as the backbone of a global height reference system
- stable, consistent, easily maintainable

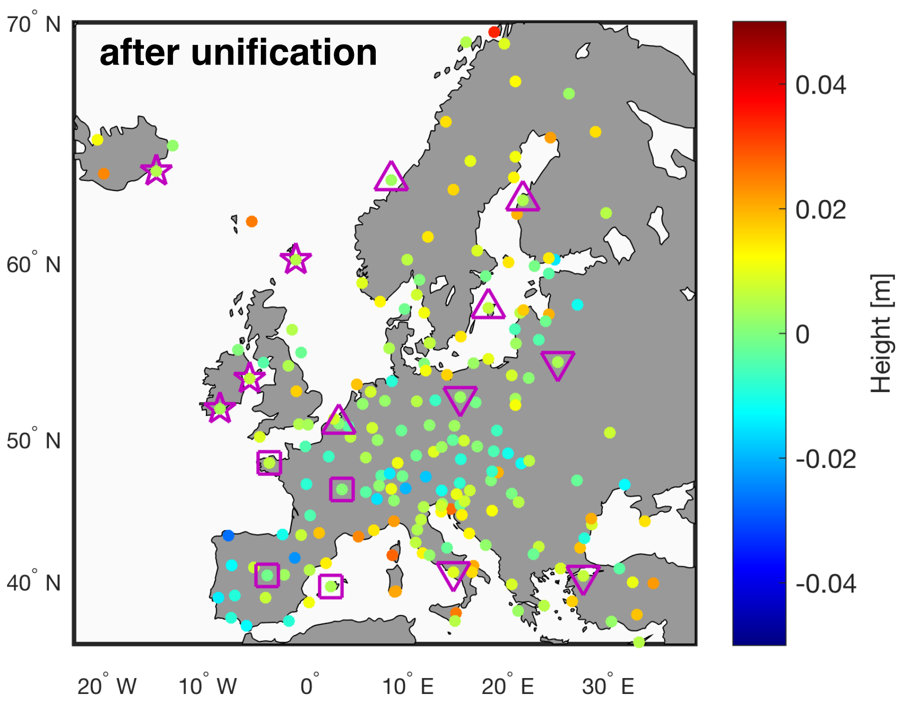
Other applications?

Unification of Local Height Systems

Clock networks can deliver accurate height differences between distant points. They are powerful for local height system unification, by identifying:

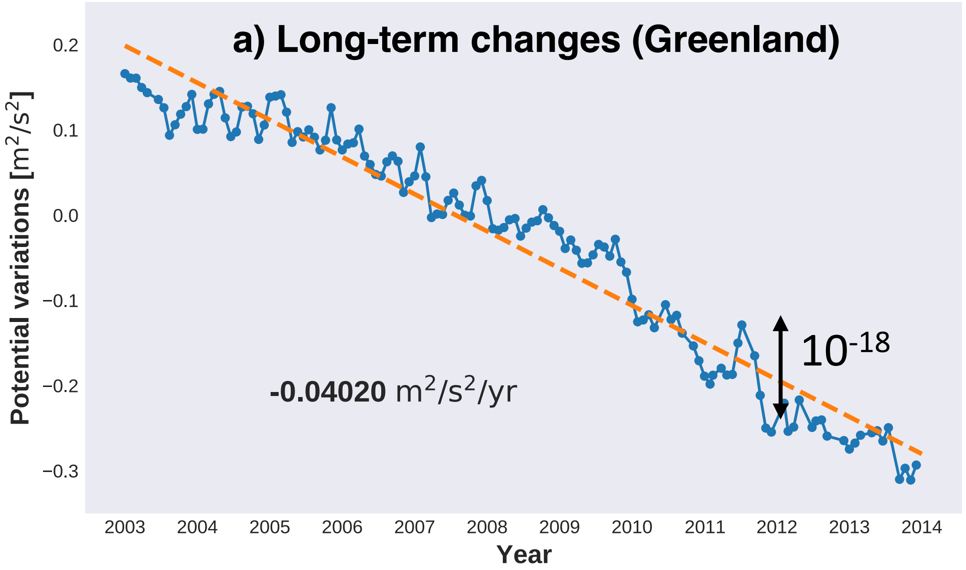
- discrepancies (offsets) between different height datums;
- systematic distortions (e.g., tilts) of regional levelling networks.

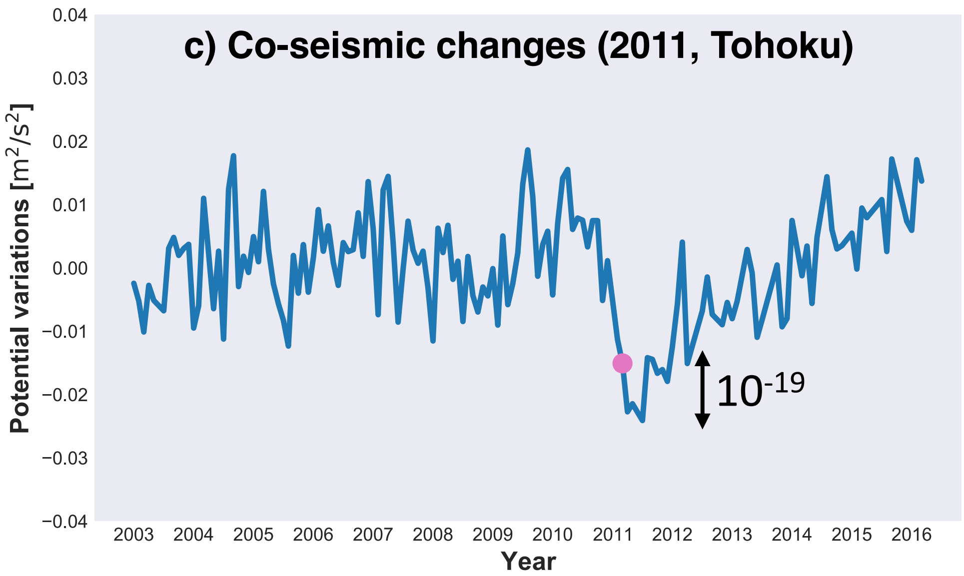


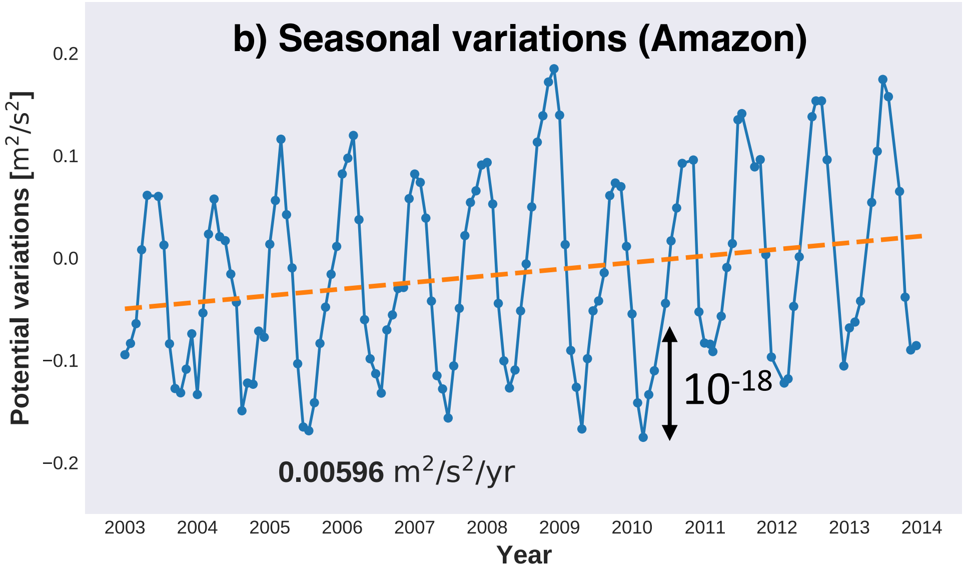


Time-Variable Gravity Signals Detection

- **Long-term signals**: gravity potential changes in Greenland (ice melting) reach **0.5 m^2/s^2** in 12 years;
- **Seasonal variations**: changes caused by annual and semi-annual hydrological cycles in Amazon reach **0.4 m^2/s^2**;
- **Short-term signals**: co-seismic changes of the great Tohoku earthquake in March 2011 (*M_W 9.0-9.1*) reach about **0.03 m^2/s^2**



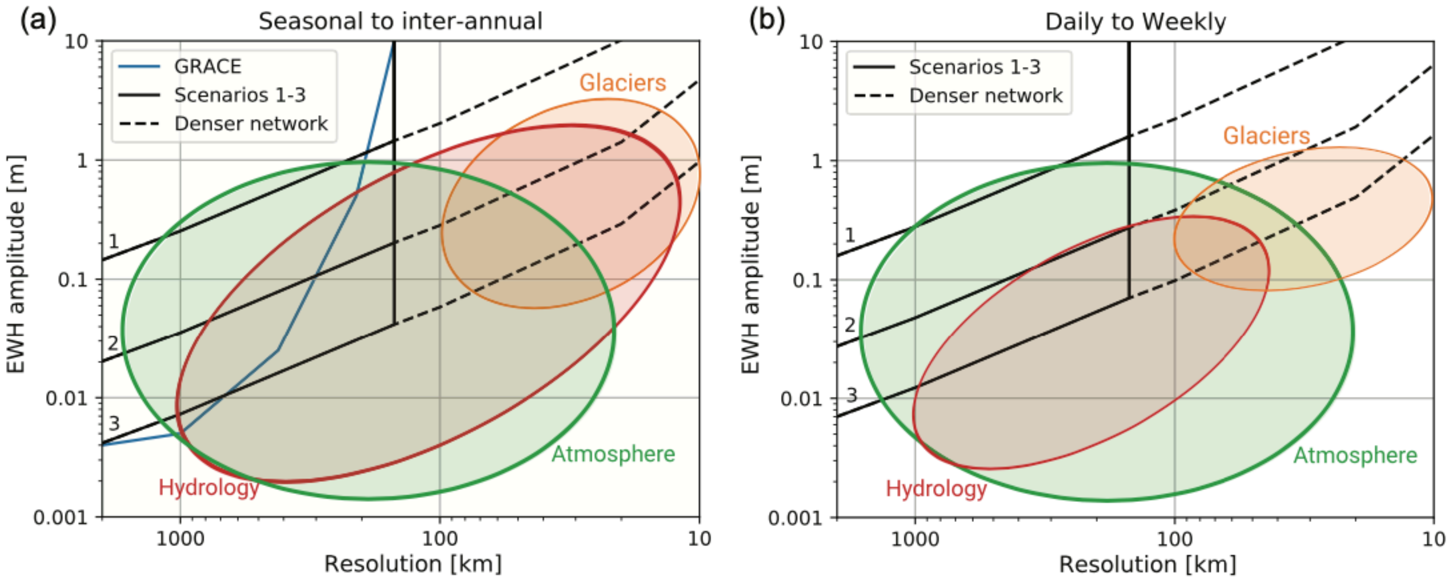




Validation of satellite-based time-variable gravity data

CloNetS-DS (Clock Network Services Design Study)

Clock networks would enable to determine temporal variations of the Earth's gravity field **at timescales of days and beyond**, e.g., the hydrological, glacial and atmospheric variations in Europe.

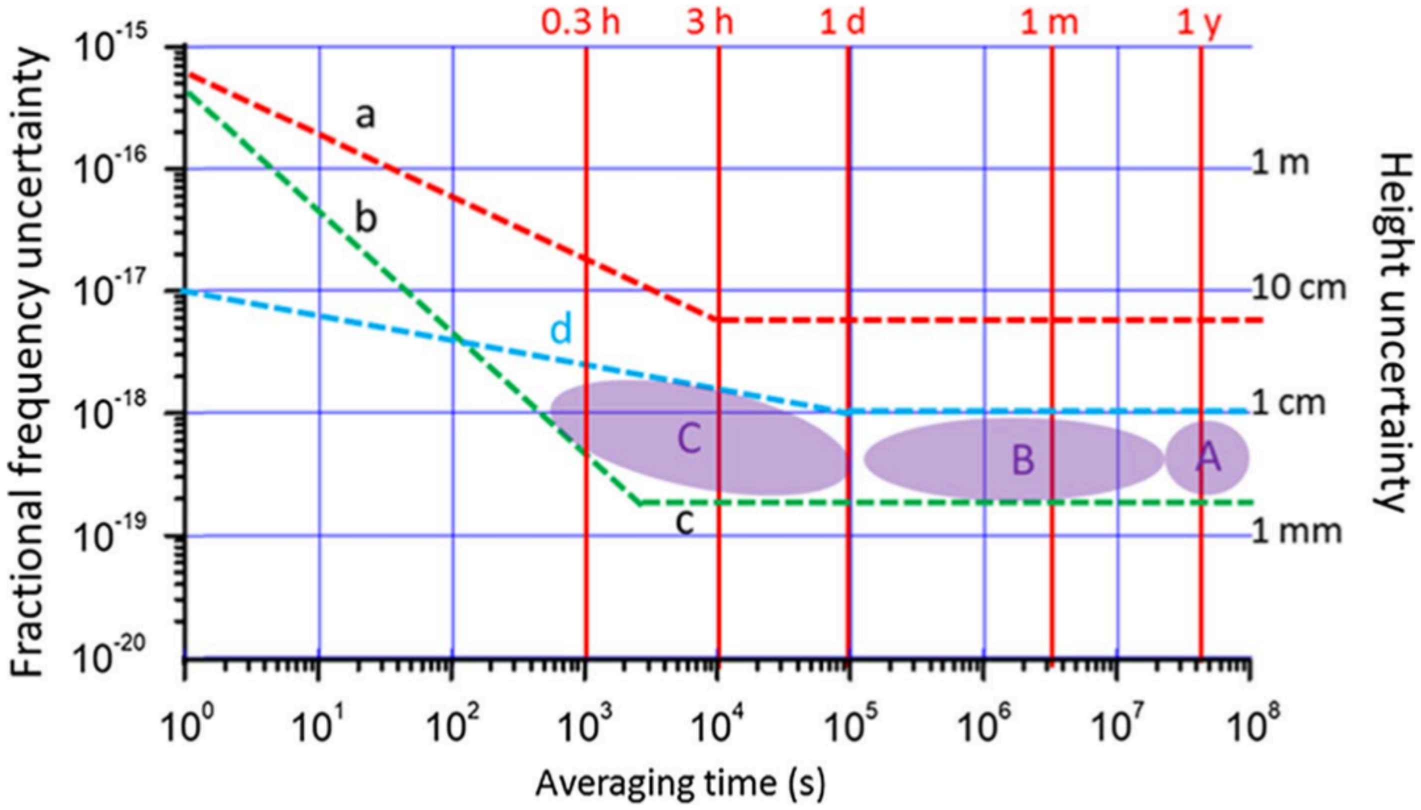


Stefan Schröder, Simon Stellmer, Jürgen Kusche (2021). Potential and scientific requirements of optical clock networks for validating satellite-derived time-variable gravity data. *Geophys. J. Int.*

Potential Applications in Plate Subduction Zone

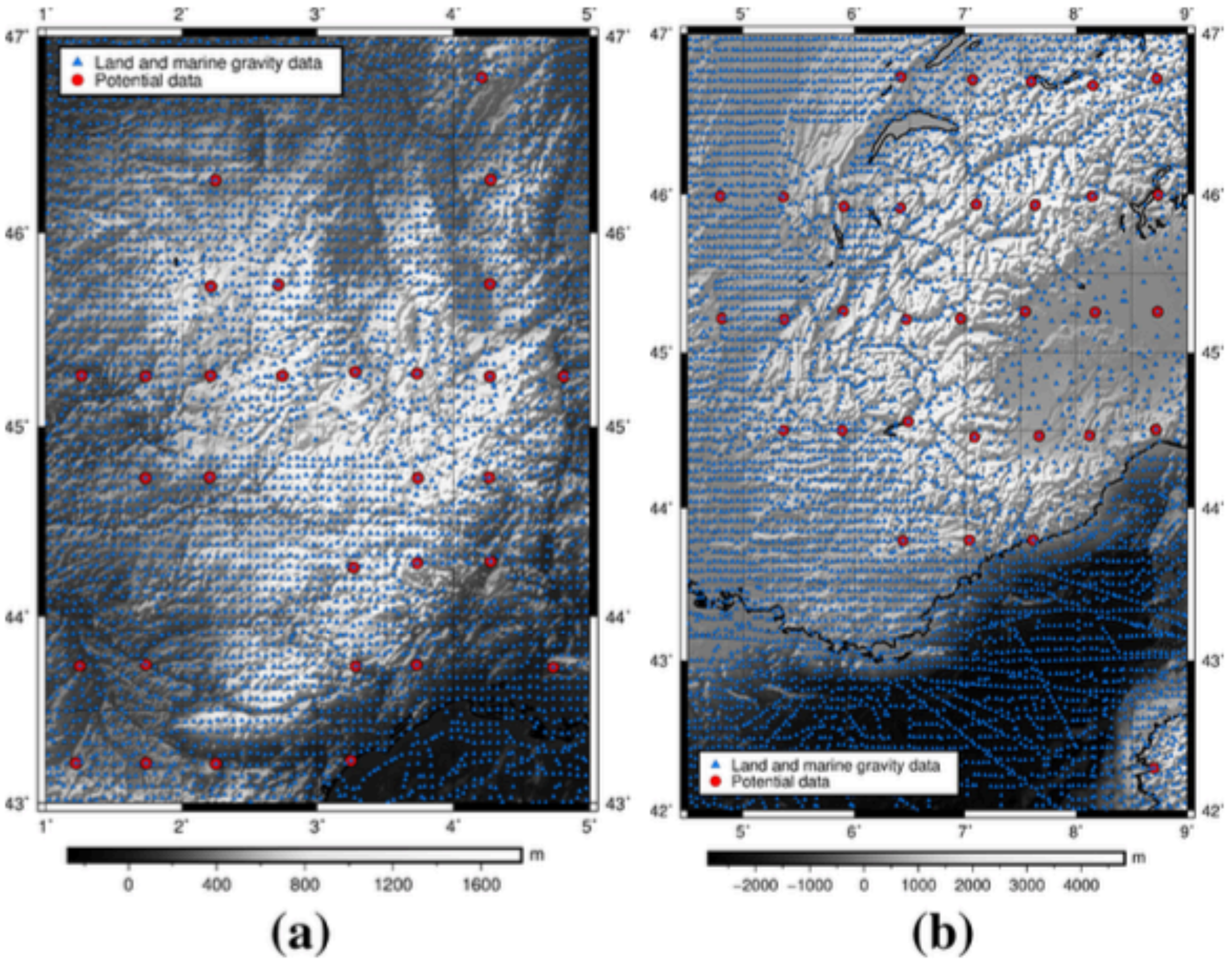
Clock networks are expected to (more precisely) detect

- A: seasonal crustal movements
- B: slow slip events
- C: solid earth and ocean tides which are not yet discovered



Yoshiyuki Tanaka and Hidetoshi Katori (2021). Expolring potential applications of optical lattice clocks in a plate subduction zone. *J Geod.*

High-spatial resolution geopotential model



Clock measurements can contribute to determine the regional gravity field model with a high spatial resolution:

- in particular in mountainous regions where the gravity coverage is sparsely distributed;
- stabilize the solution and reduce the modelling errors;

G. Lion et al. (2016). Determination of a high spatial resolution geopotential model using atomic clock comparisons. *J Geod*

Summary

- Space-borne clocks are capable to **directly acquire** the Earth's gravity potential values, which contribute most to the determination of the **very long-wavelength gravity field signals**;
- A hybrid clock network (including reference clocks in high orbits) is potentially to realize a **consistent, precise and stable** global height reference system;
- Clock networks are also powerful for the **unification of local height systems**, and might be **complementary** to satellite gravity missions like GRACE for the determination of **mass variations.**

TerraQ – Relativistic and Quantum-based Geodesy

QuantumFrontiers

Thanks for your attention!

wuhu@ife.uni-hannover.de