Lecture 3

Radiation Damping

Contents

Radiation Damping

In this lecture we will investigate the impact that the emission of synchrotron radiation has on the motion of the circulating electrons in all three planes.

We will find that when combined with the re-acceleration in the RF cavities, the energy loss leads to a damping of the oscillations in each plane.

With the emission of synchrotron radiation, the system is not conservative; the Courant-Snyder Invariant (the area of the ellipse in phase space) is no-longer a constant of motion.

Because of the dependence on the rest mass of the particle, the emission of synchrotron radiation and subsequent radiation damping tends only to be significant for electron or position machines.

Radiation damping has many important consequences:

- It gives rise to a stable equilibrium distribution in each plane (emittance and energy spread), independent from the injected beam distribution
- It permits multi-cycle, off-axis injection, since the beam damps down towards the stored beam distribution before the next injection cycle
- It helps to counteract intra-beam scatter and instabilities

Recap: energy loss due to synchrotron radiation

Substitution of the Liénard-Wiechert (retarded) potentials into Maxwell's Equations gives rise to the Liénard-Wiechert fields:

$$\mathbf{E}(t) = \frac{e}{4\pi\varepsilon_0\gamma^2}\left(\frac{(\mathbf{n}-\boldsymbol{\beta})}{r^2(1-\mathbf{n}\cdot\boldsymbol{\beta})^3}\right)_{ret} + \frac{e}{4\pi\varepsilon_0 c}\left(\frac{\mathbf{n}\times[(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}]}{r(1-\mathbf{n}\cdot\boldsymbol{\beta})^3}\right)_{ret}$$

$$\mathbf{B}(t) = \frac{\mathbf{n}\times\mathbf{E}(t)}{c}$$

The fields have two components, a velocity field and an acceleration field. At large distances (the 'far-field'), only the acceleration field is significant.

Using just the acceleration field, the power radiated by the particle per unit solid angle is

$$\frac{dP}{d\Omega} = \frac{e^2}{(4\pi)^2\epsilon_0 c}\frac{\left[\mathbf{n}\times\left[(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right]\right]^2}{(1-\mathbf{n}\cdot\boldsymbol{\beta})^5}$$

The radiation is strongly peaked in the forwards direction (parallel with the velocity and perpendicular to the acceleration), with opening angle $\sim 1/\gamma$

Recap: energy loss due to synchrotron radiation

Considering the motion in a uniform bending magnet, we have for the instantaneous power radiated by a single electron:

$$P = \frac{e^2}{6\pi\epsilon_0 c^7}\frac{1}{m^4}\frac{E^4}{\rho^2}$$

To get the total energy radiated per turn (U_0), this expression needs to be multiplied by the time spend in the bending magnets, i.e.

$$T_B = \frac{2\pi\rho}{c}$$

giving

$$U_0 = \frac{e^2}{3\epsilon_0 c^8}\frac{1}{m^4}\frac{E^4}{\rho}$$

The average power (in Watts) radiated by an electron beam of current I_b (in Amps) with energy loss per turn U_0 (in eV) is

$$P_{tot} = U_0 I_b$$

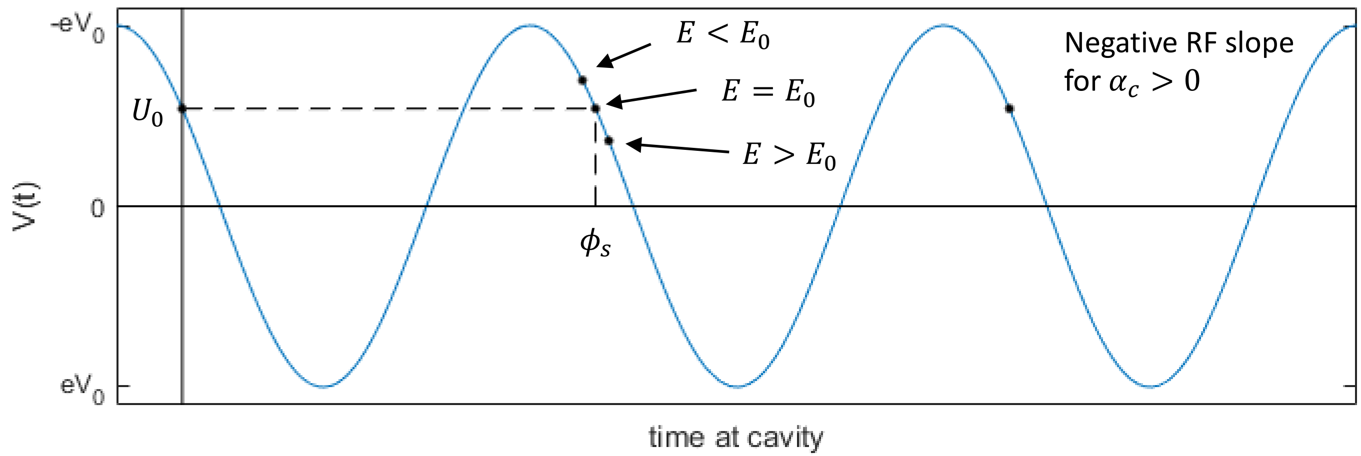
RF Cavity

RF cavities are used to supply energy to the beam. The longitudinal electric field oscillates with time, such that there is one phase per cycle that gives stable acceleration to the beam. The energy gain of the particle is

$$\Delta E = eV(t) = eV_0 \sin(\omega_{RF} t + \phi_s)$$

Assuming highly relativistic motion (i.e. above transition), the stable synchronous phase ϕ_s is the one that restores exactly the energy lost to dissipative processes

$$\Delta E = U_0 = eV_0 \sin(\phi_s)$$



Particles which are high in energy travel further in single revolution and will arrive later
Particles which are low in energy travel less distance and will arrive earlier

Energy Oscillations

We will use the notation

$$\tau = -(s - s_0)/c$$
$$\epsilon = E - E_0$$

where the subscript 0 refers to the synchronous (on-energy) particle. Using this notation, a particle at the head of the bunch will arrive early (negative τ), and a positive energy deviation ϵ implies a longer orbit length L and longer revolution period T. In this case, assuming changes in time and energy occur slowly with respect to the revolution time, using the definition of the momentum compaction factor α_c we can write ($v \sim c$)

$$\frac{d\tau}{dt} = -\alpha_c \frac{\epsilon}{E_0}$$

The rate of change of energy is the difference between energy lost and energy gained

$$\frac{\mathrm{d}\epsilon}{dt} = \frac{eV(\tau) - U(\epsilon)}{T_0}$$

which has the derivative

$$\frac{d^2\epsilon}{dt^2} = \frac{e}{T_0}\frac{dV}{d\tau}\frac{d\tau}{dt} - \frac{1}{T_0}\frac{dU}{d\epsilon}\frac{d\epsilon}{dt}$$

Energy Oscillations

This can be re-expressed as an equation of simple harmonic motion with an additional damping term

$$\frac{d^2\epsilon}{dt^2} + \frac{2}{\tau_\epsilon}\frac{d\epsilon}{dt} + \omega_s^2\epsilon = 0$$

where the synchrotron frequency ω_s and longitudinal damping time τ_ϵ are given by

$$\omega_s = \sqrt{\frac{e}{T_0}\frac{\alpha_c}{E_0}\frac{dV}{d\tau}}$$

$$\frac{1}{\tau_\epsilon} = \frac{1}{2T_0}\frac{dU}{d\epsilon}$$

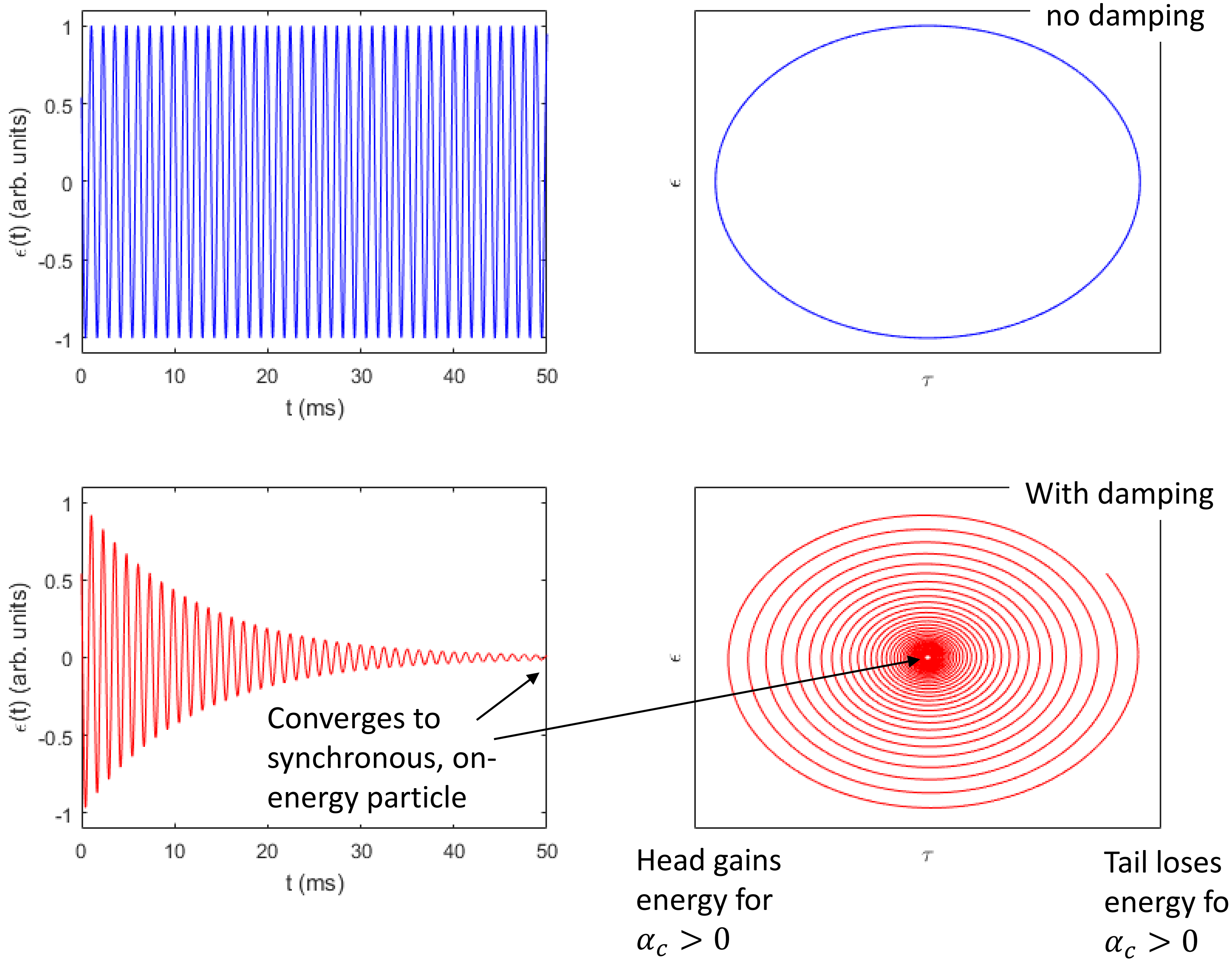
The solution of the damped oscillator equation is

$$\epsilon(t) = Ae^{-t/\tau_\epsilon}\cos(\omega_s t - \phi)$$

$$\tau(t) = \frac{A\alpha_c}{E_0\omega_s}e^{-t/\tau_\epsilon}\sin(\omega_s t - \phi)$$

where the constant A depends upon the particle's initial conditions

Energy Oscillations



Damping of Oscillations

From the previous analysis, the origin of the damping can be seen to have come from the change in energy loss per turn with energy:

$$\frac{1}{\tau_\epsilon} = \frac{1}{2T_0}\frac{dU}{d\epsilon}$$

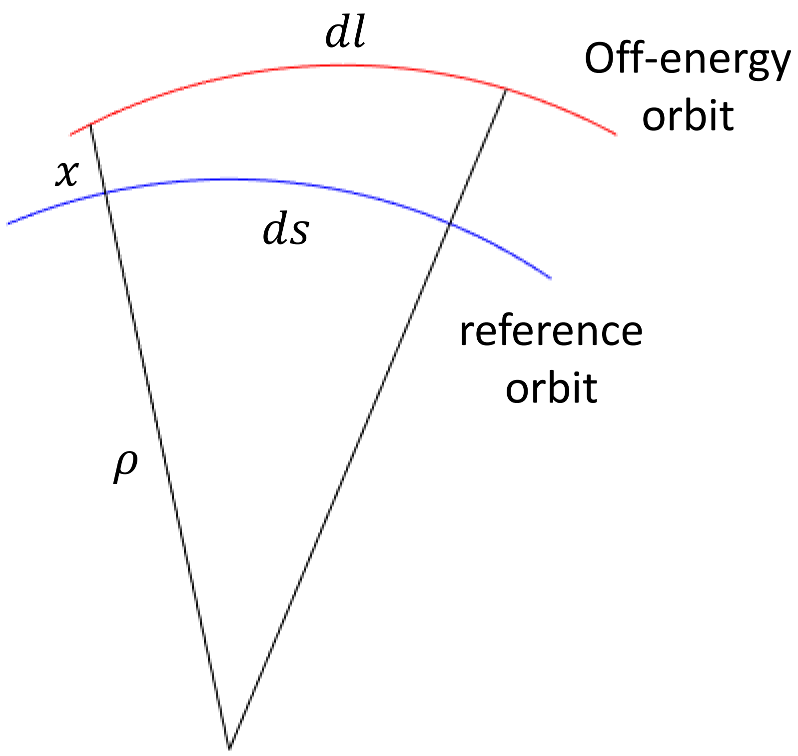
The energy loss per turn changes with particle energy for two reasons. Firstly, the power radiated is a function of the energy of the particle, and secondly because an off-energy particle will travel along a different path through the magnets. This different path changes the amount of time spent emitting radiation, and the magnetic field strength may well be different along the different orbit.

In order to progress, we need to determine the dependence of the energy loss per turn on energy and position:

$$U(\epsilon) = \oint P(\epsilon, x)dt$$

Damping of Oscillations

The difference in time spent in the dipoles depending upon energy can be found from purely geometrical considerations.



From the bend angle, the difference in path length between the on and off-momentum orbits is

$$dl = \left(1 + \frac{x}{\rho}\right) ds$$

Using the dispersion function $D(s)$, this can be re-written as

$$dl = \left(1 + \frac{D}{\rho}\frac{\epsilon}{E_0}\right) ds$$

Or in the time domain:

$$dt = \frac{1}{c}\left(1 + \frac{D}{\rho}\frac{\epsilon}{E_0}\right) ds$$

Damping of Oscillations

We now have for the energy loss per turn

$$U(\epsilon) = \frac{1}{c}\oint P(\epsilon, x)\left(1 + \frac{D}{\rho}\frac{\epsilon}{E_0}\right) ds$$

The next step is to Taylor expand $P(\epsilon, x)$ for small ϵ and x. From the lecture on synchrotron radiation, we know

$$P = \frac{e^2}{6\pi\epsilon_0 c^7}\frac{1}{m^4}\frac{E^4}{\rho^2} \propto E^2 B^2$$

since the bend radius is inversely proportional to $B(x)/E$. We can therefore say

$$P(\epsilon, x) = P_0 + \frac{2P_0}{E_0}\epsilon + \frac{2P_0}{B_0}\frac{dB}{dx}x$$

where P_0 is the power radiated by the on-energy particle following the design trajectory. This can be inserted into the equation for the energy loss (keeping terms up to first order in ϵ and expressing x in terms of the dispersion):

$$U(\epsilon) = \frac{1}{c}\oint\left(P_0 + \frac{2P_0}{E_0}\epsilon + \frac{2P_0}{B_0}\frac{dB}{dx}\frac{D}{E_0}\epsilon + \frac{P_0}{\rho}\frac{D}{E_0}\epsilon\right) ds$$

Damping of Oscillations

The derivative with energy deviation is therefore

$$\frac{dU(\epsilon)}{d\epsilon} = \frac{1}{c}\oint\left(\frac{2P_0}{E_0} + \frac{2P_0}{B_0}\frac{dB}{dx}\frac{D}{E_0} + \frac{P_0}{\rho}\frac{D}{E_0}\right)ds$$

Using the standard notation for a magnetic gradient $k = \frac{1}{B_0\rho}\frac{dB}{dx}$ we have for the final result

$$\frac{dU(\epsilon)}{d\epsilon} = \frac{2U_0}{E_0} + \frac{1}{cE_0}\oint P_0 D\left(\frac{1}{\rho} + 2k\rho\right)ds$$

and we can write the damping time as

$$\frac{1}{\tau_\epsilon} = \frac{1}{2T_0}\frac{dU}{d\epsilon} = \frac{1}{2T_0}\frac{U_0}{E_0}(2 + \mathcal{D})$$

where $\mathcal{D}$ is the integral

$$\mathcal{D} = \frac{1}{cU_0}\oint P_0 D\left(\frac{1}{\rho} + 2k\rho\right)ds$$

Damping of Oscillations

Given P_0 depends upon $1/\rho^2$, the $\mathcal{D}$ can also be expressed purely in terms of integrals involving standard lattice functions

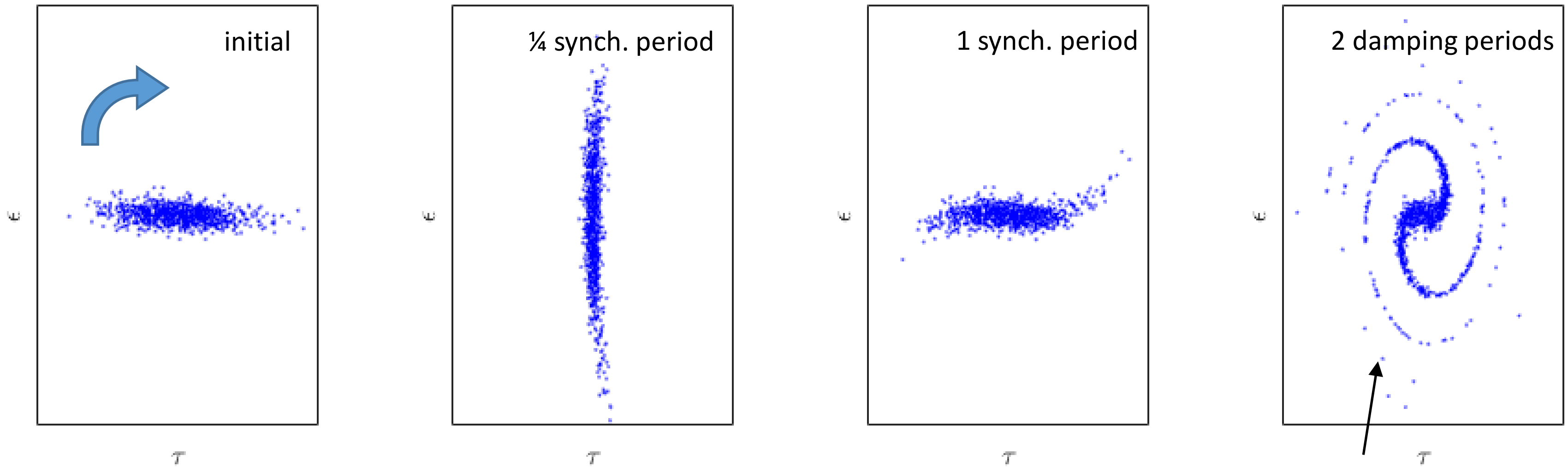
$$\mathcal{D} = \frac{\oint \frac{D}{\rho}\left(\frac{1}{\rho^2} + 2k\right) ds}{\oint \frac{1}{\rho^2} ds}$$

$\mathcal{D}$ is a dimensionless number that has contributions only in the bending magnets where ρ is non-zero, and the second term takes into account bending magnets with non-zero transverse gradient. It is typically a small positive number, and for stable motion (positive damping times) we have the condition $\mathcal{D} > -2$.

Roughly speaking, τ_ϵ is the time it would take for the particle to radiate away all its energy assuming a constant energy loss per turn of U_0

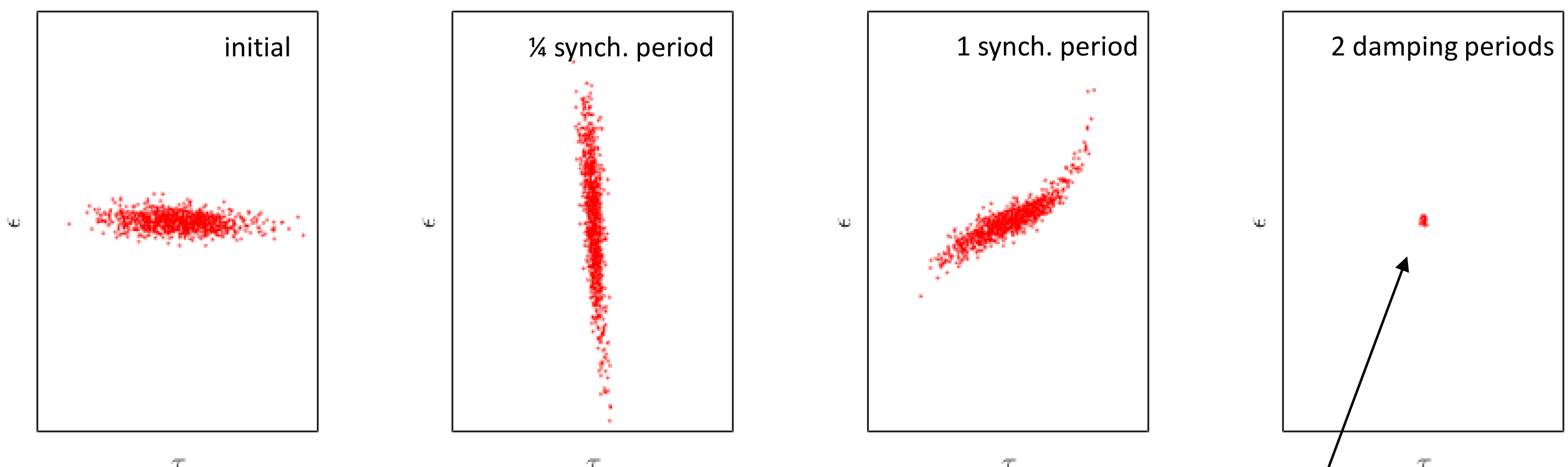
$$\tau_\epsilon = \frac{2T_0E_0}{U_0(2+\mathcal{D})} \approx \frac{T_0E_0}{U_0}$$

Without Radiation Damping



Strong filamentation from RF curvature

With Radiation Damping



Injected beam dimensions reduce in both τ and ϵ

Radiation Damping in the Vertical Plane

We now wish to consider what happens to particle oscillations in the vertical plane.

As in the longitudinal plane, the emission of synchrotron radiation causes the motion to become non-symplectic; the area of the phase-space ellipse is no-longer constant, and the Courant-Snyder Invariant changes on each turn.

For the analysis, it is convenient to define the particle coordinates as

$$y = A\cos(\phi(s) + \phi_0)$$
$$y' = -\frac{A}{\beta}\sin(\phi(s) + \phi_0)$$

and to assume we are looking at a position where the beam is focussed to a waist (i.e. at a location where α_y is zero). In this case, the distribution in phase space will form an upright ellipse, and we can write the Courant Snyder Invariant as

$$A^2 = y^2 + (\beta y')^2$$

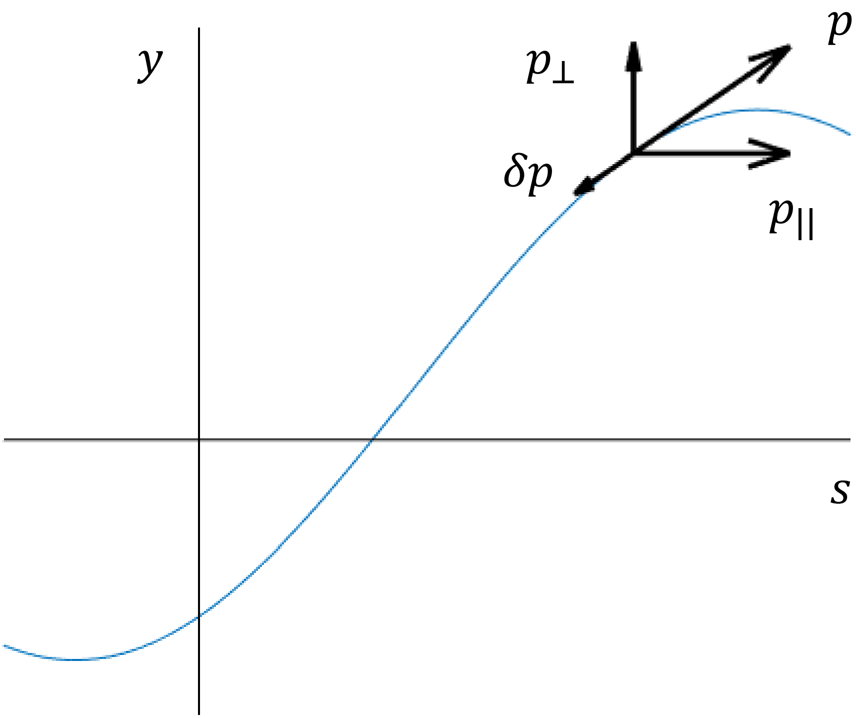
with A being the normalised amplitude.

Radiation Damping in the Vertical Plane

Emission of a photon:

When an electron emits a photon, its momentum is decreased by an amount δp. However, neither the position or the angle of the particle's motion change (y and y' constant).

Since the vertical dispersion is zero, the trajectory of the particle is also unchanged, and so at this point the Courant Snyder Invariant is also unchanged.



Radiation Damping in the Vertical Plane

In the RF cavity, the accelerating is purely in the longitudinal direction. As such, only the longitudinal component of the particle's momentum changes, giving rise to a change in the angle of the trajectory. This is the source of the damping in the vertical plane.

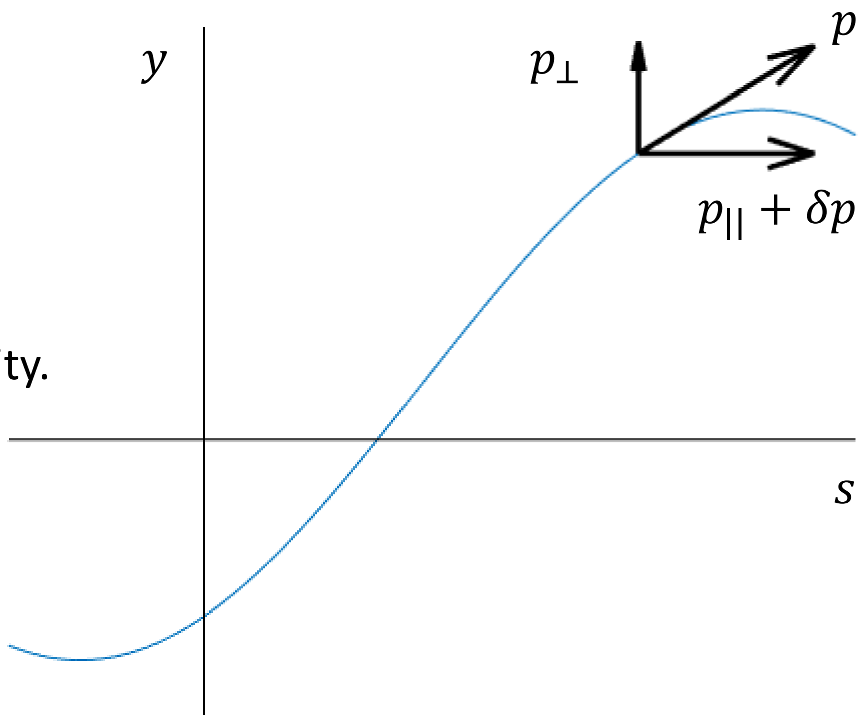
The slope of the trajectory is $y' = p_\perp / p_\parallel$, so after passing the cavity we have

$$y' + \delta y' = \frac{p_\perp}{p_\parallel + \delta p} \approx y' \left(1 - \frac{\delta p}{p} \right)$$

and so

$$\delta y' = -y' \frac{\delta \epsilon}{E_0}$$

where $\delta \epsilon$ is the energy gained in the cavity.



Radiation Damping in the Vertical Plane

After the particle has gained energy in the RF cavity, the Courant Snyder Invariant becomes

$$(A + \delta A)^2 = y^2 + \beta^2 (y' + \delta y')^2$$

which after expanding and keeping only first-order terms in $\delta y'$and δA leads to the result

$$A\delta A = \beta^2 y' \delta y' = -\beta^2 y'^2 \frac{\delta \epsilon}{E_0}$$

Clearly, the change in the invariant depends upon the particular angle y' for this particle. In order to determine what happens to the beam as a whole, we need to consider the full distribution of particles travelling on the ellipse which therefore all have the same initial invariant, A. Since all particles have different angles y', the change in the invariant will also be different. Averaging over all phases, and assuming a uniform distribution of particles, we have from the definition of y' ($y' = -\frac{A}{\beta}\sin(\phi(s) + \phi_0)$) the result

$$\langle y'^2 \rangle = \frac{A^2}{2\beta^2}$$

and we can write

$$\langle \delta A \rangle = -\frac{A}{2}\frac{\delta \epsilon}{E_0}$$

Radiation Damping in the Vertical Plane

Having established how the invariant changes due to the emission of a single photon, the next step is to calculate how the invariant changes due to the emission of many photons over the whole ring. In this case we have for the total energy lost

$$U_0 = \sum \delta\epsilon$$

The RF cavity replenishes the total energy lost. We find that in one turn

$$\Delta A = -\frac{AU_0}{2E_0}$$

So

$$-\frac{1}{A}\frac{dA}{dt} = \frac{U_0}{2E_0T_0} = \frac{1}{\tau_y}$$

The average invariant decreases exponentially with a characteristic time-constant that is approximately twice the longitudinal damping time.

Radiation Damping in the Vertical Plane

Due to the emission of synchrotron radiation, betatron oscillations are damped over time, and we can write the general result

$$y(t) = y_0 e^{-t/\tau_y} \sin(\omega_\beta t + \phi_0)$$

Since the emittance of the bunch of particles depends upon the square of the amplitude of oscillation

$$\varepsilon_y = \frac{\langle y^2 \rangle}{\beta}$$

We can see that the emittance will reduce with a time-constant that is half the radiation damping time, i.e.

$$\varepsilon_y(t) = \varepsilon_y(0) e^{-2t/\tau_y}$$

Radiation Damping in the Horizontal Plane

The treatment for the analysis of radiation damping in the horizontal plane follows the same steps as in the vertical plane, namely:

1. Consider an electron travelling on an ellipse in phase space with invariant A
2. Calculate the change in coordinates due to the emission of a single photon
3. Calculate the change in coordinates due to travelling through an RF cavity
4. Average over a distribution of particles which all sit on the same ellipse
5. Calculate the change in the invariant summing together the total number of photons emitted over one turn

The main difference between the two planes is the impact of dispersion:

- in the vertical plane the dispersion is zero, so the trajectory does not change after the emission of a photon
- In the horizontal plane, there is finite dispersion at the location where the photon is emitted, causing the particle to follow a new trajectory around the new off-energy orbit

Radiation Damping in the Horizontal Plane

After the emission of a photon, the physical position and angle of the particle in space does not change ($\delta x = \delta x' = 0$). However, because of the finite dispersion, the reference orbit that the particle follows does change.

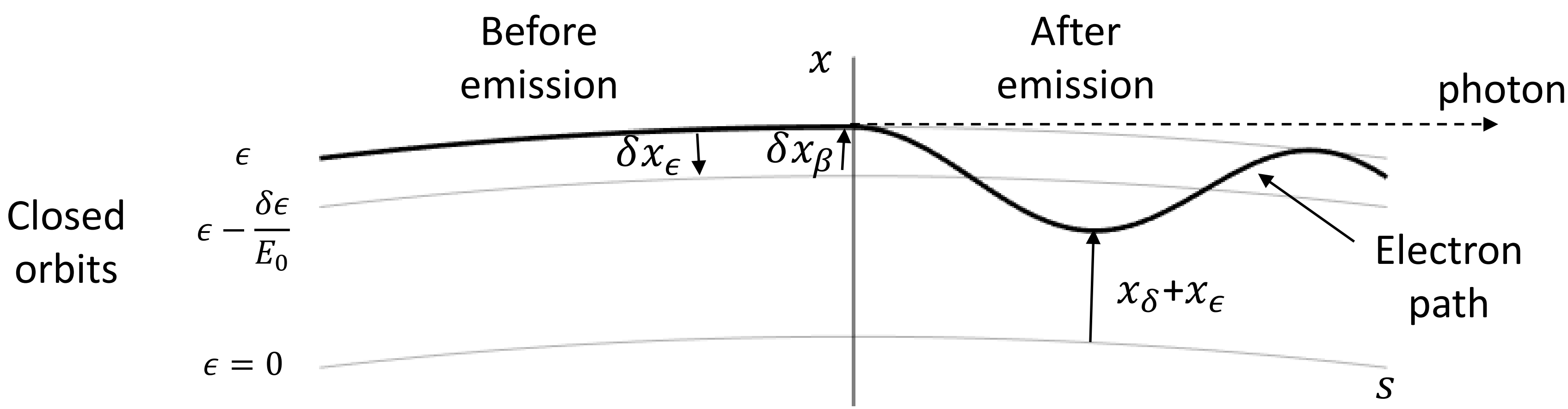
The particle will now follow a new off-energy orbit corresponding to the new energy of the particle. We have:

$$x = x_\epsilon + x_\beta$$

$$x_\epsilon = D\frac{\epsilon}{E_0}$$

So

$$\delta x_\beta = -\delta x_\epsilon = -D\frac{\delta\epsilon}{E_0}$$



Radiation Damping in the Horizontal Plane

We proceed as in the vertical plane. To begin, the equations for the horizontal position and angle are

$$x = A\cos(\phi(s) + \phi_0)$$
$$x' = -\frac{A}{\beta}\sin(\phi(s) + \phi_0)$$

And the invariant of motion is

$$A^2 = x^2 + (\beta x')^2$$

After the emission of a photon we have the new invariant

$$(A + \delta A)^2 = \left(x + \delta x_\beta\right)^2 + \beta^2\left(x' + \delta x'_\beta\right)^2$$

So to first order in δx, $\delta x'$ and δA we have

$$\begin{aligned} A\delta A &= x\delta x_\beta + \beta^2 x'\delta x'_\beta \\ &= -(Dx + \beta^2 D'x')\frac{\delta\epsilon}{E_0} \end{aligned}$$

Radiation Damping in the Horizontal Plane

Having determined how the Courant Snyder Invariant changes as a function of initial position and angle of the particle, the next step is to average over all possible values for particles with equivalent values of A. I.e., to calculate

$$\langle A\delta A\rangle = -\left(D\frac{\langle x\delta\epsilon\rangle}{E_0} + \beta^2 D'\frac{\langle x'\delta\epsilon\rangle}{E_0}\right)$$

If the value of $\delta\epsilon$ was independent of x and x' the averages in the above equation would be zero. However, just as we found for the analysis in the longitudinal plane, $\delta\epsilon$ depends on the trajectory through the bending magnets in two ways:

1) An offset in position and angle results in a longer path through the magnet, increasing the time spent radiating photons
2) The bending magnet may have a transverse gradient, giving rise to a change in magnetic field that depends on position

Let us now consider how this additional dependence on x and x' affects the analysis compared to the vertical plane.

Radiation Damping in the Horizontal Plane

The energy lost by the particle is given by

$$\delta\epsilon = -P(x)dt$$

and from the earlier analysis we have

$$dt = \frac{1}{c}\left(1 + \frac{x}{\rho}\right)ds$$

We also have the expression for the power

$$P(x) = P_0 + \frac{2P_0}{B_0}\frac{dB}{dx}x = P_0(1 + 2k\rho x)$$

So to first order in x we can write

$$\delta\epsilon = -\frac{P_0}{c}\left(1 + 2k\rho x + \frac{x}{\rho}\right)ds$$

Radiation Damping in the Horizontal Plane

The expression for the energy loss can now be inserted into the equation for the change in the Courant Snyder Invariant

$$\langle A\delta A\rangle = -\left(D\frac{\langle x\delta\epsilon\rangle}{E_0} + \beta^2 D'\frac{\langle x'\delta\epsilon\rangle}{E_0}\right)$$

Since $\langle x\rangle = \langle x'\rangle = \langle xx'\rangle = 0$ and $\langle x^2\rangle = A^2/2$, we are left with

$$\frac{\langle \delta A\rangle}{A} = \frac{P_0}{2cE_0} D\left(\frac{1}{\rho} + 2k\rho\right) ds$$

Integrating the energy lost over a whole turn gives

$$\frac{\Delta A}{A} = \frac{P_0}{2cE_0}\oint D\left(\frac{1}{\rho} + 2k\rho\right) ds$$

Which using the earlier definition of $\mathcal{D}$ can be re-written as

$$\frac{\Delta A}{A} = \frac{U_0\mathcal{D}}{2E_0}$$

Radiation Damping in the Horizontal Plane

As noted earlier, the quantity $\mathcal{D}$ is a small dimensionless number and is usually positive. As such, the emission of radiation in the dipoles would tend to *increase* the oscillation amplitude.

However, the change in the invariant from passing the RF cavity must also be taken into account (c.f. the vertical plane), i.e.

$$\frac{\Delta A}{A} = \frac{U_0 \mathcal{D}}{2E_0} - \frac{U_0}{2E_0}$$

Leading to the final result

$$-\frac{1}{A}\frac{dA}{dt} = \frac{U_0}{2E_0 T_0}(1 - \mathcal{D}) = \frac{1}{\tau_x}$$

As with the vertical plane, the average invariant decreases exponentially with a characteristic time-constant that is approximately twice the longitudinal damping time.

Radiation Damping in the Horizontal Plane

Just as we saw in the vertical plane, horizontal betatron oscillations are damped over time, and we can write the general result

$$x(t) = x_0 e^{-t/\tau_x} \sin\left(\omega_\beta t + \phi_0\right)$$

and since the emittance of the bunch of particles depends upon the square of the amplitude of oscillation

$$\varepsilon_x = \frac{\langle x^2 \rangle}{\beta}$$

we find that the emittance will reduce with a time-constant that is half the radiation damping time

$$\varepsilon_x(t) = \varepsilon_x(0) e^{-2t/\tau_x}$$

Damping Partition Numbers

The damping in the three places can be summarised as

$$\frac{1}{\tau_i} = \frac{J_i U_0}{2E_0 T_0}$$

The J_i are know as the damping partition numbers,

$$J_x = 1 - \mathcal{D} \qquad J_y = 1 \qquad J_\epsilon = 2 + \mathcal{D}$$

The sum of the damping partition numbers is constant for any lattice, i.e.

$$J_x + J_y + J_\epsilon = 4$$

This is known as the Robinson Theorem.

By adjusting $\mathcal{D}$ it is possible to transfer some damping from the longitudinal to the horizontal plane or vice versa. In order to have stable motion in all three places we require

$$J_i > 0$$

$$-2 < \mathcal{D} < 1$$

Synchrotron Radiation Integrals

Many important properties of the stored beam in an electron storage ring are determined using integrals of the lattice parameters taken around the whole ring. These are known as the synchrotron radiation integrals:

$$I_1 = \oint \frac{D_x(s)}{\rho(s)} ds$$

$$I_2 = \oint \frac{1}{\rho^2(s)} ds$$

$$I_3 = \oint \frac{1}{|\rho^3(s)|} ds$$

$$I_4 = \oint \frac{D_x(s)}{\rho(s)} \left(\frac{1}{\rho^2(s)} + 2k(s) \right) ds$$

$$I_5 = \oint \frac{H_x(s)}{|\rho^3(s)|} ds$$

where H_x is the so-called chromatic (dispersion) invariant

$$H_x(s) = \gamma_x(s) D_x^2(s) + 2\alpha_x(s) D_x(s) D_x'(s) + \beta_x D_x'^2(s)$$

Synchrotron Radiation Integrals

Using these integrals, the calculation of many storage ring parameters becomes simplified. For example, we have:

Energy loss per turn:

$$U_0 = \frac{e^2}{6\pi\epsilon_0}\gamma^4 I_2$$

Damping Partition Numbers:

$$\mathcal{D} = \frac{I_4}{I_2} \qquad J_x = 1 - \frac{I_4}{I_2} \qquad J_\epsilon = 2 + \frac{I_4}{I_2}$$

Damping times:

$$\frac{1}{\tau_x} = \frac{r_0\gamma^3}{3T_0}(I_2 - I_4) \qquad \frac{1}{\tau_y} = \frac{r_0\gamma^3}{3T_0}I_2 \qquad \frac{1}{\tau_\epsilon} = \frac{r_0\gamma^3}{3T_0}(2I_2 + I_4)$$

where r_0 is the classical electron radius

Adjusting the damping rates

There are many potential reasons for adjusting the damping rates:

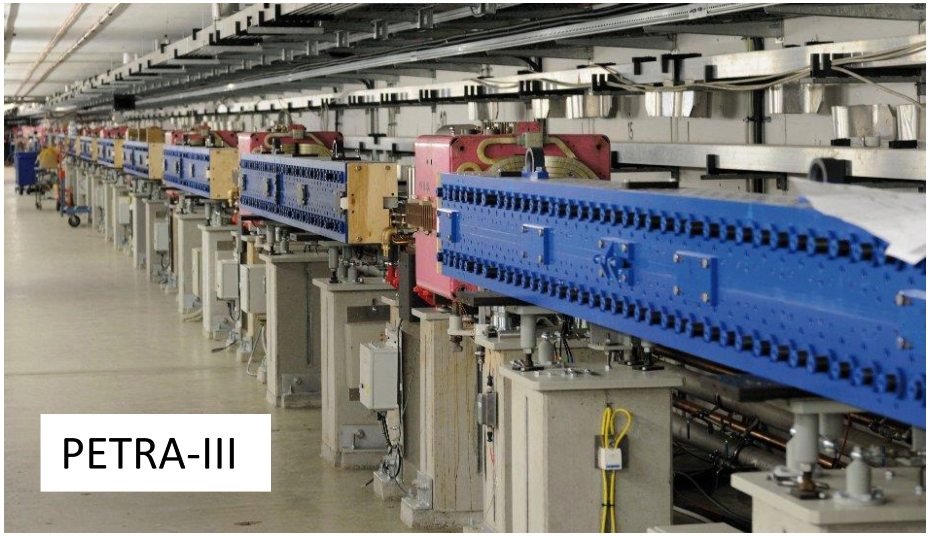
Reducing the horizontal damping time:

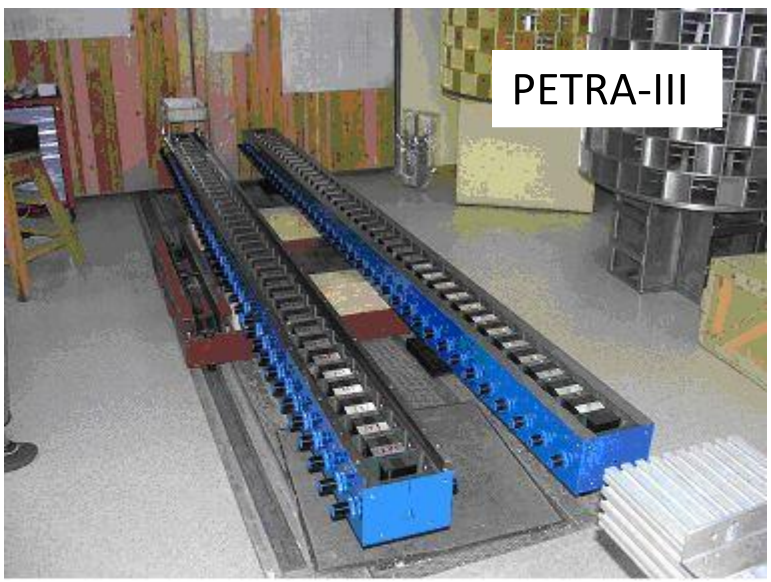
- Reduces the equilibrium electron beam emittance (damping rings, light sources)
- Eases off-axis injection or increases rep-rates for damping rings
- Increases the longitudinal damping time, causing the bunch length to increase (longer lifetime)
- Overcomes radial anti-damping present in some early synchrotron designs (e.g. CEA)

There are several techniques which can be employed to achieve this:

- Damping wigglers in dispersion-free straights (increases U_0, reducing all damping times)
- Combined-function bending magnets
- Robinson wigglers
- Change in RF frequency to displace the beam in the quadrupole magnets

Damping Wigglers





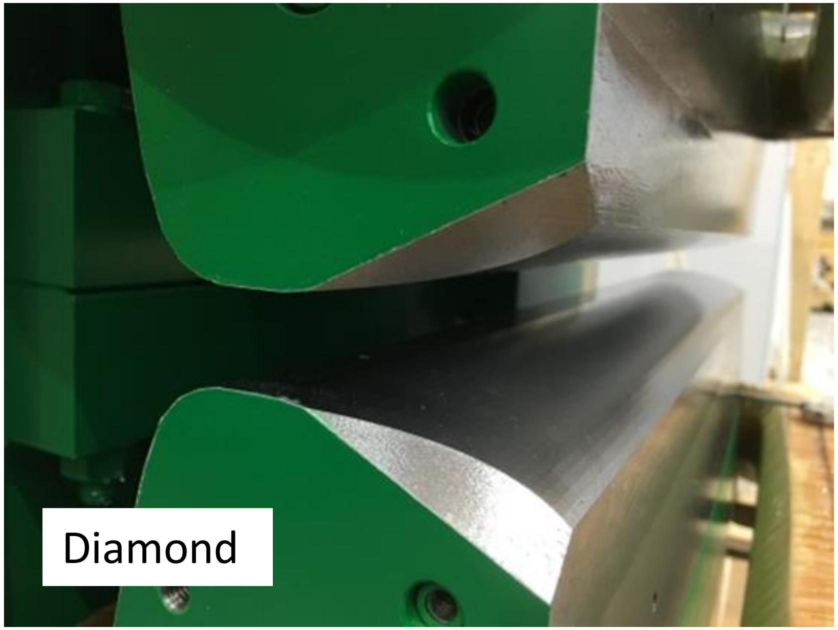
Damping wigglers can be one way of increasing the synchrotron radiation emission.

They have the effect of lowering the equilibrium emittance in electron storage rings (e.g. PETRA-III, NSLS-II) and reducing the damping times in damping rings (many designs, e.g. NLC, ILC, CLIC, …)

They reduce the damping times in all three planes simultaneously by increasing the I_2 synchrotron radiation integral:

$$\Delta I_2 = \frac{L_{wig}}{2\rho_{wig}^2}$$

Combined-function Bending Magnets



Combined-function bending magnets have both a main dipole field for steering the beam, with a gradient super-imposed on top.

Varying the gradient alters the damping partition numbers, transferring damping from the longitudinal to the transverse plane or vice versa

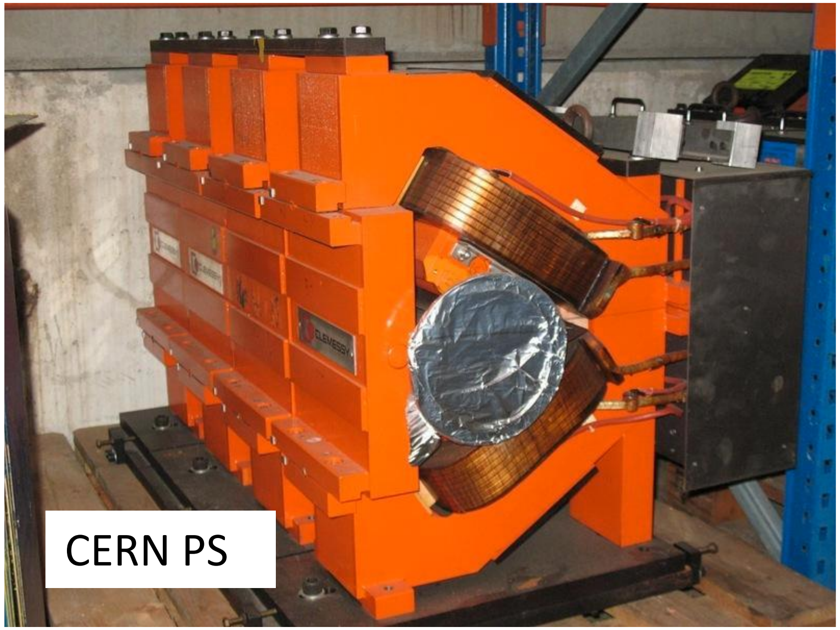
$$I_4 = \oint \frac{D_x(s)}{\rho(s)} \left(\frac{1}{\rho^2(s)} + 2k(s) \right) ds$$

Damping Partition Numbers:

$$\mathcal{D} = \frac{I_4}{I_2} \qquad J_x = 1 - \frac{I_4}{I_2} \qquad J_\epsilon = 2 + \frac{I_4}{I_2}$$

i.e. a dipole with negative gradient $k(s)$ will reduce I_4, increasing (decreasing) the horizontal (longitudinal) partition number and lowering (increasing) the emittance (energy spread)

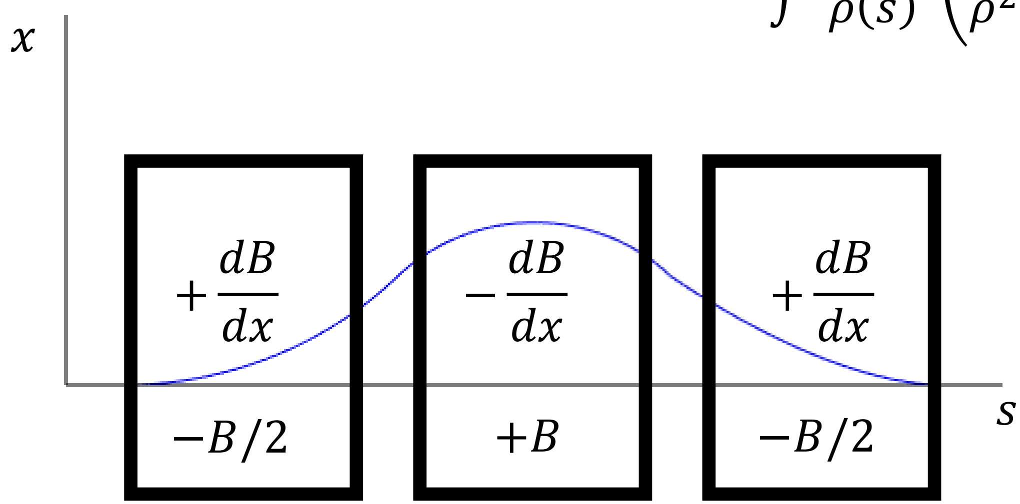
Robinson Wigglers



Robinson Wigglers originally proposed as a way of overcoming the horizontal anti-damping present in combined-function strong focussing lattices such as CEA ring.

Acts to reduce I_4 integral by ensuring product of k/ρ is negative, increasing the damping in the horizontal plane (needs dispersion!)

$$I_4 = \oint \frac{D_x(s)}{\rho(s)}\left(\frac{1}{\rho^2(s)} + 2k(s)\right) ds$$



Change in RF frequency

Altering the RF frequency can also be used as a method for modifying the damping partition numbers.

Changing the RF frequency has the effect of changing the orbit length, forcing the electrons to move to an off-energy orbit:

$$\frac{\Delta f_{RF}}{f_{RF}} = -\frac{\Delta L}{L} = -\alpha_c \frac{\epsilon}{E_0}$$

If the dispersion is non-zero, the change in energy results in a change in position $x(s) = D(s)\epsilon/E_0$, and at the quadrupoles this gives an additional dipole field component for the electrons

$$\frac{1}{\rho_{quad}} = k(s)D(s)\frac{\epsilon}{E_0}$$

The result of which is a change in $\mathcal{D}$

$$\Delta\mathcal{D} \approx \frac{\oint 2D^2k^2\epsilon/E_0 ds}{\oint 1/\rho^2\, ds}$$

Summary

Emission of synchrotron radiation combined with (longitudinal) energy gain in the RF cavities causes a damping of particle oscillations in all three planes of motion

The damping times are inversely proportional to (energy)^3

Since it depends upon the emission of synchrotron radiation, damping is usually only significant for light particles (electrons and positrons)

The dependence on the magnetic lattice parameters can be usefully summarised with the synchrotron radiation integrals

Assuming constant energy loss, the sum of the damping partition numbers is conserved

Damping times can be modified for a given storage ring design, either by increasing the energy loss per turn (damping wigglers), or by transferring damping from one plane to another (i.e. by altering the damping partition numbers)

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