

## ***Accelerator Physics Exercises No. 4***

***- Answers to be handed in on 25 November 2021***

### **Question 4.1**

Show that the RF power loss in the conducting walls of a cavity is given by the relation

$$\frac{R_{surf}}{2} \int_S |H|^2 ds$$

where  $R_{surf}$  is the surface resistance and  $S$  is the inner surface of the structure.

### **Question 4.2**

Assuming a simple pill-box cavity, show that the Quality Factor is related to the surface  $S$ , volume  $V$  and skin depth  $\delta$

$$\frac{2}{\delta} K \frac{V}{S}$$

where  $K$  is a form factor.

### **Question 4.3**

A new 50 GeV (kinetic energy) proton synchrotron, the PS2 accelerator, has been proposed to replace the CERN PS. The new PS2 will be in a new ring tunnel of mean radius 215 m. and will receive an injected beam at 4 GeV (kinetic energy) from a new linear accelerator - the Superconducting Proton Linac (SPL). The 1.8 T magnetic field of the bending magnets is excited by a sine wave oscillating between injection and top energy at a frequency of 0.3 Hz. Given that the mass of the proton is 0.9383 GeV:

- a) What is the revolution frequency at 4 GeV, 20 GeV and 50 GeV?
- b) Assuming the revolution frequency at 20 GeV, calculate the voltage per turn necessary to match the maximum rate of the rise of the field.
- c) If  $\sin \varphi_s = \sin 60^\circ$ , what is the peak voltage necessary in the cavity? Note that  $\varphi_s=0$  corresponds to the zero crossing of the accelerating voltage and the particle is not accelerated.
- d) Given a harmonic number of 32, what are the RF frequencies at 4 GeV and 50 GeV for  $\varphi_s=0$ ?

#### Question 4.4

Design a pill-box cavity 50 cm. long for a representative 10 MHz frequency for the PS2 accelerator. What is the resonant frequency for the next highest mode ( $TM_{011}$ ) in the cavity? What is the transit-time factor for protons of 4 GeV (kinetic energy) for the PS2 accelerator?

#### Question 4.5

How strong must the electric field intensity of a travelling plane wave be to accelerate electrons with an energy gradient of 10 MeV/m? (Hint: Use the Poynting vector).

#### Question 4.6

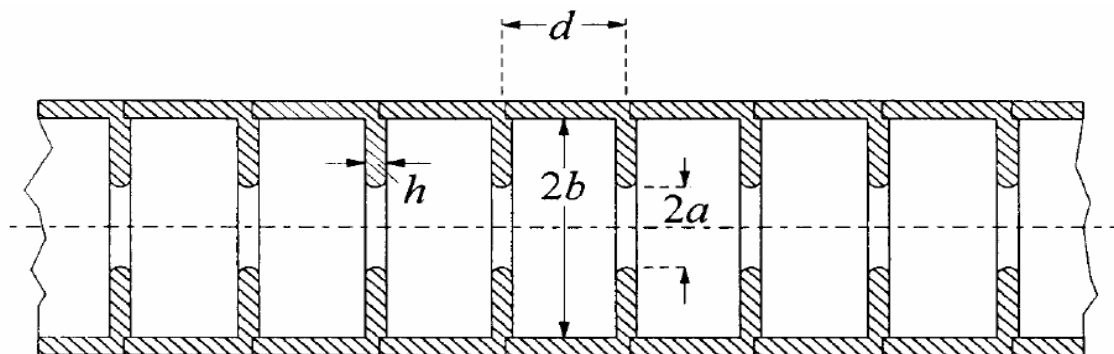
The SLAC accelerating linac structure has the design shown in the figure below with the following dimensions:

$$2b = 82.474 \text{ mm.}$$

$$2a = 22.606 \text{ mm.}$$

$$h = 5.842 \text{ mm.}$$

$$d = 35.001 \text{ mm.}$$



Assuming that the cavities are operated in the  $2\pi/3$  mode with a phase velocity  $\beta_z = 1$  and a supplied power of 35 MW, what is the total accelerating voltage and energy gain per metre for a SLAC structure of length  $l = 3$  m.?