Longitudinal Problem Set

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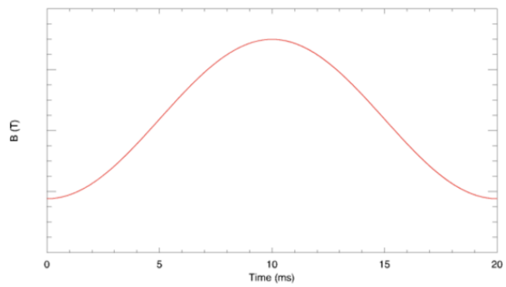
Due in 10^{th} November 2021

(a) ISIS Synchrotron



(b) ISIS dipole field as a function of time

Problem 1.1. ISIS is a proton synchrotron that operates from 70 to 800 MeV on a 50 Hz sinusoidal main magnet field (see figure b) above). Given the main dipole field at 70 MeV is 0.17639 T calculate the magnetic rigidity at that energy, the bending radius ρ and the magnetic rigidity and dipole field at top energy (800 MeV).

Problem 1.2. If 26.8% of the circumference is taken up with dipoles what is the mean radius R of the synchrotron? Calculate the revolution frequency at 0 ms (70 MeV) and 10 ms (extraction at 800 MeV).

Problem 1.3. Calculate the following parameters at 0, 5 and 10 ms

a. Momentum

b. Kinetic Energy

c. Relativistic parameters β, γ

d. What does γ_t have to be for ISIS to remain below transition throughout acceleration?

Problem 1.4. What is the minimum RF voltage required as a function of time (0 - 10 ms) to accelerate a proton at ISIS? Why do we need more?

Problem 1.5. Given a mean dispersion of 1 m, what is the γ_t? What transition kinetic energy does that correspond to? Calculate the slip factor η at 0, 5 and 10 ms.

From the longitudinal equation of motion on slide 29 one can derive the symplectic mapping equations:

$$\Delta E_{n+1} = \Delta E_n + V_1(\sin\phi_n - \sin\phi_s) \tag{1}$$

$$\phi_{n+1} = \phi_n - \frac{2\pi h\eta}{E_0\beta^2\gamma}\Delta E_{n+1} \tag{2}$$

from the n^{th} to the $(n+1)^{\text{th}}$ turn where V_1 is the peak gap voltage per turn and E_0 is the rest energy. These can be used to follow a particle's trajectory in the longitudinal phase space $(\Delta E, \phi)$ turn by turn.

Problem 2.1. Write a simulation program in the programming language of your choice that assumes several initial particle co-ordinates $(\Delta E_0,\ \phi_0) = (0,\ \phi)$ in the range $-\pi < \phi < \pi$ and calculates and applies the mapping equations over n turns. Assume we're half way through acceleration (5 ms) and use the parameters you calculated in exercise 1.3 and 1.5. (Other key parameters: $h = 2$, $V_1 = 150$ kV per turn and a sensible number for n). You'll need to calculate ϕ_s. Ensure you track a particle with start co-ordinates (0, $\pi - \phi_s - 0.02$) to trace out the separatrix. Plot out all the particles' longitudinal phase space trajectories over n turns on one graph.

Problem 2.2. Calculate the RF bucket height (energy acceptance) with the equation on slide 47 for the case in 2.1 and compare with the bucket size from your trajectories.

Hand in a copy of your program(s) with its (their) output.

OPTIONAL PROBLEM...if you're interested

Problem 2.3. What if we now introduce a second harmonic RF system at twice the frequency of the first ($h = 4$) such that we now have the energy gain per turn

$$\Delta E_{\text{turn}} = e\left(V_1\sin\phi_s + V_2\sin(2\phi_s + \theta)\right), \tag{3}$$

where V_2 is the second harmonic RF voltage and θ is the phase difference between the two RF systems. How does this change our mapping equations? Try putting this into your tracking code with $V_2 = V_1$ for the case $\phi_s = 0$. How does the RF bucket change with

a. $\theta = \pi$?

b. $\theta = 0$?