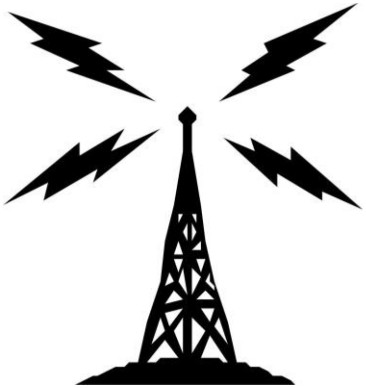
Lecture 1

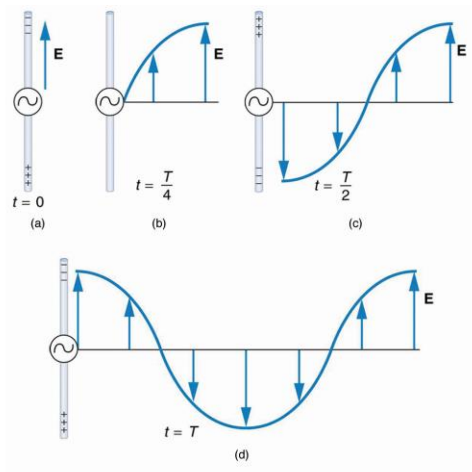
Properties of synchrotron radiation

Contents

Synchrotron Radiation

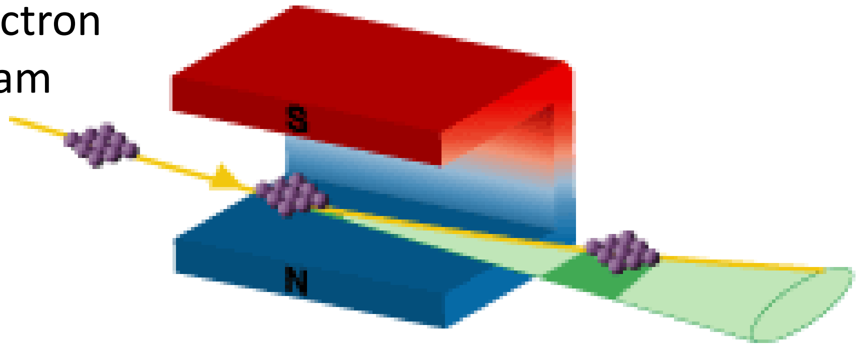
Electromagnetic radiation is produced by charged particles when accelerated





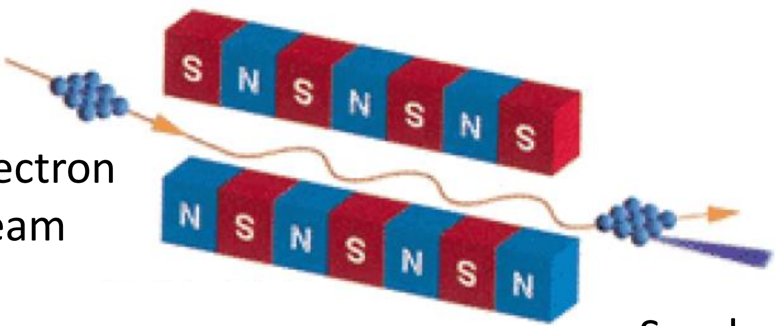
Synchrotron radiation is the name given to the electromagnetic radiation when the charged particle is accelerated in an external magnetic field (i.e. perpendicular to direction of motion)

Electron Beam



Synchrotron radiation

Electron Beam



Synchrotron radiation

Properties of Synchrotron Radiation

Covers a wide spectrum: (infra-red through to x-ray)

High flux: (flux = photons / second; allows short acquisition times or to use weakly scattering crystals)

High brightness: (highly collimated beams with small divergence and small source size; ~10^{12}-10^{14} brighter than a hospital x-ray tube)

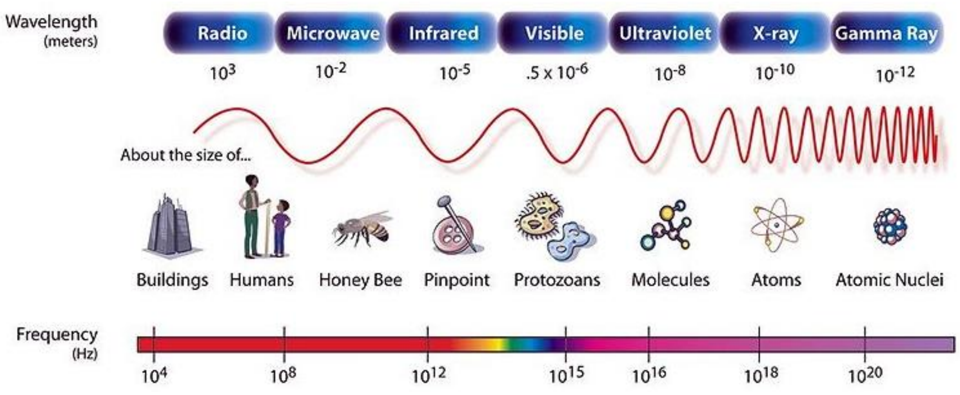
Small spot size: (good spatial resolution when scanning across sample)

Polarised: (linear and circular)

Pulsed time structure: (from few ps to 10s ps; can track physical processing occurring on same timescales)

Tuneable: (the user can select the wavelength)

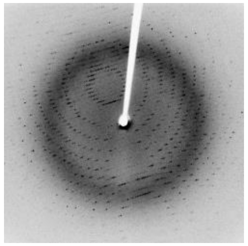
High stability: (sub-micron stability on the sample)



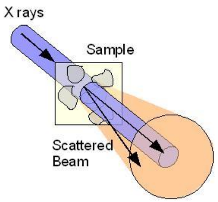
Uses of Synchrotron Radiation

Experimental tool:

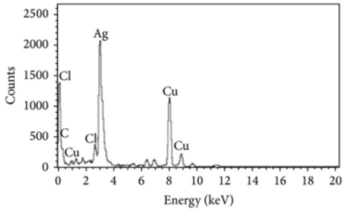
x-ray diffraction: X-rays are reflected of the regular planes of atoms in crystals. Diffraction patterns can be used to infer the underlying atomic structure



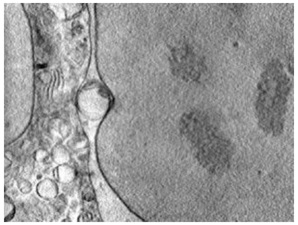
Scattering: Provides information on structure of large molecular assemblies in low-ordered systems



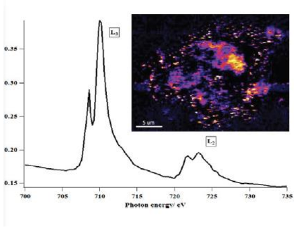
Spectroscopy: Reveals elemental composition and chemical states. The photon energy is scanned, quantifying the absorption, reflectivity or fluorescence of the sample



Imaging: X-rays pass through a sample and detector records those that pass through. The strength of the x-rays provides information on the varying density of the sample

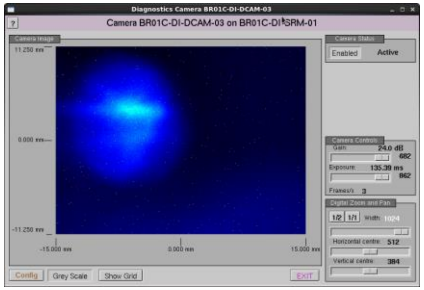


Microscopy: Allows objects to be imaged at different wavelengths of light, since different elements have different absorption wavelengths

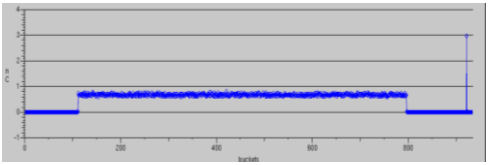


Uses of Synchrotron Radiation

Diagnostic tool:

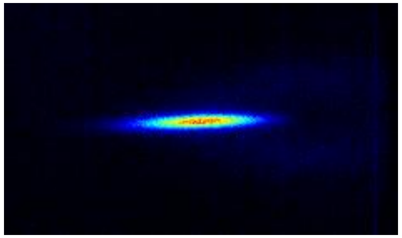


Imaging the beam use x-ray pinhole cameras to get information about the beam size

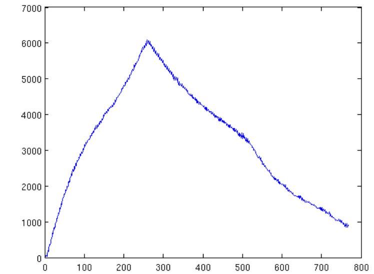


Charge distribution can count the individual photons emitted by an electron beam to infer how much charge is in each bunch

Provides damping:



emittance the competition between radiation damping and excitation defines the equilibrium emittance of an electron beam



Natural damping of instabilities radiation emission causes particles to lose momentum in the transverse plane and damp to the reference energy in the longitudinal plane

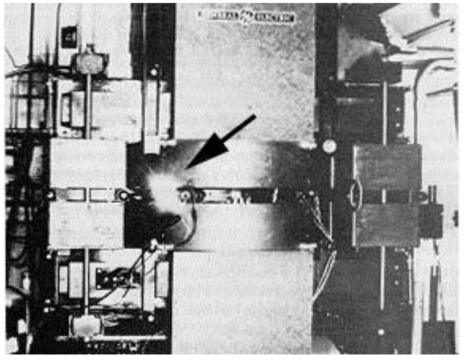
Brief History of Synchrotron Radiation Sources

First theoretical work:

1898 – Liénard (energy lost by charged particles on circular path)
1900 – Wiechert (EM potentials for point charge in arbitrary motion)
1902 – Schott (main properties of radiation, such as frequency, angular distribution)

First observation:

1947 – General Electric 70 MeV synchrotron



First user experiments:

1956 – Cornell 320 MeV synchrotron

1st generation sources:

Accelerators mainly used for high energy physics. Parasitic use of synchrotron radn.

2nd generation sources:

Purpose built facilities. SRS at Daresbury first dedicated machine (1981-2008)

3rd generation sources:

Optimised for brightness (low emittance beams and insertion devices)

Present day:

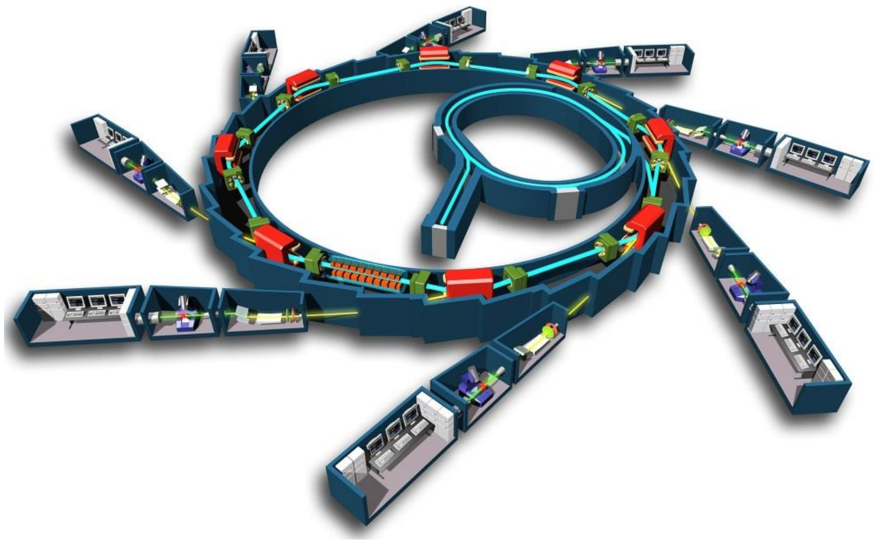
Diverse sources (low-emittance storage rings, linac-based free-electron lasers, light sources based on plasma wakefield accelerators,…)

Synchrotron Radiation Sources

Synchrotron radiation (SR) facilities fall into 2 main categories, namely storage ring based sources and linac based sources.

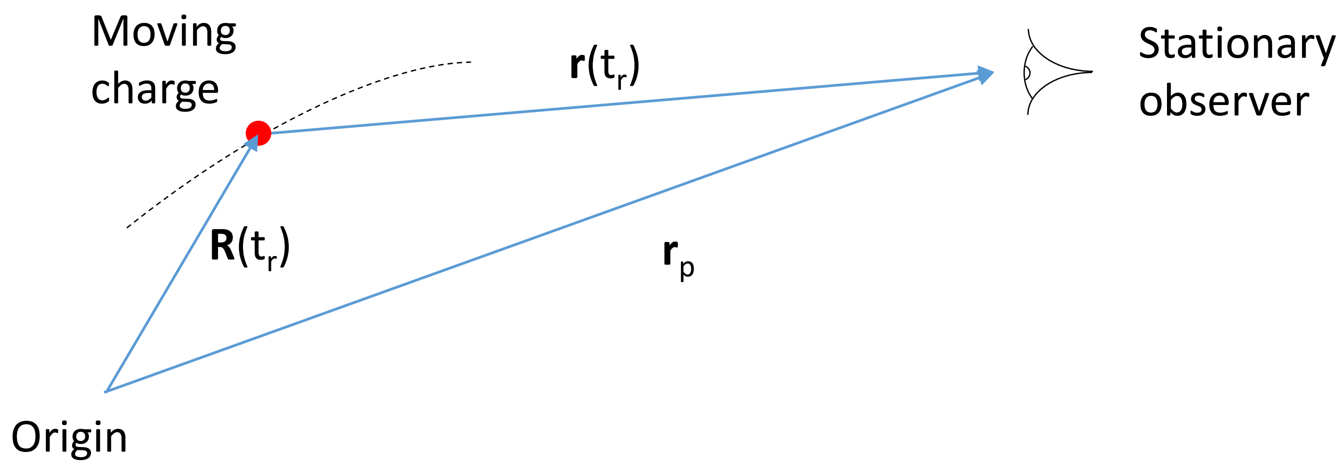
Storage ring facilities contain:

- **Injector complex** (provides high-energy electrons)
- **Storage ring** (to contain and store the electron beam for many hours / days at a time)
- **Magnetic devices** to generate SR from the electron beam
- **Beamlines** to focus to SR onto a sample



Electromagnetic fields due to a moving point charge

Consider an observer at point **r**, measuring the radiation at time t.



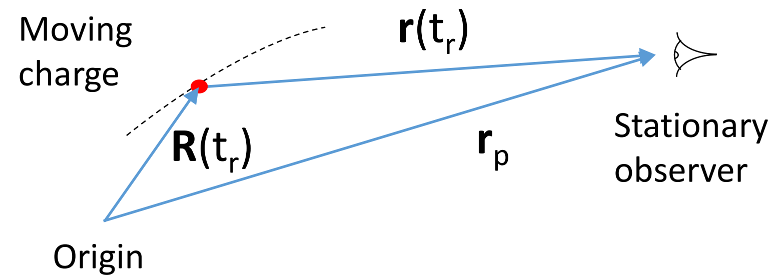
If the charge was at point **R** at the time of emission, then due to the time it takes for the radiation to propagate the distance **r** = $|\mathbf{r}_p - \mathbf{R}|$, the *actual* time of emission (t_r) will have been

$$\boxed{t_r = t - \frac{|\boldsymbol{r}(t_r)|}{c}}$$

t_r is known as the **retarded time**.

Electromagnetic fields due to a moving point charge

Retarded time:



$$t_r = t - \frac{|\boldsymbol{r}(t_r)|}{c}$$

Note that the distance between the point of emission and observation depends upon t_r.

A time interval for the observer will be different from the time interval for the emitter. This can be seen from the derivative of t with respect to t_r

$$dt = \left(1 + \frac{1}{c}\frac{dr}{dt_r}\right)dt_r$$

$$= (1 - \mathbf{n} \cdot \boldsymbol{\beta})dt_r$$

Where we have used $\mathbf{r}(t_r) = \mathbf{r}_p - \mathbf{R}(t_r)$, $\mathbf{r}_p$ is constant in time, and $\boldsymbol{\beta}$ is the normalised velocity of the particle $\boldsymbol{\beta} = \mathbf{v}/c$ (= $d\mathbf{R}/dt_r$). The unit vector **n** points in the direction $\mathbf{r}(t_r)$.

Two photons emitted by a relativistic particle received by an observer with a short time interval Δt will have been emitted over a much longer time interval $\Delta t_r = \Delta t/(1-\mathbf{n}\cdot\boldsymbol{\beta})$
This result has an important consequence:

- pulse compression (spectrum)
- intense pulses

Electromagnetic fields due to a moving point charge

We would like to calculate the electromagnetic fields at the observer caused by a moving point charge. In the **Lorentz Gauge**, the scalar and vector potentials are given by:

$$\emptyset(\mathbf{r}_{\mathrm{p}}, t) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\mathbf{R}, t_r)}{|\mathbf{r}(t_r)|} d^3r'$$

$$\mathbf{A}(\mathbf{r}_{\mathrm{p}}, t) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{R}, t_r)}{|\mathbf{r}(t_r)|} d^3r'$$

where the integrals are computed at the retarded time t_r, and $\rho(\mathbf{R}, t_r)$ and $\mathbf{J}(\mathbf{R}, t_r)$ are the charge and current densities respectively.

Integrating these equations over space, and including the time correction factor, leads to the so-called **Liénard-Wiechert Potentials**:

$$\boxed{\begin{aligned} \emptyset(\mathbf{r}_{\mathrm{p}}, t) &= \frac{e}{4\pi\varepsilon_0}\left(\frac{1}{r(1-\mathbf{n}\cdot\boldsymbol{\beta})}\right)_{ret} \\ \mathbf{A}(\mathbf{r}_{\mathrm{p}}, t) &= \frac{e}{4\pi\varepsilon_0 c}\left(\frac{\boldsymbol{\beta}}{r(1-\mathbf{n}\cdot\boldsymbol{\beta})}\right)_{ret} \end{aligned}}$$

Electromagnetic fields due to a moving point charge

The electromagnetic fields can be computed from the Liénard-Wiechert potentials using Maxwell's equations

$$\mathbf{E} = -\nabla\emptyset - \mu_0 \frac{\partial \mathbf{A}}{\partial t}$$

$$\mathbf{B} = -\mu_0[\nabla \times \mathbf{A}]$$

As with the derivation of the Liénard-Wiechert potentials, these equations need to be evaluated after taking into account the difference between emitter and observer time.

After substituting the potentials and some maths (see [1, 2]), we arrive at the Liénard-Wiechert fields:

$$\boxed{\begin{aligned} \mathbf{E}(t) &= \frac{e}{4\pi\varepsilon_0\gamma^2}\left(\frac{(\mathbf{n}-\boldsymbol{\beta})}{r^2(1-\mathbf{n}\cdot\boldsymbol{\beta})^3}\right)_{ret} + \frac{e}{4\pi\varepsilon_0 c}\left(\frac{\mathbf{n}\times[(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}]}{r(1-\mathbf{n}\cdot\boldsymbol{\beta})^3}\right)_{ret} \\ \mathbf{B}(t) &= \frac{\mathbf{n}\times\mathbf{E}(t)}{c} \end{aligned}}$$

Electromagnetic fields due to a moving point charge

$$\mathbf{E}(t) = \frac{e}{4\pi\varepsilon_0\gamma^2}\left(\frac{(\mathbf{n}-\boldsymbol{\beta})}{r^2(1-\mathbf{n}\cdot\boldsymbol{\beta})^3}\right)_{ret} + \frac{e}{4\pi\varepsilon_0 c}\left(\frac{\mathbf{n}\times[(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}]}{r(1-\mathbf{n}\cdot\boldsymbol{\beta})^3}\right)_{ret}$$

$$\mathbf{B}(t) = \frac{\mathbf{n}\times\mathbf{E}(t)}{c}$$

The Liénard-Wiechert fields have a number of interesting properties:

1) The electric and magnetic fields are perpendicular
2) For a stationary charge, β = 0, γ = 1 and we recover Coulomb's Law
3) For a charge moving with constant velocity, $\dot{\boldsymbol{\beta}}$ = 0, and only the first terms for E(t) and B(t) are nonzero. These are termed the velocity fields.
4) The second term is known as the acceleration field. Since it depends upon 1/r (compared to $1/r^2$ for the velocity field), it dominates the first term at large distances, and so is often known as the 'far-field' term.

From here on, we will concentrate on the far-field (acceleration) contribution.

Power Radiated By the Particle

To calculated the power radiated by the particle, we need to first consider the Poynting vector:

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$$

This gives the energy *passing through a unit area*, per unit time *t* (observer time!)

For the far-field term, $\mathbf{E} \cdot \mathbf{n} = 0$ and so $\mathbf{S} = \frac{E^2}{\mu_0 c}\mathbf{n}$.

To calculate the power *radiated by the particle* using emitter time, we therefore have the general expression:

$$P = \int \mathbf{S} \cdot \mathbf{n}(1 - \mathbf{n} \cdot \boldsymbol{\beta})r^2 d\Omega$$

$$\frac{dP}{d\Omega} = \frac{E^2}{\mu_0 c}(1 - \mathbf{n} \cdot \boldsymbol{\beta})r^2$$

$$\boxed{\frac{dP}{d\Omega} = \frac{e^2}{(4\pi)^2 \epsilon_0 c} \frac{\left[\mathbf{n} \times \left[(\mathbf{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\right]\right]^2}{(1 - \mathbf{n} \cdot \boldsymbol{\beta})^5}}$$

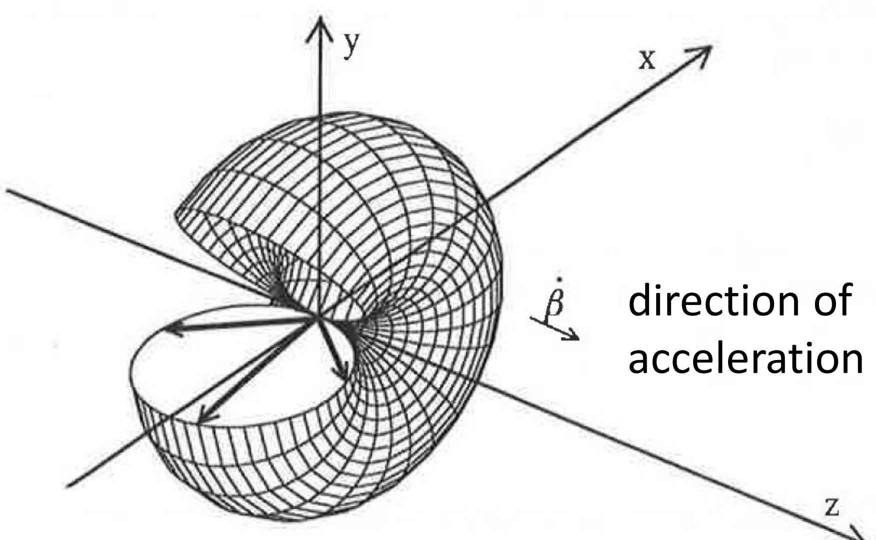
Spatial Distribution of Radiation

In the non-relativistic case, $\beta \approx 0$, and so the power distribution reduces to

$$\frac{dP}{d\Omega} = \frac{e^2}{(4\pi)^2 \epsilon_0 c} \left[\mathbf{n} \times (\mathbf{n} \times \dot{\boldsymbol{\beta}})\right]^2$$

Taking the angle between the direction of acceleration and observation as Θ,

$$\frac{dP}{d\Omega} = \frac{e^2}{(4\pi)^2 \epsilon_0 c} \left|\dot{\boldsymbol{\beta}}\right|^2 \sin^2 \Theta$$



to observer

H. Wiedemann [4]

Significant spatial dependence:

- Concentrated in x/y plane (perpendicular to direction of acceleration)
- Proportional to $\sin^2 \Theta$
- Radiation has rotational symmetry about the z axis

Power Radiated By the Particle

In the non-relativistic case, the angular distribution is given by

$$\frac{dP}{d\Omega} = \frac{e^2}{(4\pi)^2 \epsilon_0 c} \left|\dot{\boldsymbol{\beta}}\right|^2 \sin^2 \Theta$$

In order to calculate the total power radiated, we need to integrate over all angles.

Using the relation $d\Omega = \sin(\theta)\, d\theta d\Phi$, where Φ is the azimuthal angle with respect to the direction of acceleration, the total power is given by

$$\boxed{P = \frac{e^2}{6\pi \epsilon_0 c} \left|\dot{\boldsymbol{\beta}}\right|^2}$$

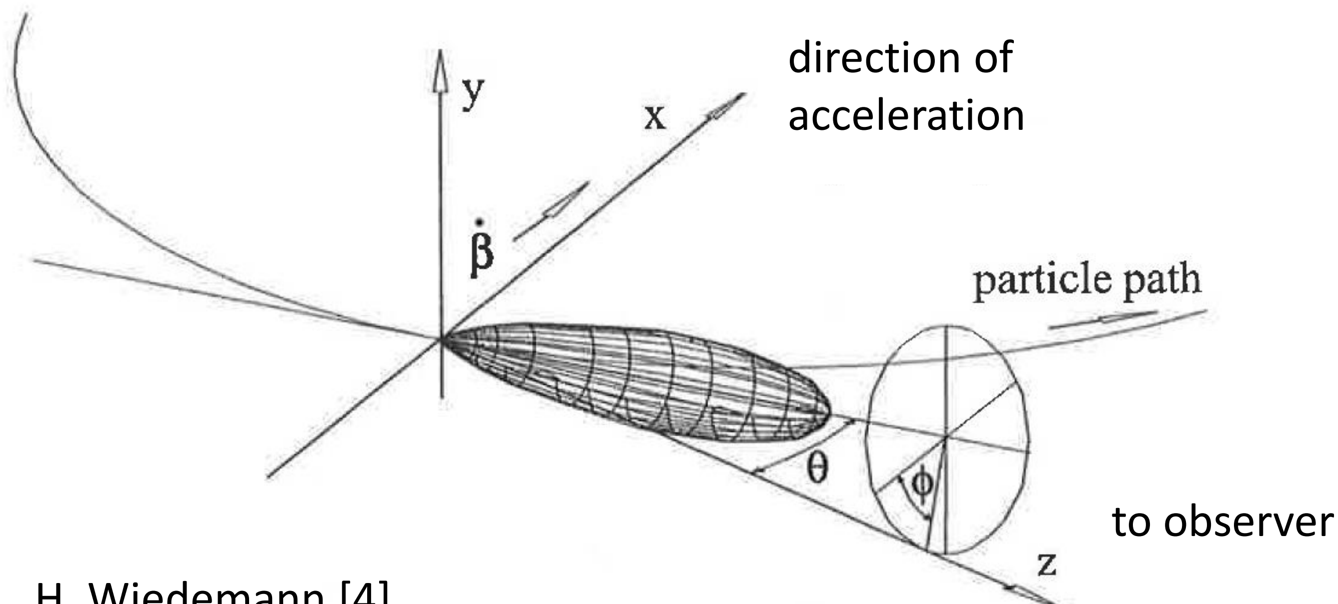
This result is known as the Lamor formula.

<u>Spatial Distribution of Radiation</u>

In the relativistic case, the term $(1 - \mathbf{n}\cdot\boldsymbol{\beta})^5$ in the denominator dominates, causing the radiation to become strongly peaked in the forwards direction

$$\frac{dP}{d\Omega} = \frac{e^2}{(4\pi)^2\epsilon_0 c}\frac{\left[\mathbf{n}\times\left[(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right]\right]^2}{(1-\mathbf{n}\cdot\boldsymbol{\beta})^5}$$

The radiation is contained within a cone with opening angle $\sim 1/\gamma$



H. Wiedemann [4]

Power Radiated By the Particle

The total power radiated in the relativistic case can be found either by integrating the previously obtained angular distribution over all angles, or by making a Lorentz transformation of the Lamor formula (see Jackson [3], Wiedemann [4]).

This results in the Liénard formula

$$\boxed{P = \frac{e^2}{6\pi\epsilon_0 c}\gamma^6\left[\dot{\boldsymbol{\beta}}^2 - \left(\boldsymbol{\beta}\times\dot{\boldsymbol{\beta}}\right)^2\right]}$$

This equation generalises into two components:

- a component where the velocity is parallel to the direction of acceleration ($\boldsymbol{\beta}\times\dot{\boldsymbol{\beta}} = 0$)
- a component where the velocity is perpendicular to the direction of acceleration

$(\dot{\boldsymbol{\beta}}^2 - \left(\boldsymbol{\beta}\times\dot{\boldsymbol{\beta}}\right)^2 = \left|\dot{\boldsymbol{\beta}}^2\right|(1-\beta^2) = \left|\dot{\boldsymbol{\beta}}^2\right|/\gamma^2)$

$$P_{\parallel} = \frac{e^2}{6\pi\epsilon_0 c}\gamma^6\dot{\boldsymbol{\beta}}_{\parallel}^{\;2} = \frac{e^2}{6\pi\epsilon_0 c}\frac{1}{m^2c^2}\left(\frac{d\mathbf{p}}{dt}\right)^2$$

$$P_{\perp} = \frac{e^2}{6\pi\epsilon_0 c}\gamma^4\dot{\boldsymbol{\beta}}_{\perp}^{\;2} = \frac{e^2}{6\pi\epsilon_0 c}\frac{\gamma^2}{m^2c^2}\left(\frac{d\mathbf{p}}{dt}\right)^2$$

where the relativistic momentum $\mathbf{p} = \gamma\boldsymbol{\beta}mc$, $\mathrm{m}\dot{\mathbf{v}}_{\parallel} = \frac{1}{\gamma^3}\frac{d\mathbf{p}_{\parallel}}{dt}$ and $\mathrm{m}\dot{\mathbf{v}}_{\perp} = \frac{1}{\gamma}\frac{d\mathbf{p}_{\perp}}{dt}$

Power Radiated By the Particle

$$P_{\parallel} = \frac{e^2}{6\pi\epsilon_0 c}\gamma^6 \dot{\boldsymbol{\beta}}_{\parallel}^{\,2} = \frac{e^2}{6\pi\epsilon_0 c}\frac{1}{m^2c^2}\left(\frac{d\mathbf{p}}{dt}\right)^2$$

$$P_{\perp} = \frac{e^2}{6\pi\epsilon_0 c}\gamma^4 \dot{\boldsymbol{\beta}}_{\perp}^{\,2} = \frac{e^2}{6\pi\epsilon_0 c}\frac{\gamma^2}{m^2c^2}\left(\frac{d\mathbf{p}}{dt}\right)^2$$

From these expressions it is clear that, for a given applied force ($d\mathbf{p}/dt$), the power radiated will be a factor γ^2 larger if that force is perpendicular to the direction of motion.

In the case of circular motion with bend radius ρ, we have

$$\frac{dp}{dt} = \frac{\gamma mc^2\beta^2}{\rho}$$

And we have for the instantaneous power radiated by a single electron of mass m and energy E

$$P_{\perp} = \frac{e^2c}{6\pi\epsilon_0}\frac{\gamma^4\beta^4}{\rho^2}$$

$$P_{\perp} = \frac{e^2}{6\pi\epsilon_0 c^7}\frac{1}{m^4}\frac{E^4}{\rho^2}$$

Power Radiated By the Particle

Instantaneous power radiated by a single particle:

$$\boxed{P_\perp = \frac{e^2}{6\pi\epsilon_0 c^7}\frac{1}{m^4}\frac{E^4}{\rho^2}}$$

Total energy loss per turn (one electron):

$$U_0 = \int P dt = PT_B \,, T_B = \frac{2\pi\rho}{c}$$

$$\boxed{U_0(keV) = 88.46\frac{E^4(GeV)}{\rho(m)}}$$

Power radiated by electron beam:

$$\boxed{P_{tot} = U_0 I_b}$$

Summary:

- Radiated power depends strongly on the rest mass of the particle ($\propto {}^{1}/_{m^4}$)
- Proportional to the square of the dipole field ($B \propto {}^{1}/_{\rho}$)
- Power radiated can be very high:
  - cool vacuum chamber surface
  - replace with RF cavity
  - Limits upper energy for circular accelerators

Radiation Spectrum

In order to calculate the frequency spectrum of the radiation, we need to take the Fourier transform of the time-varying electric field:

$$\mathbf{E}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathbf{E}(t) e^{i\omega t} dt$$

This can then be inserted into the expression for the total energy received by the observer during one passage, per unit solid angle (from Poynting vector):

$$\frac{dW}{d\Omega} = \int \frac{dP}{d\Omega} dt = \frac{1}{\mu_0 c} \int_{-\infty}^{\infty} \left(r\mathbf{E}(\mathrm{t})\right)^2 \mathrm{dt} = \frac{2}{2\pi\mu_0 c} \int_{0}^{\infty} \left(r\mathbf{E}(\omega)\right)^2 \mathrm{d}\omega$$

The angular and frequency distribution received by the observer is therefore

$$\frac{d^2W}{d\Omega d\omega} = \frac{2}{2\pi\mu_0 c} \left| \int_{-\infty}^{\infty} \left(r\mathbf{E}(t)\right) e^{i\omega t} dt \right|^2$$

Taking the far-field term only (i.e. neglecting the velocity field)

$$\frac{d^2W}{d\Omega d\omega} = \frac{e^2}{16\pi^3 \epsilon_0 c} \left| \int_{-\infty}^{\infty} \left(\frac{\mathbf{n} \times [(\mathbf{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{(1 - \mathbf{n} \cdot \boldsymbol{\beta})^3} \right)_{ret} e^{i\omega t} dt \right|^2$$

Radiation Spectrum

The energy radiated per unit solid angle, per unit frequency can also by expressed directly using the retarded time as

$$\frac{d^2W}{d\Omega d\omega} = \frac{e^2}{16\pi^3\epsilon_0 c}\left|\int_{-\infty}^{\infty}\left(\frac{\mathbf{n}\times[(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}]}{(1-\mathbf{n}\cdot\boldsymbol{\beta})^2}\right)e^{i\omega\left(t'+\frac{r(t')}{c}\right)}dt'\right|^2$$

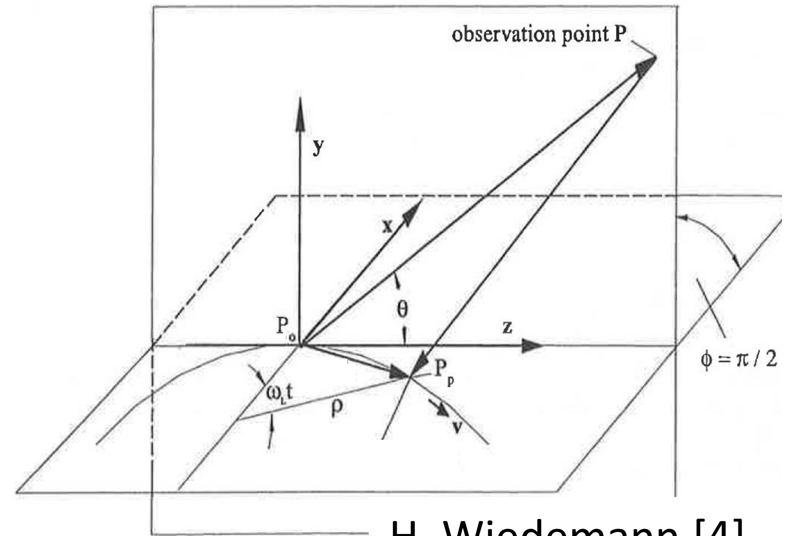
Or re-expressed more simply as (see Jackson [1], Hoffman [3])

$$\boxed{\frac{d^2W}{d\Omega d\omega} = \frac{e^2}{16\pi^3\epsilon_0 c}\omega^2\left|\int_{-\infty}^{\infty}(\mathbf{n}\times(\mathbf{n}\times\boldsymbol{\beta}))e^{i\omega\left(t'+\frac{r(t')}{c}\right)}dt'\right|^2}$$

Radiation Spectrum for a Particle in Circular Motion

If the particle is following a circular path (as is the case of an electron accelerated in a magnetic field), the particle coordinates are given by

$$\mathbf{R}(t) = \left(\rho\left(1 - \cos\frac{\beta ct}{\rho}\right), 0, \rho\sin\frac{\beta ct}{\rho}\right)$$



H. Wiedemann [4]

Computing the cross-product:

$$\mathbf{n} \times (\mathbf{n} \times \boldsymbol{\beta}) = \beta\left(-\sin\frac{\beta ct}{\rho}\mathbf{n_z} + \cos\frac{\beta ct}{\rho}\,\sin\theta\,\mathbf{n_y}\right)$$

Where we have introduced the unit vectors $\mathbf{n_z}$ and $\mathbf{n_y}$.

Substituting back into the expression for the angular and frequency distribution, and simplifying using modified Bessel functions, we have

$$\boxed{\frac{d^2W}{d\Omega d\omega} = \frac{e^2}{16\pi^3\epsilon_0 c}\gamma^2\left(\frac{\omega}{\omega_c}\right)^2(1+\gamma^2\theta^2)^2\left[K_{2/3}^2(\xi) + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}K_{1/3}^2(\xi)\right]}$$

where θ is the angle of observation in the vertical plane and we have introduced the quantities

$$\xi = \frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{3/2} \qquad \omega_c = \frac{3c\gamma^3}{2\rho}$$

Radiation Spectrum for a Particle in Circular Motion

$$\frac{d^2W}{d\Omega d\omega} = \frac{e^2}{16\pi^3\epsilon_0 c}\gamma^2\left(\frac{\omega}{\omega_c}\right)^2(1+\gamma^2\theta^2)^2\left[K_{2/3}^2(\xi) + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}K_{1/3}^2(\xi)\right]$$

Because of the circular path, the energy distribution only depends upon the angle of observation in the vertical plane. The two terms in the above expression corresponds to radiation polarised in the horizontal and vertical planes respectively.

From the properties of the Modified Bessel functions, the energy radiated can be seen to fall away quickly at frequencies above the critical frequency

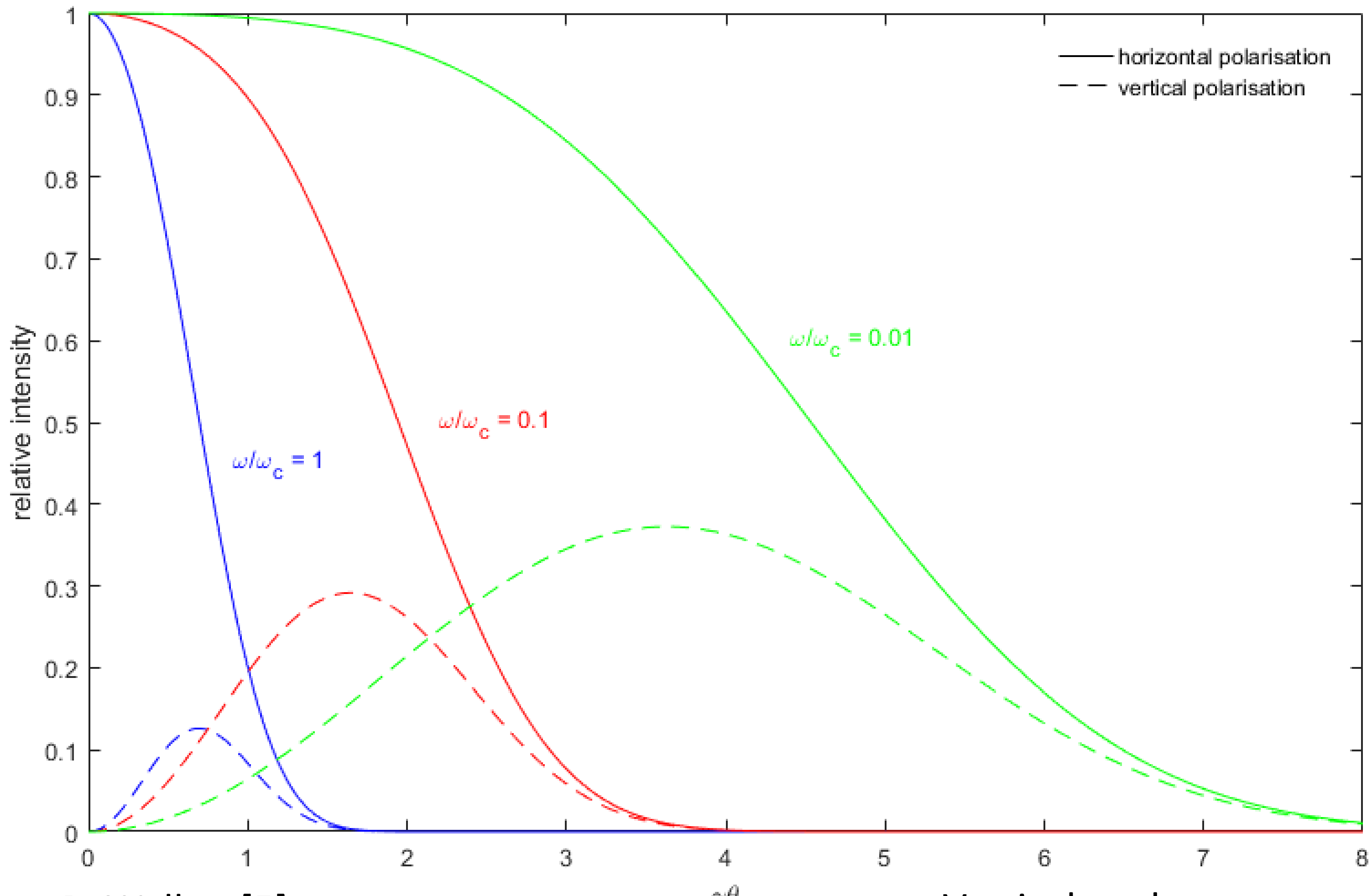
$$\boxed{\omega_c = \frac{3c\gamma^3}{2\rho}}$$

A critical angle can also be defined, given by

$$\boxed{\theta_c = \frac{1}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3}}$$

As with the critical frequency, the power falls away rapidly at angles larger than the critical angle.

Angular distribution for polarised light



R. Walker [5]

Vertical angle

1. The light is horizontally polarised in the plane of the accelerator, and becomes increasingly circularly polarised at larger vertical angles
2. The opening angle of the radiation decreases with increasing frequency

Radiation Spectrum for a Particle in Circular Motion

$$\frac{d^2W}{d\Omega d\omega} = \frac{e^2}{16\pi^3\epsilon_0 c}\gamma^2\left(\frac{\omega}{\omega_c}\right)^2(1+\gamma^2\theta^2)^2\left[K_{2/3}^2(\xi) + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}K_{1/3}^2(\xi)\right]$$

In the plane of the accelerator, only the horizontal polarisation term remains.

In this case, we may write

$$\frac{d^2W}{d\Omega d\omega} = \frac{e^2}{16\pi^3\epsilon_0 c}\gamma^2\left(\frac{\omega}{\omega_c}\right)^2 K_{2/3}^2\left(\frac{\omega}{2\omega_c}\right)$$

Or, more simply, the peak intensity on-axis as a function of frequency is

$$\boxed{\frac{d^2W}{d\Omega d\omega} = \frac{e^2}{16\pi^3\epsilon_0 c}\gamma^2 H_2\left(\frac{\omega}{\omega_c}\right)}$$

where

$$H_2\left(\frac{\omega}{\omega_c}\right) = \left(\frac{\omega}{\omega_c}\right)^2 K_{2/3}^2\left(\frac{\omega}{2\omega_c}\right)$$

Radiation Spectrum for a Particle in Circular Motion

Integrating over all angles, the total energy radiated per unit frequency, per turn is given by

$$\boxed{\frac{dW}{d\omega} = \frac{\sqrt{3}e^2}{4\pi\epsilon_0 c}\gamma\frac{\omega}{\omega_c}\int_{\omega/\omega_c}^{\infty} K_{5/3}(x)dx}$$

This is often written in terms of a universal function scaling $S\left(\frac{\omega}{\omega_c}\right)$

$$\frac{dW}{d\omega} = \frac{2e^2}{9\epsilon_0 c}\gamma S\left(\frac{\omega}{\omega_c}\right)$$

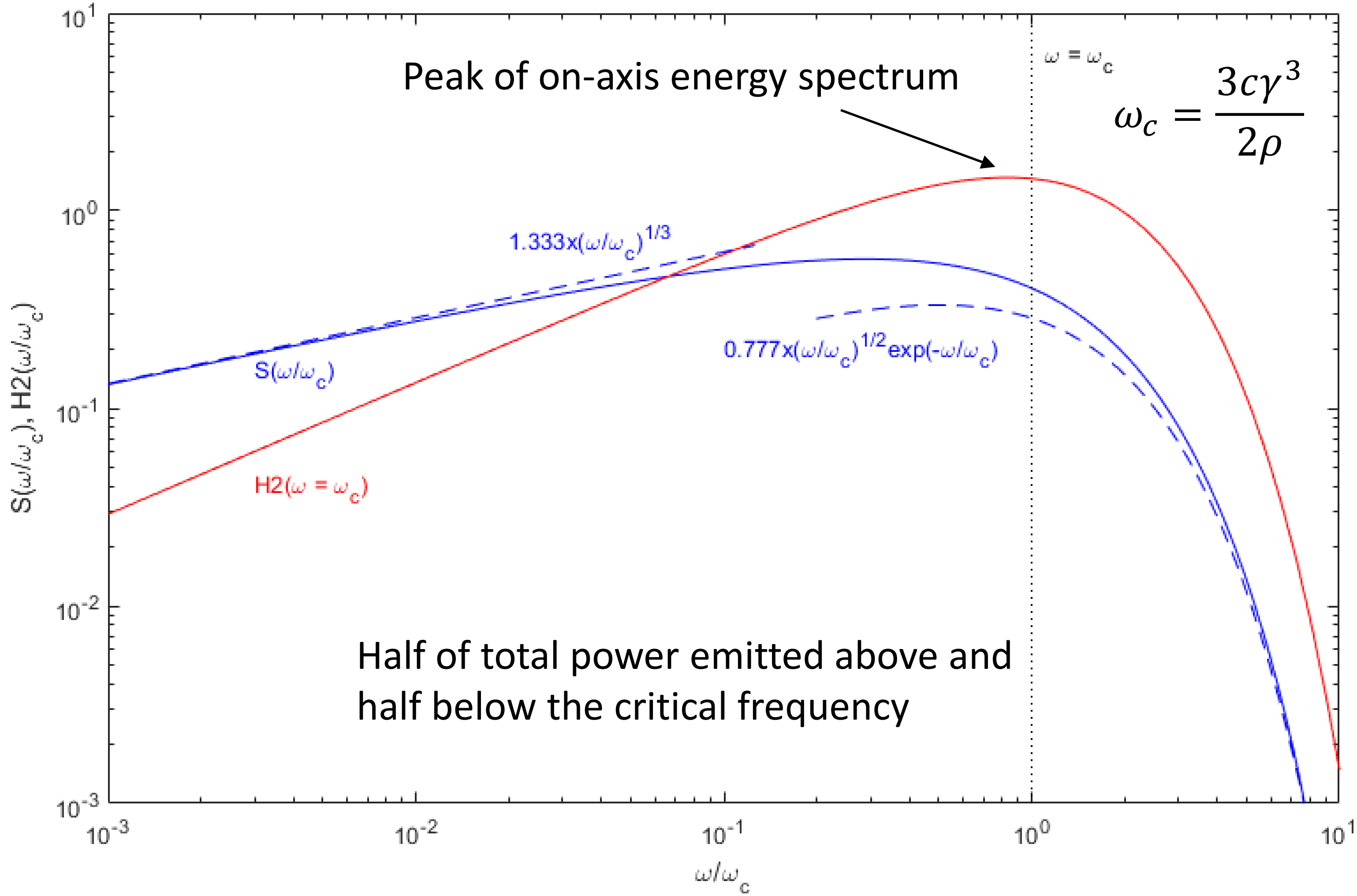
where

$$S\left(\frac{\omega}{\omega_c}\right) = \frac{9\sqrt{3}}{8\pi}\frac{\omega}{\omega_c}\int_{\omega/\omega_c}^{\infty} K_{5/3}(x)dx \qquad \int_0^{\infty} S\left(\frac{\omega}{\omega_c}\right)d\left(\frac{\omega}{\omega_c}\right) = 1, \int_0^{1} S\left(\frac{\omega}{\omega_c}\right)d\left(\frac{\omega}{\omega_c}\right) = 0.5$$

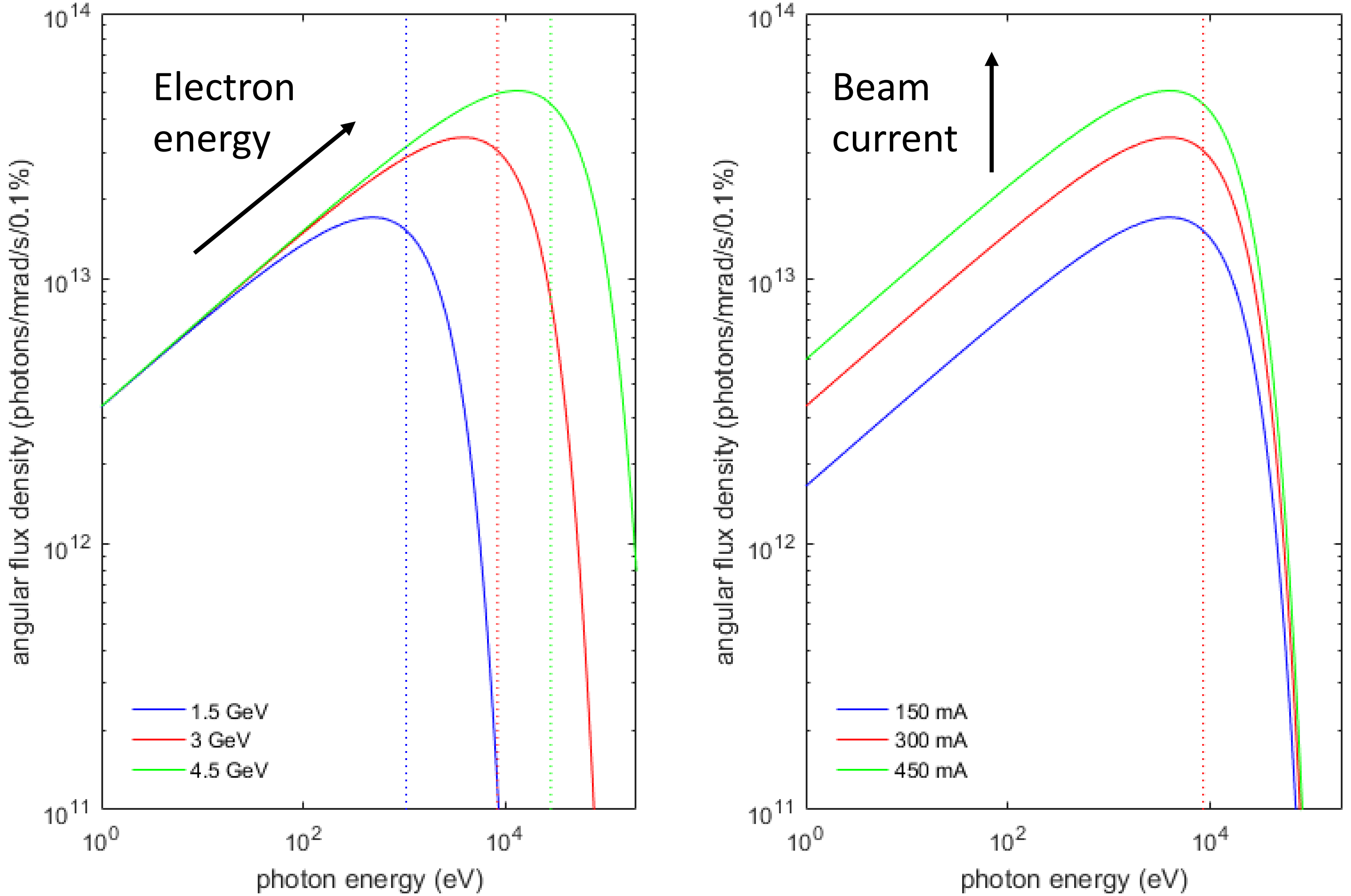
At the extremes, the energy distribution tends to:

$$\frac{dW}{d\omega} \approx \frac{e^2}{4\pi\epsilon_0 c}\left(\frac{\omega\rho}{c}\right)^{1/3} \quad (\omega \ll \omega_c) \qquad \frac{dW}{d\omega} \approx \sqrt{\frac{3\pi}{2}\frac{e^2}{4\pi\epsilon_0 c}\gamma}\left(\frac{\omega}{\omega_c}\right)^{1/2} e^{-\omega/\omega_c} \quad (\omega \gg \omega_c)$$

Radiation Spectrum for a Particle in Circular Motion



<u>Radiation Spectrum Scaling</u>



Summary

Synchrotron radiation is emitted by charged particles when accelerated by a magnetic field (i.e. perpendicular to the direction of motion)

The radiation power scales with (mass)^4, so is usually only significant for electrons and positrons

Within a constant magnetic field (circular trajectory), the light is emitted in a narrow cone at a tangent to the trajectory with opening angle $\sim 1/\gamma$

The radiation spectrum is characterised by a continuous distribution centred around a critical frequency

Synchrotron radiation has unique characteristics and many applications

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