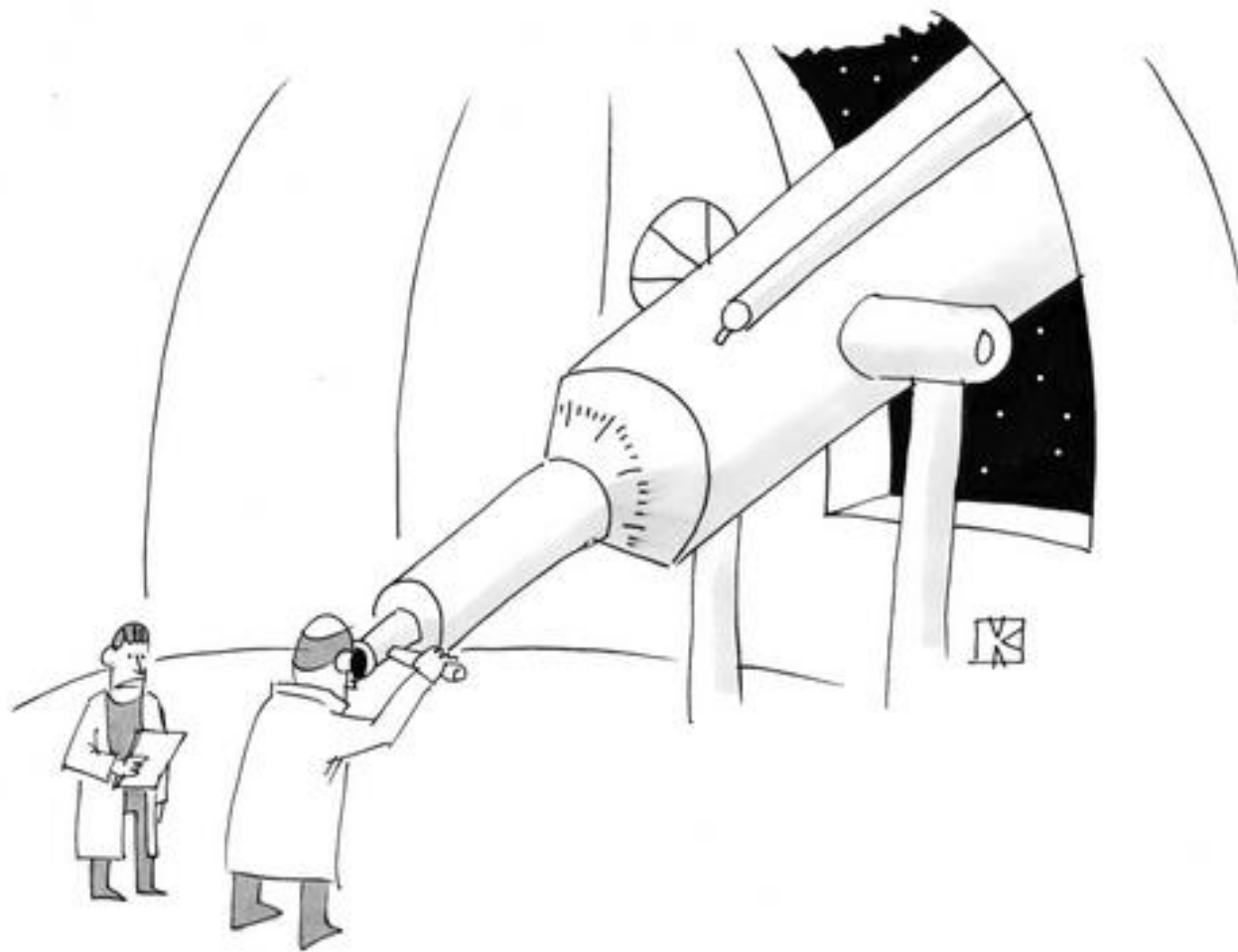
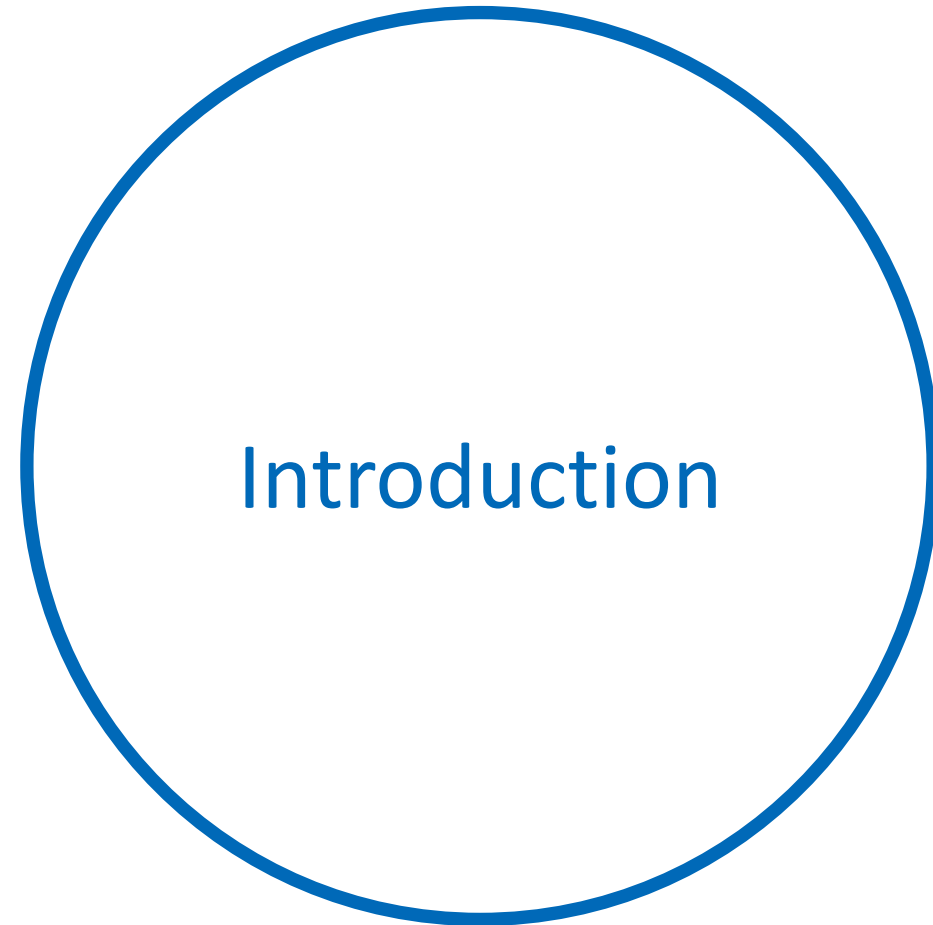


# Probing Dark Matter with Gravitational Waves



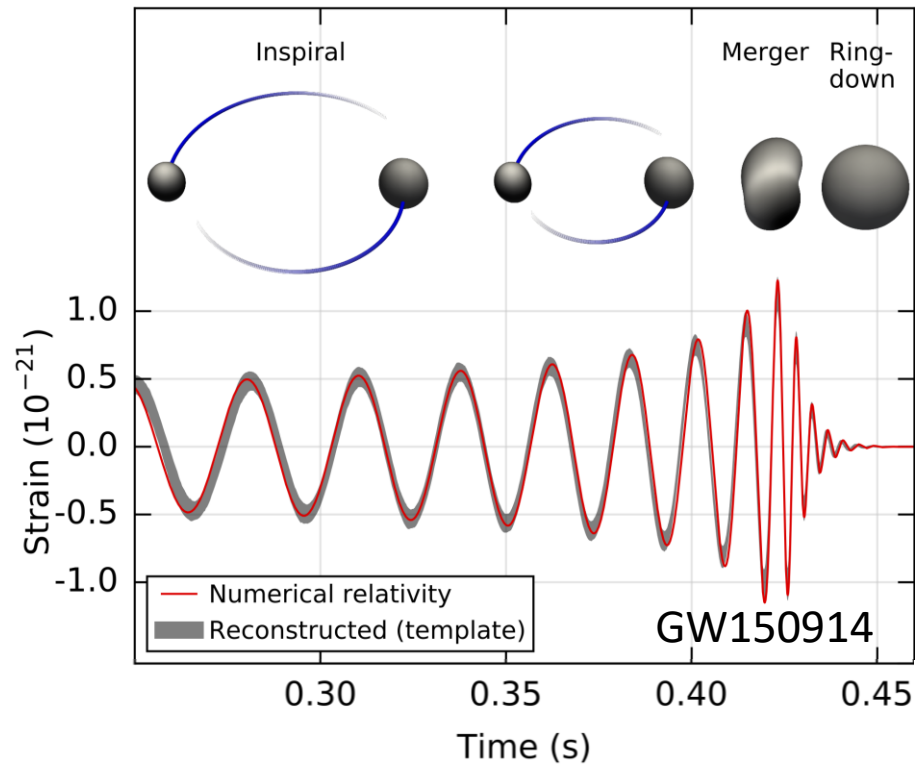
*“That isn’t dark matter, sir—you just forgot to take off the lens cap.”*

[Gregory Kogan]

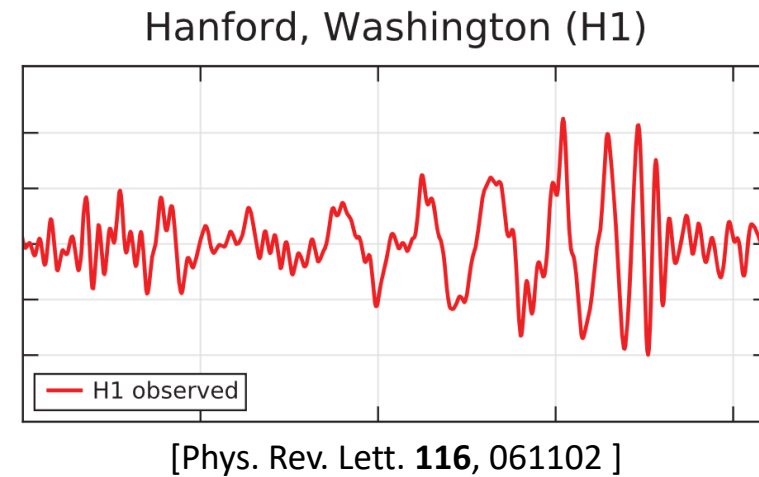


Introduction

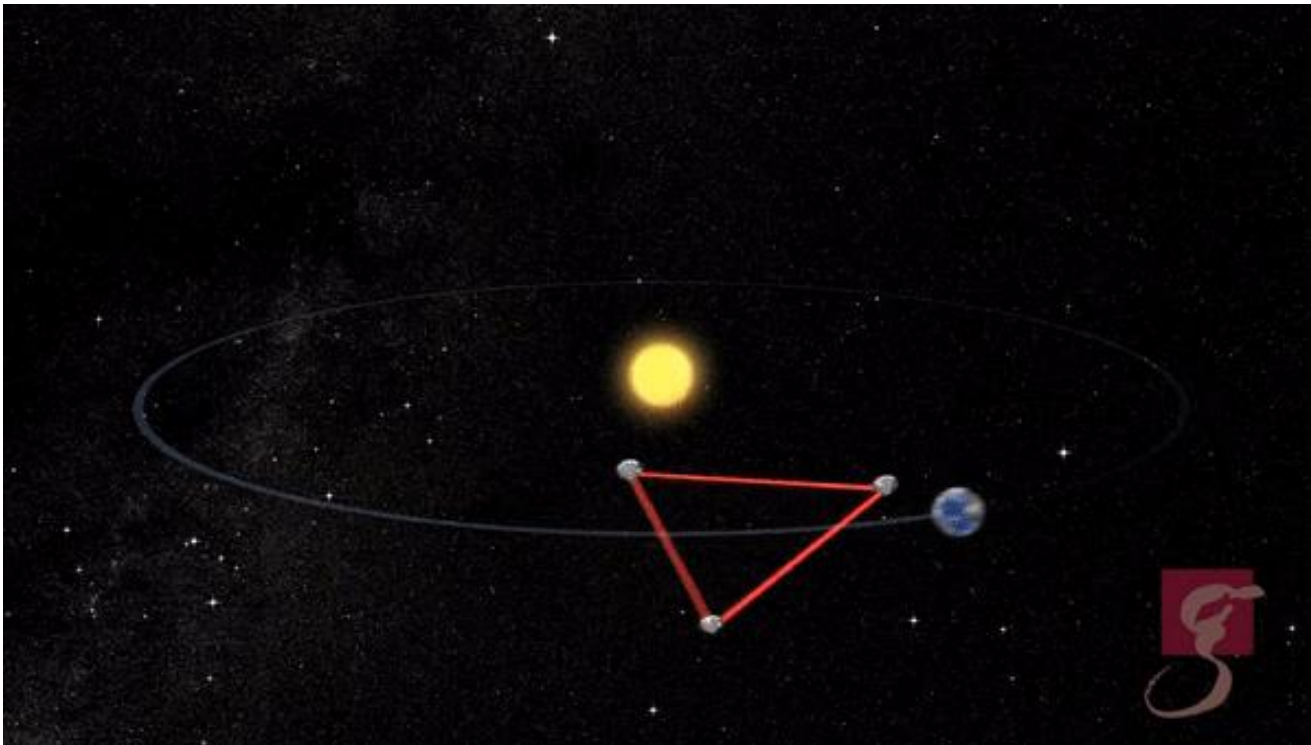
# Gravitational Waves



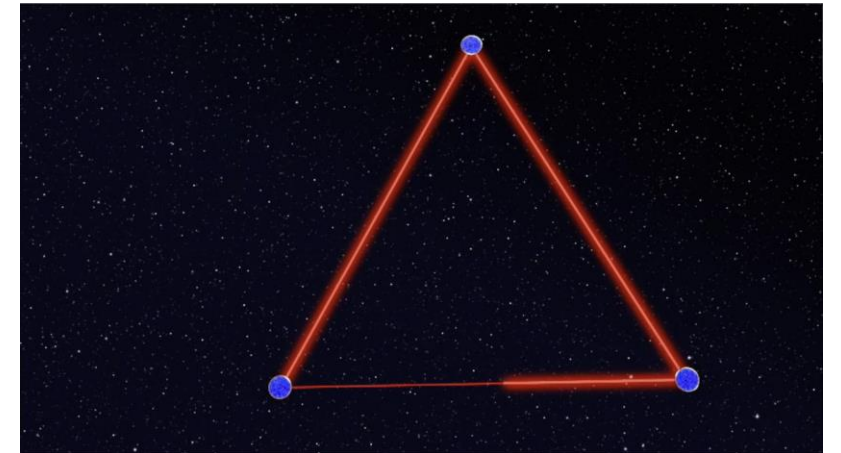
[[www.ligo.org/science/faq.php](http://www.ligo.org/science/faq.php)]



## LISA Detector: How does it work?

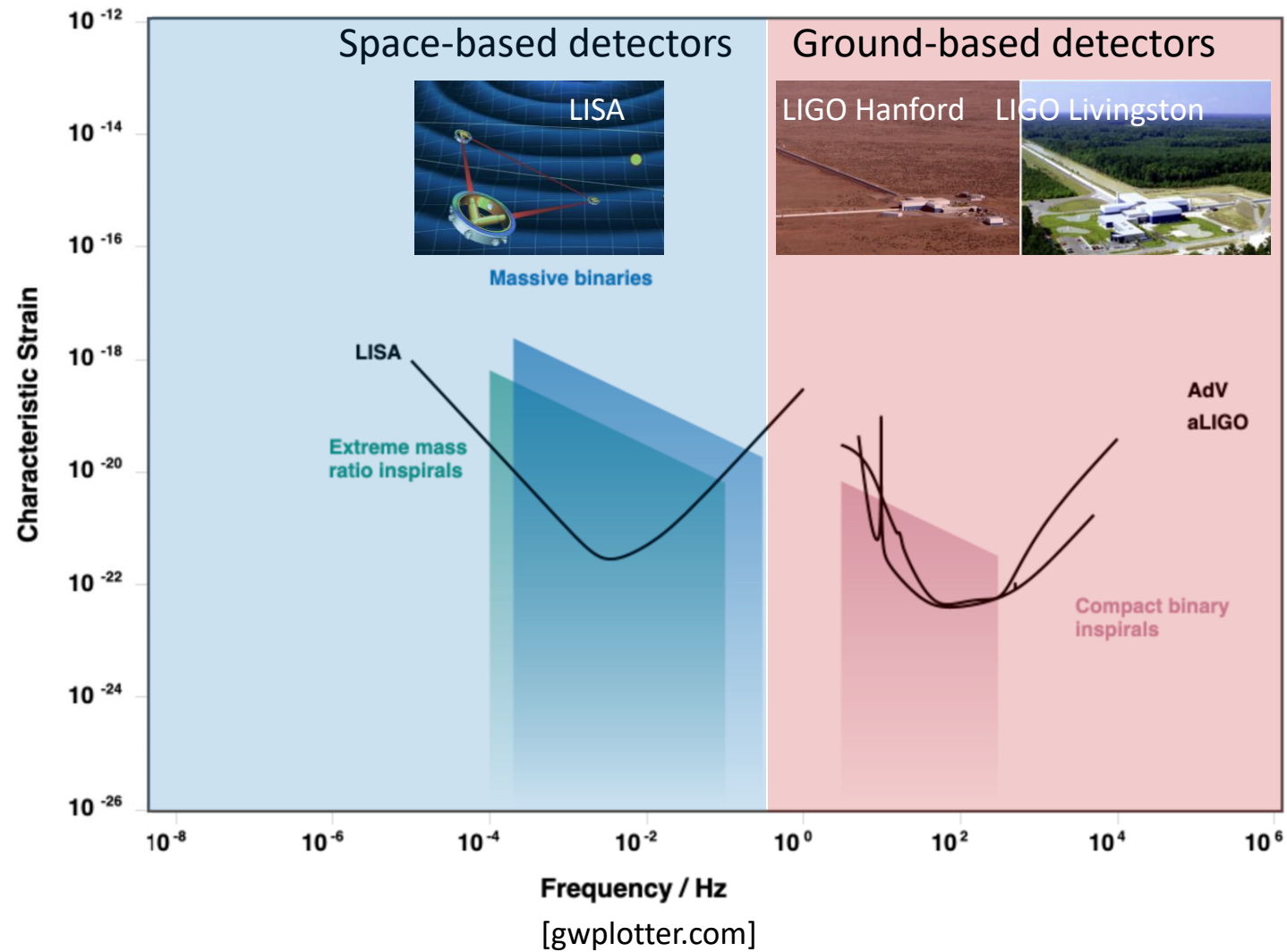


[AEI/Milde Marketing]



[Max Planck Institute for Gravitational Physics (Albert Einstein Institute) / Milde Marketing Science Communication / Exozet Effects]

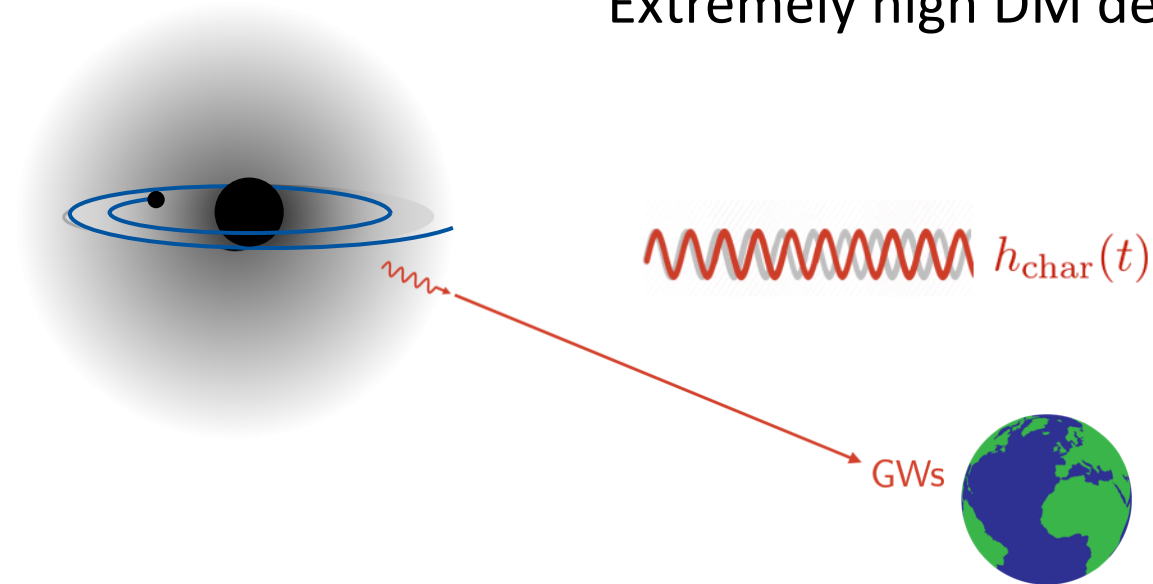
# LISA Detector: Frequency Band



## The source: IMRIs with DM density spikes

Intermediate mass-ratio inspirals:  
 $10^{-3}$

Extremely high DM density



Why does the inspiral object lose energy?

Hint: There are 2 reasons

[Hannuksela et al., '19]

[Edwards et al., '19]

## The source: IMRIs with DM density spikes

- Gravitational waves
- Additional energy loss through dynamical friction:

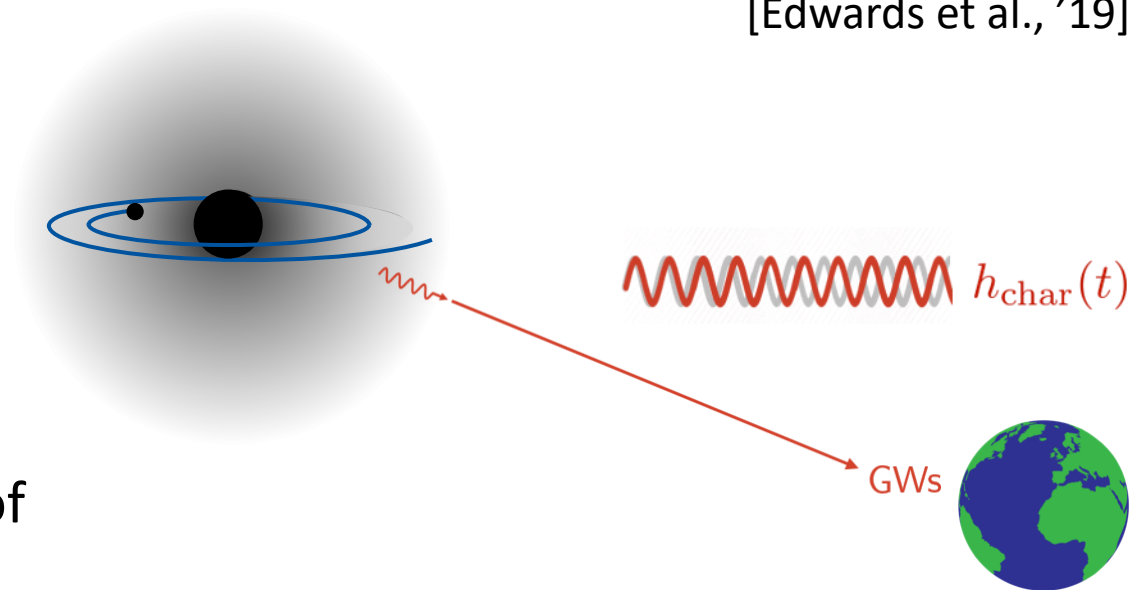
$$-\frac{dE}{dt} = \frac{dE_{\text{GW}}}{dt} + \frac{dE_{\text{friction}}}{dt}$$

- Modified dynamics: **dephasing** of GW signal

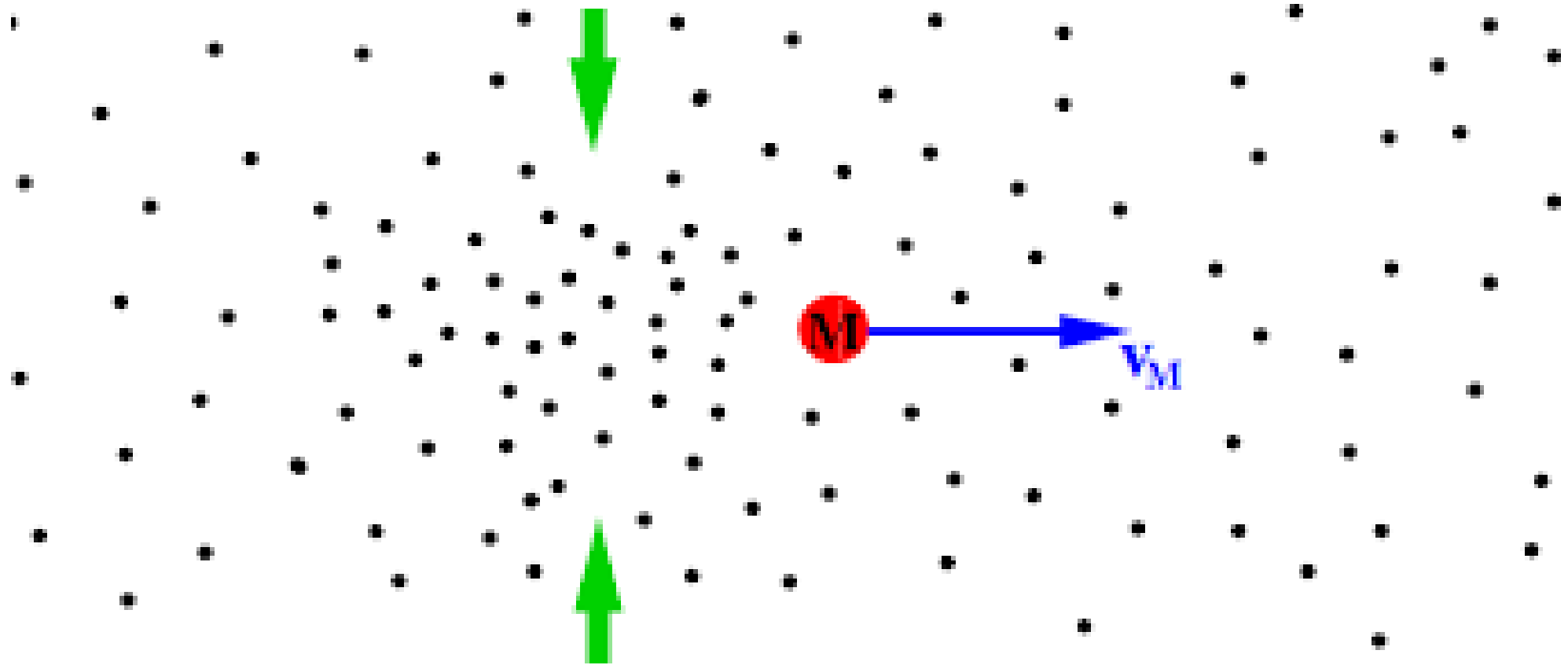
→ Probe DM with GWs!

[Hannuksela et al., '19]

[Edwards et al., '19]

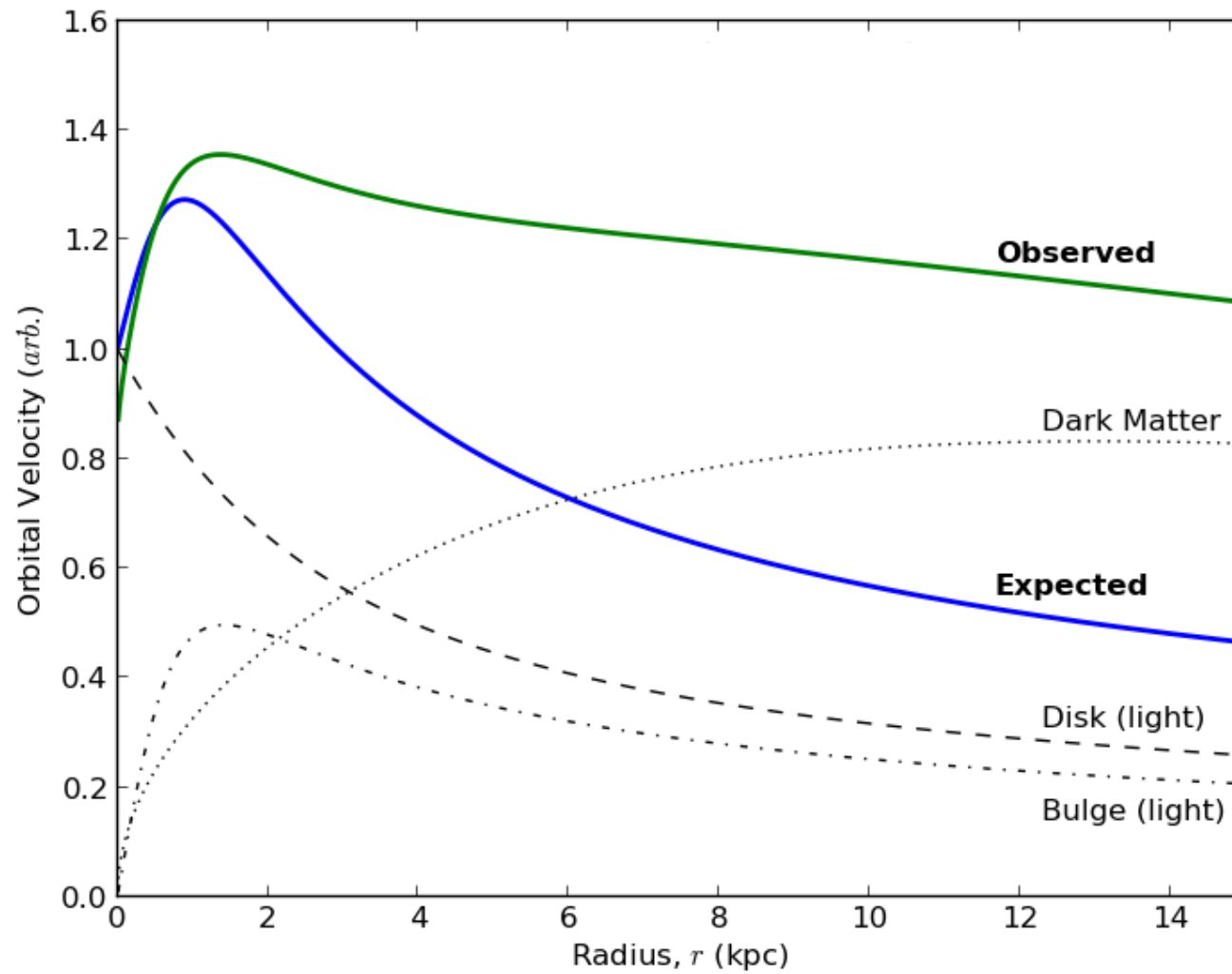


## Dynamical Friction



[[http://www.astro.yale.edu/vdbosch/astro610\\_lecture14.pdf](http://www.astro.yale.edu/vdbosch/astro610_lecture14.pdf)]

# Rotation curves



[adapted from Matthew Newby, Milkyway@home]

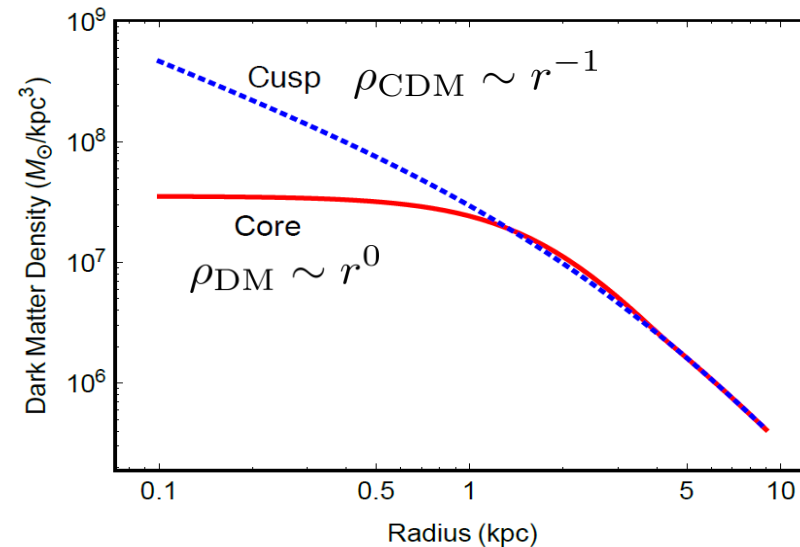
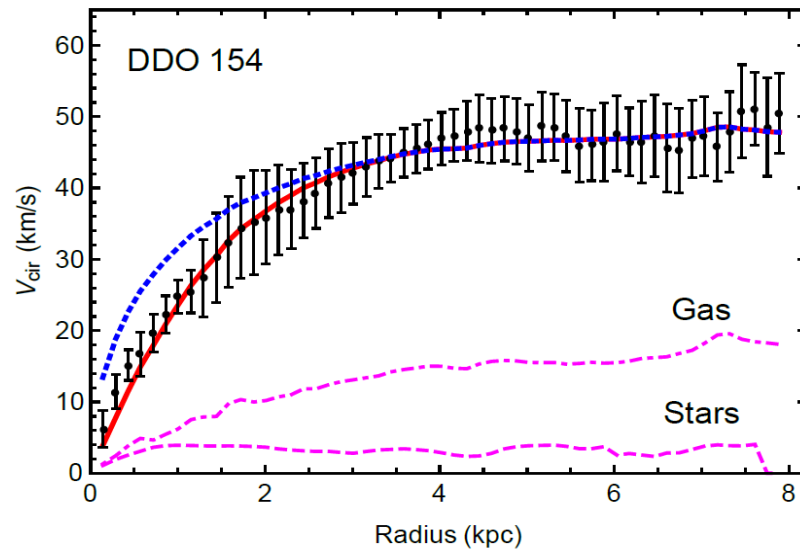
# Rotation curve of dwarf galaxies

Core-cusp problem:

[Moore, '94][Flores, Primack, '94]

DM density profile: core  $\leftrightarrow$  cusp (cold, collisionless dark matter)

→ Small-scale crisis



Adapted from: [Tulin, Yu, '17]

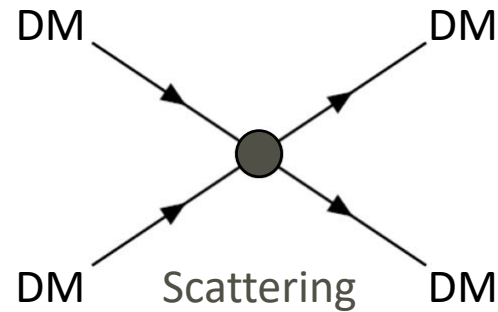
## Self-interacting dark matter (SIDM)

CDM = cold, collisionless dark matter

SIDM = cold, collisional dark matter

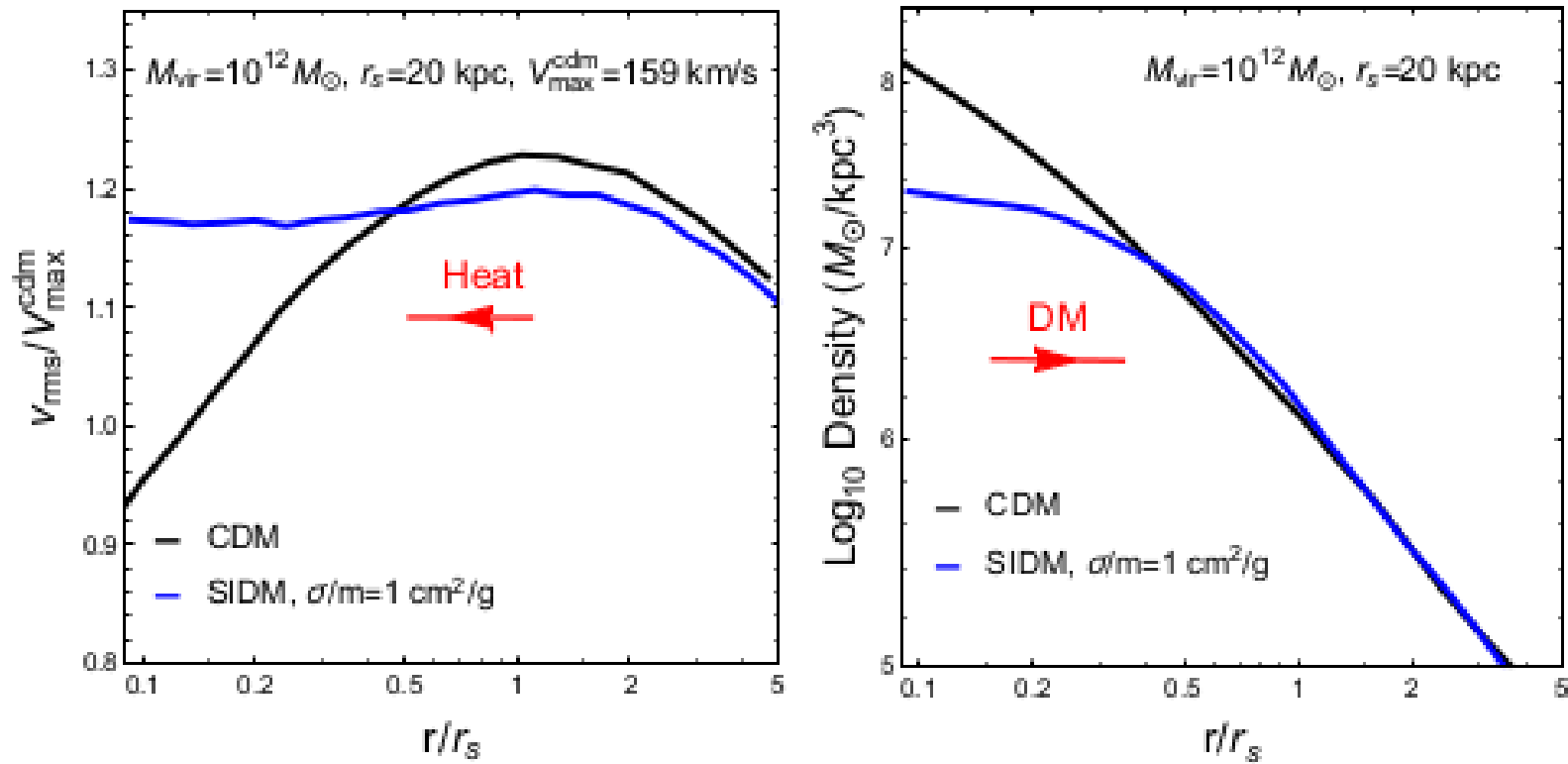
[Spergel, Steinhardt, '99]

→ DM particles self-interact:



$$\sigma/m \sim 1 \text{ cm}^2/\text{g} \sim 2 \text{ barns}/\text{GeV}$$

## Effect of Self-interactions



[Credit: Tulin & Yu (2017)]



Methods

# Describing SIDM

Hybrid approach:

- **NFW profile** in outer region
- **Isothermal profile** in inner region

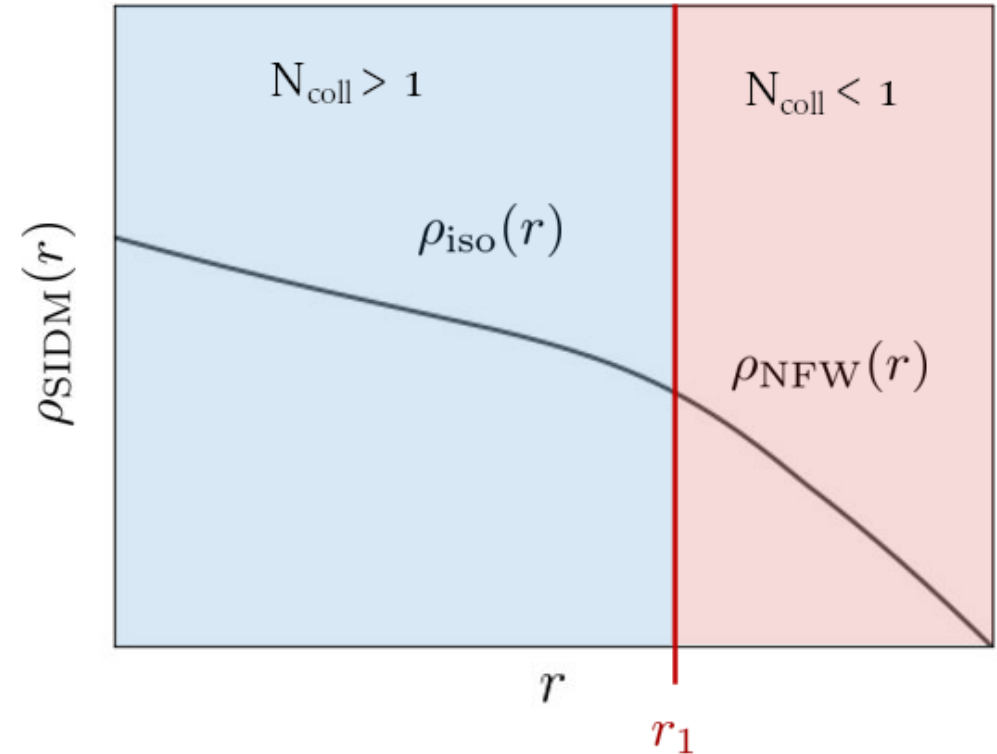
Goal:

- Match profiles continuously

NFW profile:

$$\rho_{\text{NFW}} = \frac{\rho_s}{\left(\frac{r}{r_s}\right) \left(1 + \frac{r}{r_s}\right)^2}$$

Q: how can we figure out the matching radius,  $r_1$ ?



# Describing SIDM

Hybrid approach:

- **NFW profile** in outer region
- **Isothermal profile** in inner region

Goal:

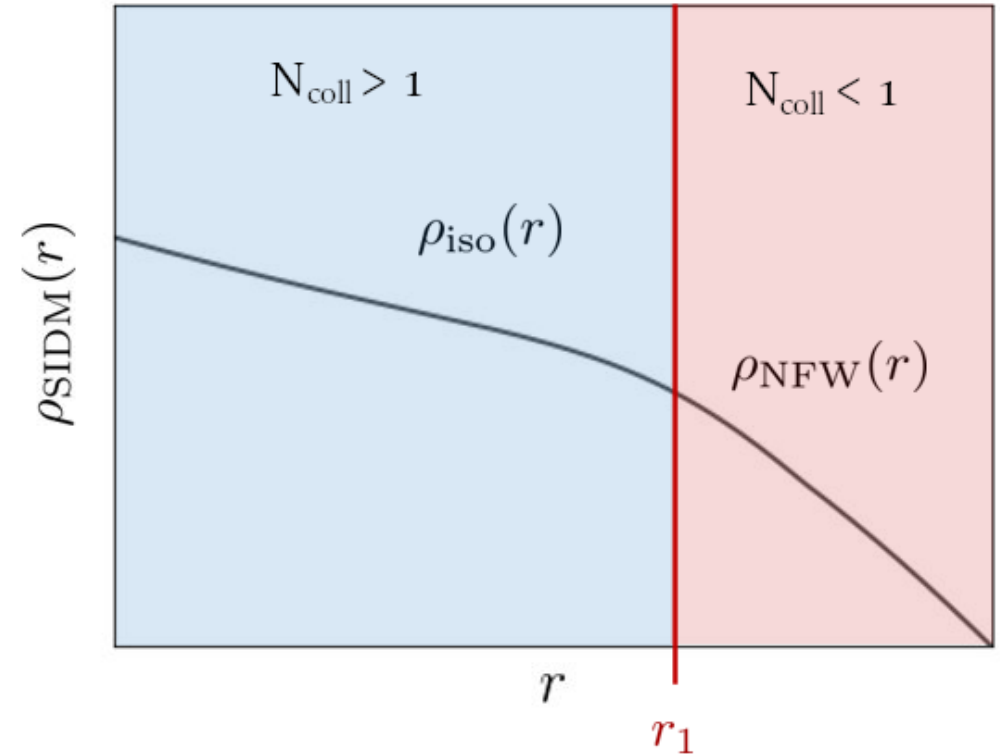
- Match profiles continuously

Matching condition:

$$N_{\text{coll}} = 1 \text{ at } r_1$$

$$\frac{N_{\text{coll}}}{t_{\text{age}}} \cdot t_{\text{age}} = 1$$

$$\rho_{\text{SIDM}}(r_1) \frac{\langle \sigma v \rangle}{m} \cdot t_{\text{age}} = 1$$



# Describing SIDM: Isothermal Profile

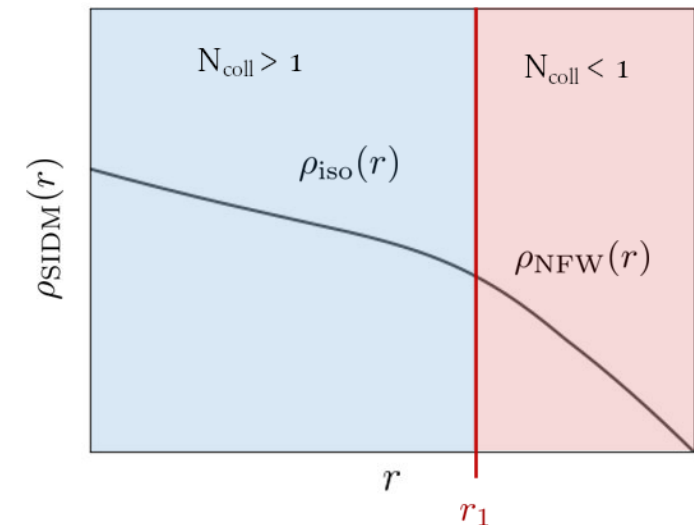
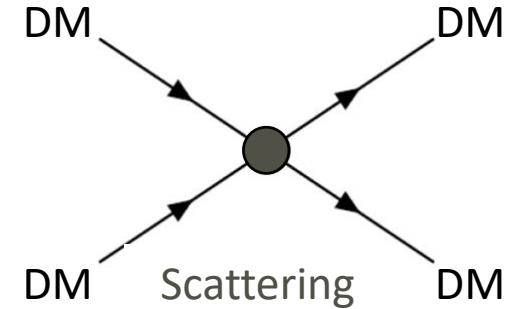
Jeans model

- Isothermal gas
- Hydrostatical equilibrium

Solve Jeans equations:

$$I. \quad \frac{d\rho_{\text{iso}}}{dr} = -\frac{G\rho_{\text{iso}}(r)}{\sigma_0^2 r^2} M_{\text{iso}}$$

$$II. \quad \frac{dM_{\text{iso}}}{dr} = 4\pi \rho_{\text{iso}}(r)$$

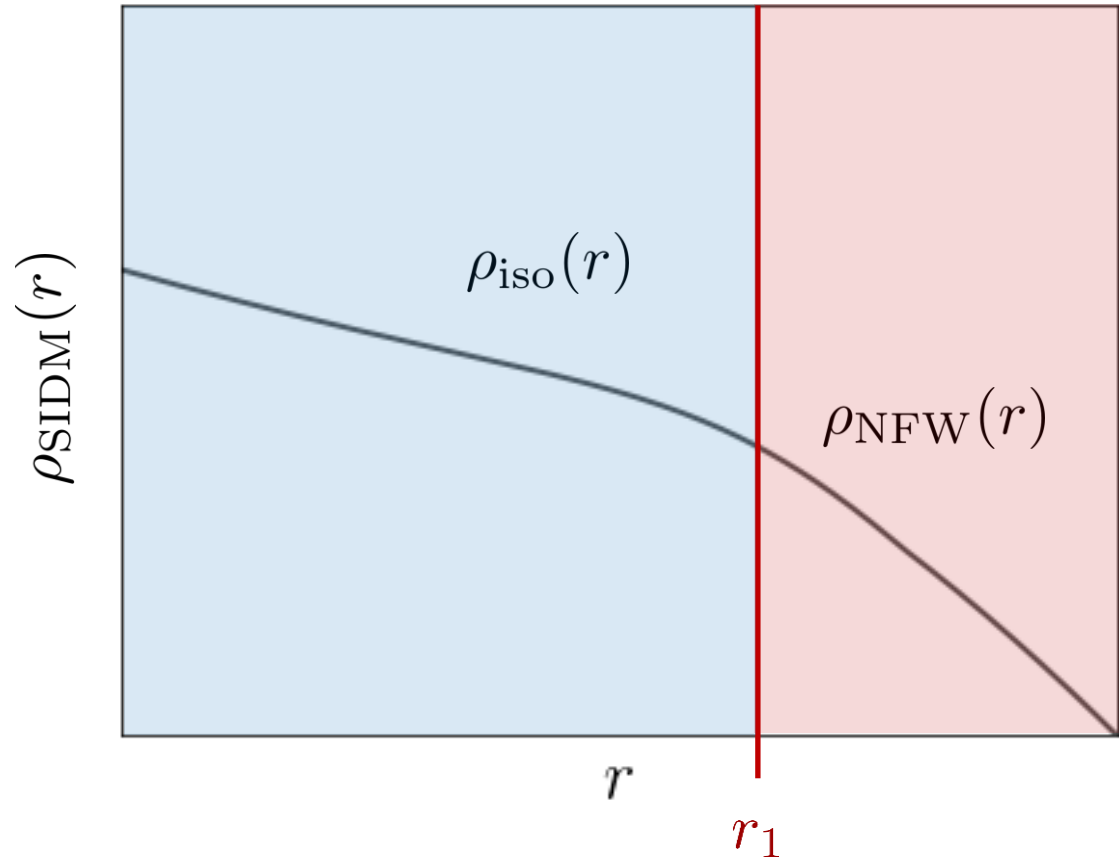


## Summary so far

SIDM density profile:

$$\rho_{\text{SIDM}}(r) = \begin{cases} \rho_{\text{iso}}(r), & r < r_1 \\ \rho_{\text{NFW}}(r), & r \geq r_1 \end{cases}$$

So what is the effect of the BH?



[Navarro, Frenk, White, '96]

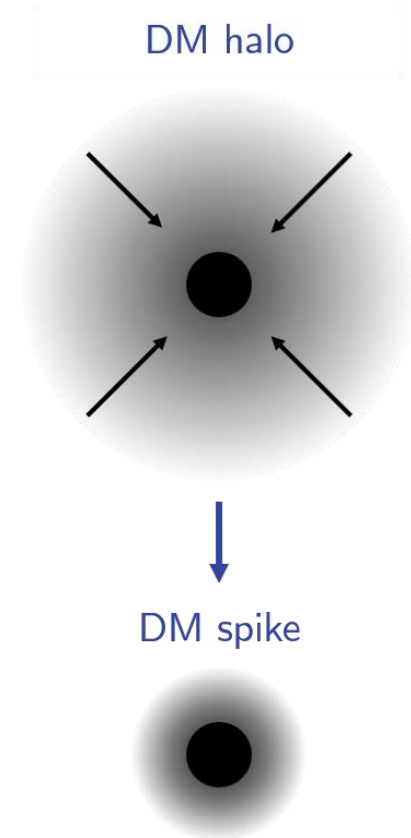
[Kaplinghat, Tulin, Yu, '15][Alvarez, Yu, '21]

# DM halos around BHs

## DM density spikes

- “Dressed” black hole in dark matter halo
- Creates dark matter spike with **extremely** high density

[Gondolo, Silk, '99][Eda et al., '13]



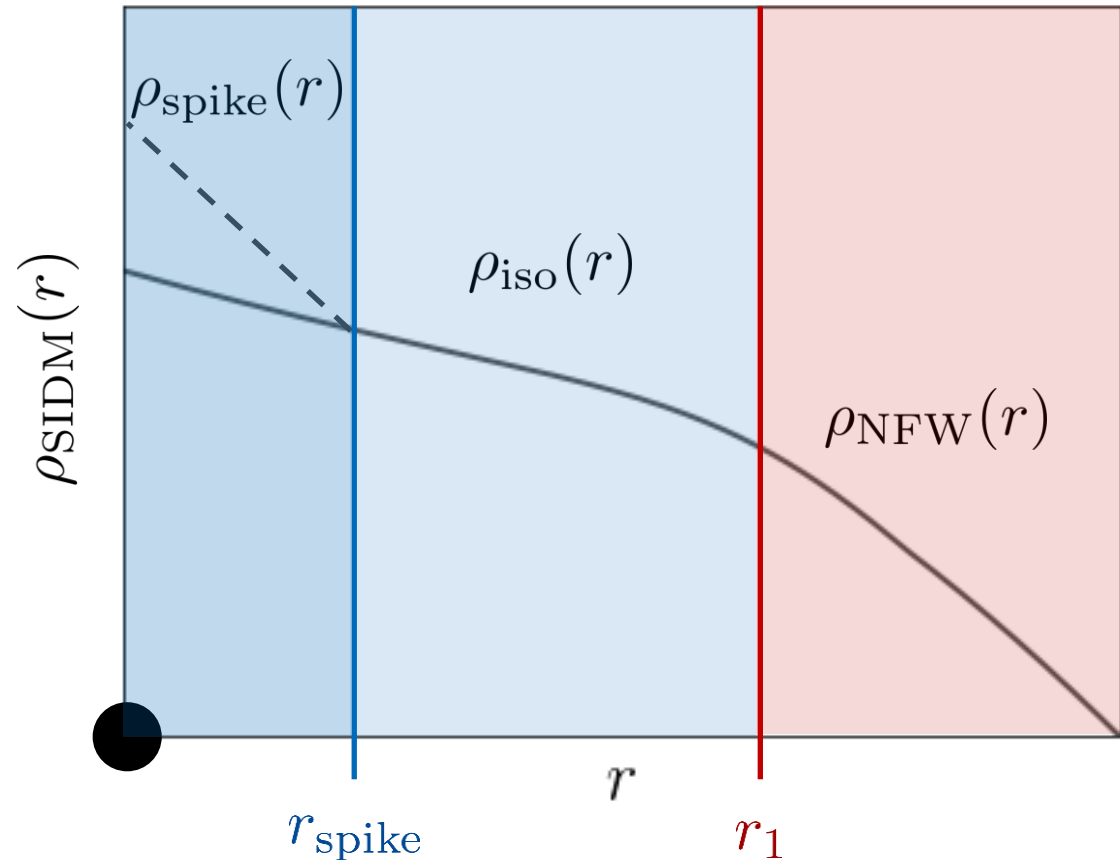
# Dark matter density spikes

SIDM density profile:

$$\rho_{spike}(r) = \rho(r_{spike}) \left( \frac{r_{spike}}{r} \right)^\gamma$$

$$r_{spike} = \frac{Gm_{BH}}{\sigma_0^2}$$

$\gamma = \text{varies between } \frac{7}{4} \text{ \& } \frac{7}{3}$



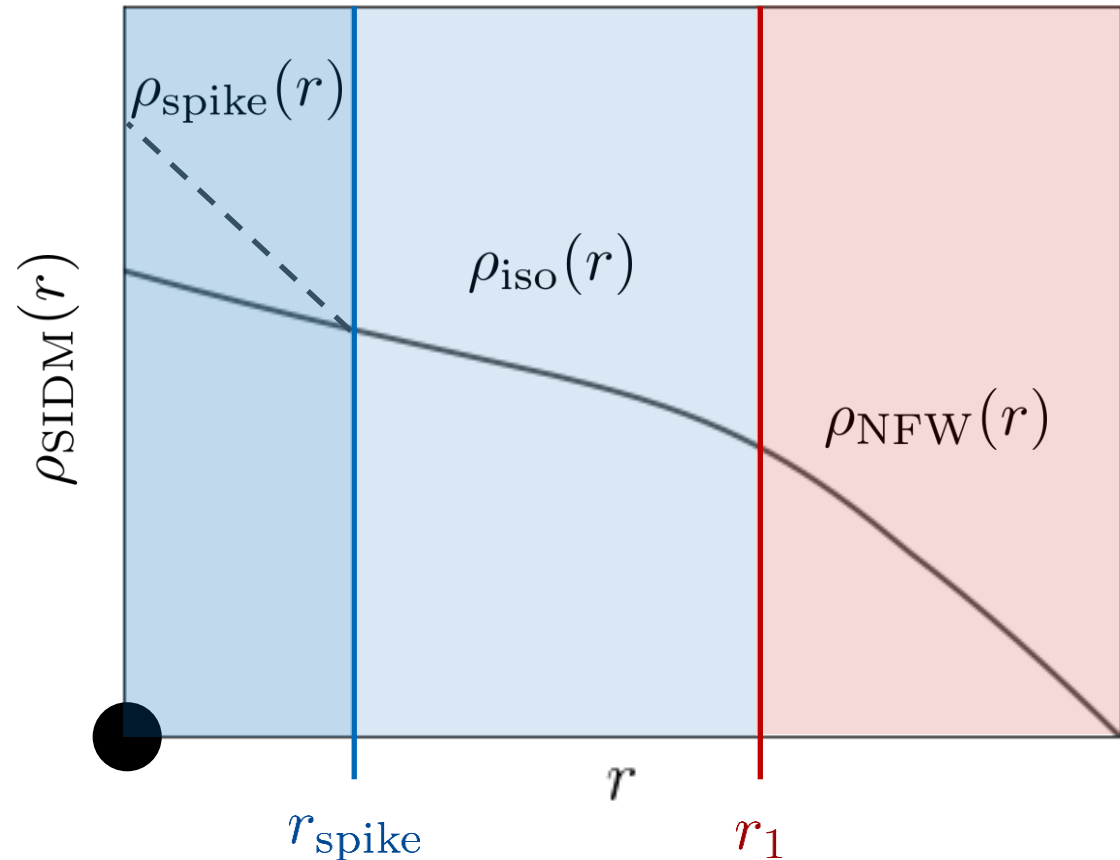
[Navarro, Frenk, White, '96]

[Kaplinghat, Tulin, Yu, '15][Alvarez, Yu, '21]

# Dark matter density spikes

SIDM density profile:

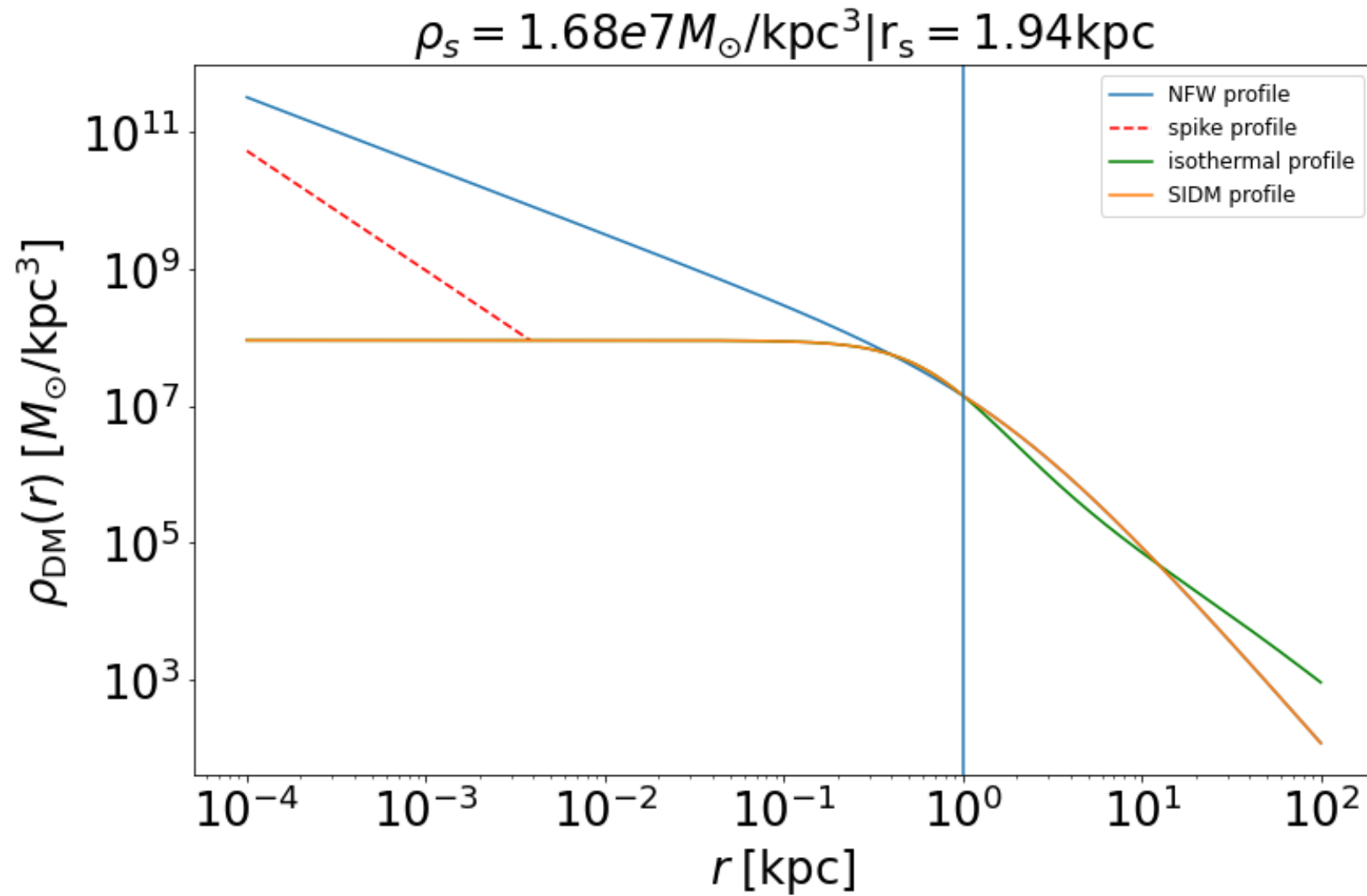
$$\rho_{\text{SIDM}}(r) = \begin{cases} \rho_{\text{spike}}(r), & r \leq r_{\text{spike}} \\ \rho_{\text{iso}}(r), & r_{\text{spike}} < r \leq r_1 \\ \rho_{\text{NFW}}(r), & r > r_1 \end{cases}$$



[Navarro, Frenk, White, '96]

[Kaplinghat, Tulin, Yu, '15][Alvarez, Yu, '21]

## Our try on this



## Footprint of the SIDM profile

> Extract it from the GW signal

Use the energy balance equation:

$$-\frac{dE_{\text{orbit}}(R)}{dt} = \frac{dE_{\text{GW}}(R)}{dt} + \frac{dE_{\text{DF}}(R)}{dt}$$

Calculate the time evolution of the orbital radius.

More about the Calculations of the GW signal in the next talk.

## Phase difference

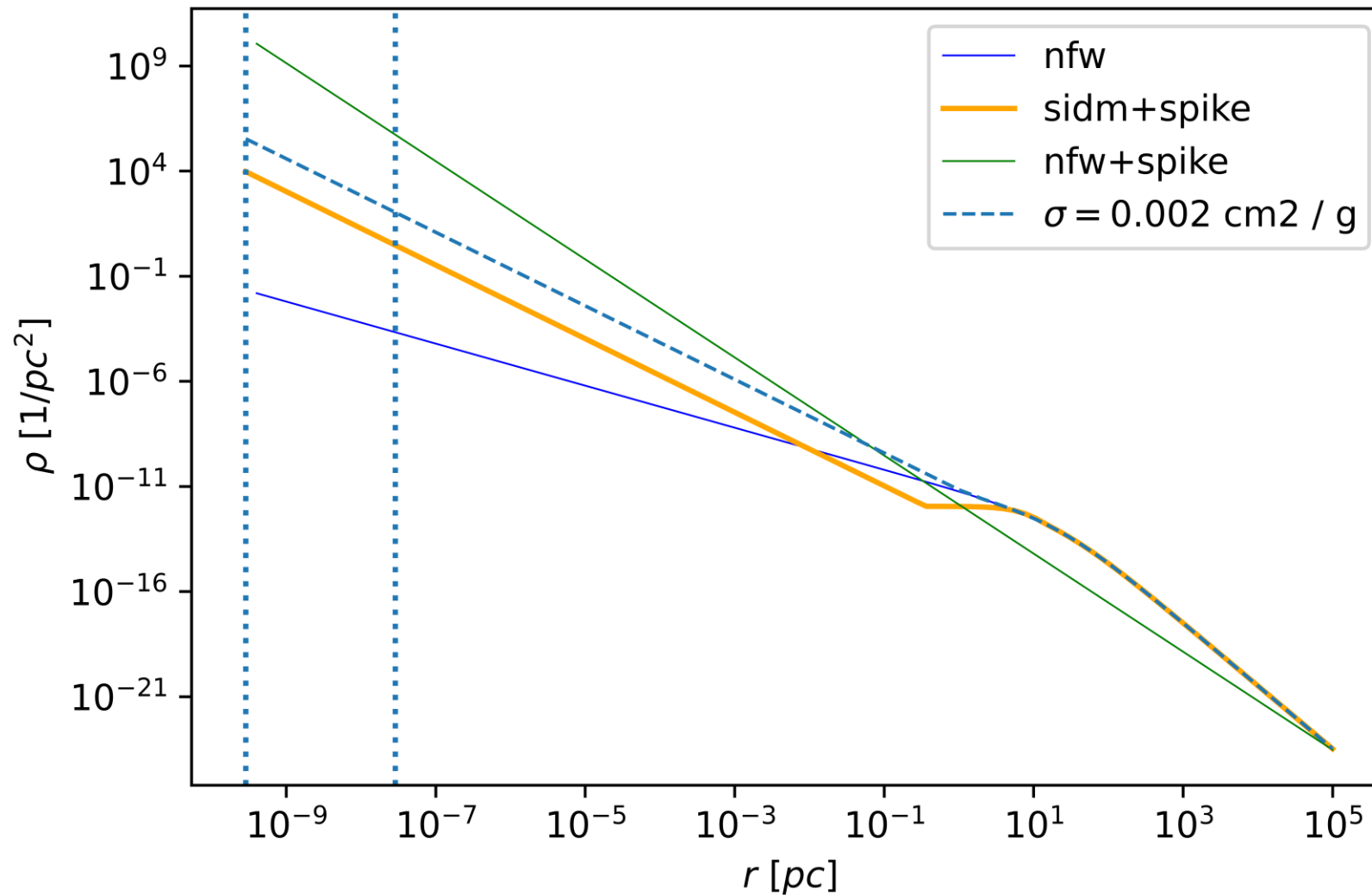
$$\Delta\tilde{\Phi}(f) \equiv \tilde{\Phi}(f) - \tilde{\Phi}_0(f)$$

## Characteristic strain

> Intensity (amplitude\*f\*2) of the wave signal



Results

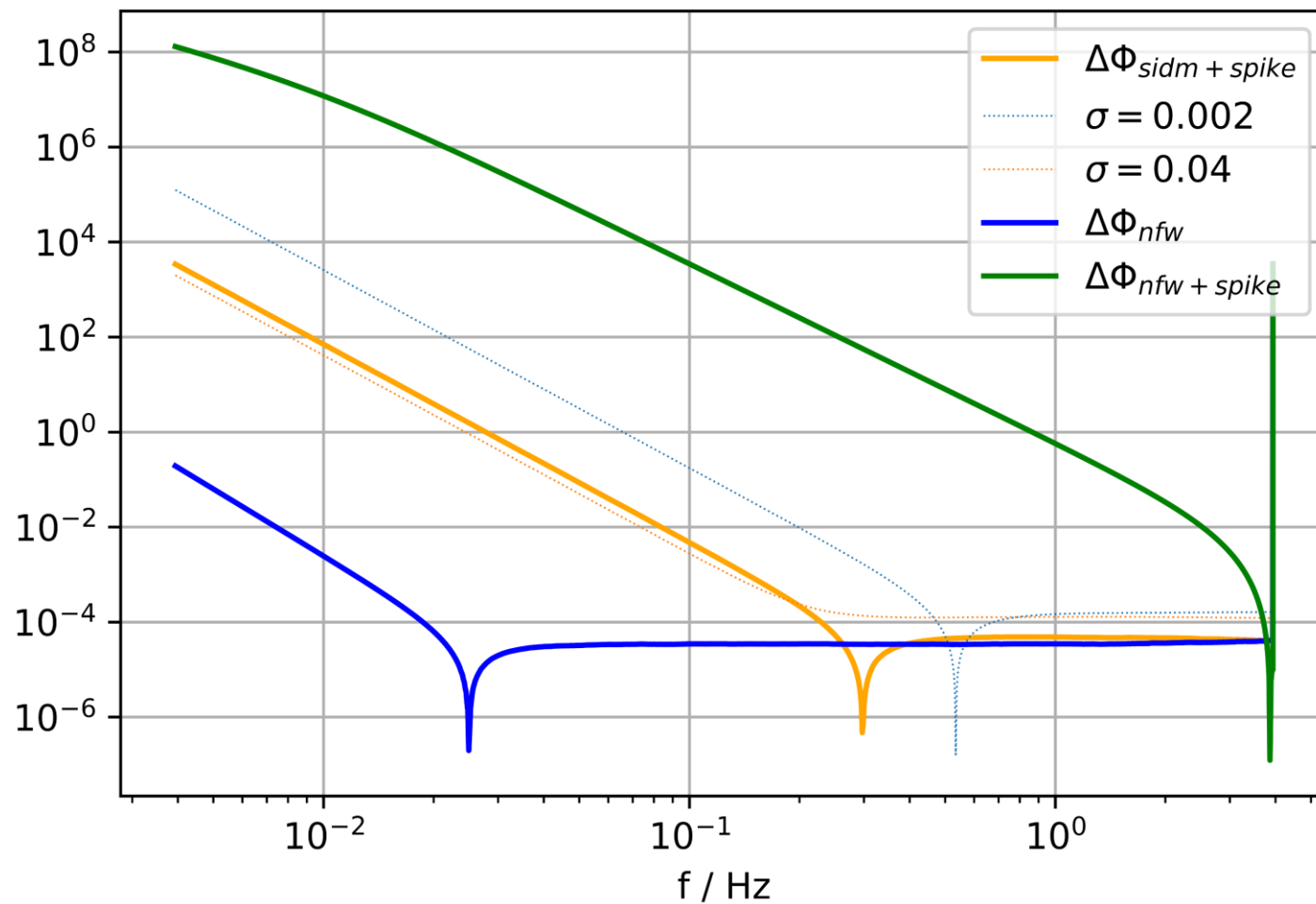
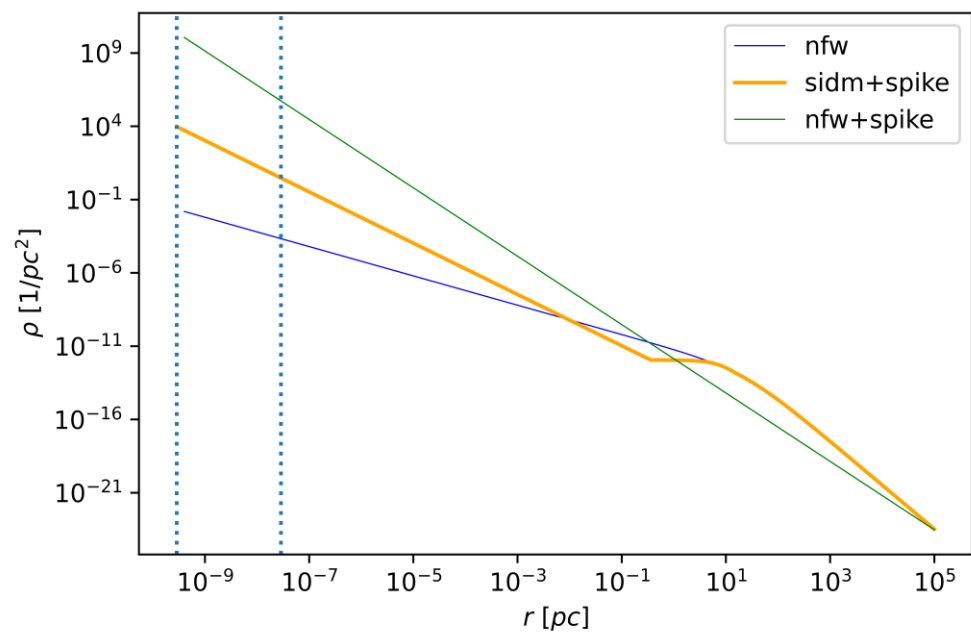


$$M_{bh} = 10^6 M_{\odot}$$

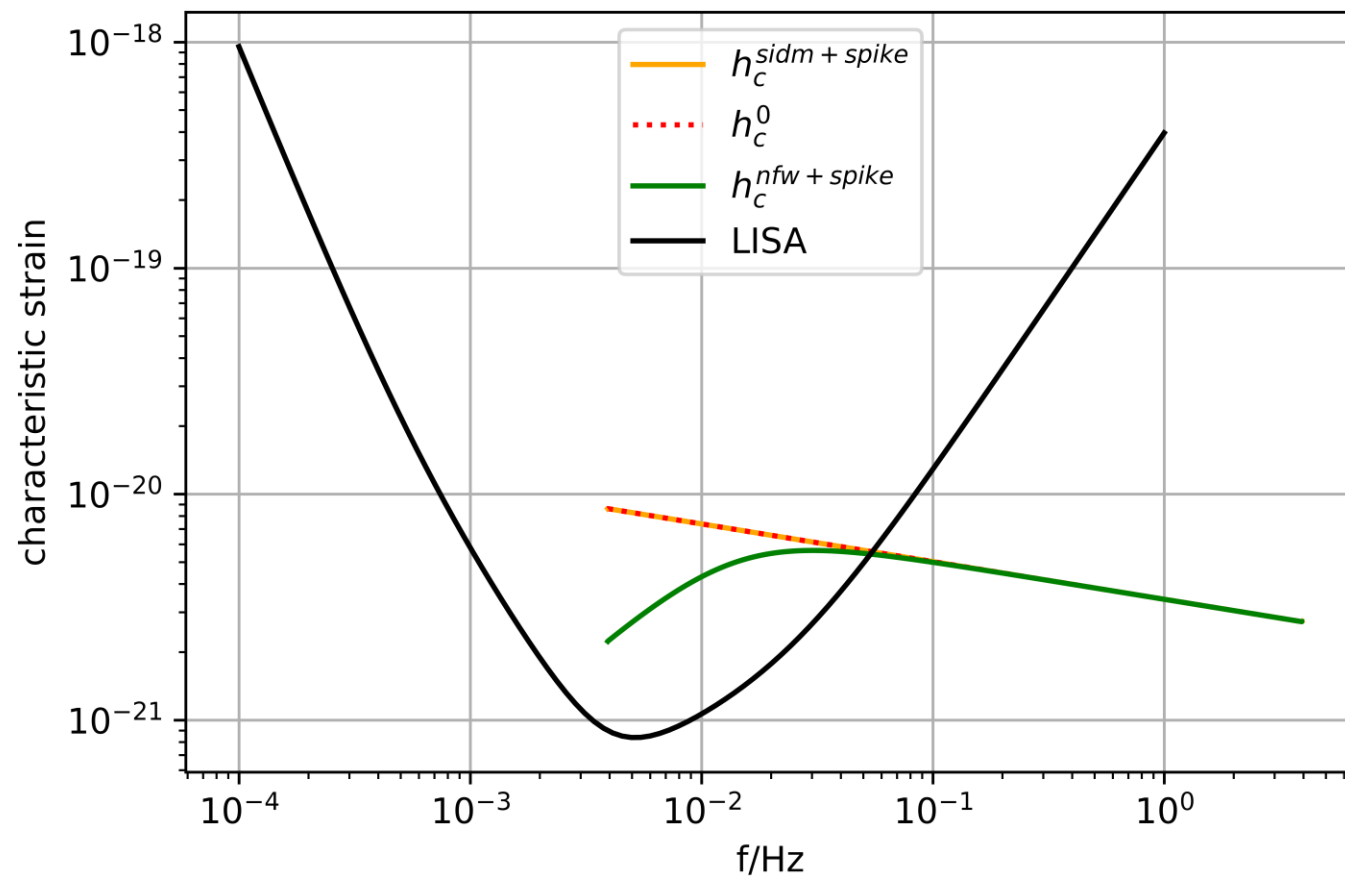
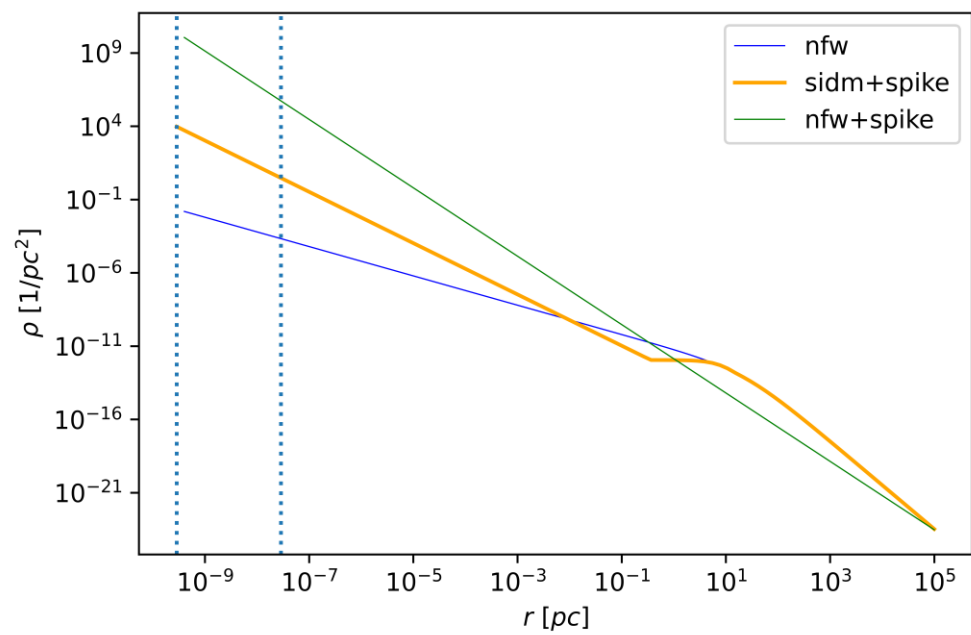
$$M_1 = 10^3 M_{\odot}$$

$$\sigma/m = 0.02 \frac{cm^2}{g}$$

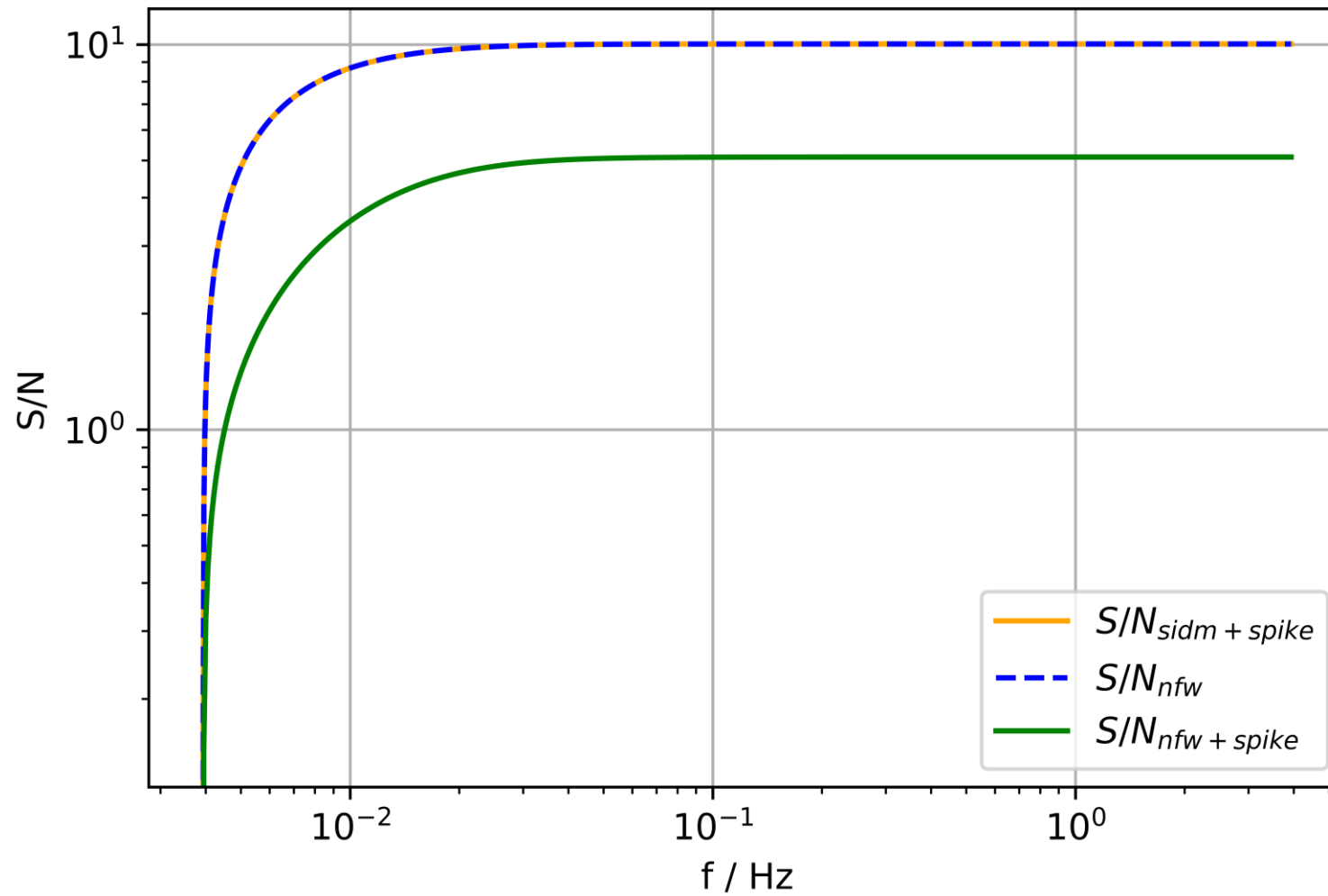
## Phase difference $\Delta\varphi$



# Characteristic Strain



## Signal-to-noise-ratio





Summary  
And  
Outlook

## Summary

- Detectable difference in the characteristic strain plot only found for cuspy-spiked profiles  
→ NFW + spike
- SIDM profiles with a spike leave no trace in characteristic strain

# Outlook

Possible improvements:

- Multiple solutions to Jeans equations
  - Explore core-collapse vs. core growing solutions
  - Include baryonic contributions
- Introduce velocity-dependent cross sections
- Investigate edge case for SIDM profile with spike
  - Possibly more distinct GW signal?
- Improved calculation of the orbits/GW signal
  - [See GW group!](#)

Thank you for your attention!