## Exploring Heterotic Models with Reinforcement Learning and Genetic Algorithms



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#### based on: 2103.04759, 2108.07316, 2110.14029, 2111.07333

## <u>Outline</u>

- Introduction
- Reinforcement learning
- Line bundles and RL a toy example
- Monad string models and RL
- Genetic algorithms
- Conclusion

## Introduction

String theory and machine learning (ML)

Much of the work so far for string theory and supervised ML

A remarkable variety of different types of data turns out to be susceptible to supervised ML methods:

Line bundle cohomology, Calabi-Yau Hodge numbers, properties of hadrons, data for toric varieties, certain classes of string models, AdS/CFT models, knot data, . . . For review see: F. Ruehle, Phys. Rep. 839, 1–117, 2020

Large string data sets require effective search algorithms:

Reinforcement learning (RL) known to be efficient for large environments (AlphaGo Zero)

J. Halverson, B. Nelson, F. Ruehle, 1911.07835

M. Larfors, R. Schneider, 2003.04817

S. Krippendorf, R. Kroepsch, M. Syvaeri, 2107.04039

String theory and genetic algorithms (GAs)

Long history outside our field starting in 1960's J. Holland, "Adaption in Natural and Artificial Systems", 1975

GAs are known to lead to efficient search algorithms

So far, only sporadically used in our field: Abel, Rizos, 1404.7359, Abel, Cerdeno, Robles 1805.03615, Cole, Schachner, Shiu, 1907.10072, Abel, Constantin, Harvey, Lukas 2110.14029 Cole, Krippendorf, Schachner, Shiu, 2111.11466 Q: Can RL/GA be used for string model building?

- Can RL/GAs ``construct" models with prescribed properties?
- Can RL/GAs discover model building strategies?
- Can RL/GAs be used for a comprehensive search?
- How do the two methods compare?

Study these questions for heterotic vacua based on a CY manifold X and a vector bundle  $V\to X\,$  constructed from a monad.

# <u>Reinforcement learning</u>

#### The cartoon



#### Application to model building

Goal: Explore large classes of string models and find those with certain prescribed properties, such as having a SM spectrum

Environment	 family of (QFT) string models
states	 specific models
action	 typically small modification of model
reward	 measure for how much desirable features of model improve

#### Two examples:

• Froggatt-Nielsen models for quark masses T. Harvey, A.L. 2103.04759

——— here

Heterotic CY models with monad bundles

## Algorithm:

REINFORCE and Actor-Critic algorithms, realised as Mathematica packages

# Line bundles and RL – a toy example

#### Mathematical background

Lines bundles  $L = O(\mathbf{k}) \to X$  on CY X classified by integer vectors  $\mathbf{k} = (k^1, \dots, k^h)$  where  $h = h^{1,1}(X)$ .

index of line bundle:

$$\begin{split} & \mathrm{ind}(L) = \frac{1}{12} (2\,d_{ijl}k^ik^jk^l + c_{2i}(TX)k^i) \\ & \text{CY triple intersection} \\ & \text{numbers} \\ \end{split} \begin{array}{l} & \text{second Chern class} \\ & \text{of CY tangent bundle} \\ \end{split} \end{split}$$

idea: would like to train a neural network to find line bundles with a given target index

## RL setup for line bundles

## Goal: Find line bundle with target index.

MDP	 line bundles on a fixed CY
Environment	 all $\mathbf{k} \in \mathbb{Z}^h$ with $ k^i  \leq k_{ ext{max}}$
states	 $\mathbf{k} \in \mathbb{Z}^h$
action	 $k^i  ightarrow k^i \pm 1$ (deterministic)
discount	 $\gamma=0.98$
reward	 value $\mathcal{V} = - ind(\mathcal{O}(\mathbf{k})) - ind_{target} $ reward function of change in value

#### <u>Results</u>

# example bi-cubic $X \in \begin{bmatrix} \mathbb{P}^2 & | & 3 \\ \mathbb{P}^2 & | & 3 \end{bmatrix}$ with h = 2, $\operatorname{ind}_{\operatorname{target}} = 18$



#### The trained policy network

- leads to terminal states for 100% of 32 step episodes
- has an average episode length of 5.7 steps
- has already found all terminal states during training

#### Plot of environment



Two sample episodes from trained net



#### basins of attraction for terminal states



#### Scaling of search algorithms:

- $\bullet$  systematic scan  $\sim$  total number of states
- $\bullet$  RL  $\sim$  number of terminal states

## Monad string models and RL

#### How large is the monad environment?

Definition of monads, based on line bundle sums:

$$0 \to V \to B \xrightarrow{f} C \to 0$$
  $V \cong \operatorname{Ker}(f)$ 

$$B = \bigoplus_{a=1}^{r_B} \mathcal{O}_X(\mathbf{b}_a) \qquad C = \bigoplus_{\alpha=1}^{r_C} \mathcal{O}_X(\mathbf{c}_\alpha) \quad \sim 10^{h(r_B + r_C - 1)} \text{ states}$$

Even small cases unexplored, for example

$$h = 2, r_B = 6, r_C = 1 \sim 10^{12}$$
 states

$$h = 3, r_B = 6, r_c = 1 ~ \sim 10^{18}$$
 states

I am not aware of a ``trick" that would allow for a systematic scanning in these cases . . .

Only a few phenomenologically promising models known . . .

RL setup for heterotic monads

Goal: Explore monads on a given CY and find standard models.

monads on a fixed CY with  $c_1(B) = c_1(C)$ MDP  $\longrightarrow \quad \text{all}(B,C) \text{ with } \begin{cases} b_{\min} \leq b_a^i \leq b_{\max} \\ c_{\min} < c_a^i < c_{\max} \end{cases}$ Environment (B,C)states  $\longrightarrow \begin{cases} b_a^i \to b_a^i \pm 1 \\ c_i^i \to c_i^i \pm 1 \end{cases} \quad \text{(deterministic)}$ action  $\longrightarrow \gamma = 0.98$ discount value = -deviation from desired reward properties, reward from value terminal state a standard model

a) Bi-cubic, SO(10) GUT,  $\Gamma = \mathbb{Z}_3 \times \mathbb{Z}_3$  Wilson line, negative entries  $b_{\min} = -3, \ b_{\max} = 5, \ c_{\min} = 0, \ c_{\max} = 5, \ r_B = 6, \ r_c = 2 \qquad \rightarrow \quad \sim 10^{12} \text{ states}$ 

#### Training measurements (about 1h on single CPU)



- training based on tiny sample of environment
- 100% of all length 32 episodes terminal, average length 15.5
- find few x 10 terminal states, 10–20 stable, with no mirror-families

#### Use trained network for systematic search

Carry out many episodes with random initial states (35 core days)



- Saturation suggests all models are found (about 600)
- Many new standard-like models found
- Difficult to find these models with a systematic scan
- Includes the one known standard model (L. Anderson, J. Gray, Y.-H. He, A.L., 0911.1569)

b) tri-linear, SO(10) GUT,  $\Gamma = \mathbb{Z}_3 \times \mathbb{Z}_3$  Wilson line, negative entries

$$X \sim \begin{pmatrix} \mathbb{P}^2 & | & 1 & 1 & 1 \\ \mathbb{P}^2 & | & 1 & 1 & 1 \\ \mathbb{P}^2 & | & 1 & 1 & 1 \end{pmatrix}$$

 $b_{\min} = -3, \ b_{\max} = 5, \ c_{\min} = 0, \ c_{\max} = 5, \ r_B = 6, \ r_c = 2 \qquad \rightarrow \quad \sim 10^{19} \text{ states}$ 

- training about 1d on single CPU
- training based on tiny sample of environment
- 100% of all length 32 episodes terminal, average length 22
- 13000 terminal states, 7500 without mirror-families, O(500) stable

-> a new database of standard-like models, virtually impossible to find with standard scanning methods

#### Model building strategies suggested by RL



Figure 7: The different contributions to the intrinsic value for  $(r_b, r_c) = (6, 2)$  bicubic models. This data is averaged over 1000 termianl states using the trained network.

# Genetic algorithms

How do genetic algorithms perform on a monad environment? (S. Abel, A. Constantin, T. Harvey, AL, 2110.14029, 211107333)

- monad (B, C) \_\_\_\_\_ convert to binary  $\rightarrow$  010010 $\cdots$ 0110
- assign ``fitness" to each monad = value function of RL
- create population (250 individuals in our case)

 goal: after a number of generations the population has many fit individuals (= terminal states)

#### Example bicubic



Produces some terminal states extremely efficiently (minutes!)

Example tri-linear



Systematic GA search

#### Repeated GA evolutions for bi-cubic (10 core days)



- Saturation suggest all terminal states found
- Terminal states largely the same as found from RL
- GA more efficient than RL (for our realisations) for finding a few states quickly and for comprehensive search

## **Conclusion**

- RL can engineer topological properties and can``learn" the rules for geometrical string model building
- RL suggests model building strategies
- At small  $h^{1,1}(X)$  RL can be used for a comprehensive search within environments too large for a systematic scan
- GAs work extremely efficiently as well confirm RL results
- Many new standard-like models found in this way
- Questions/extensions:

SU(5) models, including other model-building constraints, different string constructions, including gravitational sector, . . .

How does computing/training time scale with  $h^{1,1}(X)$ ? Can RL or GA be used for a complete scan of the landscape?



#### Value=fitness and reward

Value measures degree of deviation from desired properties:

- $c_1(V) = c_1(B) c_1(C) \stackrel{!}{=} 0$  (built into environment)
- anomaly  $c_2(V) \stackrel{!}{\leq} c_2(TX)$
- ``bundleness": V a bundle rather than a sheaf
- very basic check for bundle supersymmetry
- number of chiral families of ok,  $\operatorname{ind}(V)/|\Gamma| \stackrel{!}{=} -3$
- basic check for bundle  $\Gamma$ -equivariance

Anything that involves calculating cohomology too slow during training -> needs checking later

Reward: some function of the change in value + terminal bonus