Heavy four-quark states from lattice QCD

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Collaborators: P. Junnarkar and M. Padmanath

Mini-workshop on $T_{cc}^+$ and beyond, CERN, September 14, 2021
<table>
<thead>
<tr>
<th>Relatively Easy</th>
<th>Moderately difficult</th>
<th>Difficult</th>
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**Hadron Spectra: What can Lattice QCD do?**
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- **Relatively Easy**
  - Ground state masses of strong-interactions stable single hadrons (pion, nucleon, ground state charmonia etc.) can reliably be obtained (provided reliable control over statistical and systematics errors are achieved).

- **Moderately difficult**

- **Difficult**
Hadron Spectra: What can Lattice QCD do?

Ground state masses of strong-interactions stable single hadrons (pion, nucleon, ground state charmonia etc.) can reliably be obtained (provided reliable control over statistical and systematics errors are achieved).

Excited state masses not very closed to thresholds (excited states of a few mesons, heavy baryons)
Ground state masses of strong-interactions stable single hadrons (pion, nucleon, ground state charmonia etc.) can reliably be obtained (provided reliable control over statistical and systematics errors are achieved).

Excited state masses not very closed to thresholds (excited states of a few mesons, heavy baryons)

Hadrons
- particularly close to thresholds (like deuteron, X(3872), Z(4430), $T_{cc}$ pentaquarks, H-dibaryon etc)
- hadrons decaying/mixing to multi-hadrons ($1^{-+}$ light exotics, light baryon resonances, glueballs etc)
- nuclei
Heavy Four-quark states

• Fourquark states have been observed experimentally with heavy quark contents. ....LHC, Belle, BES

• Is their possibility to find more of those? And other multiquark states?

• What can lattice studies do?
LQCD for heavy quark physics

Requirement: lattice quark mass $ma \ll 1$

$$\frac{1}{L} \ll m_{\pi} \ll m_{H} \ll \frac{1}{a}$$
LQCD for heavy quark physics

Requirement: $ma \ll 1$

- **Charm**: $ma = 1.275$ GeV,
  
  $ma = 0.5 \rightarrow a \sim 0.075$ fm
  
  $ma = 0.3 \rightarrow a \sim 0.046$ fm

- **Bottom**: $ma = 4.66$ GeV
  
  $ma = 0.5 \rightarrow a = 0.021$ fm
  
  $ma = 0.3 \rightarrow a = 0.013$ fm

- **Heavy light hadrons**: 
  \[ \frac{1}{L} \ll \frac{1}{m_H} \ll m_{\pi} \ll \frac{1}{a} \]

  - Being heavy, lattice correlation functions for heavy quarks decay rapidly.
  
  - Relativistic charm quark calculations are now possible.
  
  - However, relativistic bottom-quark is still very costly. Only a few groups have recently initiated to utilize relativistic bottom quarks.
  
  - Most of the lattice calculations with bottom quarks utilize NRQCD.
Lattice study of heavy exotics

• Nonrelativistic $b$ quark with relativistic other quarks (calculations with relativistic $b$ quarks are starting)

• Potential:
  • Static quark potential (Born-Oppenheimer)…
  • HALQCD lattice potential ..HAL QCD (’16,’18)
Energy excitations can be obtained from Euclidean two-point correlation functions

\[ \langle C^{2pt}_{ab}(t, \vec{P}) \rangle = \sum_n Z_{b,n} Z^\dagger_{a,n} e^{-E_n} \]
Energy excitations can be obtained from Euclidean two-point correlation functions

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\langle C_{ab}^{2pt}(t, \vec{P}) \rangle = \sum_n Z_{b,n} Z_{a,n}^* e^{-E_n}
\]

Suitable quark smearing (including distillation method) can improve overlap to the desired energy levels

\[
q(x, t) = \sum_y S(x, y) q_b(y, t)
\]
**Hadron Spectra: What can Lattice QCD do?**

- Energy excitations can be obtained from Euclidean two-point correlation functions:
  \[
  \left< C_{ab}^{2pt}(t, \bar{P}) \right> = \sum_n Z_{b,n} Z_{a,n}^* e^{-E_n}
  \]

- Suitable quark smearing (including distillation method) can improve overlap to the desired energy levels:
  \[
  q(x, t) = \sum_y S(x, y) q_b(y, t)
  \]

- Correlation matrices of a large basis of hadronic interpolating fields can provide multiple energy levels.

- Variational analysis with a good basis of operators can be utilized to obtain energy excitations.

- Finite volume analysis using Luscher formula can help to find out pole structure along with scattering information from the extracted Euclidean energy levels.
Energy spectra within finite volume

Infinite-volume bound state

\[ [E(L) - E(\infty)] \propto \frac{e^{-\gamma L}}{\gamma L} \]

Huang, Yang, Phys. Rev. 105 (1957)


... Recent review:

Briceño, Dudek, Young, Rev. Mod. Phys. 90 (2018)

Infinite-volume scattering state

\[ [E(L) - E(\infty)] \propto \frac{a}{ML^3} \]

Finite volume Euclidean time spectra

Infinite volume real time scattering amplitudes

Mini-workshop on $T_{cc}^+$ and beyond, CERN
Heavy four-quark states

\[ \begin{align*}
\frac{q}{q} & \frac{\bar{Q}}{\bar{Q}} \\
[Q\bar{Q}]_3 & \rightarrow Q \\
\frac{q}{q} & \frac{Q}{Q}
\end{align*} \]

Diquark properties:

- \[ [Q\bar{Q}]_{mq^\rightarrow\infty} \rightarrow \text{compact} \]
- \( \{qq'\}^3 \rightarrow \text{attractive} \)
- \( \{qq'\} \equiv (qC\gamma_5q') \rightarrow \text{lightest} \)
- \( m(\{ud\}) < m(\{us\}) \)

A possible structure:

\[ (qC\gamma_5q')(\bar{Q}C\gamma_i\bar{Q}') \]

How about?

\[ \{qq'\} \]

\[ \{\bar{Q}Q'\} \]

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How to build heavy tetraquarks? A way

- Two heavy quarks with two light quarks
- $c_l = 3$, good light diquark
  - $F = 3, J_l = 0 \Rightarrow J_h = 1, C_h = 3, J^P = 1^+$
  
  $$\downarrow$$
  $$\{3, J = 0\} \quad \{3, J = 1\}$$
  
- Spin dependent interaction $\propto 1/m_h$. For threshold $J^P = 1^+$ states, like $B^*B, B^*D, B_s^*B_s, D^*D$, this interaction will be suppressed.

- With $C_h = 3$, colour Coulomb attraction, this is not present for two-meson thresholds.

Possible states? : $\bar{b}b\bar{u}d, \bar{b}b\bar{u}s, \bar{b}\bar{b}u\bar{c}, \bar{b}\bar{b}sc, \\ \bar{b}\bar{c}ud, \bar{b}\bar{c}us$ etc.

$J = 1, l_1l_2\bar{Q}\bar{Q}$  $\quad J = 0, ll\bar{Q}\bar{Q}$

@Francis et al (2016)
Interpolating Fields

\[ J = 1, l_1 l_2 \overline{Q} \overline{Q} \]

\[(l_1, l_2) \rightarrow (3_c, 0, F_A), \quad (\overline{Q}, \overline{Q}) \rightarrow (3_c, 1, F_s)\]
\[(l_1, l_2; l_1 \neq l_2) \subset (u, d, s, c). \quad Q \neq l_1 \neq l_2 \subset (\bar{c}, \bar{b})\]

- **Tetraquark type:**

\[ T^1(x) = (l_1)_\alpha^a(x) (C\gamma_5)_\alpha\beta (l_2)_\beta^b(x) \overline{Q}_\alpha^a(x) (C\gamma_5)_\kappa\rho \overline{Q}_\rho^b(x) \]

- **Two mesons type:**

\[ \mathcal{M}^1(x) = M_1(x)M_2^*(x) - M_2(x)M_1^*(x) \]
\[ M_{1,2}(x) = (l_{1,2})_{\alpha}^a(x) (\gamma_5)_{\alpha\beta} \overline{Q}_\beta^a(x) \]
\[ M_{1,2}^*(x) = (l_{1,2})_{\alpha}^a(x) (\gamma_5)_{\alpha\beta} \overline{Q}_\beta^a(x). \]

\[ C_{ij}(t) = \sum_x \langle 0 | \mathcal{O}_i(x, t) \mathcal{O}_j^\dagger(0, 0) | 0 \rangle \]
\[ \mathcal{O}_i(x, t) \in \{ T(x, t), M(x, t) \} \]

**Phys. Rev. D99 034507 (2019),**

<table>
<thead>
<tr>
<th>( (l_1l_2\overline{Q}\overline{Q}) )</th>
<th>( [(M_1M_2^<em>) (M_2M_1^</em>)] )</th>
<th>( I )</th>
<th>( m_\pi ) (MeV)</th>
</tr>
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<tbody>
<tr>
<td>( u\overline{d}\overline{b}\overline{b} )</td>
<td>( (BB^0*) (B^0 B^*) )</td>
<td>0</td>
<td>(257 - 688)</td>
</tr>
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<td>( u\overline{s}\overline{b}\overline{b} )</td>
<td>( (BB^<em>_s) (B_s B^</em>) )</td>
<td>( \frac{1}{2} )</td>
<td>(186 - 688)</td>
</tr>
<tr>
<td>( u\overline{c}\overline{b}\overline{b} )</td>
<td>( (BB^<em>_c) (B_c B^</em>) )</td>
<td>( \frac{1}{2} )</td>
<td>(153 - 688)</td>
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<tr>
<td>( u\overline{d}\overline{c}\overline{c} )</td>
<td>( (DD^0*) (D^0 D^*) )</td>
<td>0</td>
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Mini-workshop on \( T_{cc}^+ \) and beyond, CERN
Mini-workshop on $T^+_c$ and beyond, CERN
Junnarkar et al,

Mini-workshop on $T_{cc}^+$ and beyond, CERN
Heavier the heavy quark masses, deeper the binding

Lighter the light quark masses, deeper the binding
$J = 0, \bar{u}Q\bar{Q}$

$$(l,l) \rightarrow (6_c, 0, F_S), \quad (\bar{Q}\bar{Q}) \rightarrow (\bar{6}_c, 0, F_S).$$

$$l \in (u, s, c) \quad h \in (c, b).$$

**Tetraquark-type:**

$$\mathcal{T}^0(x) = l^a_\alpha(x)(C\gamma_5)_{\alpha\beta}l^b_\beta(x) \bar{Q}_K^b(x)(C\gamma_5)_{\kappa\rho}\bar{Q}^a_\rho(x).$$

**Two mesons-type:**

$$\mathcal{M}^0(x) = \bar{Q}_\alpha^a(x)(\gamma_5)_{\alpha\beta}l^a_\beta(x) \bar{Q}_K^b(x)(\gamma_5)_{\kappa\rho}l^b_\rho(x).$$

$$C_{ij}(t) = \sum_x \langle 0|\mathcal{O}_i(x, t)\mathcal{O}^\dagger_j(0, 0)|0\rangle$$

$$\mathcal{O}_i(x, t) \in \{T(x, t), M(x, t)\}$$

<table>
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<tr>
<th>$(l_1 l_2 \bar{Q}\bar{Q})$</th>
<th>$(M_1 M_2)$</th>
<th>$I$</th>
<th>$m_\pi$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u\bar{u}b\bar{b}$</td>
<td>$(B B)$</td>
<td>1</td>
<td>(337 - 688)</td>
</tr>
<tr>
<td>$u\bar{u}c\bar{c}$</td>
<td>$(D D)$</td>
<td>1</td>
<td>(297 - 688)</td>
</tr>
<tr>
<td>$s\bar{s}b\bar{b}$</td>
<td>$(B_s B_s)$</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>$c\bar{c}b\bar{b}$</td>
<td>$(B_c B_c)$</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>$s\bar{s}c\bar{c}$</td>
<td>$(D_s D_s)$</td>
<td>0</td>
<td>-</td>
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</table>

Finite volume effects for heavy hadrons:

$$\Delta_{FV} = E_{FV} - E_{\infty} \propto O(e^{-k_{\infty}L})/L,$$

with

$$k_{\infty} = \sqrt{(m_1 + m_2)B_{\infty}},$$

Huang, Yang, Phys. Rev. 105 (1957)


Recent review:
Briceño, Dudek, Young, Rev. Mod. Phys. 90 (2018)
$u \bar{d} b \bar{b} (1^+) $

![Graph showing binding energy vs. lattice NRQCD, lattice QCD static potentials, and model calculations.](image)


Mini-workshop on $T_{cc}^+$ and beyond, CERN

<table>
<thead>
<tr>
<th>State</th>
<th>$\Delta E^1$ [MeV]</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$u\bar{d}\bar{b}\bar{b}$</td>
<td>-143(34)</td>
<td>$u\bar{s}\bar{b}\bar{b}$</td>
<td>-87(32)</td>
</tr>
<tr>
<td>$u\bar{c}\bar{b}\bar{b}$</td>
<td>-6(11)</td>
<td>$s\bar{c}\bar{b}$</td>
<td>-8(3)</td>
</tr>
<tr>
<td>$u\bar{d}\bar{c}\bar{c}$</td>
<td>-23(11)</td>
<td>$u\bar{s}\bar{c}\bar{c}$</td>
<td>-8(8)</td>
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<tr>
<td>$u\bar{u}\bar{b}\bar{b}$</td>
<td>-5(18)</td>
<td>$u\bar{w}\bar{e}$</td>
<td>26(11)</td>
</tr>
<tr>
<td>$s\bar{s}\bar{b}\bar{b}$</td>
<td>3(9)</td>
<td>$s\bar{s}\bar{e}\bar{e}$</td>
<td>14(4)</td>
</tr>
<tr>
<td>$c\bar{c}\bar{b}\bar{b}$</td>
<td>16(1)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
$\bar{c}cq_1q_2$
What about $T_{bc}: \bar{b}\bar{c}q_1q_2$?

Various models predicted mixed results for $ud\bar{b}\bar{c} (1^+)$:

- HQ-symmetry inspired and non-chiral models: mostly unbound or very weekly bound

- QCD sum rule, chiral models: a bound state (both for 0 and 1-isospins) with binding over a wide range ~ 20-400 MeV!

Hudspith et al, Phys. Rev. D102, 114506 (2020)
Lattice QCD: $u d b c (1^+)$

$-61 \text{ MeV} < \Delta E_{u d b c} \sim -15 \text{ MeV}$

$\bar{D}B^*$ threshold

Pflaumer, Lattice’21

Interpolating fields for $u\bar{d}\bar{b}\bar{c} \ (1^+)$

**Diquark-Anti-diquark type**

<table>
<thead>
<tr>
<th>Field Configuration</th>
<th>Result</th>
</tr>
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<tbody>
<tr>
<td>$[uC\gamma_5 d][\bar{c}C\gamma_i \bar{b}]$</td>
<td></td>
</tr>
<tr>
<td>$[uC\gamma_0 \gamma_5 d][\bar{c}C\gamma_i \gamma_0 \bar{b}]$</td>
<td></td>
</tr>
<tr>
<td>$[uC\gamma_i d][\bar{c}C\gamma_5 \bar{b}]$</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{ijk}[uC\gamma_j d][\bar{c}C\gamma_k \bar{b}]$</td>
<td></td>
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**Meson-Meson type**

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<tr>
<td>$[uC\gamma_i \bar{b}][dC\gamma_5 \bar{c}]$</td>
<td></td>
</tr>
<tr>
<td>$[uC\gamma_5 \bar{b}][dC\gamma_i \bar{c}]$</td>
<td></td>
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Mini-workshop on $T_{cc}^+$ and beyond, CERN
New:

- We use more number of operators including also the diquark-antidiquark type operators with the light diquark having a spin 1 keeping in mind that the charm quark is much lighter compared to the bottom quark.

Preliminary Findings:

- Multiple energy levels around elastic threshold.

- At least one energy level below two-meson thresholds.
ud\bar{b}c \ (1^+ )

Preliminary
Using only meson-meson operators on finer (~ 0.058 fm) lattice
• Need to do analysis with the full operator set
• Near closeness to thresholds demands a finite volume analysis to conclude

More work is in progress on to cross-check this finding with more statistics and another volume ---stay tuned

Mini-workshop on $T_{cc}^+$ and beyond, CERN
Recent calculations on charmed resonances

Hadspec@JHEP 02 (2021) 100
Hadspec@arXiv:2102.04973
What to do next? When can we be sure?

- Perform an infinite volume analysis (Luscher method) with multi operators and multi-volume (computationally intensive procedure).

- Do a calculation with relativistic bottom quarks just to make sure no problem with NRQCD (working on this).

- Calculate form factors and/or distribution functions to find out the structure of these states (computationally intensive procedure).
Conclusions and Outlooks

- Exotic multiquark states with heavy quark contents have been observed having four and five valence quark configurations.

- Lattice QCD calculations are playing a crucial role in studying these states.

- Multiple lattice QCD studies suggest the existence of a deeply bound doubly-bottom four-quark states. More works using finite volume analysis are needed to identify their pole structures.

- Lattice results on $T_{cc}$ and $T_{bc}$ are mixed. Because of less binding energy, checking through finite volume analysis is essential.

- More comprehensive lattice results on four and other multiquark configurations will come in the next five years or so.

- Will also look forward to new discoveries.