A CONTINUUM SOLUTION TO THE HIERARCHY PROBLEM

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A warped solution to the hierarchy problem

- A warped extra dimension was proposed in 1999 by Randall and Sundrum (RS)

L. Randall, R. Sundrum, 9905221

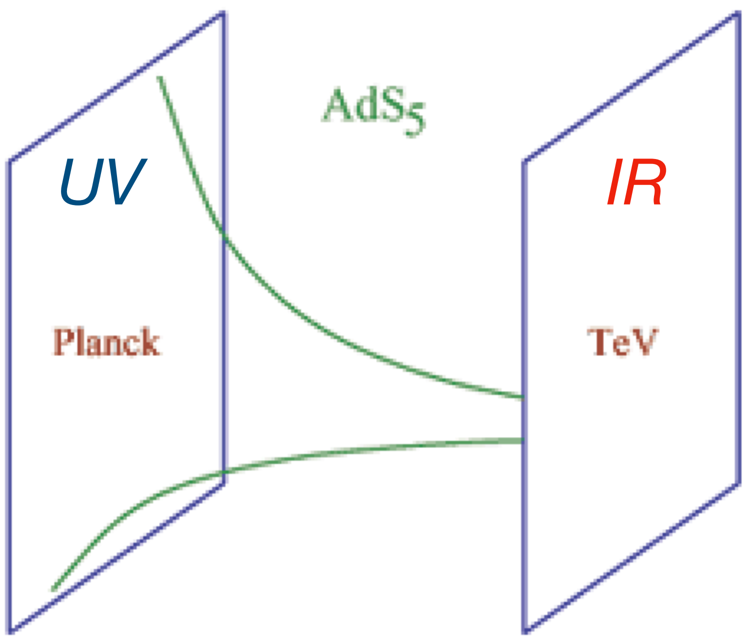
- It was based on an AdS_5 space with line element

$$ds^2 = e^{-2A}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2,\ A = ky$$

RS

$$\rho \simeq e^{-A(y_1)}k, \quad k \sim M_{Pl}, \quad A(y_1) \sim 35$$

- With two branes at $(0,y_1)$

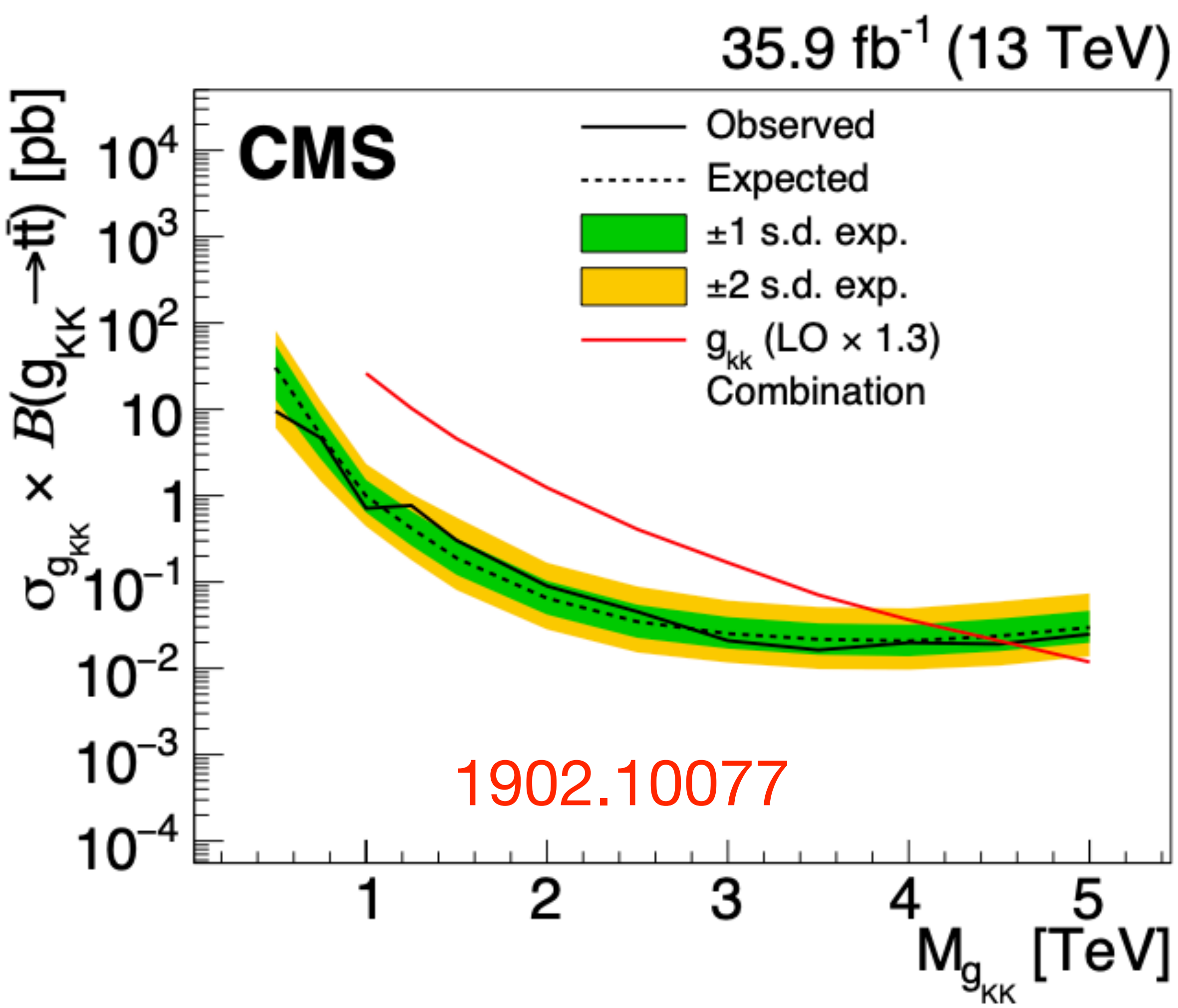


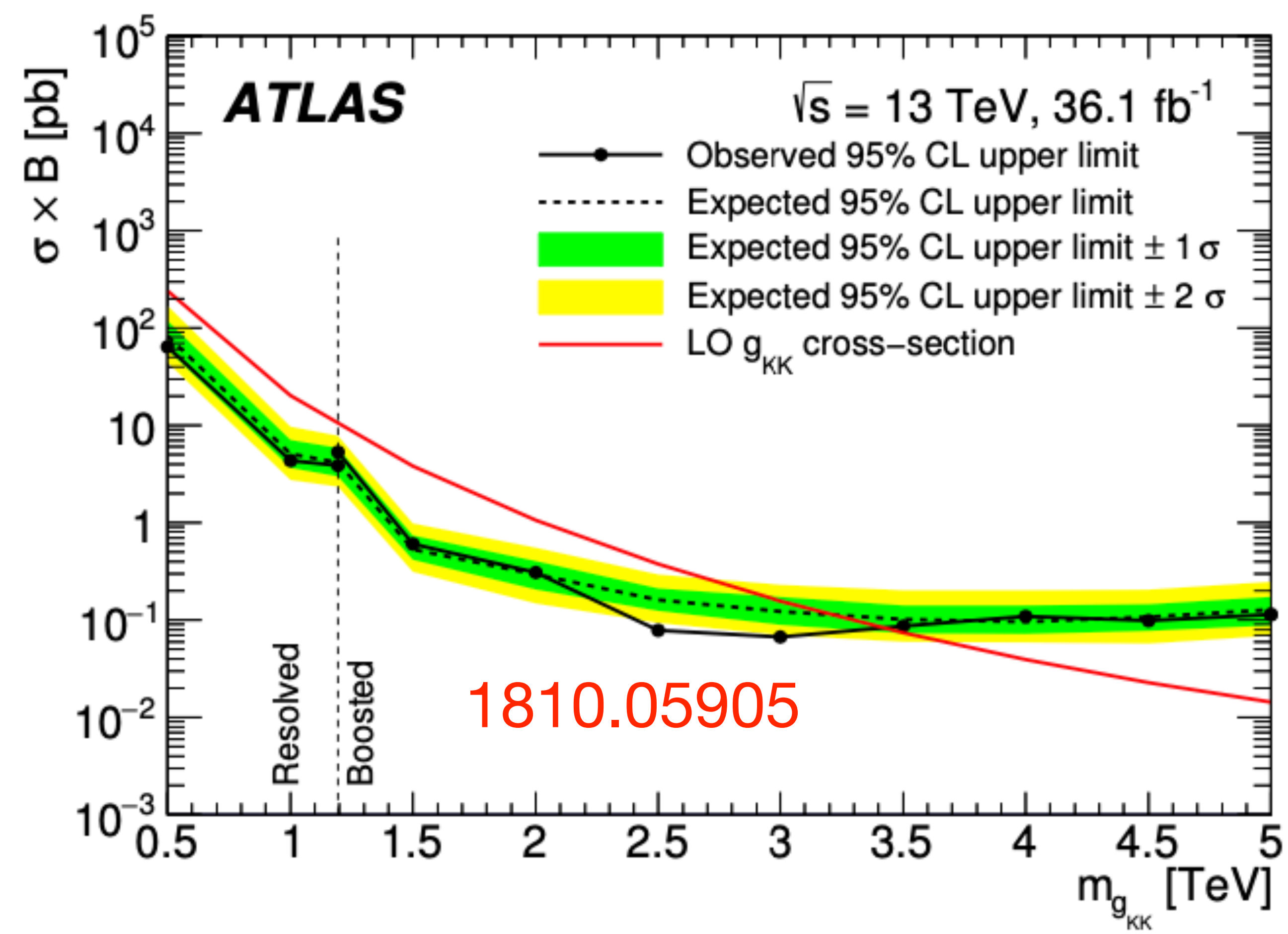
$AdS \Leftrightarrow CFT \quad correspondence$

- The Higgs is localized toward the IR brane (composite):
- Heavy (light) fermions are localized toward the IR (UV) brane: composite (elementary)
- Zero mode gauge bosons are flat
- KK resonances are localized toward the IR brane (composite)

Collider challenges: $t\bar{t}$ production

- The LHC data are putting severe bound on the mass of the lightest KK resonances, e.g. for KK gluons:





$$g_{g_{KK}qq} = -0.2 g_s, \; g_{g_{KK}t_L t_L} = g_s, \; \Gamma_{g_{KK}} = 30\,\%$$

- These limits point toward the possibility that nature might have chosen values of $\rho \gg TeV$, or that we have to reformulate our model. Two ways out:

- i) Way out 1: Strongly coupled (gluon) resonances are not narrow, but they are **broad resonances**: Bumps flatten in detection!

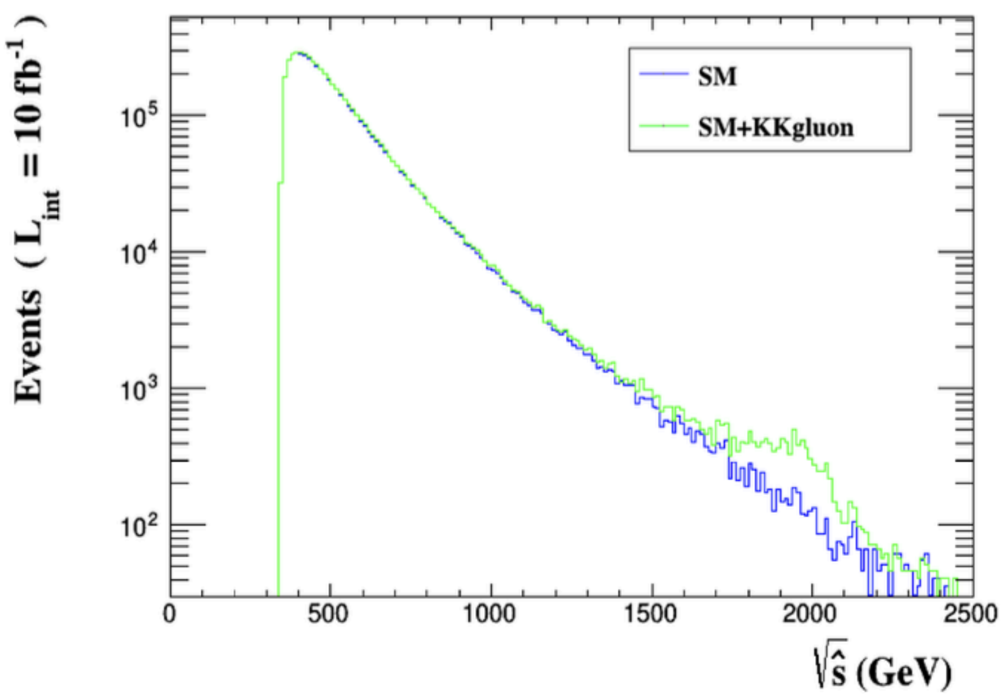
- ii) Way out 2: Resonances are a **gapped continuum**: no bumps at all!

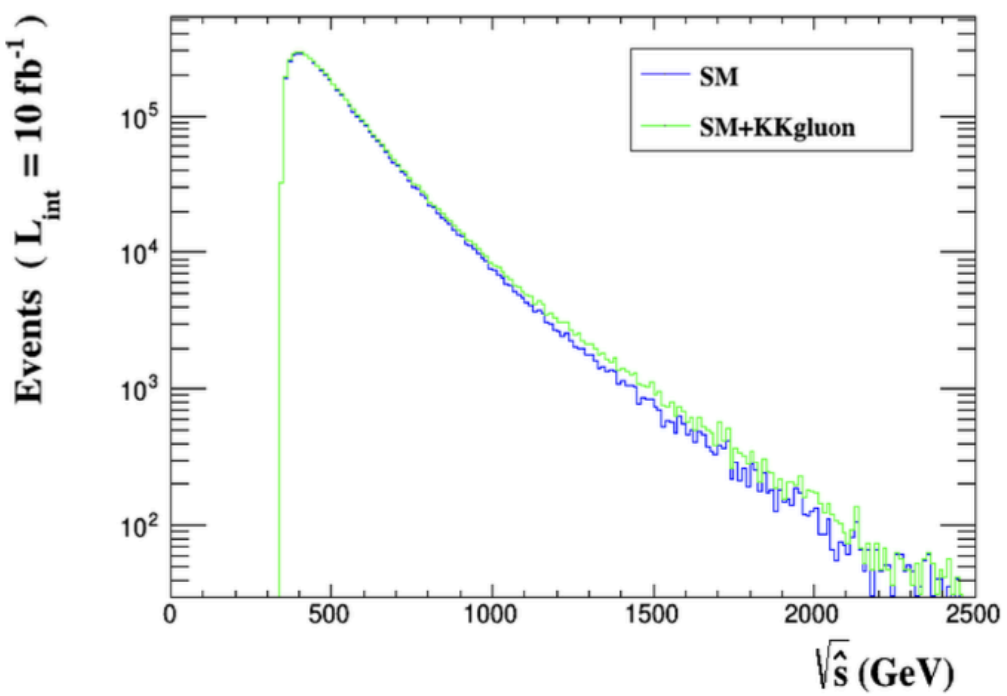
Broad resonances

- If some fermions (t_R) are localized toward the IR brane, where KK resonances G^* are localized, the latter can be very broad and difficult to detect R. Escribano et al., 2102.11241

$\Gamma/M = 0.1, \quad \Gamma = 200\,GeV$

$\Gamma/M = 0.8 \quad \Gamma = 1.6\,TeV$





$\sigma(pp \to t\bar{t})$

M=2 TeV

Gapped continuum KK resonances

- Experimental searches normally assume that new particles are isolated (narrow) resonances

- Another possible "explanation" for elusiveness: KK states are a TeV gapped continuum of states instead of isolated particles   C. Csaki et al., 1811.06019

G. Giudice et al., 1711.08437

- A theory in that direction is the clockwork mechanism, or its 5D version, where TeV is the fundamental scale, and Planck is a derived scale. The KK modes have a TeV mass gap and a (quasi continuum) spacing of 30 GeV. Similar to Linear Dilaton scenarios, dual to Little String theories (where hierarchy problem must be solved by the string theory)

I. Antoniadis et al., 1102.4043

$$A(y) = ky - \frac{1}{\nu^2}\log(1 - y/y_s)$$

UV AdS IR deformation

- The class of 5D models we have considered here share some properties

1. They reproduce RS in the UV and therefore they can explain conventionally the hierarchy with a fundamental Planck scale and a warped TeV scale

2. For $\nu > 1$ they yield discrete KK spectra with TeV spacing

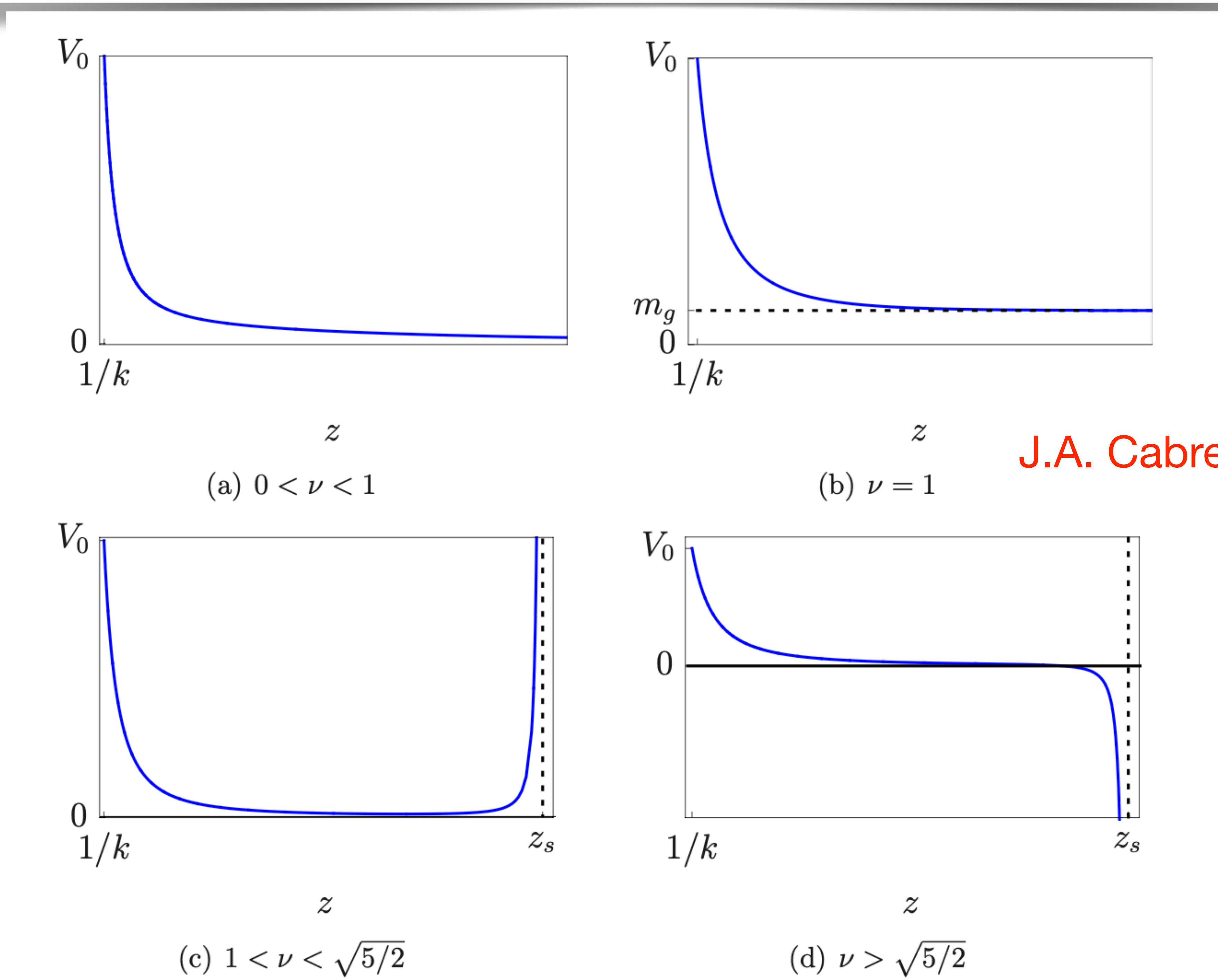
3. For $\nu < 1$ they yield ungapped continuum spectra similar to unparticles H. Georgi, 0703260

4. For critical $\nu = 1$ they yield gapped continuum spectra

A. Falkowski et al., 0806.1737 J.A. Cabrer et al., 0907.5361

Schrödinger-like potentials for the graviton

Different values of ν



J.A. Cabrer et al., 0907.5361

The critical $\nu = 1$ *case:* continuum with mass gap

- Their Green functions generalize from particle propagator with isolated poles

$$\frac{1}{p^2 - m^2 + i\epsilon} = \mathscr{P}\frac{1}{p^2 - m^2} + i\pi\delta(p^2 - m^2)$$

- … to Green functions with an isolated pole (the zero mode) and a continuum of states, instead of a discrete sum of KK modes, with a mass gap m

$$G_A(p^2, m^2) = ReG_A(p^2, m^2) + i\,ImG_A(p^2, m^2)\theta(p^2 - m^2)$$

- This is the behavior of gapped unparticles where the gap was usually produced by EW breaking

- Here the gap is TeV, and is linked to the solution of the hierarchy problem

- 5D dimension has UV boundary (y=0), IR brane $(y = y_1)$ where Higgs lives, and singularity $(y = y_s)$

- The mass gap is different for different bulk fields

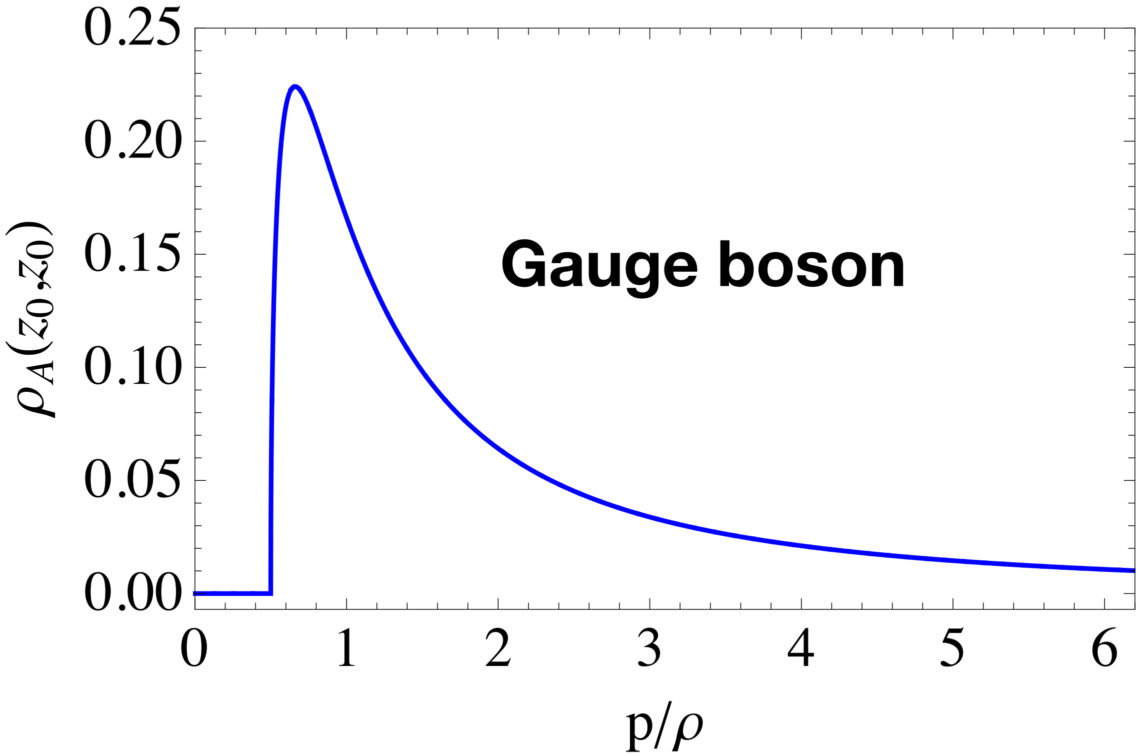
- The mass gap for the different states

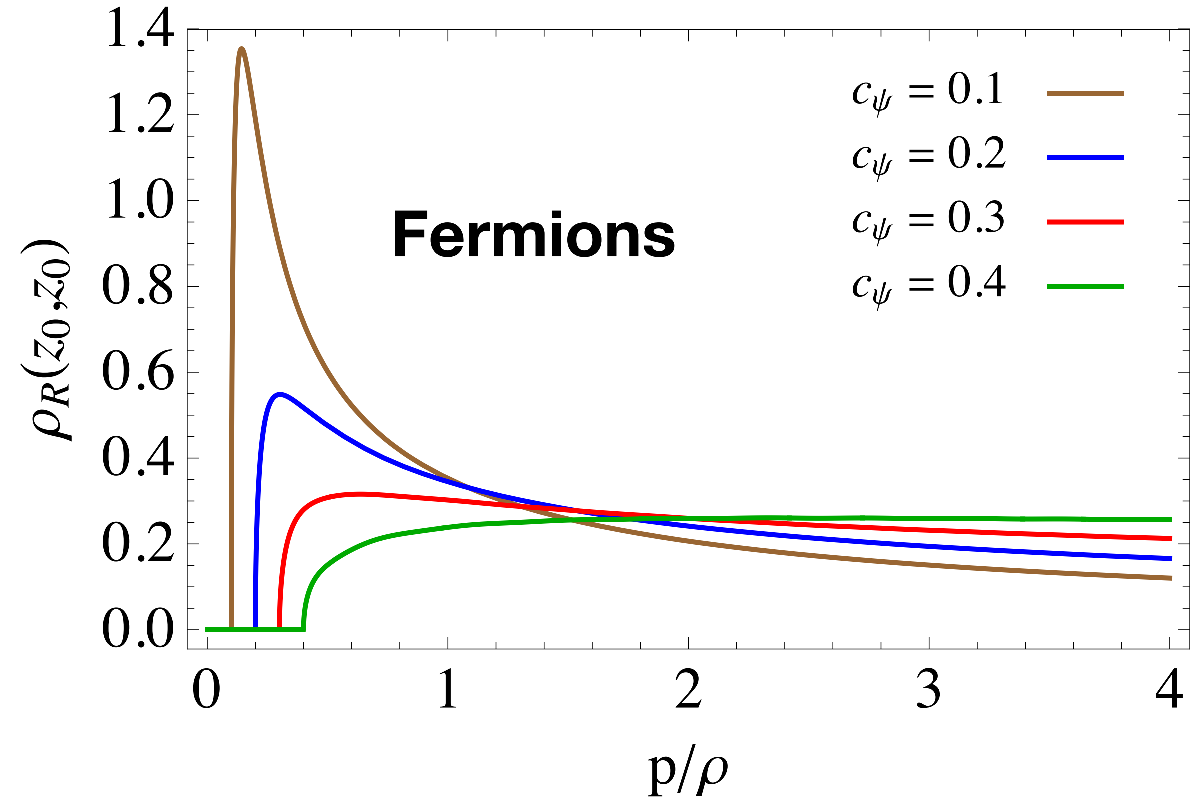
field	gauge boson	fermion	graviton radion Higgs
mass gap	$\frac{1}{2}\rho$	$\lvert c_f \rvert \rho$	$\frac{3}{2}\rho$

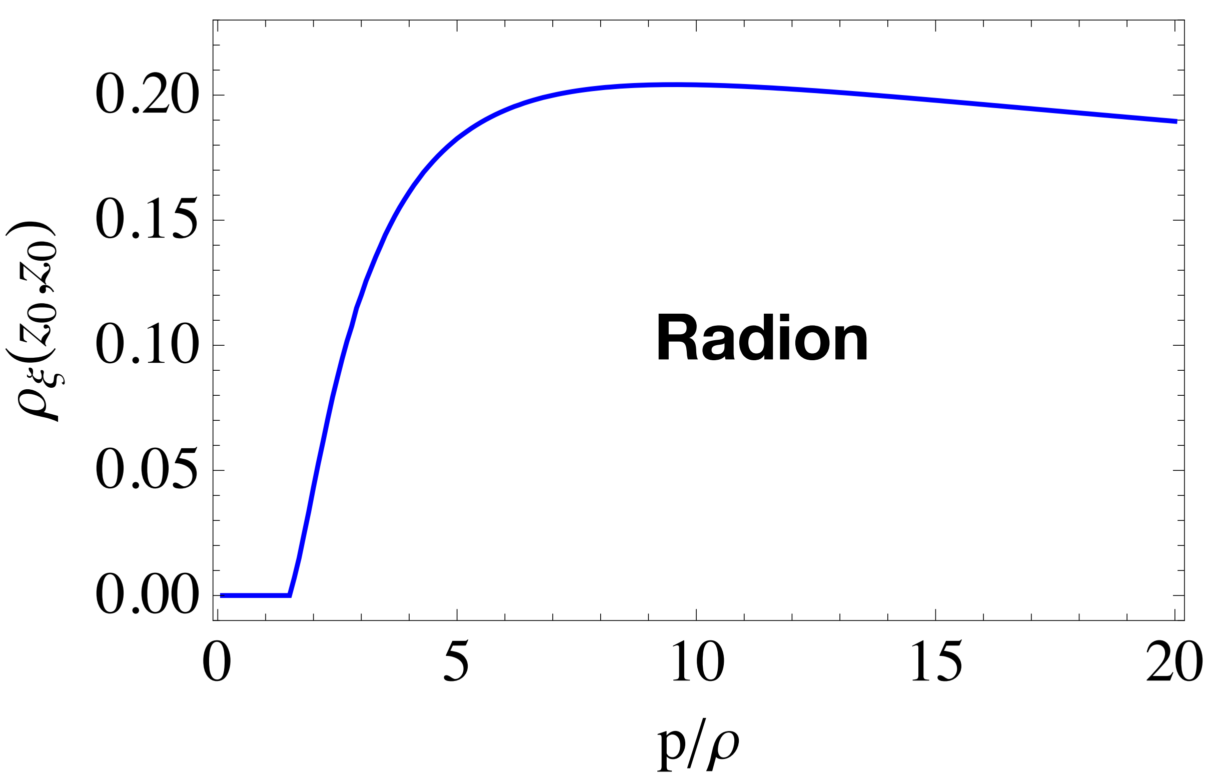
The continuum spectrum shows up in the holographic spectral density functions

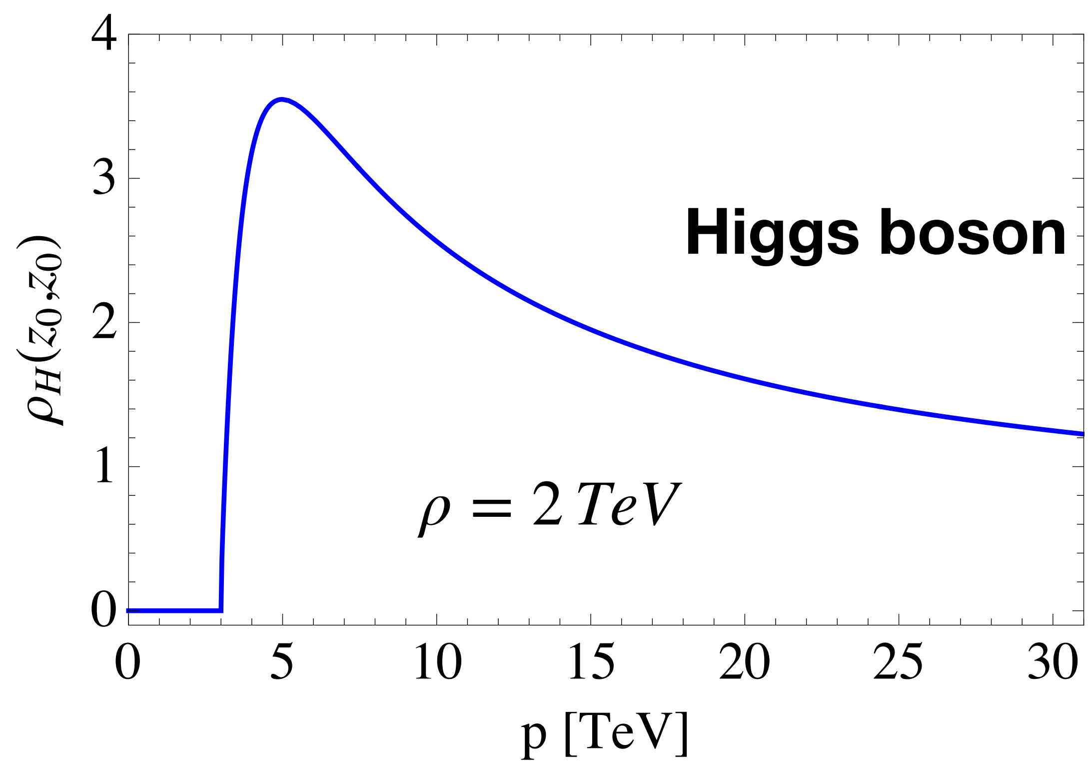
E. Megías and MQ, 1905.07364

$p \equiv \sqrt{s}, \quad m_g =$ **mass gap**









Phenomenology of continuum KK modes

- The mass gap is different for different states

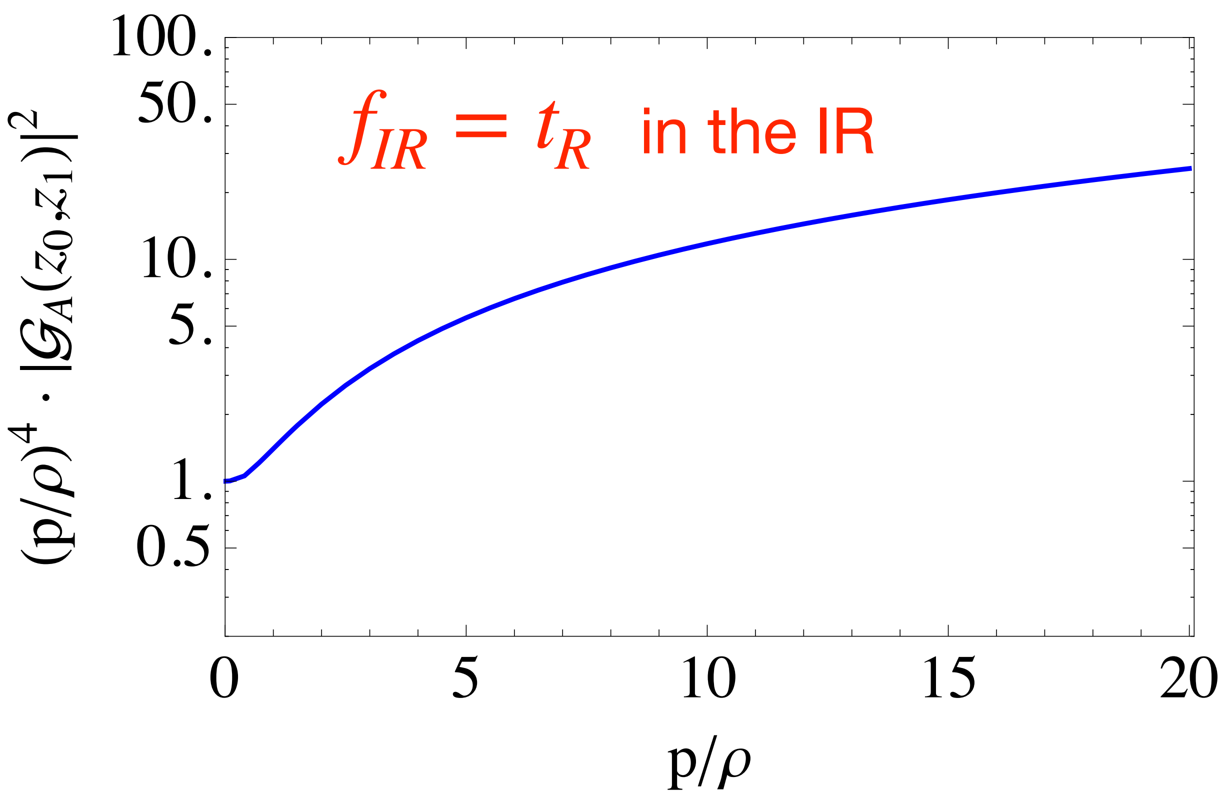
field	gauge boson	fermion	graviton radion Higgs
mass gap	$\frac{1}{2}\rho$	$\lvert c_f \rvert \rho$	$\frac{3}{2}\rho$

As for light fermions $c_f > 1/2$, *the easiest produced continuum is for gauge bosons*

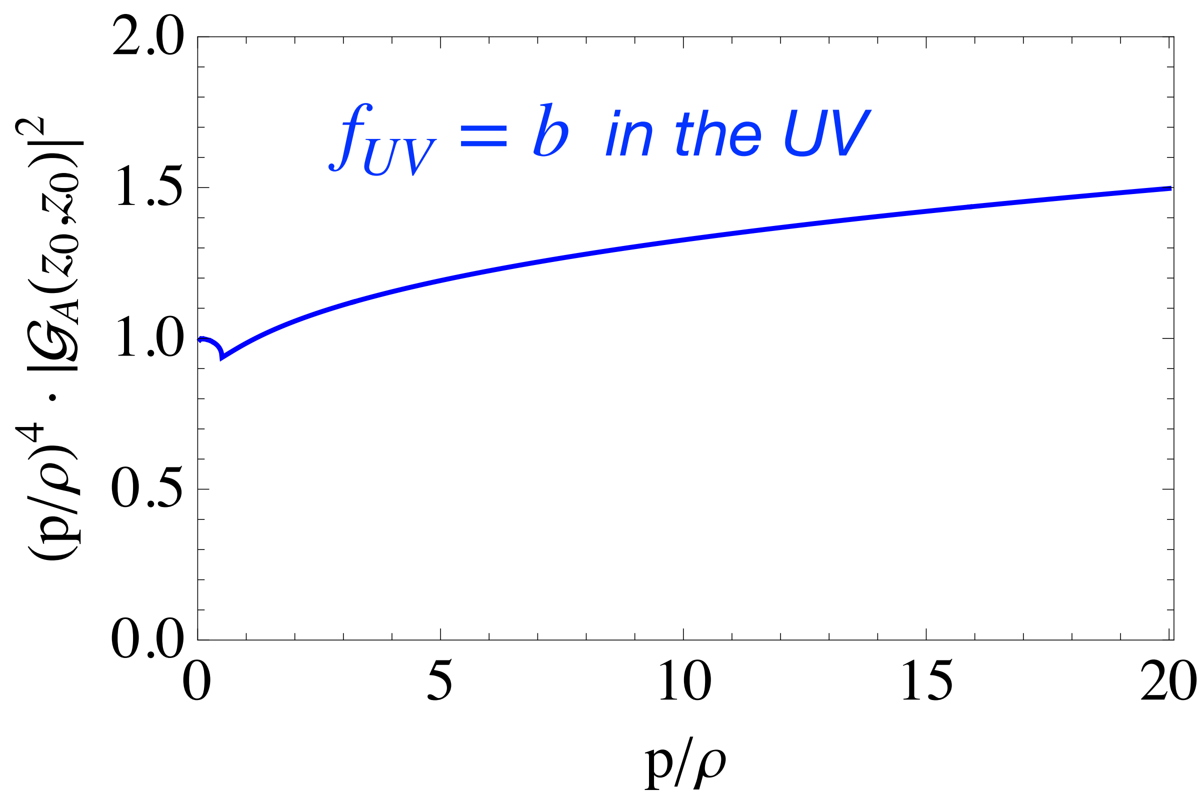
- Normal searches at LHC are based on bumps in the invariant mass of final state

- However here, in production of fermions from DY processes via gluon KK continuum, there is just an increase in the cross section: $\sigma(q\bar{q} \to g^* \to f\bar{f}), \quad p = \sqrt{\hat{s}}$

$$\sigma/\sigma_{SM}(q\bar{q} \to f_{IR}\bar{f}_{IR})$$



$$\sigma/\sigma_{SM}(q\bar{q} \to f_{UV}\bar{f}_{UV})$$



Strong increase with energy $\quad p \equiv \sqrt{\hat{s}} \quad$ **Little increase with energy**

Analytical approximation on Green's functions

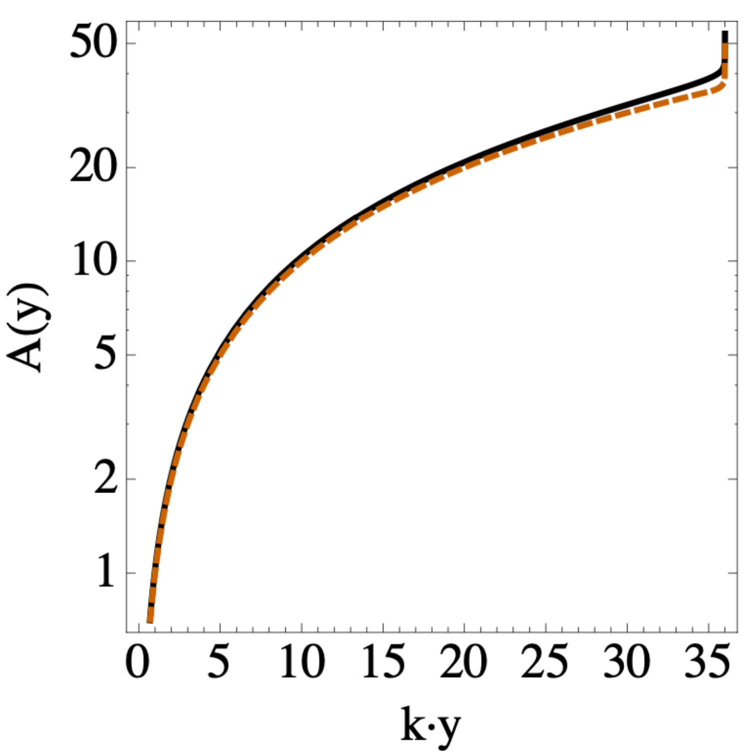
- We can get analytical insight into the structure of Green's function by approximating the exact metric (in proper coordinates) $A_{exact}(y) = ky - \log(1 - y/y_s)$

- By an approximate one (in conformal coordinates)

$$A_{app}(z) = \log kz\, \Theta(z_1 - z) + [\log kz_1 - \rho(z - z_1)]\, \Theta(z - z_1)$$

RS

LDM



Condition: $ky_s - ky_1 = 1$

E. Megías, M.Q., 2106.09598

One can explore the structure of Green's functions in the complex s-plane

- Green's functions for gauge bosons A_μ with y and/or $y' < y_1$

$$G_A(y, y'; p) \propto \frac{1}{\Phi(p)}$$

$$\Phi(p) = \mathcal{K} \cdot J_+(p/\rho) - Y_+(p/\rho)$$

$$\mathcal{K} \equiv \frac{2}{\pi}\left(\gamma_E - \log(2) + \log(p/\rho) - ky_1\right)$$

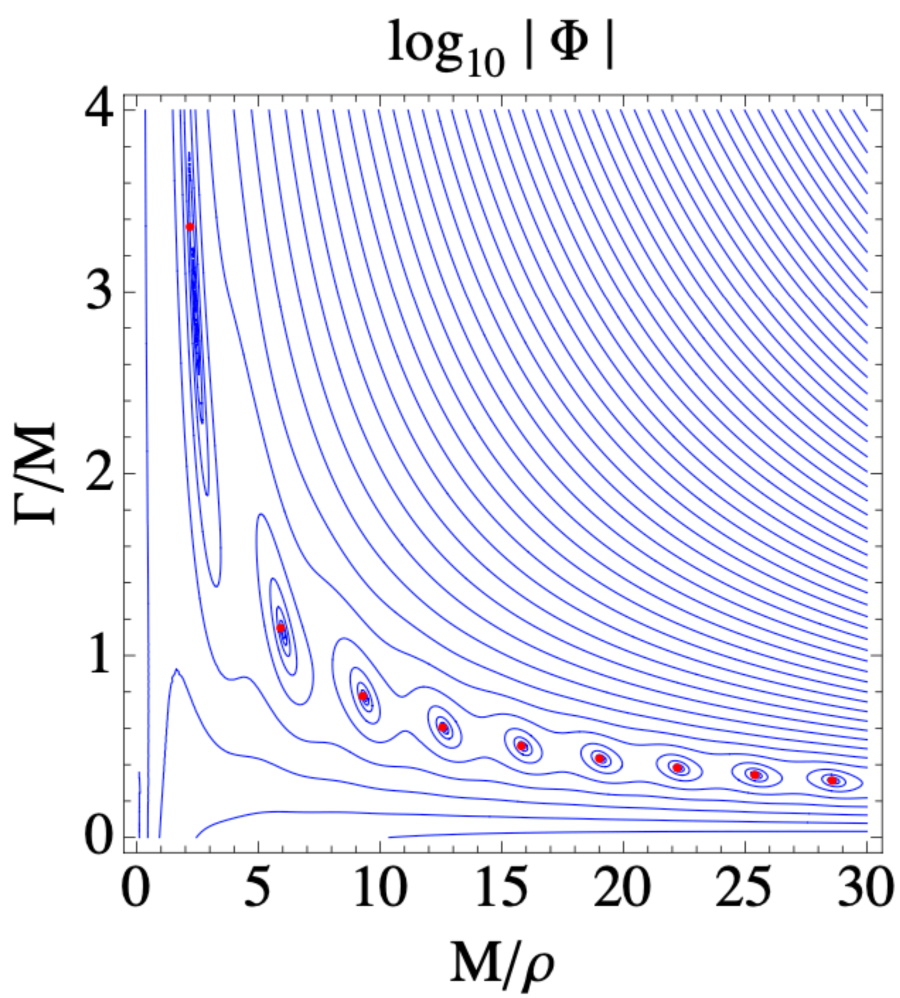
$$J_\pm(p/\rho) = 2\frac{p}{\rho}J_0(p/\rho) + \Delta_A^\pm J_1(p/\rho), \quad Y_\pm(p/\rho) = 2\frac{p}{\rho}Y_0(p/\rho) + \Delta_A^\pm Y_1(p/\rho)$$

$$\Delta_A^\pm \equiv \pm\delta_A - 1, \quad \delta_A = \sqrt{1 - 4p^2/\rho^2}$$

- The zeros of $\Phi(p)$ are poles of $G_A(y, y'; p)$

- We have looked for resonances by looking for poles in the second (unphysical) Riemann sheet of the complex s-plane

- Unexpectedly we have found a tower of broad resonances located in the unphysical Riemann sheet



n-th branch of Lambert function

$$\frac{s}{\rho^2} = -\mathscr{W}_n\left[\pm\frac{1}{4}(1+i)\right]^2, \quad n = -1, -2, -3, \ldots$$

No poles found in the first (physical) Riemann sheet

Poles interpreted as resonances !!!

$$(M/\rho, r) = (2.42, 2.87), (6.03, 1.12), (9.37, 0.768), (12.64, 0.601), (15.87, 0.500), \cdots$$

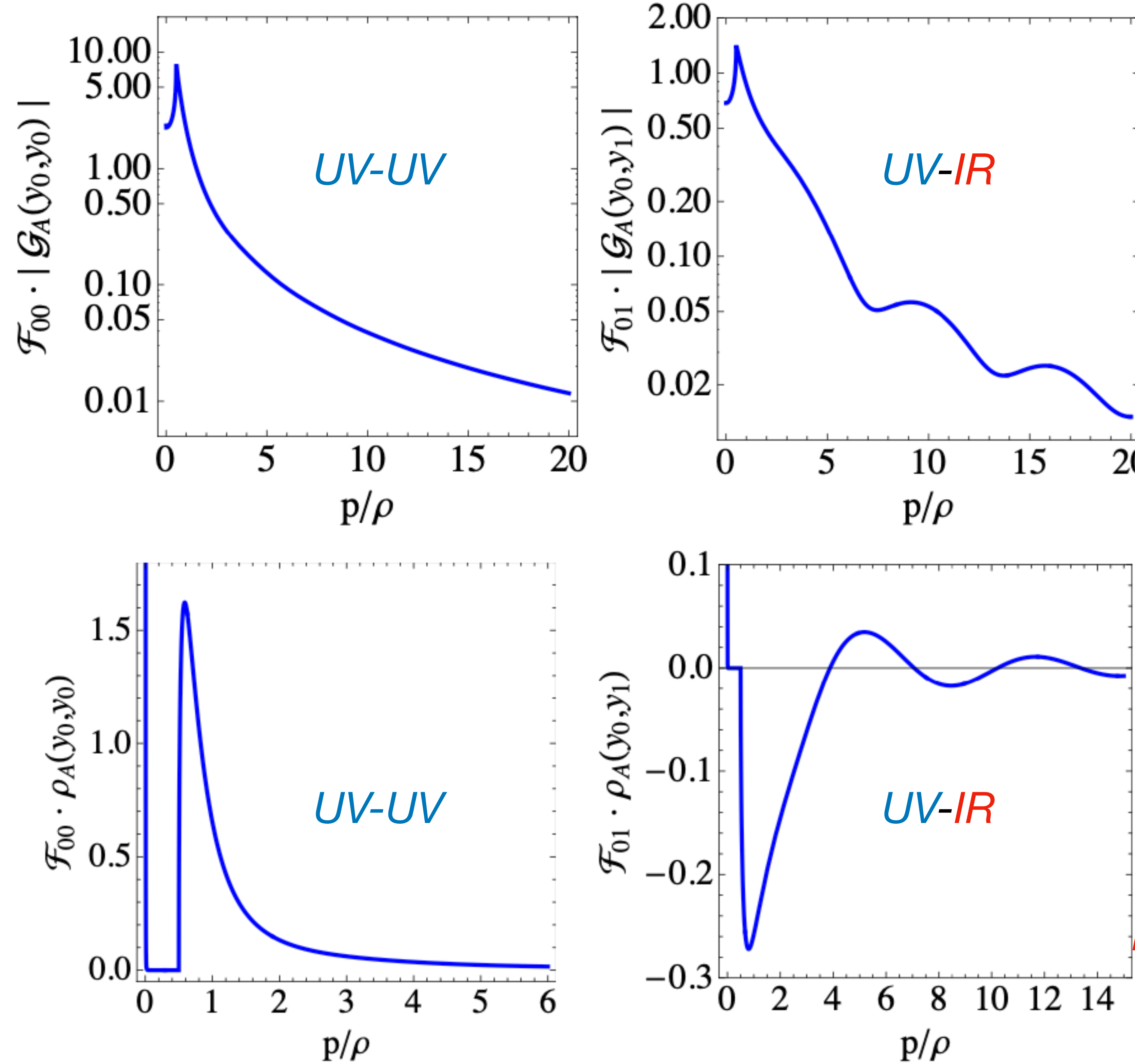
- Green's functions with y and $y' > y_1$ are decomposed as

- $G_A(y, y'; p) = G_{res}(y, y'; p) + G_{un}(y, y'; p)$

- where $G_{res}(y, y'; p) \propto \dfrac{1}{\Phi(p)}$ $\longleftarrow$ *poles*

- and $G_{un}(y, y'; p) \propto \dfrac{1}{\delta_A(p)}$ which correspond to gapped unparticles with $d_U = 3/2$

Some Green and spectral functions



$$\mathscr{F}_{00} = \frac{\rho^2}{k}(ky_s)^2$$

$$\mathscr{F}_{01} = \frac{\rho^2}{k}(ky_s)$$

Massless gauge bosons

$$\rho_A(y, y'; p) = \frac{1}{y_s}\delta(s) + \cdots$$

- Notice that while $\rho_A(y_0, y_0)$ is positive definite (as it is 4D spectral function in the UV brane), $\rho_A(y_0, y_1)$ is not that challenges the physical interpretation of the spectral function in 4D QFT

- This apparent contradiction can be resolved by noticing that $\rho_A(y, y')$ can be considered as the matrix element (y, y') of an operator $(\hat{\rho}_A)^y_{y'} \equiv \rho_A(y, y')$

- This operator acts on the infinite dimensional space parametrized by the coordinate y

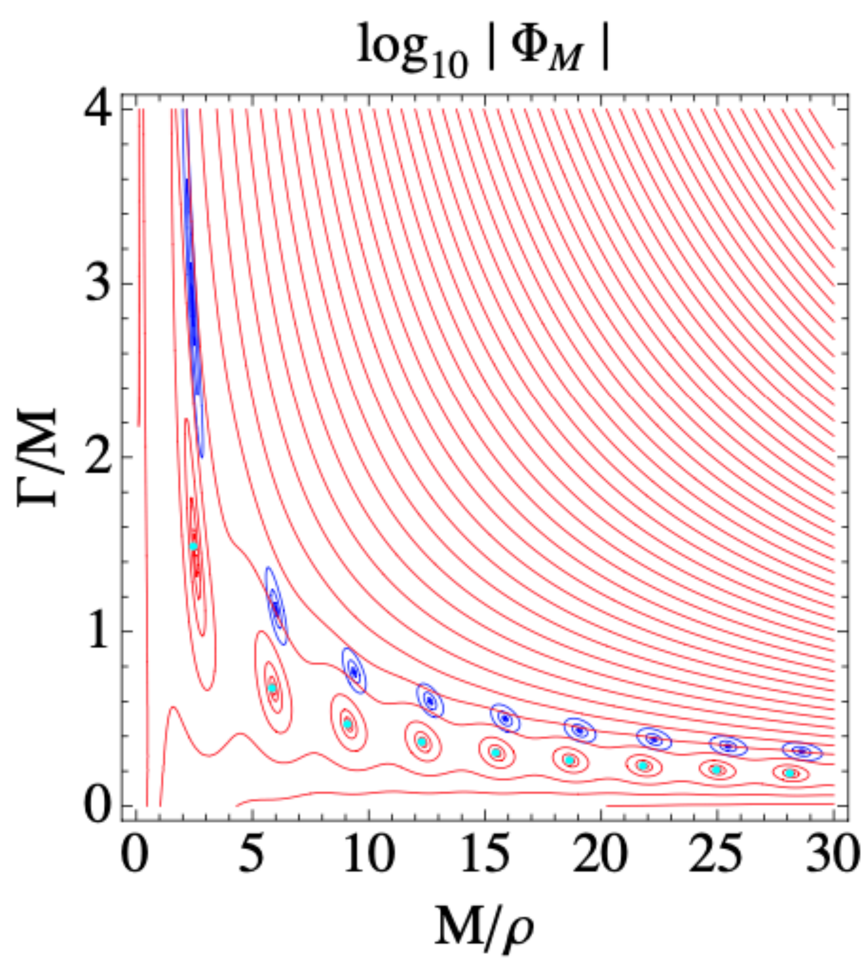
- The matrix $\hat{\rho}_A$ turns out to have a factorizable form, i.e.
  $(\hat{\rho}_A)^y_{y'} = \rho_y \rho_{y'}, \quad \rho_y \equiv \sqrt{(\hat{\rho}_A)^y_y}$

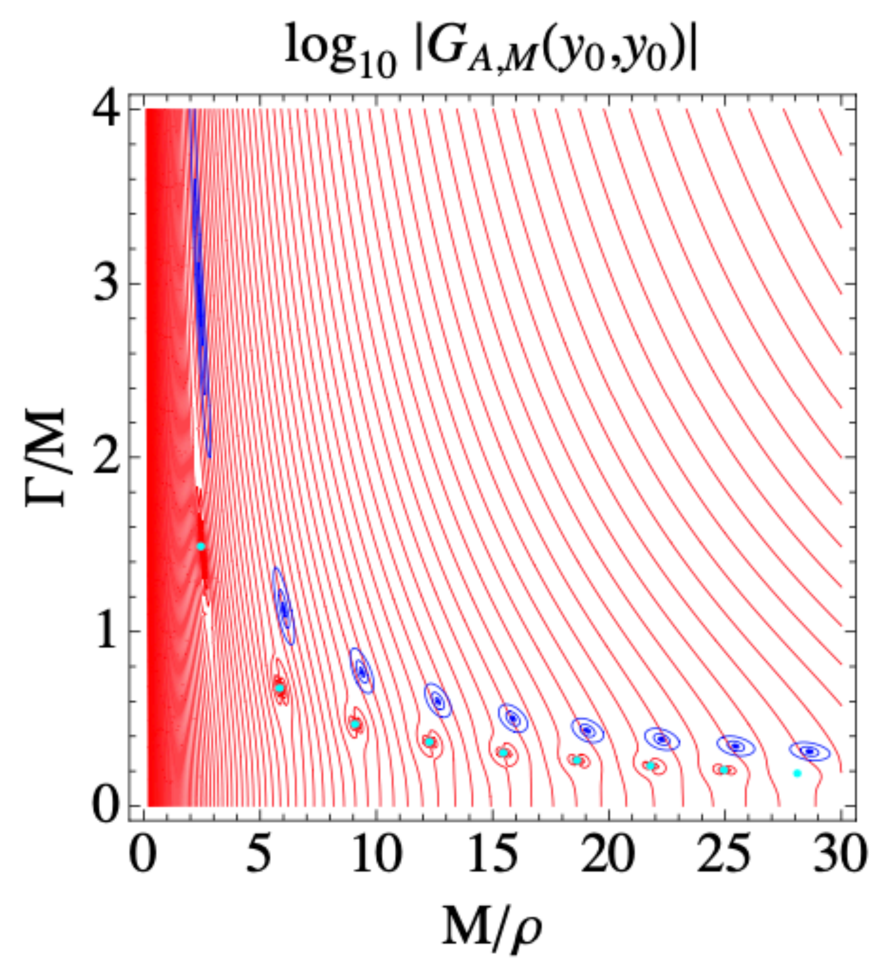
- All eigenvalues of $\hat{\rho}_A$ are zero except one which is given by
  $\lambda(p) \equiv \mathrm{tr}\, \hat{\rho}_\mathrm{A} = \int \mathrm{dy} \rho_\mathrm{A}(\mathrm{y}, \mathrm{y}; \mathrm{p}) \geq 0$

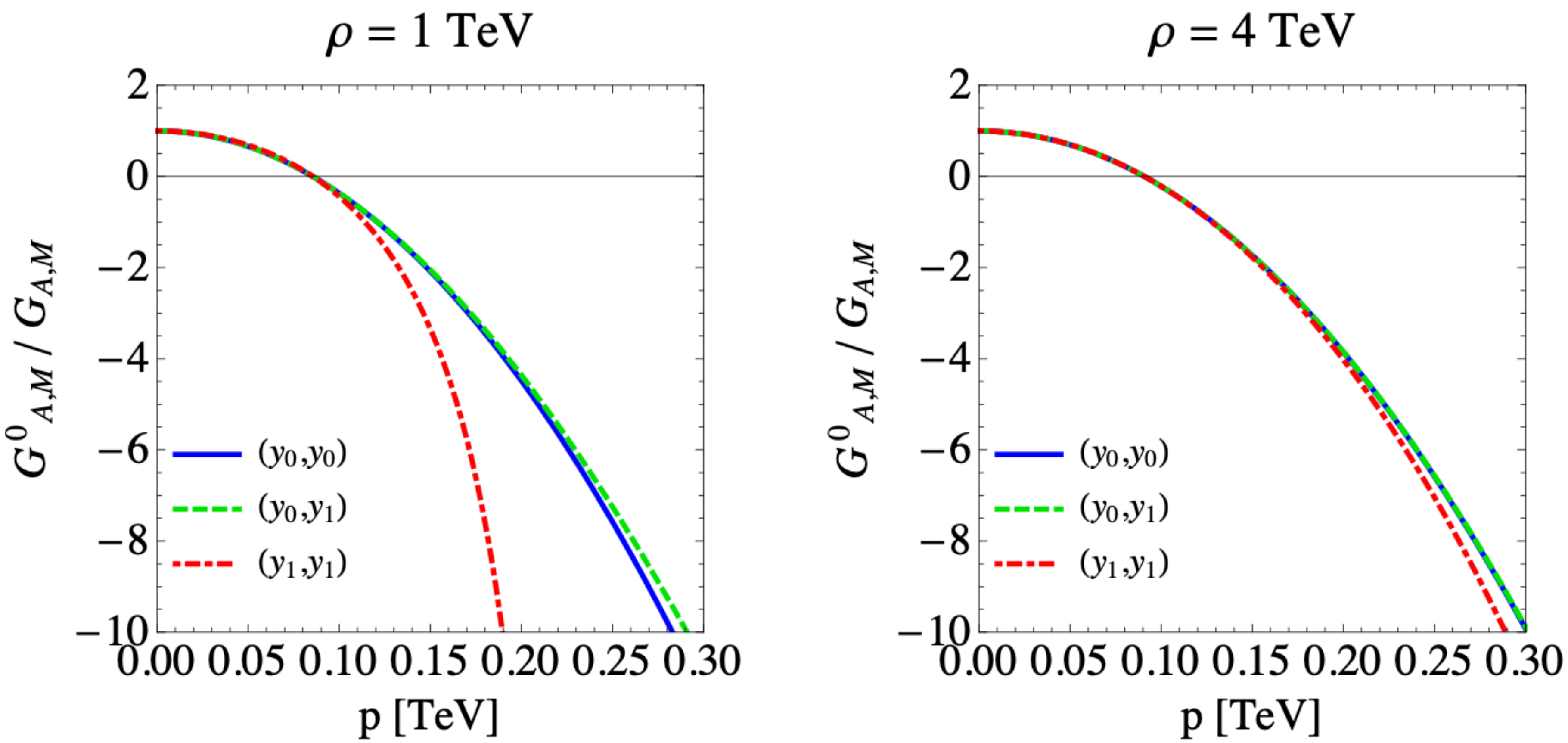
For massive Gauge bosons

W_μ, Z_μ

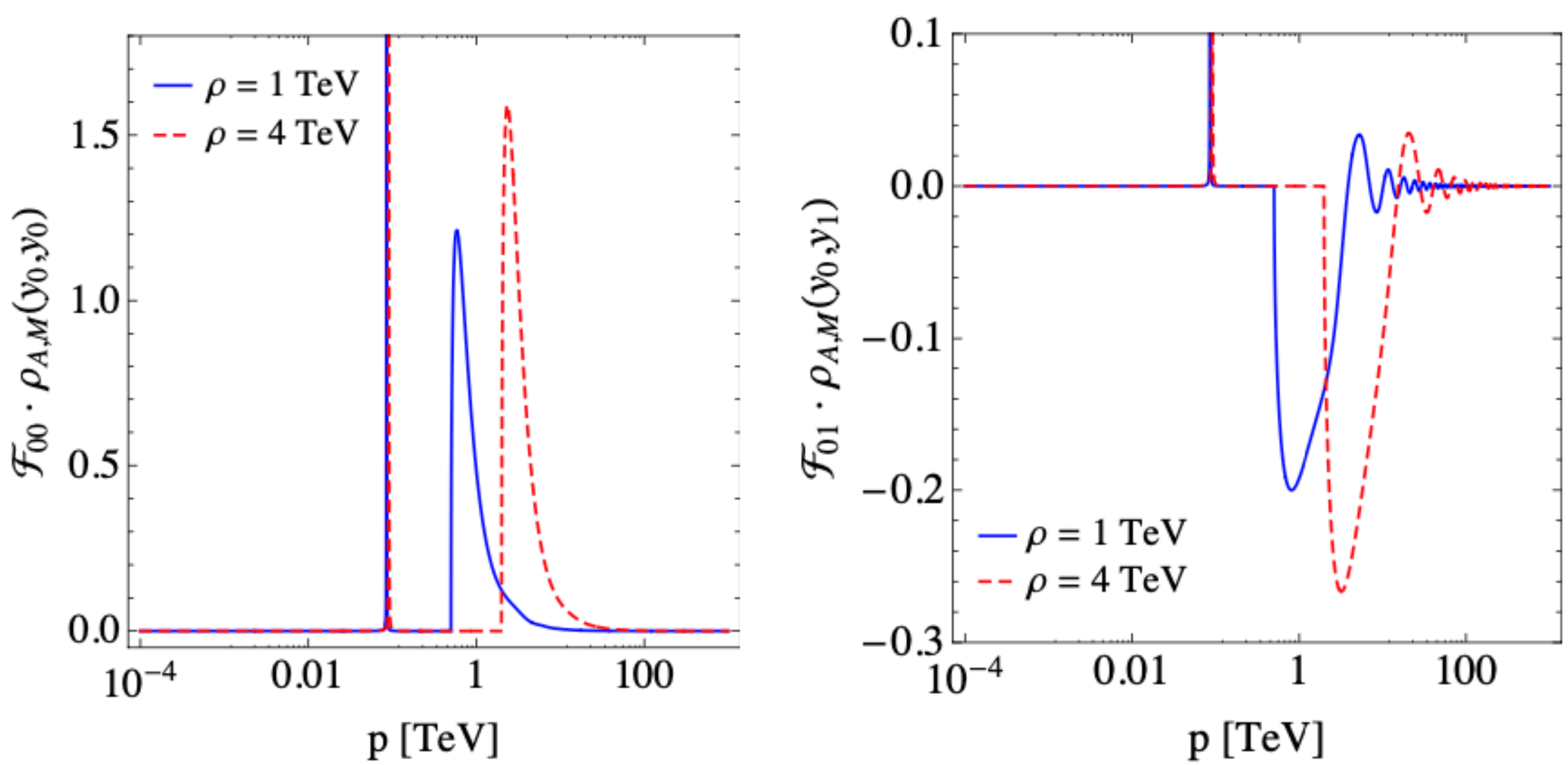
- Similar results follow by replacing the function $\Phi(p)$ by $\Phi_M(p)$ after replacing $\Delta_A^\pm \to \Delta_A^\pm + 2ky_s(m_A/\rho)^2$







$m_Z = 0.091\ TeV$

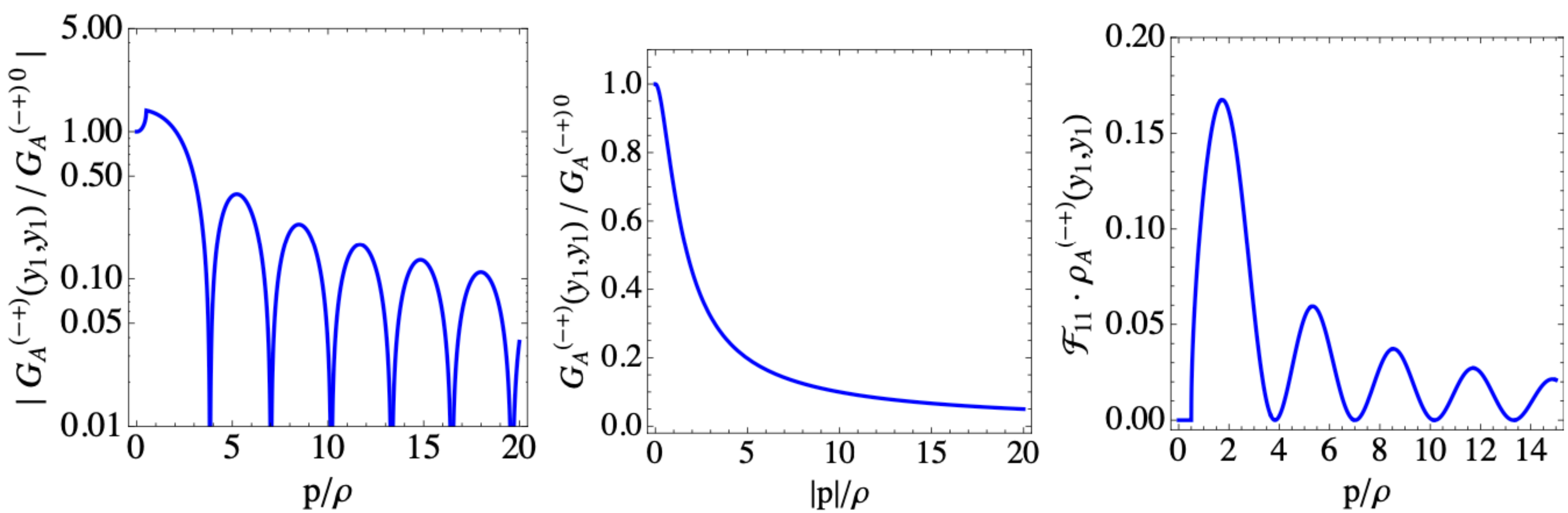


$$\rho_A(y, y'; p) = \frac{1}{y_s}\delta(s - m_A^2) + \cdot$$

For gauge bosons with Dirichlet boundary conditions i.e. $SU(2)_R$

- There are no zero modes, only the continuum and broad resonances
- The Green's functions satisfy $G^{(-,+)}(y_0, y'; p) = 0$, which follows from the Dirichlet boundary condition $G^{(-,+)}(y_0) = 0$

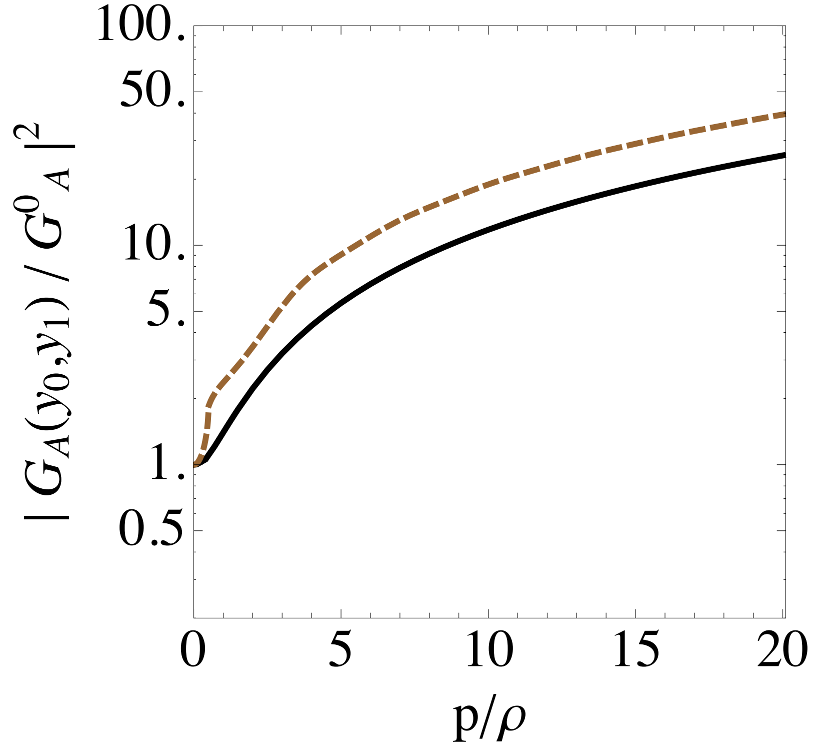
$p^2 > 0$ $\qquad$ $p^2 < 0$ $\qquad$ $G^{(-,+)0} = -\dfrac{k}{2\rho^2}$



Phenomenological applications

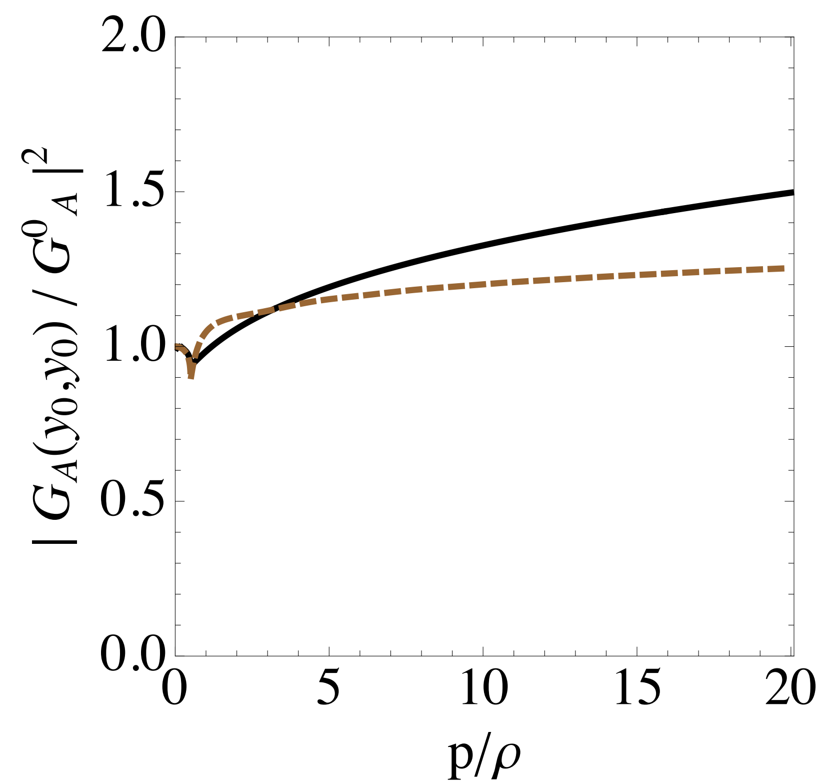
- A set of broad KK resonances or a continuum of unparticles can be produced at the LHC
- In the production of fermions from DY processes via gluon KK continuum, there is just an increase in the cross section:

$$\hat{\sigma}/\hat{\sigma}_{SM}(q\bar{q} \to f_{IR}\bar{f}_{IR}) \qquad \hat{\sigma}/\hat{\sigma}_{SM}(q\bar{q} \to f_{UV}\bar{f}_{UV})$$



num. model

anal. model



$p \equiv \sqrt{\hat{s}}$

Strong increase with energy

Little increase with energy

The hadronic cross-section can be written schematically as (MadGraph5_aMC)

$$\sigma(pp \to t\bar{t}X) = \int_0^s d\hat{s} \int_{\frac{1}{2}\log(\hat{s}/s)}^{\frac{1}{2}\log(s/\hat{s})} \frac{dy}{s} \sum_{a,b=q,\bar{q}} f_a^{(p)}(x_1) f_b^{(p)}(x_2)\, \hat{\sigma}(ab \to t\bar{t})$$

$$\hat{s} = x_1 x_2 s \qquad x_{1,2} = \sqrt{\hat{s}/s}\, e^{\pm y}$$

The partonic cross-section can be written schematically as

$$\bar{\sigma}(q\bar{q} \to G_{res} \to t\bar{t}) = \frac{g_s^4(\hat{s})}{54\pi} \hat{s}\, |G_{res}(\hat{s})|^2 \sqrt{1 - \frac{4m_t^2}{\hat{s}}} (g_{q_V}^2 + g_{q_A}^2) \left[g_{t_V}^2 \left(1 + \frac{2m_t^2}{\hat{s}} \right) + g_{t_A}^2 \left(1 - 4m_t^2/\hat{s}\right) \right]$$

To compare with experimental data one can perhaps assume dominance of the first resonance: work in preparation…

Conclusions

- Warped extra dimension is an interesting alternative to solve the hierarchy problem (dual to CFT,…)

- One exploring possibility to solve the elusiveness of signals at LHC is that KK modes are very broad, triggered by fermions very localized toward the IR brane (e.g. t_R)

- Another (most intriguing) possibility is a continuum of KK states (related to CFT, unparticles,..)

- The 5D model with a continuum of KK states is pretty unique: it corresponds to a critical value of the metric parameters

- It is a 5D modelization of a 4D conformal theory with a mass gap

- Green's functions behave like those of unparticles: i.e. they have an imaginary part not corresponding to particle creation!

- In general, Green's functions can be decomposed as unparticle component + Green function with broad resonances: i.e. with poles in the second Riemann sheet of s

- The location of masses is very similar to that in the RS model, while the width is very broad for the first resonances while for heavy resonances it is very narrow

- Confronting this theory with LHC experimental data is in progress…

- There are also plenty of theoretical problems to be understood: unitarity of the theory, spectral densities,…

E. Megias, M. Perez-Victoria, M.Q., work in progress

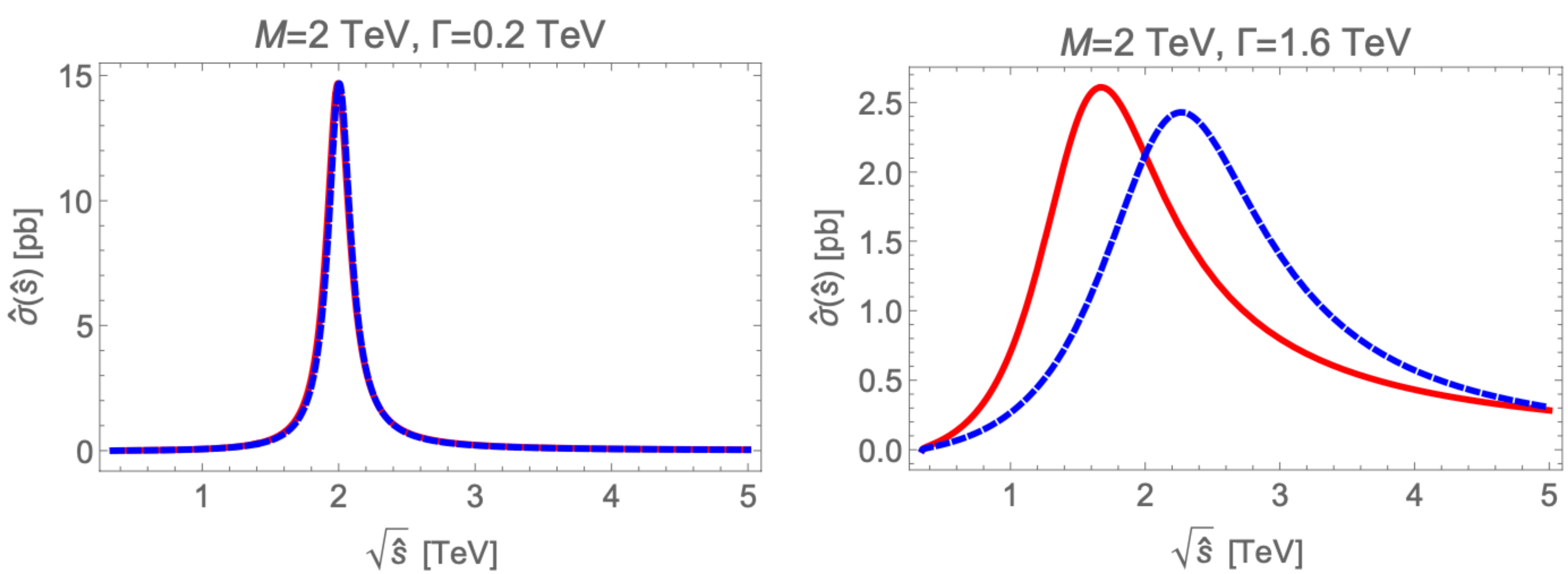
Thank you

Backup slides

Top-quark pair production

- For broad resonances the Breit-Wigner (BW) approximation is not good enough

- The partonic cross-section $q\bar{q} \to G^* \to t\bar{t}$ can be compared for the BW vs Exact propagator



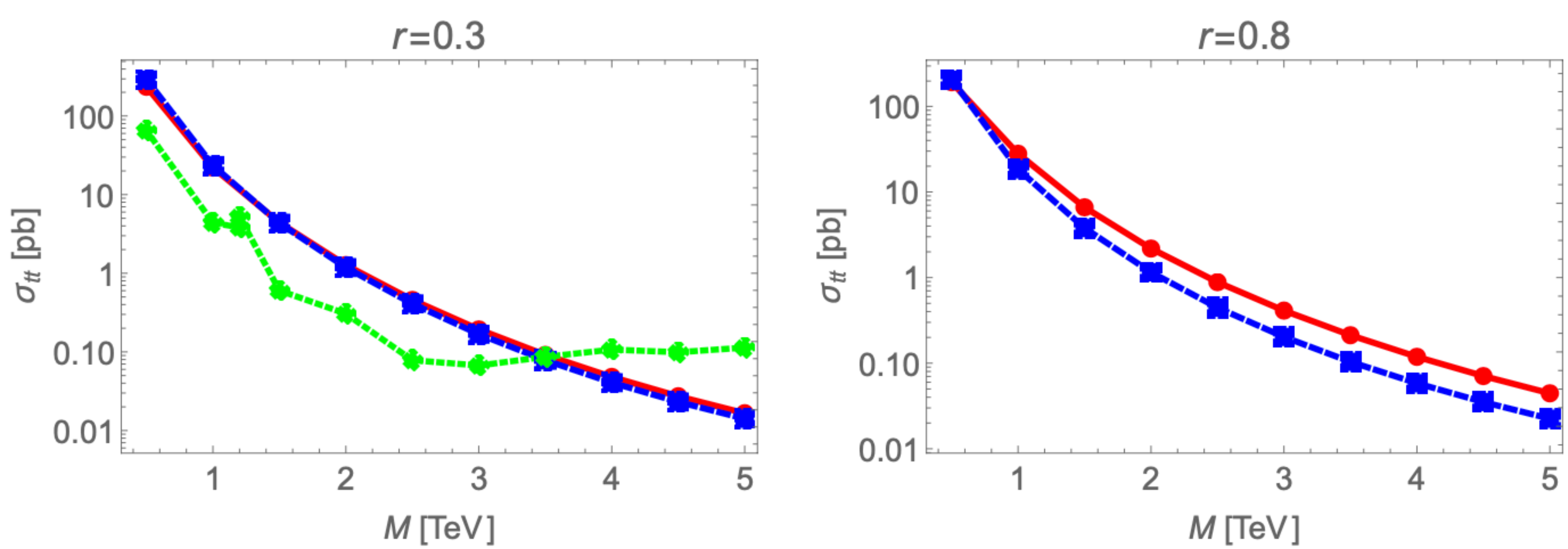
Exact propagator

BW propagator

Top-quark pair production

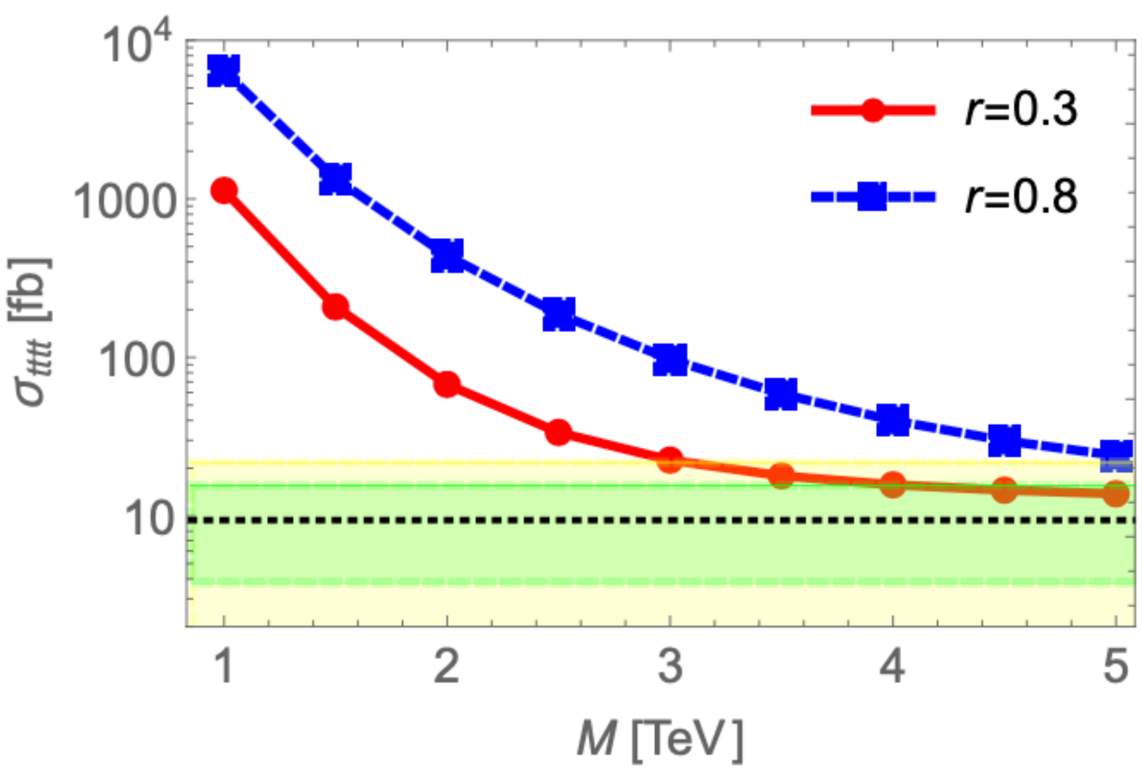
- Actual experimental limits, which cover $r \leq 0.4$, do not change much

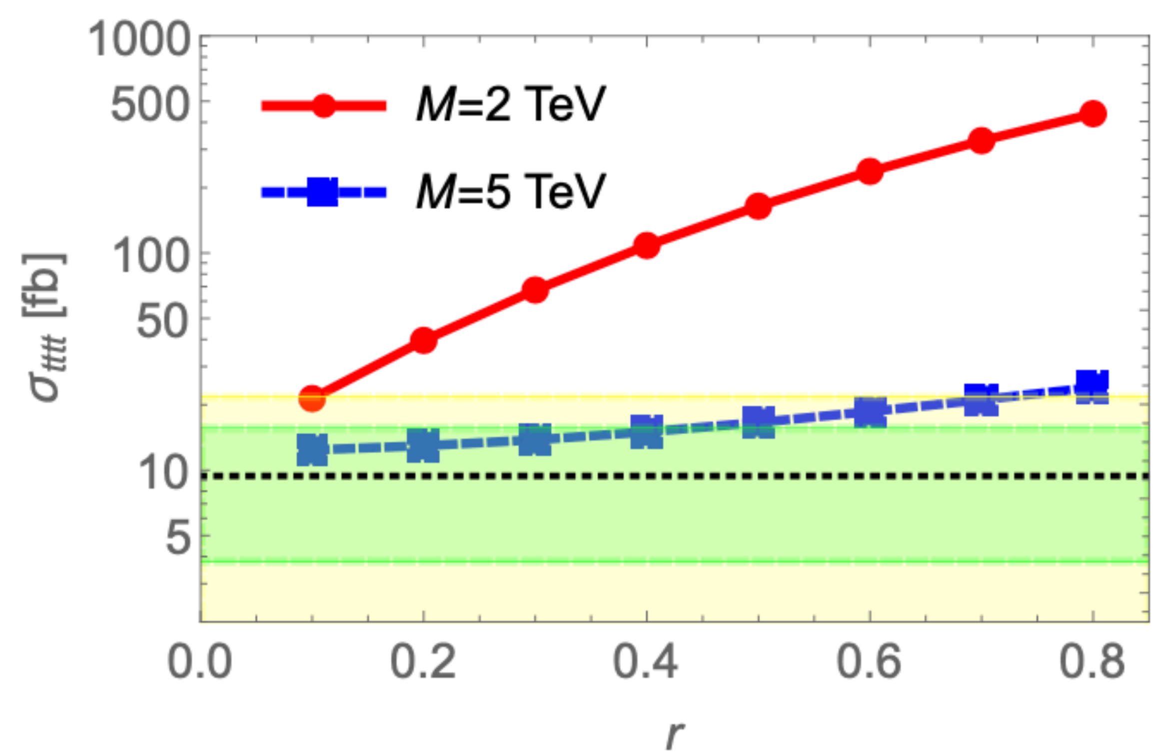
- For larger values of r experimental results should be sensitive to the propagator form $\Rightarrow \Delta M \sim 0.5\, TeV$



Four top-quark production

- Dotted line: central value observed by the CMS collaboration
- Green (yellow) bands are 1σ (2σ) deviations from central value





Four top-quark production is a current hot topic which requires experimental improvement