

Scalar Leptoquark Matching onto SMEFT

A functional approach

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Workshop on BSM physics, Corfu 2021

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- ① Introduction to Functional Matching
- ② Universal One-Loop Matching for Scalar Leptoquarks
- ③ Demonstration: the LQ-model $S_1 + \tilde{S}_2$
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- EFT is a robust way in searching for BSM physics

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- Last decade, an old **functional matching** technique [Gaillard, NPB (1986), Cheyette NPB (1988), L-H Chan, PRL (1986)] has seen a renewed interest Henning, Lu and Murayama, JHEP (2016,2018)

History of Matching to EFT - II

- First Universal results for the One-Loop Effective Action (UOLEA) [Drozd, J. Ellis, Quevillon, You, JHEP (2016); S. Ellis, Quevillon, You, Zhang, PLB (2016); JHEP (2017)] did not account for mixed statistics and open covariant derivatives.

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- SuperTrace functional technique [Cohen, Lu, Zhang, 2011.02484] establishes a cleaner way to display covariant diagrams for matching. Automated tools exist [STream, 2012.07851; SuperTracer, 2012.08506]. *It is this approach we follow in our work.*

Advances on this topic include: Finn, Karamitsos, Pilaftsis, EPJC (2021)

The STr functional matching procedure

The Basic formula for functional matching:

$$\Gamma_{\text{EFT}}[\phi] = \Gamma_{\text{L,UV}}[\phi]$$

where ϕ denotes light-fields.

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$$\Gamma_{\text{L,UV}}[\phi] \Big|_{\text{hard}} = \frac{i}{2} \text{STr} \log K \Big|_{\text{hard}} - \frac{i}{2} \sum_{n=1}^{\infty} \frac{1}{n} \text{STr} [(K^{-1} X)^n] \Big|_{\text{hard}}$$

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The expansion in $(K^{-1}X)$ can be graphed (STr diagrams), e.g., $n = 3$

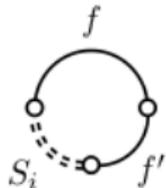


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Our study

In a recent study [[A.D., K. Mantzaropoulos, 2108.10055](#)] we derived

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- This is the first study of functional matching with multiple heavy-field decoupling.
- Support the usefulness of STr functional matching over other methods

Steps for functional matching

- 1 Find the tree level EFT, $\mathcal{L}_{\text{EFT}}^{(\text{tree})}[\phi]$

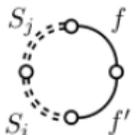
For LQs only four fermion interactions appear at tree level

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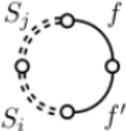
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- 2 Find the $X = [U + P_\mu Z^\mu + \bar{Z}^\mu P_\mu + \dots]$ matrices
- 3 Enumerate UOLEA-terms (19) and STr diagrams (15) up-to $d = 6$
e.g.,


$$= -\frac{i}{2} \text{STr} \left[\frac{1}{P^2 - M_i^2} U_{S_i S_j} \frac{1}{P^2 - M_j^2} U_{S_j f} \frac{1}{\not{P}} U_{f f'} \frac{1}{\not{P}} U_{f' S_i} \right] \Big|_{\text{hard}}$$

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$$= \frac{i}{2} \frac{\log M_i^2 / M_j^2}{M_i^2 - M_j^2} \text{tr} \{ U_{S_i S_j} U_{S_j f} U_{f f'} U_{f' S_i} \}$$

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The effective Lagrangian is the sum: $\mathcal{L}_{\text{EFT}}^{(\text{tree})}[\phi] + \mathcal{L}_{\text{EFT}}^{(1\text{-loop})}[\phi]$

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Two heavy LQs: $S_1 + \tilde{S}_2$

Lets consider two (out of five), heavy LQs with masses M_1 and \tilde{M}_2 :

Field/Group	SU(3)	SU(2)	U(1)
S_1	$\bar{3}$	1	$\frac{1}{3}$
\tilde{S}_2	3	2	$\frac{1}{6}$

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New Interactions with Quarks and Leptons:

$$\begin{aligned}\mathcal{L}_{S-f} = & [(\lambda_{pr}^{1L}) \bar{q}_{pi}^c \cdot \epsilon \cdot \ell_r + (\lambda_{pr}^{1R}) \bar{u}_i^c e_r] S_{1i} + \text{h.c.} \\ & + (\lambda_{pr}^{\not{B}L}) S_{1i} \epsilon^{ijk} \bar{q}_{pj} \cdot \epsilon \cdot q_{rk}^c + (\lambda_{pr}^{\not{B}R}) S_{1i} \epsilon^{ijk} \bar{d}_{pj} u_{rk}^c + \text{h.c.} \\ & + (\tilde{\lambda}_{pr}) \bar{d}_{pi} \tilde{S}_{2i}^T \cdot \epsilon \cdot \ell_r + \text{h.c.} ,\end{aligned}$$

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New Interactions with the Higgs:

$$\begin{aligned} \mathcal{L}_{S-H} = & - (M_1^2 + \lambda_{H1}|H|^2) |S_1|^2 - (\tilde{M}_2^2 + \tilde{\lambda}_{H2}|H|^2) |\tilde{S}_2|^2 + \lambda_{22} (\tilde{S}_{2i}^\dagger \cdot H) (H^\dagger \cdot \tilde{S}_{2i}) \\ & - A_{21} (\tilde{S}_{2i}^\dagger \cdot H) S_{1i}^\dagger + \frac{1}{3} \lambda_3 \epsilon^{ijk} (\tilde{S}_{2i}^T \cdot \epsilon \cdot \tilde{S}_{2j}) (H^\dagger \cdot \tilde{S}_{2k}) + \text{h.c.} \end{aligned} \quad (3.3)$$

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New Self-Interactions:

$$-\mathcal{L}_S = \frac{c_1}{2} (S_1^\dagger S_1)^2 + \frac{\tilde{c}_2}{2} (\tilde{S}_2^\dagger \cdot \tilde{S}_2)^2 + c_{12}^{(1)} (S_1^\dagger S_1)(\tilde{S}_2^\dagger \cdot \tilde{S}_2) + c_{12}^{(2)} (\tilde{S}_{2\alpha}^\dagger S_1)(S_1^\dagger \tilde{S}_{2\alpha}) \\ + c_2^{(8)} (\tilde{S}_{2i}^\dagger \cdot \tilde{S}_{2j}) (\tilde{S}_{2j}^\dagger \cdot \tilde{S}_{2i}) + \left[A' S_{1i}^\dagger \epsilon^{ijk} (\tilde{S}_{2j}^T \cdot \epsilon \cdot \tilde{S}_{2k}) + \text{h.c.} \right] .$$

Tree-level ($S_1 + \tilde{S}_2$ model)

There are **12 baryon number conserving** operators (semileptonic + four-quark)

$$[G_{\ell q}^{(1)}]_{prst}^{(0)} = \frac{(\lambda_{sp}^{1L})^* (\lambda_{tr}^{1L})}{4M_1^2},$$

$$[G_{\ell equ}^{(1)}]_{prst}^{(0)} = \frac{(\lambda_{sp}^{1L})^* (\lambda_{tr}^{1R})}{2M_1^2},$$

$$[G_{eu}]_{prst}^{(0)} = \frac{(\lambda_{sp}^{1R})^* (\lambda_{tr}^{1R})}{2M_1^2},$$

$$[G_{qq}^{(1)}]_{prst}^{(0)} = \frac{(\lambda_{rt}^{\beta L})^* (\lambda_{sp}^{\beta L})}{2M_1^2},$$

$$[G_{ud}^{(1)}]_{prst}^{(0)} = \frac{(\lambda_{tr}^{\beta R})^* (\lambda_{sp}^{\beta R})}{3M_1^2},$$

$$[G_{quqd}^{(1)}]_{prst}^{(0)} = \frac{4}{3} \frac{(\lambda_{ts}^{\beta R})^* (\lambda_{pr}^{\beta L})}{M_1^2},$$

$$[G_{\ell q}^{(3)}]_{prst}^{(0)} = -\frac{(\lambda_{sp}^{1L})^* (\lambda_{tr}^{1L})}{4M_1^2},$$

$$[G_{\ell equ}^{(3)}]_{prst}^{(0)} = -\frac{(\lambda_{sp}^{1L})^* (\lambda_{tr}^{1R})}{8M_1^2},$$

$$[G_{\ell d}]_{prst}^{(0)} = -\frac{(\tilde{\lambda}_{tp})^* (\tilde{\lambda}_{sr})}{2\tilde{M}_2^2},$$

$$[G_{qq}^{(3)}]_{prst}^{(0)} = -\frac{(\lambda_{rt}^{\beta L})^* (\lambda_{sp}^{\beta L})}{2M_1^2},$$

$$[G_{ud}^{(8)}]_{prst}^{(0)} = -\frac{(\lambda_{tr}^{\beta R})^* (\lambda_{sp}^{\beta R})}{M_1^2},$$

$$[G_{quqd}^{(8)}]_{prst}^{(0)} = -4 \frac{(\lambda_{ts}^{\beta R})^* (\lambda_{pr}^{\beta L})}{M_1^2},$$

Tree-level ($S_1 + \tilde{S}_2$ model)

and (all) 4 baryon number violating operators

$$[G_{qqq}]_{prst}^{(0)} = -2 \frac{(\lambda_{pr}^{\beta L})^* (\lambda_{st}^{1L})}{M_1^2},$$

$$[G_{qqu}]_{prst}^{(0)} = \frac{(\lambda_{pr}^{\beta L})^* (\lambda_{st}^{1R})}{M_1^2},$$

$$[G_{duq}]_{prst}^{(0)} = \frac{(\lambda_{pr}^{\beta R})^* (\lambda_{st}^{1L})}{M_1^2},$$

$$[G_{duu}]_{prst}^{(0)} = \frac{(\lambda_{pr}^{\beta R})^* (\lambda_{st}^{1R})}{M_1^2}.$$

usually not discussed or killed by extra (ad-hoc?) discrete symmetries.

Tree-level ($S_1 + \tilde{S}_2$ model)

Only four-fermion operators \rightarrow suitable for explaining possible anomalies in meson decays

Bauer and Neubert, PRL (2016); A. Crivellin, C. Greub, D. Müller and F. Saturnino, JHEP (2021); A. Angelescu, D. Bečirević, D.A. Faroughy, F. Jaffredo and O. Sumensari, 2103.12504.

Also talks by Steve King, S. Trifinopoulos, N. Mahmoudi and J. Kumar earlier in this workshop.

Renormalizable operators: e.g corrections to the Higgs mass, $\delta m^2/16\pi^2$

$$(\delta m^2) = N_c \left[\lambda_{H1} M_1^2 (1 + L_1) + (2\tilde{\lambda}_{H2} - \lambda_{\tilde{2}\tilde{2}}) \tilde{M}_2^2 (1 + L_2) + |A_{\tilde{2}1}|^2 \left(1 + \frac{M_1^2 \log \mu^2 / M_1^2 - \tilde{M}_2^2 \log \mu^2 / \tilde{M}_2^2}{\Delta_{12}^2} \right) \right].$$

where

$$L_i = \log \frac{\mu^2}{M_i^2}, \quad \Delta_{12}^2 = M_1^2 - \tilde{M}_2^2,$$

and μ is the renormalization scale.

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Perturbation theory instability is evident. The Higgs field is part of the light fields so it should be of the order of EW scale. Otherwise EFT does not make sense!

One-loop ($S_1 + \tilde{S}_2$ model)

Renormalizable operators: e.g corrections to the Higgs mass, $\delta m^2/16\pi^2$

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Possible solutions:

- 1 LQ masses M_1, \tilde{M}_2 of the order of the TeV scale and $O(1)$ couplings
- 2 LQ masses at a high scale ($\gg m_w$) but Higgs sector couplings tiny
- 3 Choose a renorm scale μ such that $L_1 = L_2 = -1$: Then $\delta m^2 = 0$!
- 4 Supersymmetrize the LQ-model

or combinations of the above four cases...

One-loop ($S_1 + \tilde{S}_2$ model) cont'd

Non-Renormalizable operators: $\mathcal{L}_{\text{EFT}} \supset G_{\ell d} \mathcal{O}_{\ell d}$

$$[G_{\ell d}]_{prst} \propto -\frac{\tilde{\lambda}_{tp}^* \tilde{\lambda}_{sr}}{2\tilde{M}_2^2} \left(1 + \frac{1}{16\pi^2} \frac{M_1^2}{\tilde{M}_2^2} (N_c c_{12}^{(1)} + c_{12}^{(2)}) (1 + L_1) \right).$$

$\mathcal{O}_{\ell d}$	$(\bar{\ell}\gamma^\mu\ell)(\bar{d}\gamma_\mu d)$
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Perturbative instability for large hierarchy of M_1 and \tilde{M}_2 .

This tuning is not usually quoted in the literature. Same solutions as before may be admitted. No problem when masses are close to each other.

One-loop ($S_1 + \tilde{S}_2$ model): Neutrino masses

Weinberg operator is radiatively induced: $\mathcal{L}_{\text{EFT}} \supset \frac{G_{\nu\nu}}{16\pi^2} \mathcal{O}_{\nu\nu}$

$$[G_{\nu\nu}]_{pr}^{(1)} = N_c A_{\tilde{2}1} \left((\lambda^{1L})^T y_D \tilde{\lambda} \right)_{pr} \frac{\log M_1^2 / \tilde{M}_2^2}{M_1^2 - \tilde{M}_2^2}.$$

$\mathcal{O}_{\nu\nu}$	$\epsilon^{\alpha\beta} \epsilon^{\alpha_1\beta_1} H^\alpha H^{\alpha_1} \bar{\ell}_{p\beta}^c \ell_{r\beta_1}$
------------------------	---

Physical neutrino masses: (in mass basis of 1704.03888) ($m_\nu / 16\pi^2$)

$$m_\nu = -\sqrt{2} \frac{v A_{\tilde{2}1}}{M_1^2 - \tilde{M}_2^2} \left[U_{\text{MNS}}^T (\hat{\lambda}^{1L})^T K_{\text{CKM}} m_d \hat{\lambda} U_{\text{MNS}} \right] \log \left(\frac{M_1^2}{\tilde{M}_2^2} \right)$$

See also, Mahanta, PRD (2000); Dorsner, PRD (2012); A. Crivellin, C. Greub, D. Müller and F. Saturnino, 2010.06593; Zhang, 2105.08670

One-loop ($S_1 + \tilde{S}_2$ model): $(g - 2)_\ell$

A recent 4.2σ anomaly $\Delta\alpha_\mu = (251 \pm 59) \times 10^{-11}$ [BNL collab., 2104.03281] has re-warmed up all BSM physics enthusiasts around the globe.

Two $d = 6$ operators are responsible in SMEFT (Warsaw basis),

\mathcal{O}_{eW}	$(\bar{\ell}\sigma^{\mu\nu}e)\sigma^I H W_{\mu\nu}^I$
\mathcal{O}_{eB}	$(\bar{\ell}\sigma^{\mu\nu}e)H B_{\mu\nu}$

$$[C_{eB}]^{(1)}(\mu) = \frac{g' N_c}{16\pi^2} \left\{ \frac{5}{24} \left[\log\left(\frac{\mu^2}{M_1^2}\right) + \frac{5}{2} \right] \frac{Y_{1U}^{1L}}{M_1^2} - \frac{1}{24} \frac{y_E \cdot \Lambda_e}{M_1^2} + \frac{1}{16} \frac{\tilde{\Lambda}_\ell \cdot y_E}{\tilde{M}_2^2} \right\}$$

$$[C_{eW}]^{(1)}(\mu) = \frac{g N_c}{16\pi^2} \left\{ -\frac{1}{8} \left[\log\left(\frac{\mu^2}{M_1^2}\right) + \frac{1}{2} \right] \frac{Y_{1U}^{1L}}{M_1^2} + \frac{1}{24} \frac{\Lambda_\ell \cdot y_E}{M_1^2} - \frac{1}{48} \frac{\tilde{\Lambda}_\ell \cdot y_E}{\tilde{M}_2^2} \right\}$$

\Downarrow

One-loop ($S_1 + \tilde{S}_2$ model): $(g - 2)_\ell$

To leading-log approximation, just set $\mu = m_t$

OR

From $\mu = M_1$ run down to m_t with RGEs [Jenkins, Manohar, Trott, 1310.4838](#)

⇓

and plug it into

$$\Delta a^\ell = \frac{4m_\ell v}{\sqrt{2}} \left[\frac{1}{g'} \Re e[C_{eB}] - \frac{1}{g} \Re e[C_{eW}] \right]_{\ell\ell}$$

see [A.D., Materkowska, Paraskevas, Suxho, Rosiek, 1704.03888](#)

⇓

One-loop ($S_1 + \tilde{S}_2$ model): $(g - 2)_\ell$

$$\begin{aligned}\Delta a_\ell^{(S_1 + \tilde{S}_2)} &= \sum_{q=u,c,t} \frac{m_\ell}{4\pi^2} \frac{m_q}{M_1^2} \left[\log\left(\frac{m_t^2}{M_1^2}\right) + \frac{7}{4} \right] \Re e(\hat{\lambda}_{q\ell}^{1L*} \hat{\lambda}_{q\ell}^{1R}) \\ &\quad - \frac{m_\ell^2}{32\pi^2 M_1^2} \left(\hat{\lambda}_{q\ell}^{1L*} \hat{\lambda}_{q\ell}^{1L} + \hat{\lambda}_{q\ell}^{1R*} \hat{\lambda}_{q\ell}^{1R} \right) + \frac{m_\ell^2}{32\pi^2 \tilde{M}_2^2} \hat{\lambda}_{q\ell}^* \hat{\lambda}_{q\ell}\end{aligned}$$

in agreement with fixed order calculations, e.g. [Bauer and Neubert, PRL \(2016\)](#)

A chiral enhancement of $O(m_t/m_\mu)$ can solve the anomaly for a TeV S_1 -mass and $O(1)$ couplings. See talk by [M. Tammaro in this workshop](#)

However, the same covariant diagram results in large contributions to the muon mass as well.

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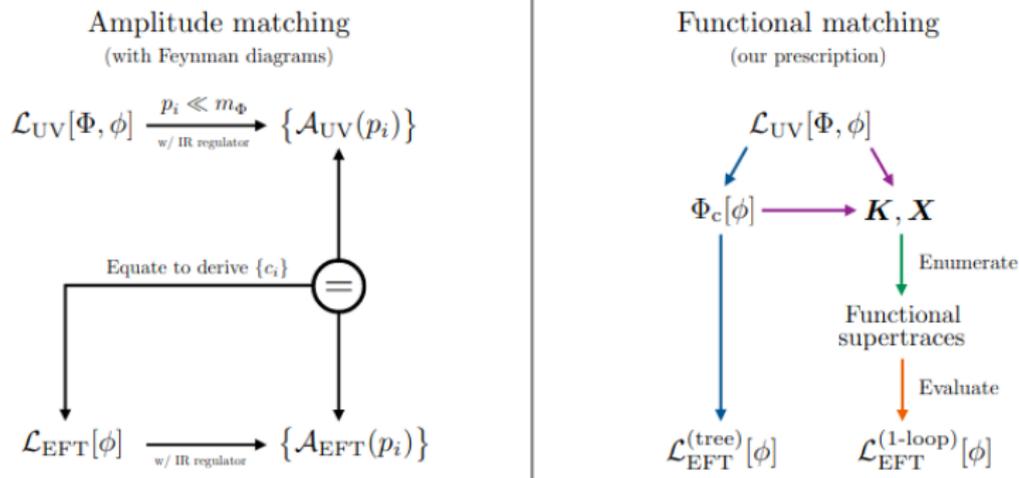
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Thank you for your attention

Backup: Feynman diagrammatic vs. functional matching

Figure taken from Cohen, Lu, Zhang, 2011.02484



Advantages of functional approach: a systematic approach, no EFT basis needed, no guess of effective operators, no calculating twice amplitudes.