

CORFU SUMMER INSTITUTE  
21ST HELLENIC SCHOOL AND WORKSHOPS ON ELEMENTARY PARTICLE PHYSICS AND GRAVITY  
CORFU, GREECE 2021

Workshop on the Standard Model and Beyond

# Some reflexions on hidden features of SM extensions with scalar triplets



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1<sup>st</sup> September, 2021

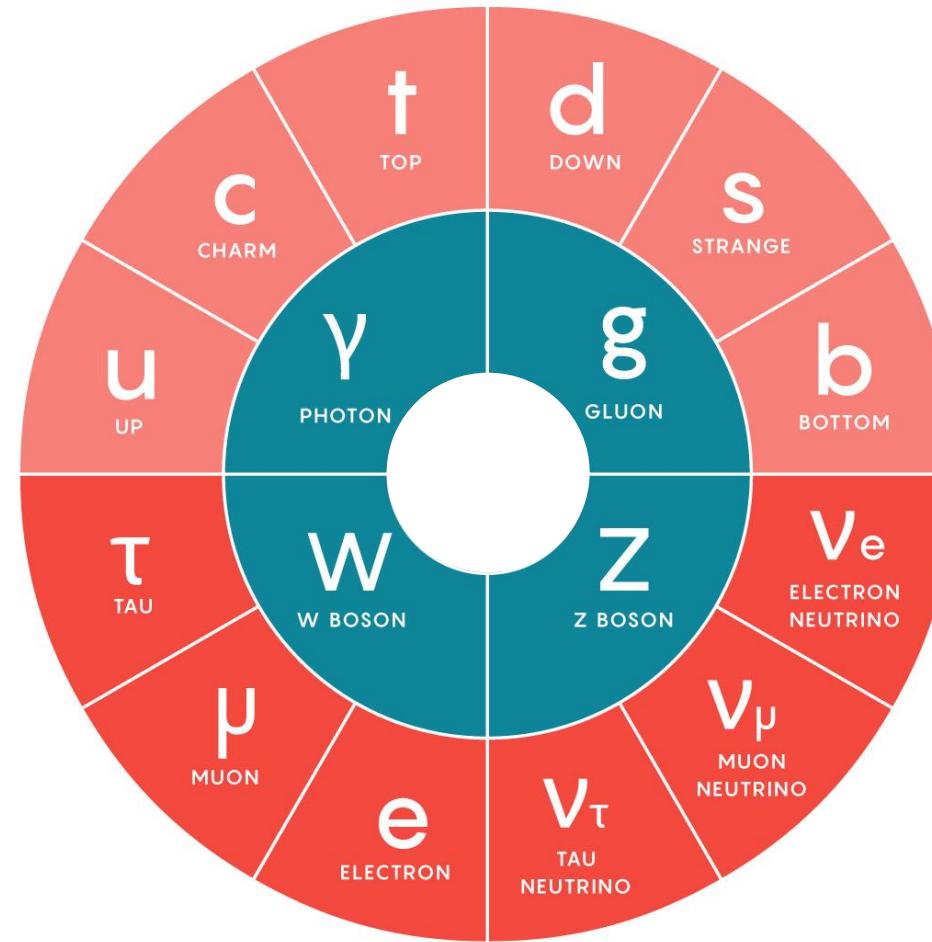
# The Standard Model of Particle Physics

On the 4<sup>th</sup> July of 2012...

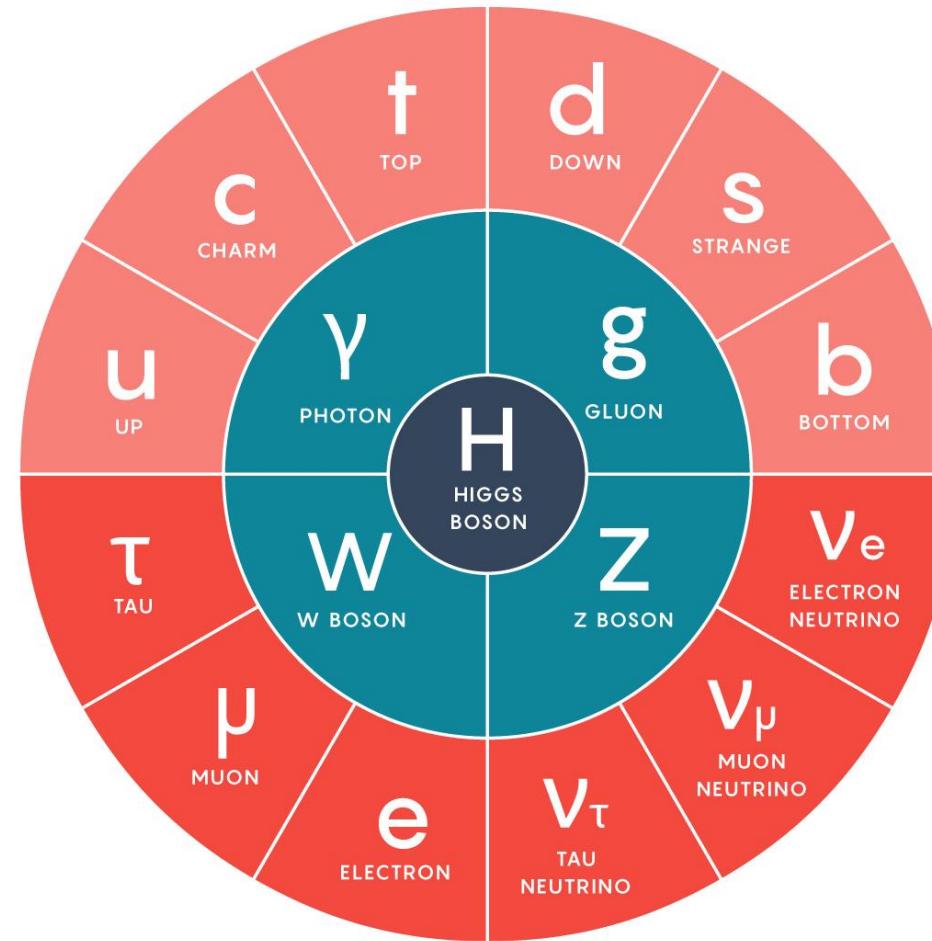


The discovery of a Higgs-like particle is announced at CERN

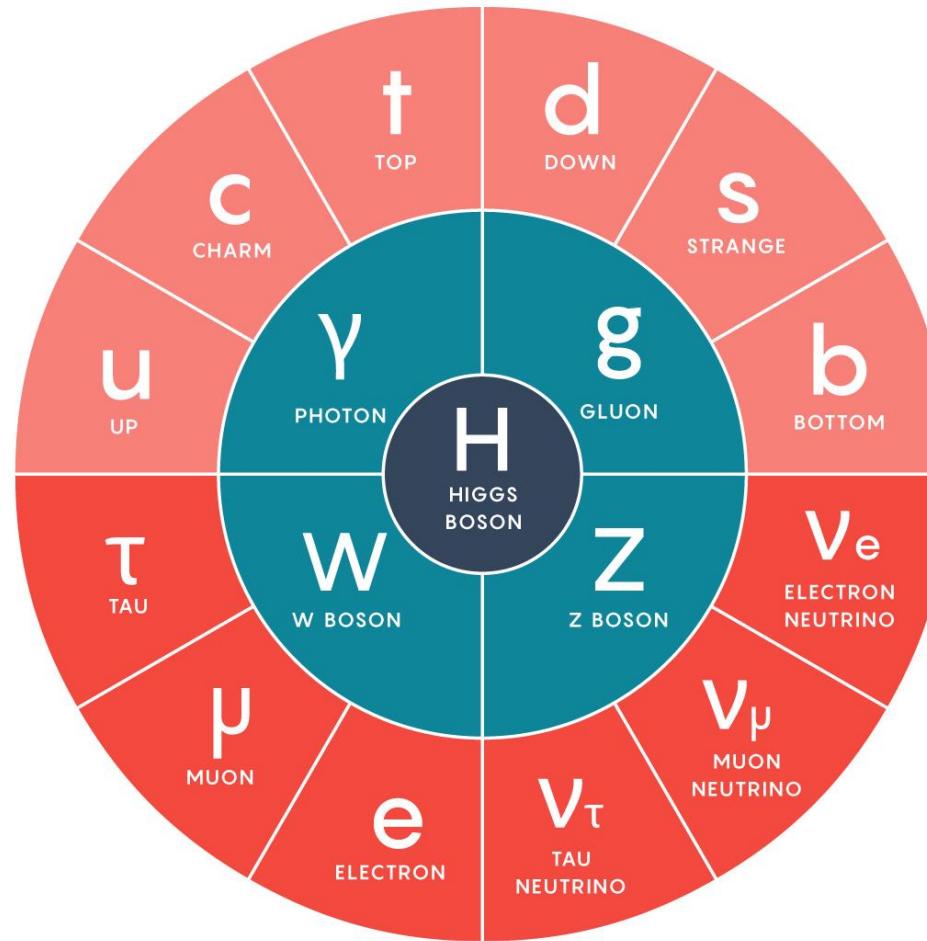
# The Standard Model of Particle Physics



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# The Standard Model of Particle Physics



However, this is not the whole story...

# Beyond the Standard Model

Scalar triplet extensions of the Standard Model

Multi-Higgs  
scenario

Higgs-triplet model  
(HTM)

Two-scalar-triplet model  
(2STM)

Motivation

Problem

# Beyond the Standard Model

Scalar triplet extensions of the Standard Model

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Two-scalar-triplet model  
(2STM)

Motivation

Neutrino masses  
in type-II seesaw  
mechanism

Problem

Are neutral minima  
stable against charge  
breaking?

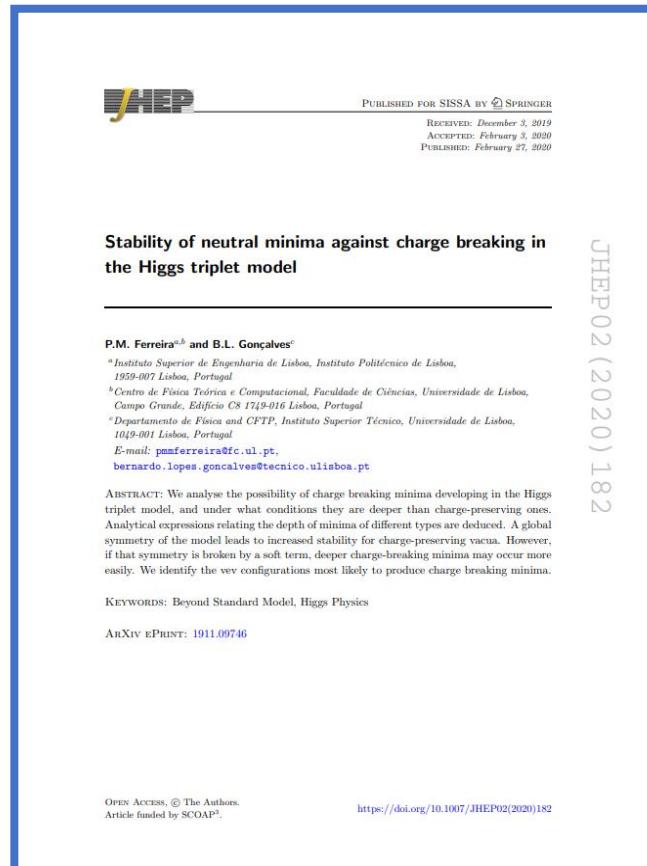
# Beyond the Standard Model

## Scalar triplet extensions of the Standard Model

### Multi-Higgs scenario

### Motivation

### Problem



[arXiv:1911.09746v3 \[hep-ph\]](https://arxiv.org/abs/1911.09746v3)

### Two-scalar-triplet model (2STM)

# The Higgs-Triplet Model

All SM fields, with the **addition of an SU(2) scalar triplet**

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} + \Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$$

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## DIFFERENT VACUUM POSSIBILITIES:

- CP-breaking vacua
- Charge-breaking (CB) vacua
- Normal (N) vacua

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Are neutral minima stable against charge breaking in the Higgs triplet model?

# The Higgs-Triplet Model

Most general gauge invariant scalar potential

$$V = m^2 \Phi^\dagger \Phi + M^2 \text{Tr}(\Delta^\dagger \Delta) + \mu \left( \Phi^T i\tau_2 \Delta^\dagger \Phi + \text{h.c.} \right)$$

$$+ \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 \left[ \text{Tr}(\Delta^\dagger \Delta) \right]^2 + \lambda_3 \text{Tr} \left[ (\Delta^\dagger \Delta)^2 \right] + \lambda_4 (\Phi^\dagger \Phi) \text{Tr}(\Delta^\dagger \Delta) + \lambda_5 \Phi^\dagger \Delta \Delta^\dagger \Phi$$

Soft-breaking term

$$\Phi \rightarrow e^{i\theta} \Phi$$

→ Potential **without** soft-breaking term  $\mu = 0$  Allows for dark matter particles

→ Potential **with** soft-breaking term  $\mu \neq 0$  Helps generate neutrino masses

# The Higgs-Triplet Model

Three possibilities for neutral vacua

$$\langle \Phi \rangle_{N1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_\Phi \end{pmatrix}, \quad \langle \Delta \rangle_{N1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix}$$

- Can occur whether the soft breaking term is present or not
- If such term is not present, we get a massless axion

$$\langle \Phi \rangle_{N2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \Delta \rangle_{N2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

- Only occurs when the soft breaking term is not present
- Good dark matter candidates

$$\langle \Phi \rangle_{N3} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{N3} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix}$$

- Can occur whether the soft breaking term is present or not
- Unphysical vacuum type (massless quarks)

# The Higgs-Triplet Model

$c_1 \neq 0$

Three possibilities for neutral vacua

$$\langle\Phi\rangle_{N1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_\Phi \end{pmatrix}, \quad \langle\Delta\rangle_{N1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix}$$

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Six different possibilities for CB vacua

$$\langle\Phi\rangle_{CB1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ c_1 \end{pmatrix}, \quad \langle\Delta\rangle_{CB1} = \frac{1}{\sqrt{2}} \begin{pmatrix} -c_3/\sqrt{2} & 0 \\ c_2 & c_3/\sqrt{2} \end{pmatrix}$$

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$$\langle\Phi\rangle_{CB5} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ c_1 \end{pmatrix}, \quad \langle\Delta\rangle_{CB5} = \frac{1}{\sqrt{2}} \begin{pmatrix} c_2/\sqrt{2} & c_3 \\ 0 & -c_2/\sqrt{2} \end{pmatrix}$$

$$\langle\Phi\rangle_{CB6} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ c_1 \end{pmatrix}, \quad \langle\Delta\rangle_{CB6} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & c_2 \\ 0 & 0 \end{pmatrix}$$

# The Higgs-Triplet Model

$$c_1 \neq 0$$

Using a bilinear formalism similar to the one developed for the 2HDM, it is possible to find analytical formulae relating the depth of the potential at different extrema of the potential

# Example

$$c_1 \neq 0$$

Stability of minima of type N1 against CB1

$$\langle \Phi \rangle_{N1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_\Phi \end{pmatrix}, \quad \langle \Delta \rangle_{N1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix}$$

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$$\mu = 0$$

$$V_{CB1} - V_{N1} = \frac{c_3^2 m_+^2}{4 \left( 1 + \frac{2v_\Delta^2}{v_\Phi^2} \right)}$$

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Squared mass calculated in  
minima of type N1

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Squared mass calculated in  
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When N1 is a minimum one always obtains  $V_{CB1} - V_{N1} > 0$

STABILITY  
GUARANTEED

# Example

$c_1 \neq 0$

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$$\mu \neq 0$$

$$V_{CB1} - V_{N1} = \frac{m_A^2}{4 \left( 1 + \frac{4 v_\Delta^2}{v_\Phi^2} \right)} (c_2 - v_\Delta)^2 \left( 1 - \frac{v_\Delta}{c_2} \frac{c_1^2}{v_\Phi^2} \right) + \frac{m_+^2 c_3^2}{4 \left( 1 + \frac{2 v_\Delta^2}{v_\Phi^2} \right)}$$

# Example

$c_1 \neq 0$

Stability of minima of type N1 against CB1

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Even if  $N1$  is a minimum it is not guaranteed that  $V_{CB1} - V_{N1} > 0$

STABILITY  
NOT  
GUARANTEED

# Stability so far...

$$c_1 \neq 0$$

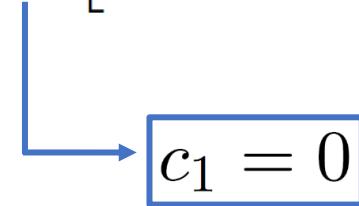
CB minima  $c_1 \neq 0$

		$\mu = 0$	$\mu \neq 0$
$N_2$ minima	STABILITY GUARANTEED	DOES NOT OCCUR	
$N_1$ minima	STABILITY GUARANTEED	STABILITY NOT GUARANTEED	

# The case of the vevless doublet

$$c_1 = 0$$

$$\frac{\partial V}{\partial c_1} = c_1 \left[ m^2 + \lambda_1 c_1^2 + \frac{\lambda_4}{2}(c_2^2 + c_3^2 + c_4^2) + \frac{\lambda_5}{2} (2c_2^2 + c_3^2) \right] = 0$$



$c_1 = 0$  disconnected solution from  $c_1 \neq 0$

# The case of the vevless doublet

$c_1 = 0$

Three possibilities for neutral vacua

$$\langle \Phi \rangle_{N1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_\Phi \end{pmatrix}, \quad \langle \Delta \rangle_{N1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix}$$

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$$\langle \Phi \rangle_{N3} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{N3} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix}$$

Six new possibilities for CB vacua

$$\langle \Phi \rangle_{CB7} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{CB7} = \frac{1}{\sqrt{2}} \begin{pmatrix} c_3/\sqrt{2} & c_2 \\ c_2 & -c_3/\sqrt{2} \end{pmatrix}$$

$$\langle \Phi \rangle_{CB8} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{CB8} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & c_2 \\ c_2 & 0 \end{pmatrix}$$

$$\langle \Phi \rangle_{CB9} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{CB9} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -c_2 \\ c_2 & 0 \end{pmatrix}$$

$$\langle \Phi \rangle_{CB10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{CB10} = \frac{1}{\sqrt{2}} \begin{pmatrix} c_3/\sqrt{2} - c_3^2/2c_2 \\ c_2 & -c_3/\sqrt{2} \end{pmatrix}$$

$$\langle \Phi \rangle_{CB11} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{CB11} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & c_4 \\ 0 & 0 \end{pmatrix}$$

$$\langle \Phi \rangle_{CB12} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{CB12} = \frac{1}{\sqrt{2}} \begin{pmatrix} c_3/\sqrt{2} & 0 \\ 0 & -c_3/\sqrt{2} \end{pmatrix}$$

# The case of the vevless doublet

$c_1 = 0$

Three possibilities for neutral vacua

$$\langle \Phi \rangle_{N1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_\Phi \end{pmatrix}, \quad \langle \Delta \rangle_{N1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix}$$

$$\langle \Phi \rangle_{N2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \Delta \rangle_{N2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

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$$\langle \Phi \rangle_{CB10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{CB10} = \frac{1}{\sqrt{2}} \begin{pmatrix} c_3/\sqrt{2} - c_3^2/2c_2 \\ c_2 & -c_3/\sqrt{2} \end{pmatrix}$$

$$\langle \Phi \rangle_{CB11} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{CB11} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & c_4 \\ 0 & 0 \end{pmatrix}$$

$$\langle \Phi \rangle_{CB12} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{CB12} = \frac{1}{\sqrt{2}} \begin{pmatrix} c_3/\sqrt{2} & 0 \\ 0 & -c_3/\sqrt{2} \end{pmatrix}$$

Again performing a bilinear formalism...

# Full analytical conclusions

CB minima  $c_1 \neq 0$

	$\mu = 0$	$\mu \neq 0$
N2 minima	STABILITY GUARANTEED	DOES NOT OCCUR
N1 minima	STABILITY GUARANTEED	STABILITY NOT GUARANTEED

CB minima  $c_1 = 0$

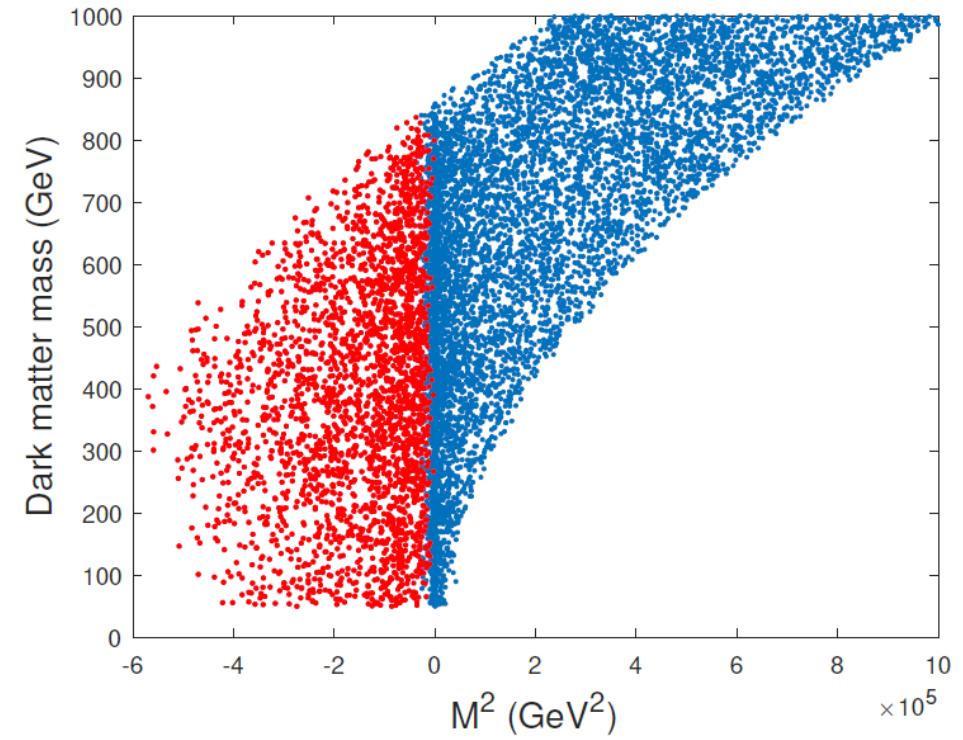
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# Numerical scan without soft-breaking

- N2 minima is stable against N1 minima type
- **N2 not necessarily stable against CB with vevless doublet**
- We have identified the most likely CB vacua as the vev combinations we dubbed CB7 and CB10

$$\langle \Phi \rangle_{CB7} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{CB7} = \frac{1}{\sqrt{2}} \begin{pmatrix} c_3/\sqrt{2} & c_2 \\ c_2 & -c_3/\sqrt{2} \end{pmatrix}$$

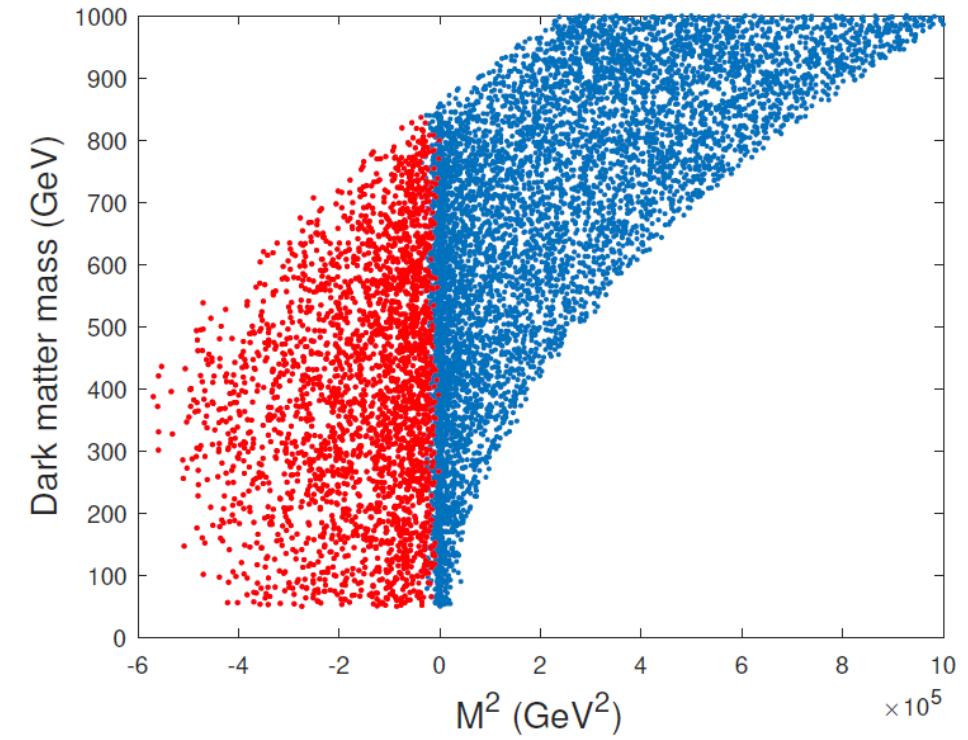
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In blue, all scanned points for a minimum of type N2; in red, those points for which there exists a CB vacuum (CB7 or CB10) lower than N2

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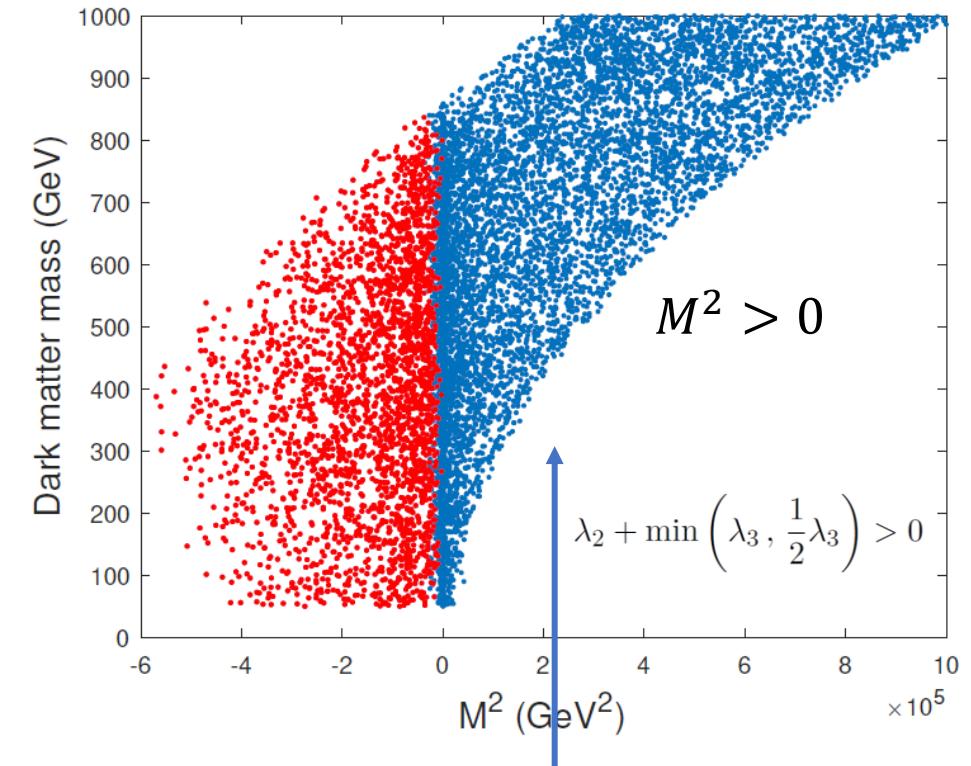


It can be shown analytically that

$$\text{An } N2 \text{ minimum is stable against charge breaking iff } M^2 > -\sqrt{\min\left(\lambda_2 + \frac{1}{2}\lambda_3, \lambda_2 + \lambda_3\right)} \frac{m_h v}{\sqrt{2}}$$

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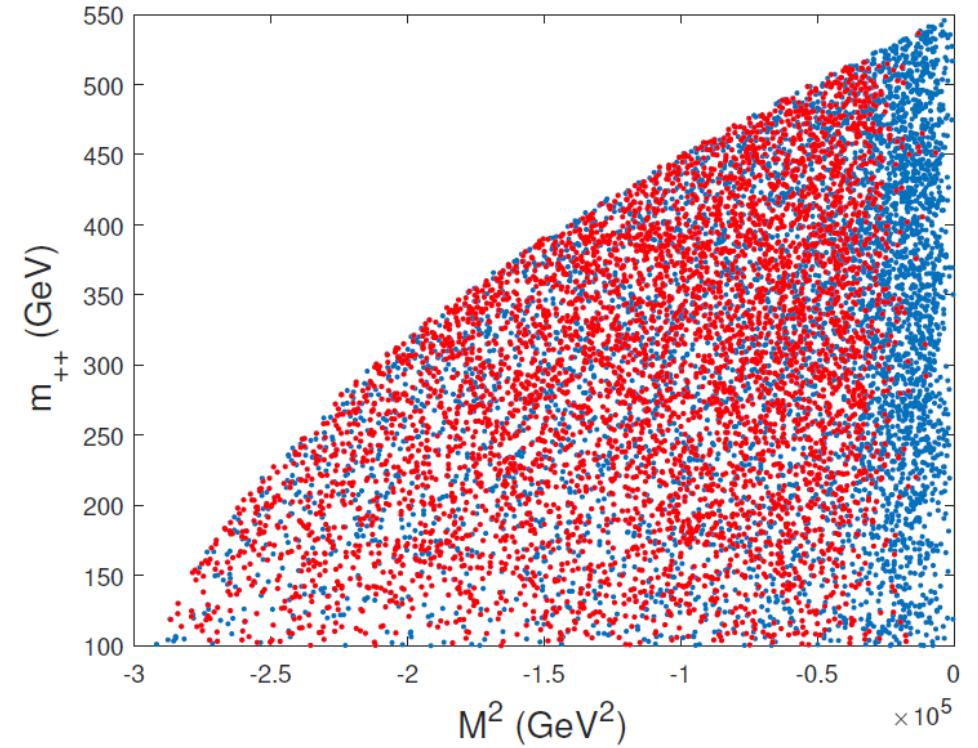


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# Numerical scan with soft-breaking

- N2 minima is not possible to occur
- **With a minimum of type N1, there are several possible deeper CB vacua**
- Large percentage of potentially-unstable neutral minima



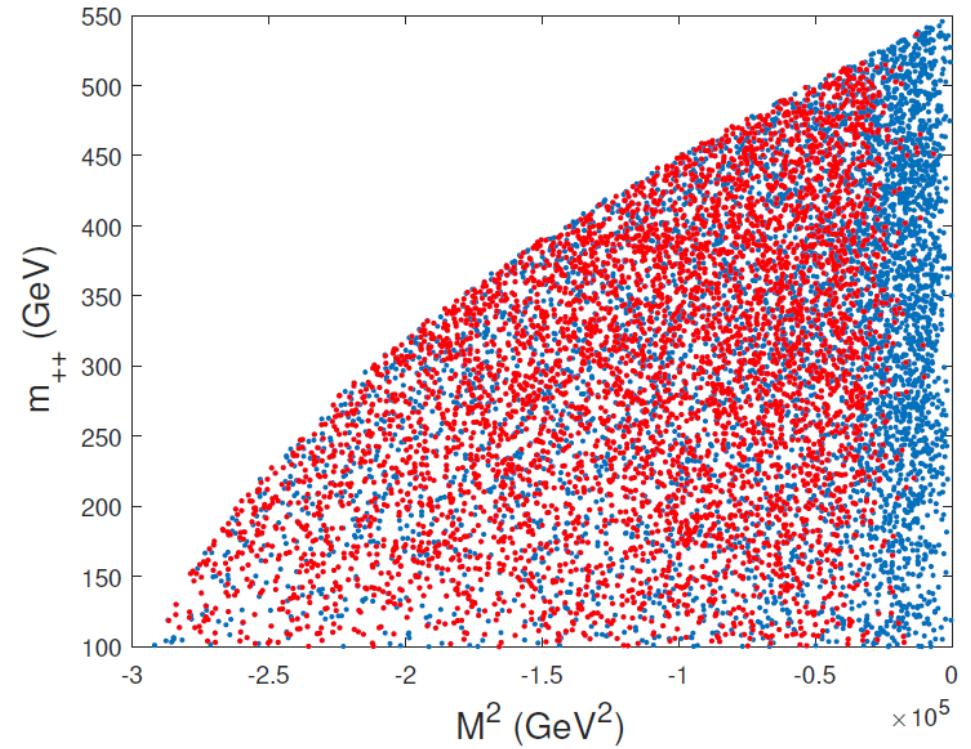
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## SOME REMARKS

- We found that for roughly 26% (48%) of the parameter space found for the globally symmetric (softly broken) potential neutral minima had deeper charge breaking ones
- CB global minima can indeed coexist, in some cases fairly frequently, with neutral minima



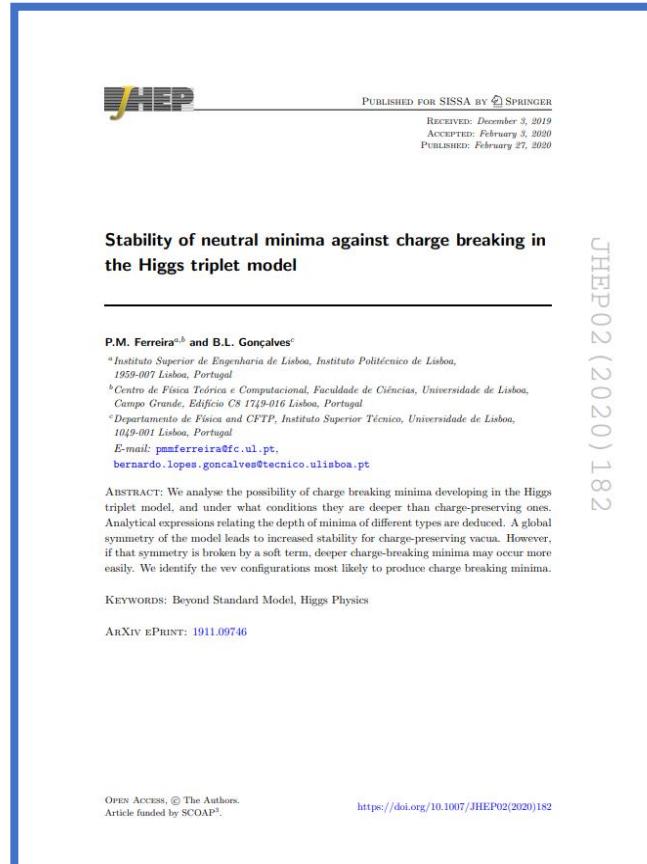
# Beyond the Standard Model

## Scalar triplet extensions of the Standard Model

### Multi-Higgs scenario

### Motivation

### Problem



[arXiv:1911.09746v3 \[hep-ph\]](https://arxiv.org/abs/1911.09746v3)

### Two-scalar-triplet model (2STM)

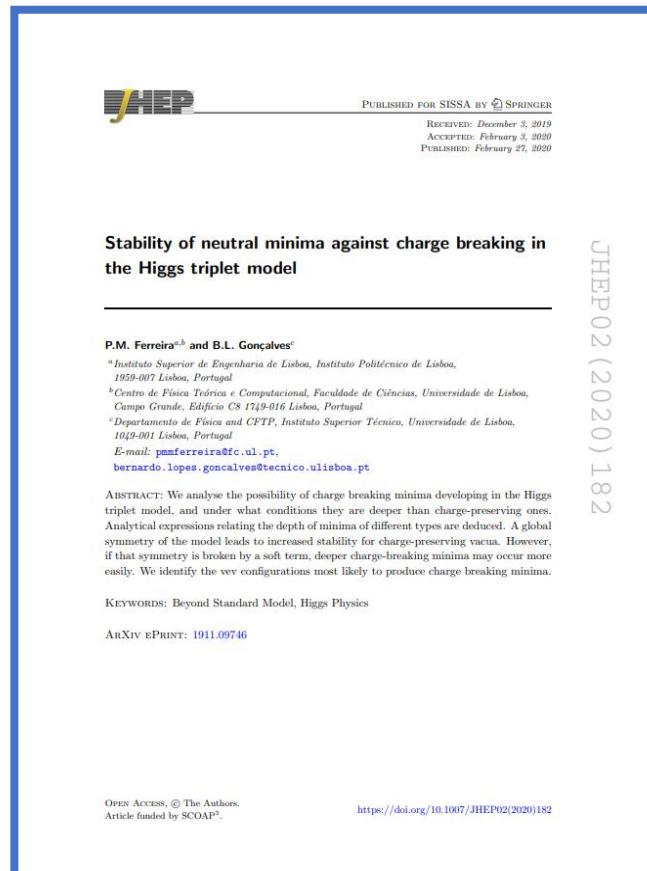
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Minimal triplet extension  
in which spontaneous CP  
violation occurs

Do we have decoupling  
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OUT  
SOON

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# Spontaneous CP violation in scalar-triplet models

In the Higgs-triplet model:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad + \quad \Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$$

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# Spontaneous CP violation in scalar-triplet models

In the two-scalar-triplet model:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad + \quad \Delta_{1,2} = \begin{pmatrix} \delta_{1,2}^+/\sqrt{2} & \delta_{1,2}^{++} \\ \delta_{1,2}^0 & -\delta_{1,2}^+/\sqrt{2} \end{pmatrix}$$

$$V = V_{U(1)} + V_{SB}$$

$$V_{U(1)} = m^2 \Phi^\dagger \Phi + M_{11}^2 \text{Tr}(\Delta_1^\dagger \Delta_1) + M_{22}^2 \text{Tr}(\Delta_2^\dagger \Delta_2) + \lambda_0 (\Phi^\dagger \Phi)^2$$

$$\text{U(1)-symmetric} \quad + \lambda_1 [\text{Tr}(\Delta_1^\dagger \Delta_1)]^2 + \lambda_2 [\text{Tr}(\Delta_2^\dagger \Delta_2)]^2 + \lambda_{21} \text{Tr}(\Delta_2^\dagger \Delta_2) \text{Tr}(\Delta_1^\dagger \Delta_1) + \lambda_{12} \text{Tr}(\Delta_1^\dagger \Delta_2) \text{Tr}(\Delta_2^\dagger \Delta_1)$$

$$\Phi \rightarrow e^{i\alpha} \Phi \quad + \tilde{\lambda}_1 \text{Tr}[(\Delta_1^\dagger \Delta_1)^2] + \tilde{\lambda}_2 \text{Tr}[(\Delta_2^\dagger \Delta_2)^2] + \tilde{\lambda}_{21} \text{Tr}(\Delta_2^\dagger \Delta_2 \Delta_1^\dagger \Delta_1) + \tilde{\lambda}_{12} \text{Tr}(\Delta_1^\dagger \Delta_2 \Delta_2^\dagger \Delta_1)$$

$$\Delta_{1,2} \rightarrow e^{i\alpha_{1,2}} \Delta_{1,2} \quad + \lambda'_1 \text{Tr}(\Delta_1^\dagger \Delta_1) \Phi^\dagger \Phi + \lambda'_2 \text{Tr}(\Delta_2^\dagger \Delta_2) \Phi^\dagger \Phi + \hat{\lambda}_1 \Phi^\dagger \Delta_1 \Delta_1^\dagger \Phi + \hat{\lambda}_2 \Phi^\dagger \Delta_2 \Delta_2^\dagger \Phi$$

$$+ V_{SB} = M_{12}^2 [\text{Tr}(\Delta_1^\dagger \Delta_2) + \text{Tr}(\Delta_2^\dagger \Delta_1)] + (\mu_1 \Phi^T i\tau_2 \Delta_1^\dagger \Phi + \mu_2 \Phi^T i\tau_2 \Delta_2^\dagger \Phi + \text{H.c.}) \quad \text{Softly-breaking terms}$$

# Spontaneous CP violation in scalar-triplet models

In the two-scalar-triplet model:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} + \Delta_{1,2} = \begin{pmatrix} \delta_{1,2}^+/\sqrt{2} & \delta_{1,2}^{++} \\ \delta_{1,2}^0 & -\delta_{1,2}^+/\sqrt{2} \end{pmatrix}$$

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \Delta_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ u_1 e^{i\theta_1} & 0 \end{pmatrix}, \quad \langle \Delta_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ u_2 e^{i\theta_2} & 0 \end{pmatrix}$$

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$$\begin{aligned} u_1 &= u c_\beta \\ u_2 &= u s_\beta \end{aligned}$$

$$uv (\mu_1 c_\beta s_{\theta_1} + \mu_2 s_\beta s_{\theta_2}) = 0$$

$$\tan \beta = u_2/u_1$$

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CP violation can be communicated to the fermion sector

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**BUT WHAT ABOUT THE SCALAR MASS SPECTRUM?**

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ u_1 e^{i\theta_1} \end{pmatrix}, \quad \langle \Delta_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ u_2 e^{i\theta_2} & 0 \end{pmatrix} \quad \theta_1 \neq \theta_2$$



$$uv (\mu_1 c_\beta s_{\theta_1} + \mu_2 s_\beta s_{\theta_2}) = 0$$

**SCPV IS POSSIBLE**

CP violation can be communicated to the fermion sector

# Scalar mass spectrum

Using results from matrix theory, it is possible to find analytical results, exact or up to a good approximation, regarding the eigenvalues of the mass matrices, thus the scalar masses

- Six neutral scalars
- Three charged scalars
- Two doubly-charged scalars

# Scalar mass spectrum

CP-conserving scenario

- Two Goldstone bosons
- One Higgs-like particle
- All the remaining particles decouple

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CP-violating scenario

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- ...

# CP-violating case

This result is exact!

$$m_{h_{2,3}^0}^2 \leq \frac{u^2}{2} \left[ \Lambda_3 + \Lambda_5 + (\Lambda_3 - \Lambda_5)c_{2\beta} \pm \sqrt{[\Lambda_3 + \Lambda_5 + (\Lambda_3 - \Lambda_5)c_{2\beta}]^2 + (\Lambda_4^2 - 4\Lambda_3\Lambda_5)s_{2\beta}^2} \right]$$

$\Lambda_i \longrightarrow$  combinations of quartic couplings

$$|u| < 8 \text{ GeV}$$

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$\Lambda_i \longrightarrow$  combinations of quartic couplings

$$|u| < 8 \text{ GeV}$$

- Two Goldstone bosons
- Two light neutral scalars
- One Higgs-like particle

...

# CP-violating case

$$m_{H_2^+}^2 \simeq \frac{m_{H_1^{++}}^2}{2} \simeq -\frac{1}{4} \frac{\hat{\lambda}_1 f_1(\beta, \theta_1, \theta_2) + \hat{\lambda}_2 f_2(\beta)}{f_1(\beta, \theta_1, \theta_2) + f_2(\beta)} v^2$$

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- Two Goldstone bosons
- Two light neutral scalars
- One Higgs-like particle
- Two charged particles at the electroweak scale

...

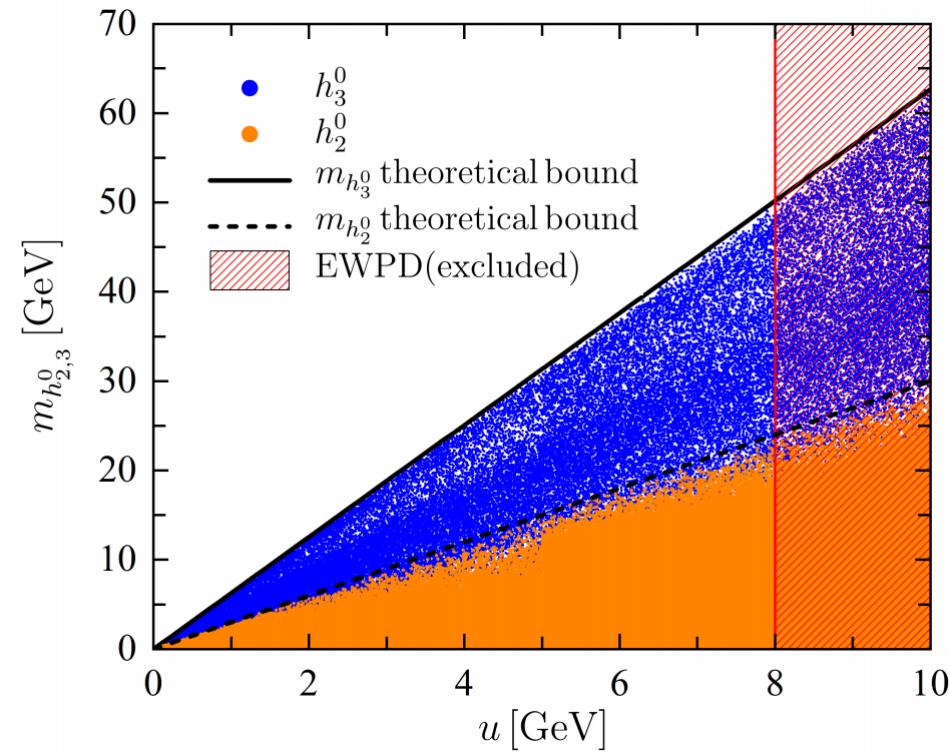
# CP-violating case

- Two Goldstone bosons
- Two light neutral scalars
- One Higgs-like particle
- Two charged particles at the electroweak scale
  - All the remaining particles decouple

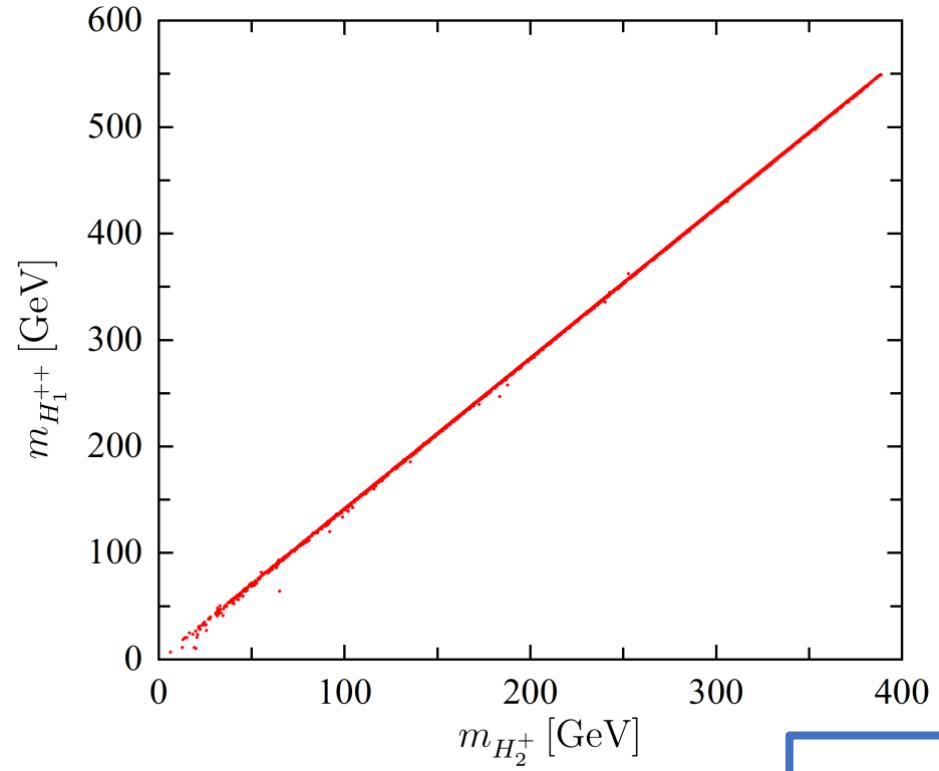
# Full analytical conclusions

	Mass spectrum	CP-Conserving	CP-Violating
Neutral	$h_1^0$	Massless - Goldstone boson	
	$h_2^0$	SM Higgs-like	
	$h_3^0$	Decoupled	Light
	$h_4^0$		SM Higgs-like
	$h_5^0$	Decoupled	
	$h_6^0$		Decoupled
Singly-charged	$H_1^+$	Massless - Goldstone boson	
	$H_2^+$	Decoupled	Electroweak
	$H_3^+$	Decoupled	Decoupled
Doubly-charged	$H_1^{++}$	Decoupled	Electroweak
	$H_2^{++}$	Decoupled	Decoupled

# Numerical scan

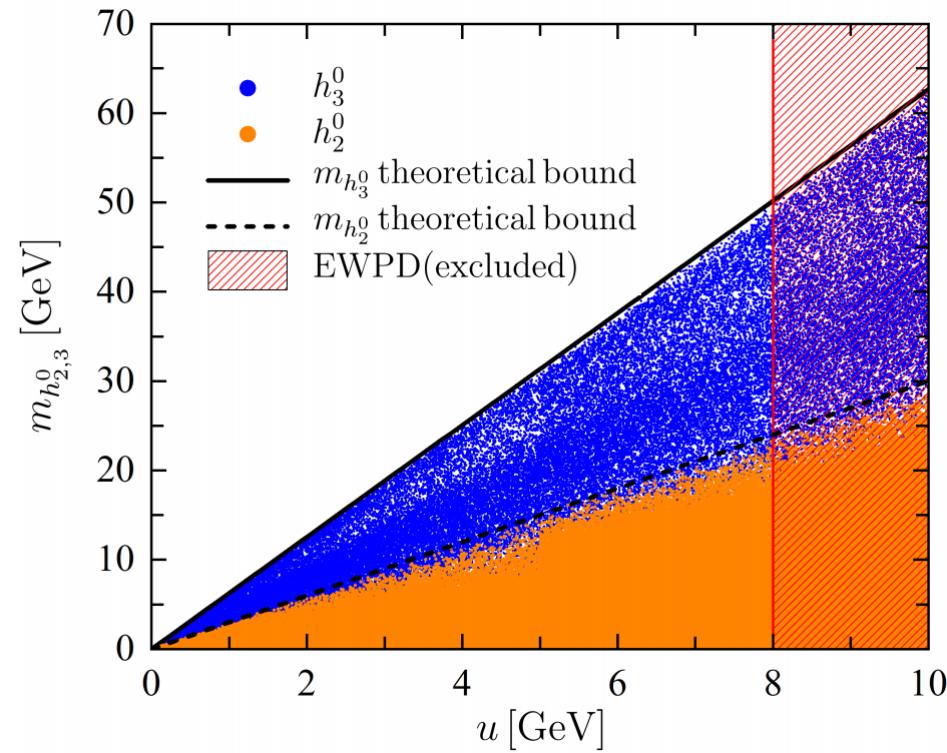


$$m_{h_{2,3}^0}^2 \leq \frac{u^2}{2} f(\Lambda, \beta)$$

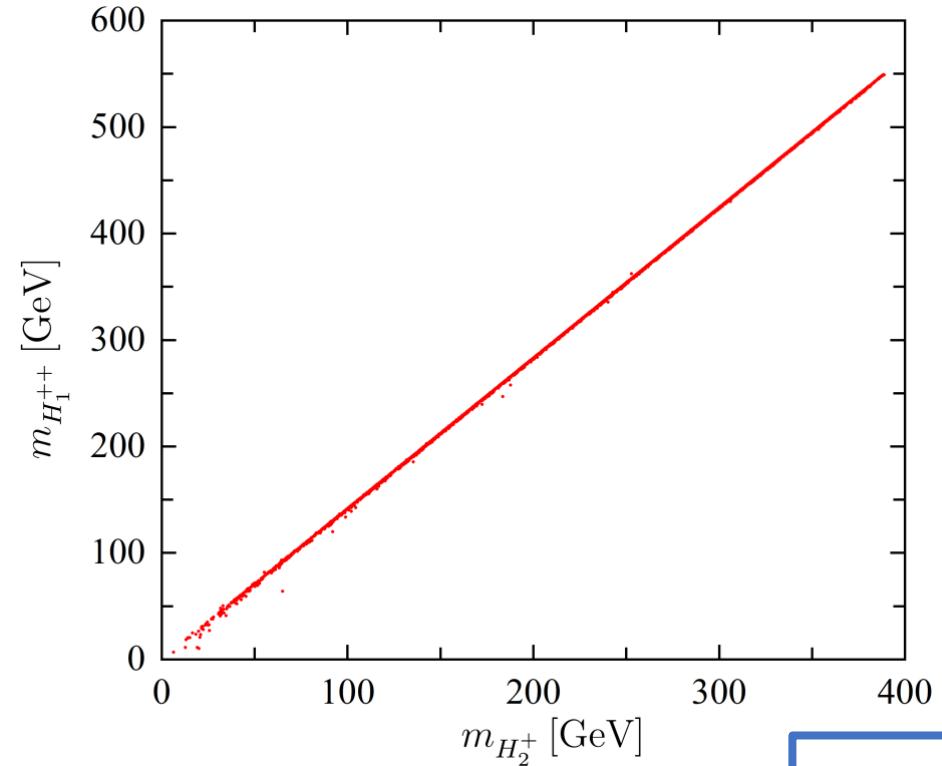


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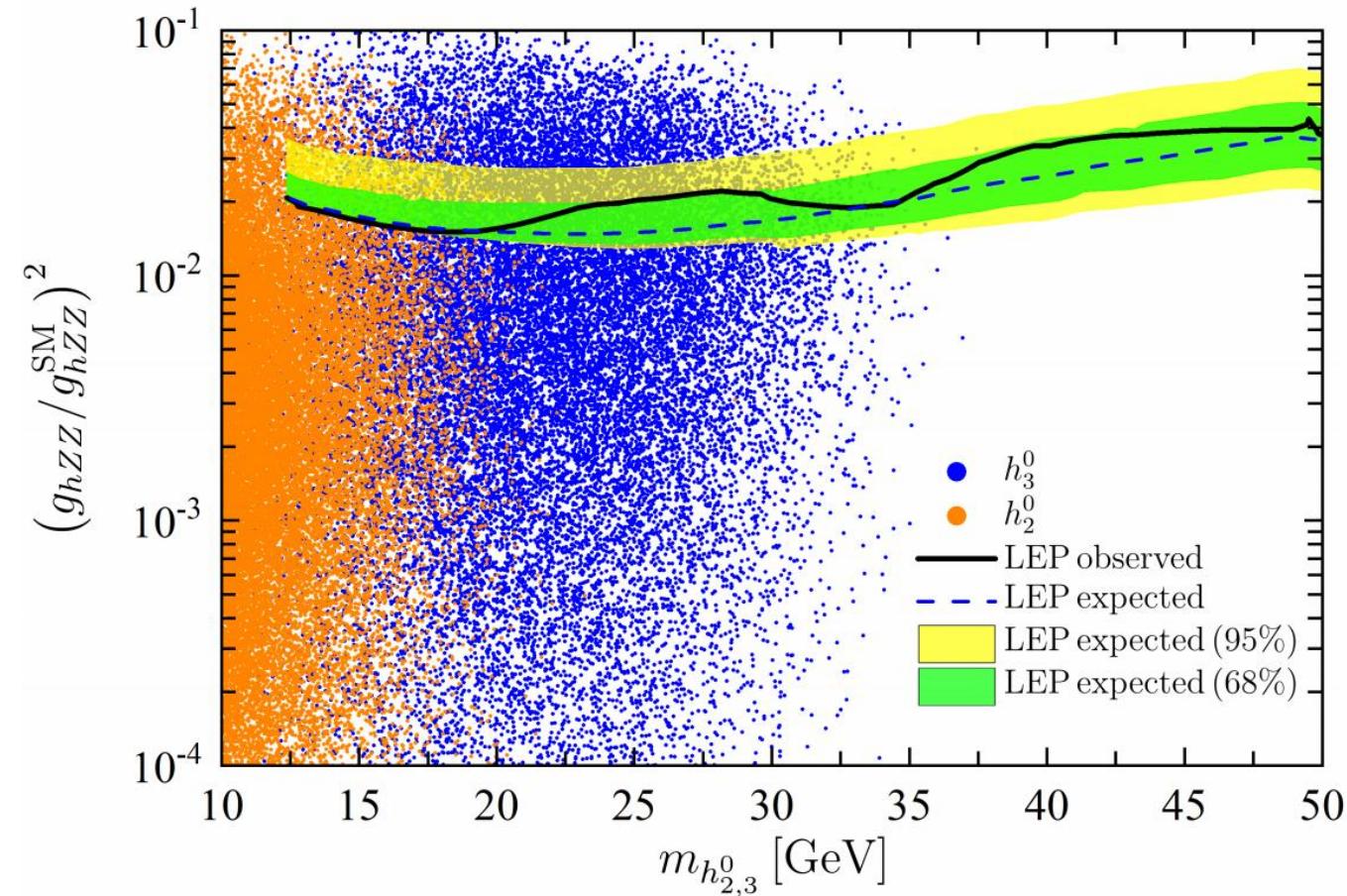
Analytical results confirmed!

# Numerical scan

Could such scalars have evaded detection?

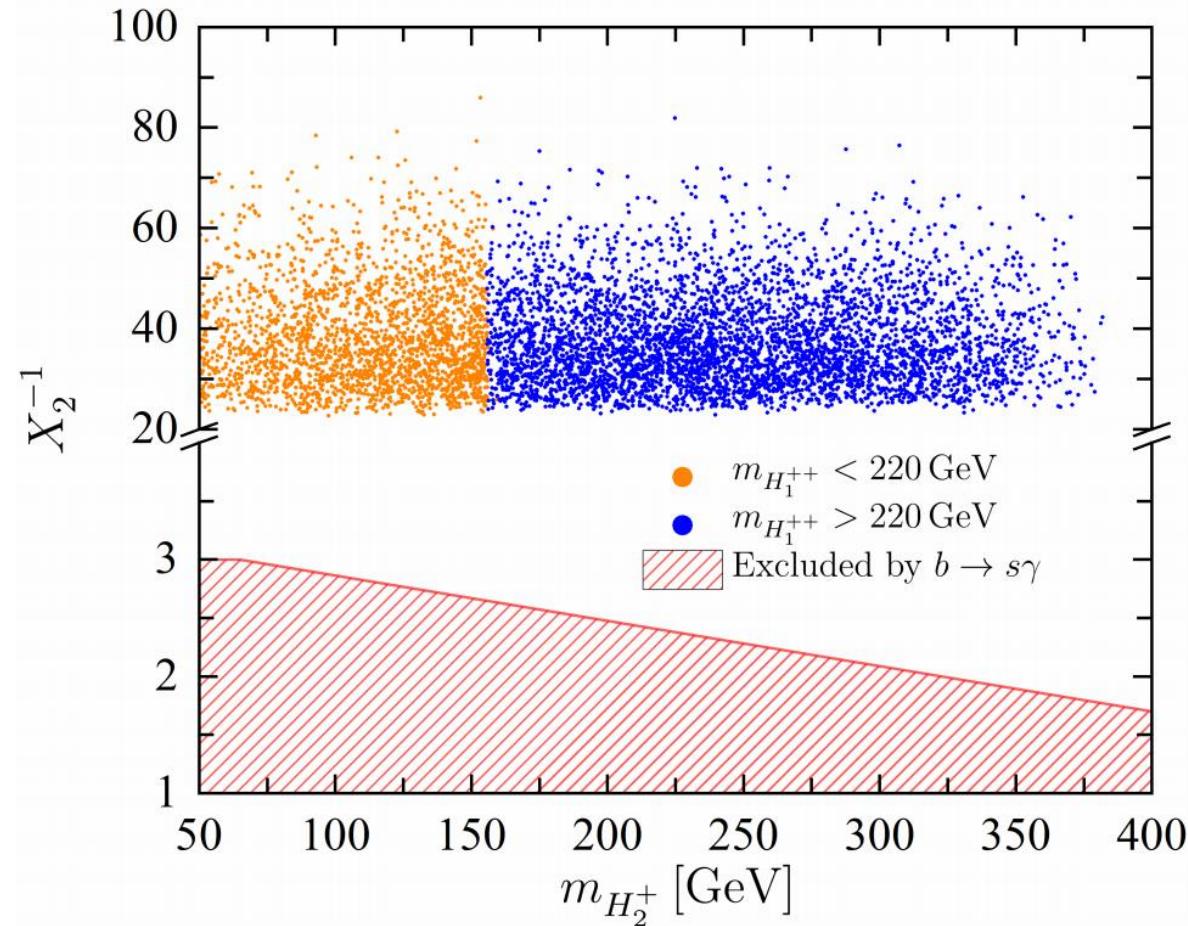
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$$\phi^+ = X_1 H_1^+ + X_2 H_2^+ + X_3 H_3^+$$

# Some remarks

BUT...

- Lepton-flavour-violation processes can impose a lower bound on the triplet's vev  $u$
- Can one simultaneously explain neutrino masses and leptonic CP violation via SCPV in the 2STM?
- Is this true for any number of triplets?

# Conclusions

Scalar triplet extensions of the Standard Model

Multi-Higgs  
scenario

Higgs-triplet model  
(HTM)

Two-scalar-triplet model  
(2STM)

Problem

Are neutral minima  
stable against charge  
breaking?

Do we have decoupling  
in the scalar mass  
spectrum?

Projects: UIDB/00777/2020, UIDP/00777/2020, UIDB/00618/2020, CERN/FISPAR/0004/2019, CERN/FISPAR/0014/2019  
PhD Grant: SFRH/BD/139165/2018

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Main take-  
home  
message

CB global minima  
can indeed coexist  
with neutral minima

When SCPV occurs,  
the 2STM has no  
decoupling limit

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