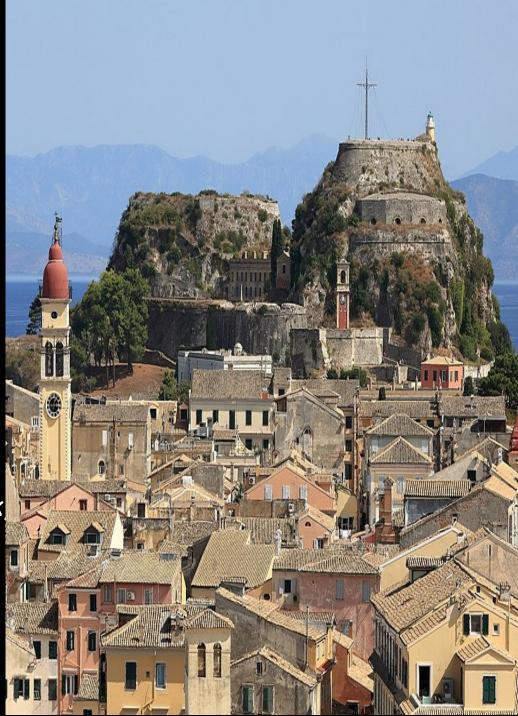
Theoretical aspects of Neutrino physics: Recent developments

A. Yu. Smirnov

Max-Planck Institut fur Kernphysik Heidelberg, Germany

Corfu, Greece, September 3, 2021



Outline:

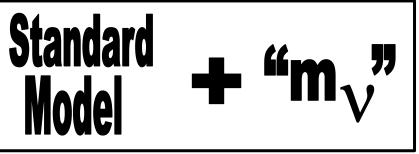
Neutrinos in the Standard model: $SM + m_v$

Neutrinos and BSM

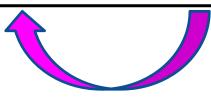
Anomalies and connections

Neutrino and new physics at the low energy scales Masses, mixing and flavor symmetries

Massive neutrinos and the Standard Model



Oscillations
Adiabatic conversion



Physics BSM responsible for m_{ν} can be introduced in such a way that feedback on SM is negligible



Á



Dirac mass

Majorana mass

 $\frac{1}{\Lambda}$ LL HH

 m_v (E, n, ..)

Effective mass

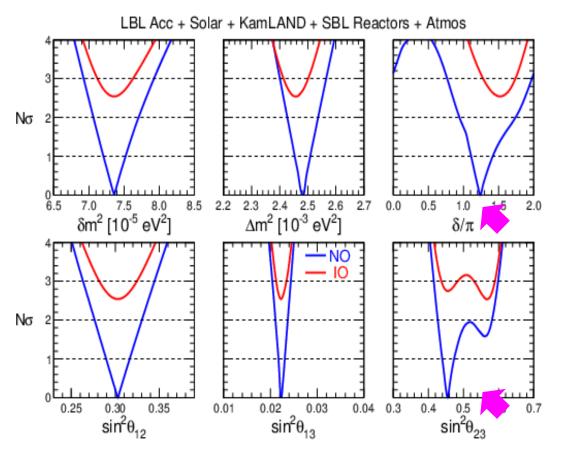
Y Lv_R H

D5 Weinberg operator

generated by interactions with medium, e.g. DM

Summary, Global fit

F Capozzi, et al 2107.00532 [hep-ph]



Data - more consistent, analysis - stable, agreed with results of NuFIT

Data agree with hard mass: no dependence on E, environment.
CPT -OK

For NO δ_{CP} - closer to π Deviation of 2-3 mixing from $\pi/4$ is smaller Preference of NO is less significant than before

Tensions inside the global?

Solar - KamLAND ∆m₂₁² - tension disappears

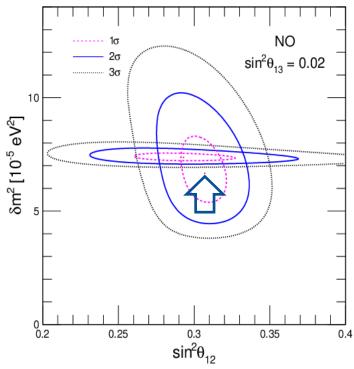
SK (also SNO+) observe the upturn of spectrum (SNO, SK)

The D-N asymmetry at SK is reduced $3.3\% \rightarrow 2.1\%$



Best fit value of Δm_{21}^2 from analysis of the solar neutrino data increased

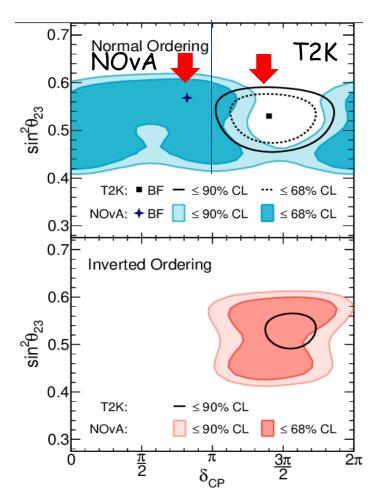
Discrepancy with KamLAND results reduced $2\sigma \rightarrow 1.2 \sigma$



1, 2, 3σ *C*L contours

F Capozzi, et al 2107.00532 [hep-ph]

NOvA - T2K tension or CP phase close to π ?



2108.08219 [hep-ex]

NOv*A*: $\delta_{CP} = 0.82\pi$

disfavors δ_{CP} = 1.5 π by 2 σ

See talk by A. De Roeck

NOvA-T2K difference can be related to different baselines and matter effects

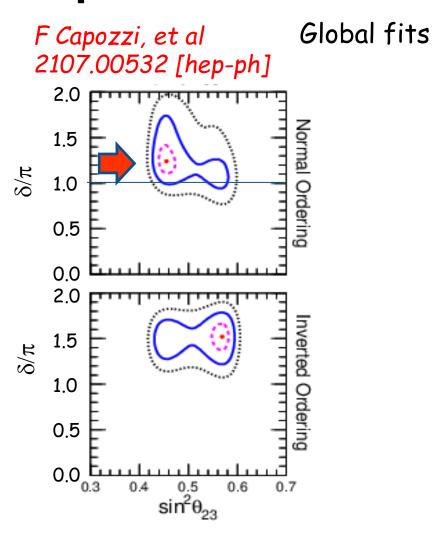
Reconcile with NSI or sterile neutrinos:

S. Chatterje, A. Palazzo, 2008.04161 [hep-ph], 2005.103338 [hep-ph]

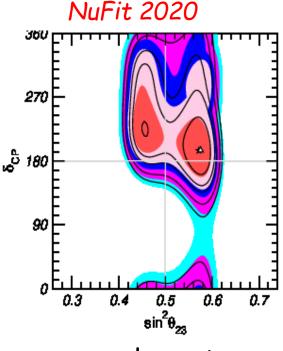
No tension in the case of inverted ordering

Global fit: $\delta_{CP} \rightarrow \pi$

... or CP-phase close to π ?



bad news for measurements of CP- asymmetry



even closer to $\boldsymbol{\pi}$

Why CP phase is large in quark sector and not in lepton sector?

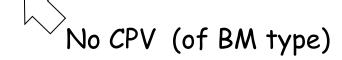
$$\delta_{CP} = \pi$$
?

B. Dasgupta, A Y.S., Nucl.Phys. B884 (2014) 357 1404.0272 [hep-ph]

Framework:

$$U_{PMNS} \sim V_{CKM}^+ U_X$$

If the only source of CP violation





$\sin\theta_{13} \sin \delta_{CP} = (-\cos \theta_{23}) \sin\theta_{13}^{q} \sin\delta_{q}$

$$\delta_{CP} \sim \lambda^3/s_{13} \sim \lambda^2 \sim 0.046$$

$$\lambda^3$$
 $\delta_q = 1.2 + /- 0.08 \text{ rad}$ $\sin \delta_q = 0.93$

$$\delta_{CP} \sim -\delta \text{ or } \pi + \delta$$

where
$$\delta$$
 = (s_{13}^q/s_{13}) $c_{23} \sin \delta_q$

Leptonic CP is small because the leptonic 1-3 mixing is large

To the theory of neutrino propagation and oscillations

Surprisingly, big activity recently

Challenging theory:

reformulations, reinterpretations, improvements, corrections, entanglement, quantumness

Changing evolution equation:

Lorentz invariance violation, metric change, Equivalence principle violation

Oscillations at extreme conditions:

high - low densities, high - low energies, dense neutrino background → collective oscillations

New interactions: NSI, long range forces, interaction with DM Searches for new neutrino states, sterile neutrinos

Coherence and de-coherence

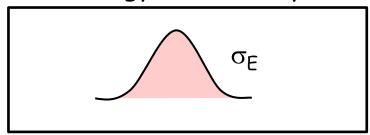




x -t space: separation of wave packets



E-p space: integration over the energy uncertainty



results in suppression of interference

L. Stodolsky

Equivalence due to

$$\sigma_{x} \sim 1/\sigma_{E}$$

Coherence length:

$$L_{coh} = \frac{\sigma_x}{\Delta v_q}$$

 Δv_a -difference of group velocities

$$v_{qi} = dH_i / dp$$

In vacuum: $\Delta v_a = \Delta m^2/2E^2$

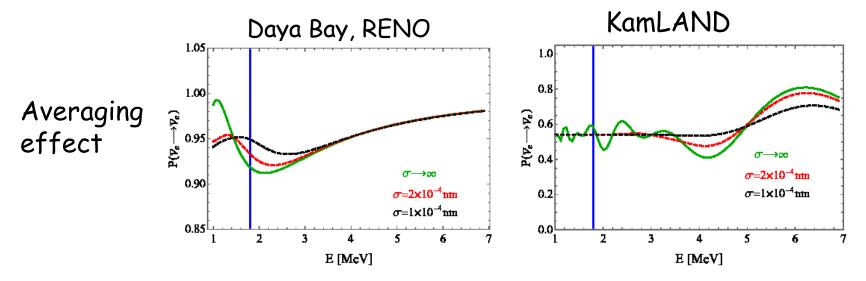


$$L_{coh} = \sigma_x \frac{2E^2}{\Delta m^2}$$

Decoherence of reactor neutrinos

A de Gouvea, V De Romeri, C.A. Termes, 2104.05806 [hep-ph]

Bound on size of the WP



Absence of decoherence (averaging) effects means

$$L \ll L_{coh}$$
 \Rightarrow $\sigma_x > L \frac{\Delta m^2}{2E^2}$

Analysis of data: $\sigma_{x} > 2.1 \times 10^{-11}$ cm

Expected: $\sigma_{x} \sim 10^{-9} \text{ cm}$

Coherence in matter

Matter changes group velocities $v_i \rightarrow L_{coh}$

S.P. Mikheyev, A.Y.S.

Y. P. Porto-Silva , A Y S 2103.10149 [hep-ph]

Constant density

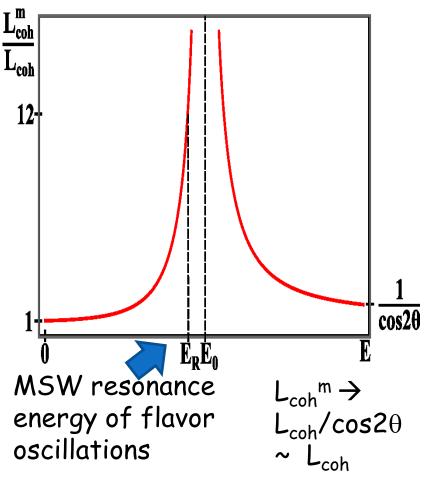
At certain E_0 $L_{coh} \rightarrow infty$

corresponds to equality of the group velocities:

$$\Delta v = v_{2m} - v_{1m} = 0$$

→ no separation of the wave packets

 E_0 coincides with the MSW resonance in oscillations of mass states

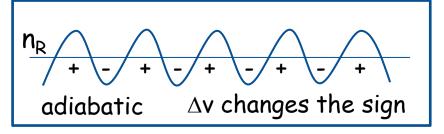


At high densities the coherence length as in vacuum

Infinite coherence

Y. P.. Porto-Silva , A Y S 2103.10149 [hep-ph]

n Periodic modulations of density

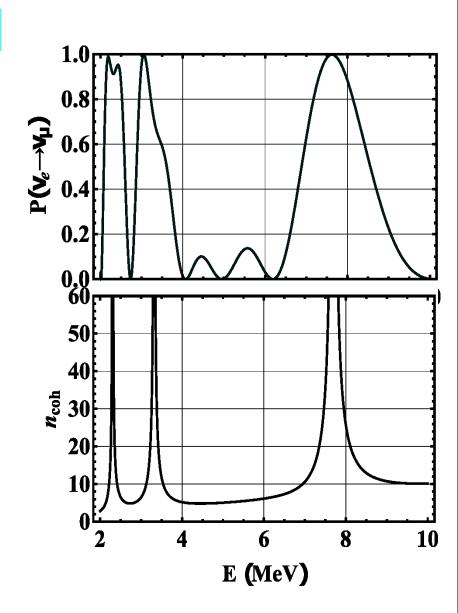


n Castle wall profile



Number of coherence periods Several E_{inf} associated with parametric resonances

Applications to SN neutrino collective oscillations



Neutrino anomalies Status & implications

```
RAA: Reactor Antineutrino Anomaly
```

Oscillatory pictures - fluctuations

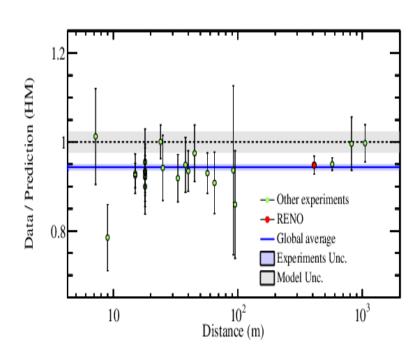
NEUTRINO-4

Gallium anomaly (Cross sections - reduced significance)

LSND

MiniBooNE

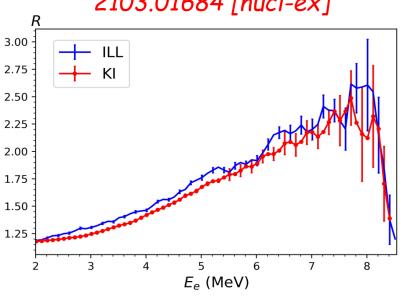
Reactor Antineutrino Anomaly and its re-evaluation



The v_e event rates as a function of the distance from a reactor, relative to the Huber-Mueller prediction based on ILL spectra.

IBD yield/HM: 0.941 ± 0.019

V. Kopeikin, et al. 2103.01684 [nucl-ex]



KI - Kurchatov institute new measurements of the ratio between 235U and 239Pu spectra

Ratio of cumulative spectra $R = S_5 / S_9$

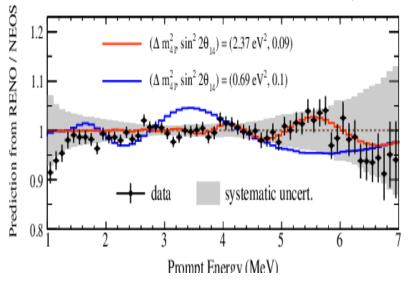
$$R(ILL) = 0.959 R(KI)$$

→ explains anomaly

Oscillations or fluctuations?

Oscillatory curve with two free parameters always gives better fit of fluctuating data points than constant

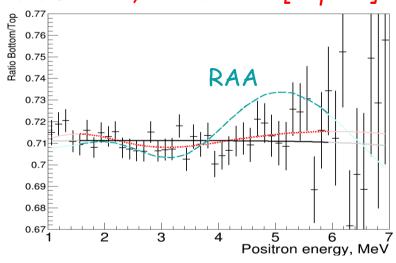
NEOS Z. Atif et al 2011.00896 [hep-ex]



Points: NEOS observed prompt spectrum over prediction for NEOS using RENO spectrum

 $\Delta m_{14}^2 = 2.37 \text{ eV}^2$, $\sin^2 2\theta_{14} = 0.09$

DANSS M. Danilov, 2012.10255 [hep-ex]



RAA (dotted): $\Delta m_{14}^2 = 2.4 \text{ eV}^2$

 $\Delta m_{14}^2 = 1.3 \text{ eV}^2, \sin^2 2\theta_{14} = 0.02$

Now different ...

Neutrino-4

Energy resolution of the detector, more reliable Monte Carlo simulation:

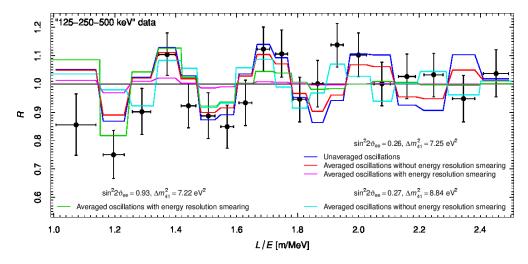
→ Significance reduces:

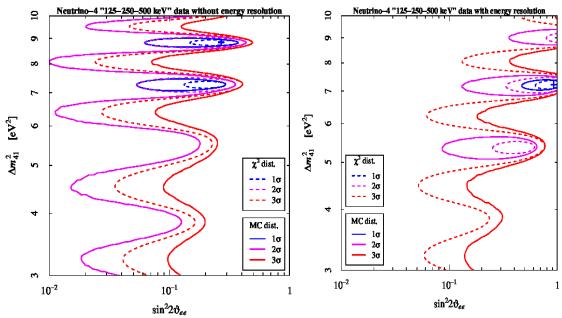
$$3\sigma \rightarrow 2.2\sigma$$

→ b.f. point moves to maximal mixing

Strong tension with the KATRIN, PROSPECT, STEREO, solar ve bounds

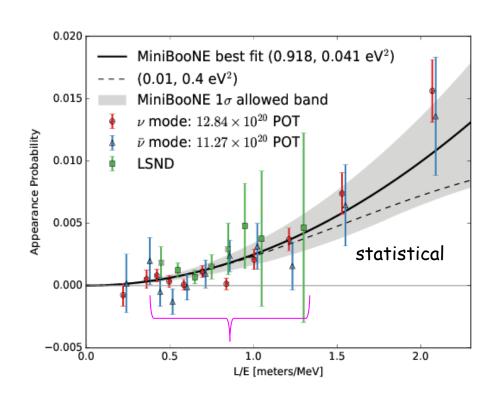
C. Giunti, et al, P.L. B816 (2021) 136214 2101.06785 [hep-ph]





MiniBooNE and LSND

A.A. Aguilar-Arevalo et al Phys.Rev.Lett. 121 (2018) no.22, 221801) 1805.12028 [hep-ex] |



L/E dependences of QE events excess in LSND and MiniBooNE

No good LSND -MB agreement in the overlapping region

b.f. line- for
$$\sin^2 2\theta = 0.918$$
excluded

1 σ line - for $\sin^2 2\theta = 0.01$
strongly
disfavored

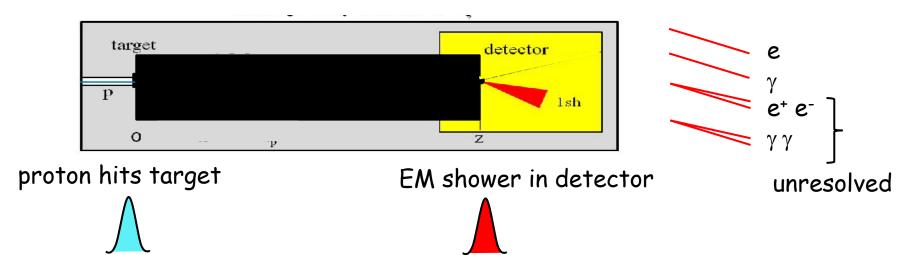
Oscillation interpretation nearly excluded by disappearance data

No oscillatory dependence: Non-oscillatory explanations are possible

Many alternative scenarios have been proposed

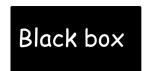
Effective theory of the MiniBooNE excess

V. Brdar, O. Fisher, A.S. 2007.14411 hep-ph



p bunches hits and appearance of showers are time correlated

Time delay is consistent with v = c, i.e. propagation of neutrinos Put upper bounds on masses of new particles excludes some scenarios



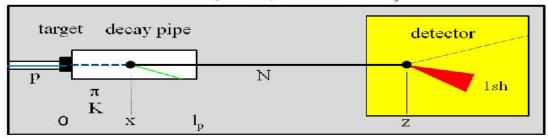
Production via Propagation
Decays
Un-scattering
of new particles

mixing with up-scattering of

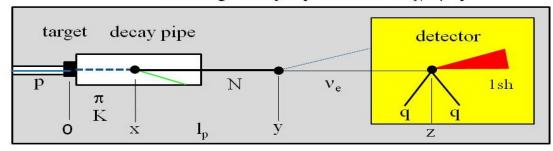
usual neutrinos

Opening black box

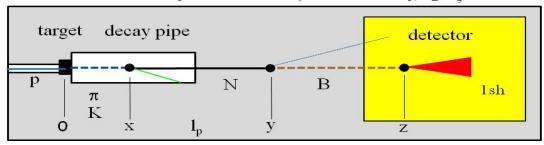
Mixing-Decay scenario MD2



Mixing-Decay ve scenario MNDvUe



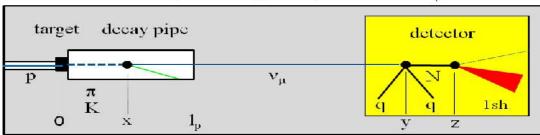
Mixing-Double Decay scenario, $M_N D_B D_{\xi}$



Opening black box

Upscattering-Decay scenario UD_{ξ}

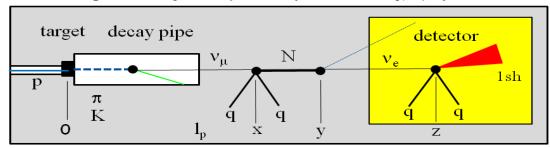
 $U_N D_\xi$



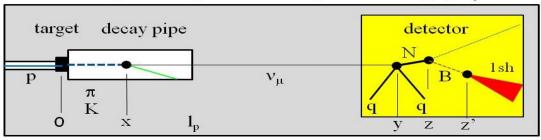
a.

C.

Upscattering-Decay into v_e scenario, $U_N D_v U_e$



Upscattering-Double Decay scenario $U_ND_BD_\xi$



b.

Predicting events in other experiments. Bounds

Excess of 1 sh events at MB

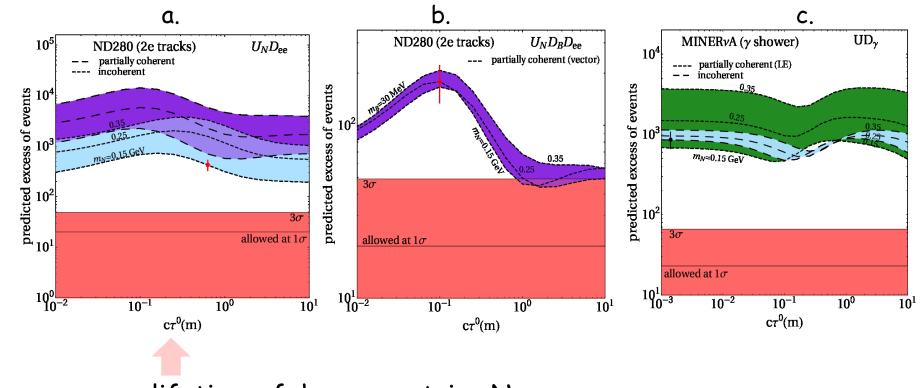


scenario



Number of events of a given type at other experiments

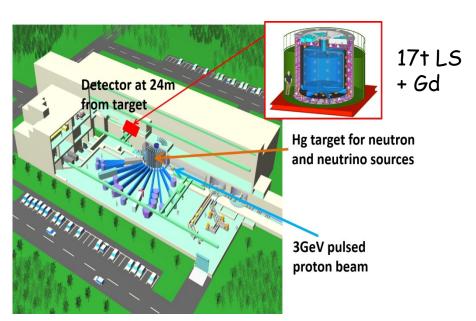
Similar setups: T2K ND280, MINERVA, NOVA, NOMAD Confront experimental experimental bounds



lifetime of heavy neutrino N

JSNS²: Ultimate tests?

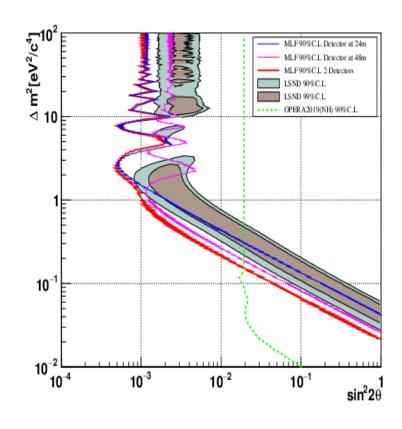
Ajimura, S. et al. 2012.10807 [hep-ex] 2104.13169 [physics.ins-det]



J-PARC Sterile Neutrino Search at J-PARC Spallation Neutron Source (at Material Life Facility MLF)

Repeating LSND: μ -decay at rest, searches for

$$\overline{v}_{\mu}$$
 - \overline{v}_{e} oscillations



Sensitivity of JSNS² and upgrade JSNS²-II: second detector at 48m

JSNS² operates now

ICARUS at Fermilab detects first events

Neutrinos and non-neutrino anomalies, connections

Neutrinos and DM

Is the (hot) component of the DM

Mechanism of generation of small neutrino masses is related to DM



RH neutrinos as DM particles

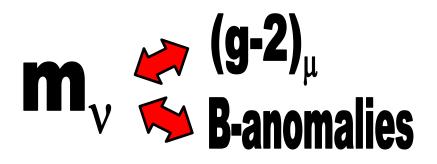
Neutrino portal connects

DM and neutrinos

DM particles participate (appear in loops) in generation of neutrino mass

The same symmetry is responsible for smallness of neutrino mass and stability of the DM

Refraction on DM produces the effective neutrino mass

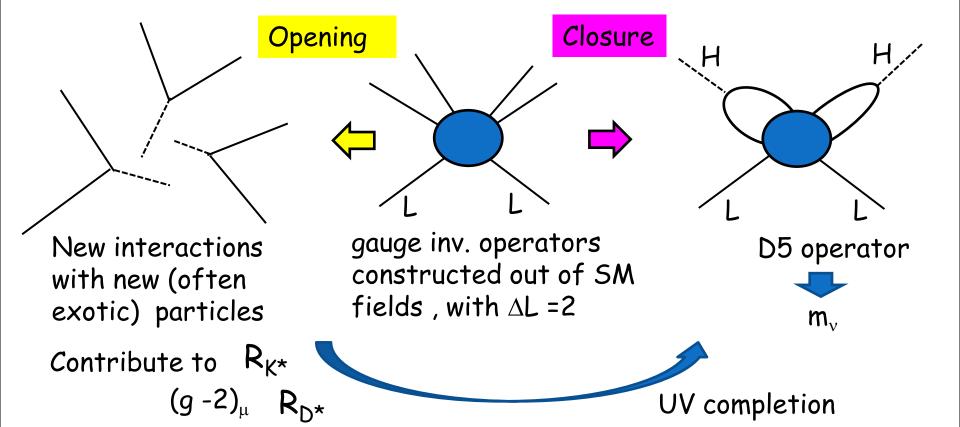


Model building without model builder

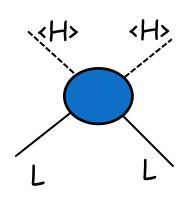
 R_{K^*} R_{D^*}

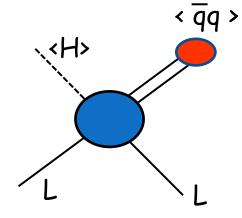
Lepton non-universality

Systematic way to search for connection is to use effective FT



In addition





Substitute Higgs VEV by quark chiral condensate

A. Babic at al, 1911.12189

following original 5 D Thomas and R.-M. Xu PLB 284,341 (1992)

D5 operator

Generated by

$$O_7 = \frac{g}{\Lambda^3} L^c L H \overline{[(Q u_R) (d_R Q)]}$$

$$m_v = g \langle H \rangle \frac{\langle \overline{q}q \rangle}{\Lambda^3}$$

$$\langle qq \rangle = 300 \text{ MeV}$$
 $\Lambda = \text{few TeV}$

$$\Lambda = few TeV$$

Issues

How D5 operator is suppressed UV completion

Neutrinos and New physics at the low energy scales

... or second desert?

1020

1010

Why not worry about the low scale desert?

14 - 17 orders Standard masses

of the SM particles

High scale desert

10°

Another hierarchy problem?

BSM origins?

10-20

10-10

Neutrino mass - physics at low scales

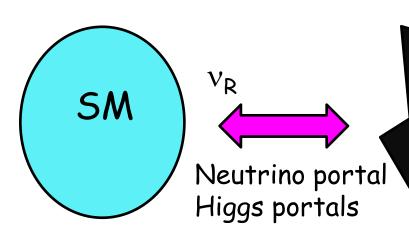
Low scale desert > 40 orders

10-30

proton decay rate $\Gamma_{\rm p}$

10-40

Neutrinos and interactions with light dark sector



Interactions affect oscillations

Neutrinos provide probes of the light dark sector

```
Dark sector

Scalars, 
Fermions, 
Wector bosons, 
Dark photons

Sterile

Axions,

Majorons,

DM
```

The simplest example

Scalar interaction

$$L = g \overline{v}_L \chi \phi + h. c.$$

where χ - fermion, ϕ - scalar

q - effective coupling pheno bound

 $q < 10^{-7}$

L can be generated via the RH neutrino portal

Rich phenomenology

Long range forces

M.Kawasaki

H. Murayama

T. Yanagida,

1991

Refraction

Effective m,

Bound neutrino systems

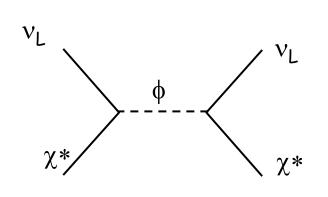
elastic forward scattering, $q^2 = 0$

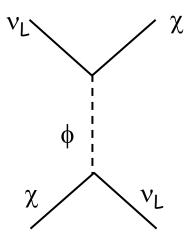
Potential $V \sim g^2 / m_{med}^2$

do not disappear when g, $m_{med} \rightarrow 0$ while inelastic interactions ~ g^2/q_{min}^2 May have important cosmological and astrophysical consequences

Resonance neutrino refraction

Neutrino scattering on background fermions χ with scalar ϕ mediator





Effective potential

$$V^{B} = \frac{1}{2}V_{0}\left[\frac{(1-\epsilon)(y-1)}{(y-1)^{2}+\xi^{2}} + \frac{1+\epsilon}{y+1}\right]$$

$$V_0 = \frac{g^2}{2m_\phi^2} (n_\chi + \overline{n}_\chi)$$

$$y = E/E_R$$

$$E_R = m_\phi^2/2m_\chi$$

Resonance: y = 1 corresponds to $s = m_0^2$

A.S., V.Valera, 2106.13829 [hep-ph] in SM: due to Z, W

C. Lunardini, A.S.

Asymmetry of bgr:

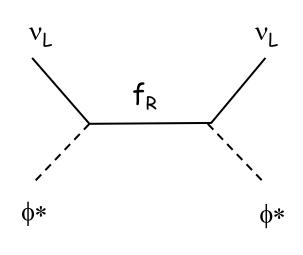
$$\varepsilon = (n_{\chi} - \overline{n}_{\chi})/(n_{\chi} + \overline{n}_{\chi})$$

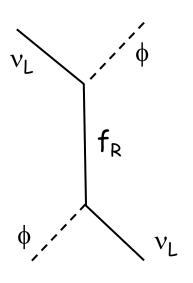
 n_{χ} and $\overline{n_{\chi}}$ – the number densities of χ and $\chi*$

Refraction index $n_{ref} = 1 + V/p$

width of resonance $\Gamma = \frac{g^2}{4\pi} m_{\phi}$ $\xi = \Gamma / E_{R}$

Neutrino refraction on scalar DM





$$V_s \sim \frac{(s - m_f^2) \overline{n}}{(s - m_f^2)^2 + s \Gamma^2}$$

$$V_u \sim \frac{n}{u - m_f^2}$$

$$\Gamma = \frac{g^2}{32\pi} \, \mathrm{m_f}$$

Resonance: $s = m_f^2 \rightarrow E_R = m_f^2/2m_\phi$

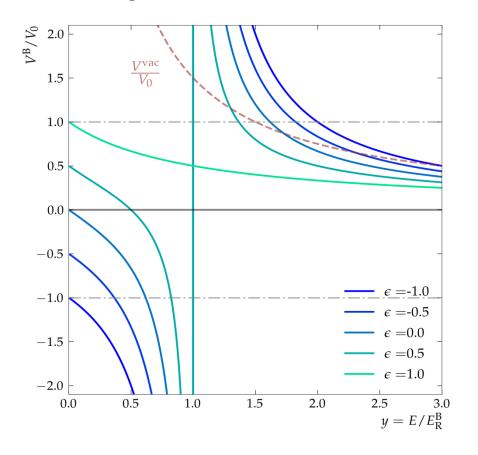
S. F Ge and H Murayama, 1904.02518 [hep-ph]

Ki-Yong Choi, Eung Jin Chun, Jongkuk Kim, 1909.10478 [hep-ph] 2012.09474 [hep-ph]

Neutrino scattering on DM particles ϕ (target) with f_R - mediator

n and \overline{n} - the number densities of ϕ and $\phi*$

Background potential



Potential depends on energy

$$V^{B} = V_{0} \frac{y - \varepsilon}{y^{2} - 1}$$

| | VB/V _O |
|---------------------|-------------------|
| y = 0 | 3 |
| $y \rightarrow inf$ | 1 /y |
| ε = 1 | 1/(y + 1) |
| ε = 0 | $y/(y^2 - 1)$ |
| ε = - 1 | 1/(y - 1) |

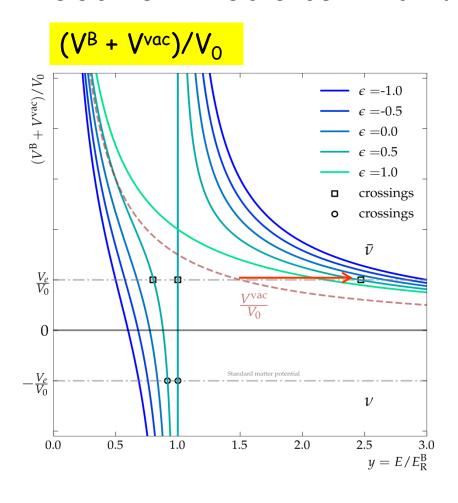
$$V^{\text{vac}} = \Delta m^2 / 2E$$
 = V_R^{vac} / y

$$V_R^{\text{vac}} = \Delta m^2 / 2E_R$$

Relative contribution of the background and vacuum terms

$$r = V_0/V_R^{\text{vac}}$$

Effective kinetic term and MSW resonances



A.S. V. Valera, 2106.13829 [hep-ph]

 $V_e = \sqrt{2}G_F n_e$ - usual matter potential

Boxes - MSW resonances

shift of the usual MSW resonance

2 new resonances in v-channel 2 new resonances in \overline{v} -channel

V^B included into effective kinetic term

Effective mass squared difference

 $\Delta m_{eff}^2 = 2E(V^{vac} + V^B) = \Delta m^2 (1 + V^B/V^{vac})$

Effective mass squared splitting

 $|\Delta m_{eff}|^2/\Delta m^2$ as function of y for different $r = V_0/V_R^{vac}$

The oscillation phase:

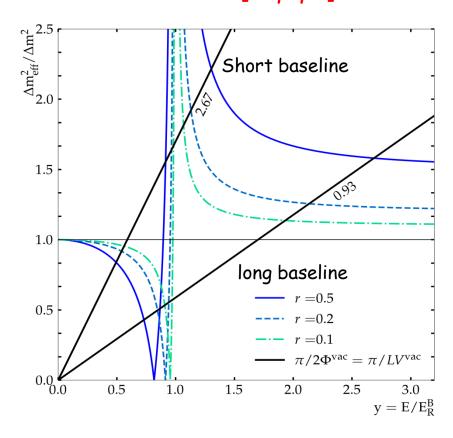
$$\Phi = \frac{\Delta m_{eff}^2}{2E} L = \frac{\Delta m_{eff}^2}{\Delta m^2} \Phi^{vac}$$

Straight lines: $1/\Phi^{\text{vac}}$

Their crossing with $\Delta m_{eff}^2/\Delta m^2$ corresponds to Φ = 1

Above the lines Φ (y) > 1 and the oscillation effect is large

A.S., V.Valera, 2106.13829 [hep-ph]



With increase of r the y-region of large phase expands

Phase factor & MiniBooNE excess

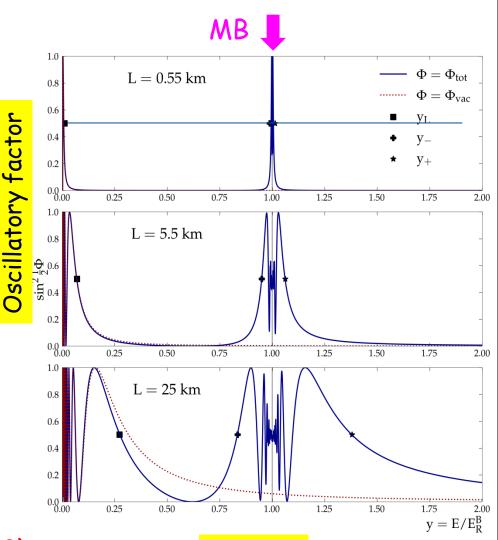
Oscillation probability

 $P = \sin^2 2\theta \sin^2 \Phi/2$

Signatures:

- dip,
- bump (after averaging fast oscillations),
- tail

MiniBooNE excess is due to bump for relatively small L, apart from resonance region 200 -400 MeV the phase and oscillations effect are small.



J. Asaadi et al., PRD 97, 7, 2470, (2018)

energy

Excluding MiniBooNE explanation

A.S., V.Valera, 2106.13829 [hep-ph]

Based on dependence on energy of

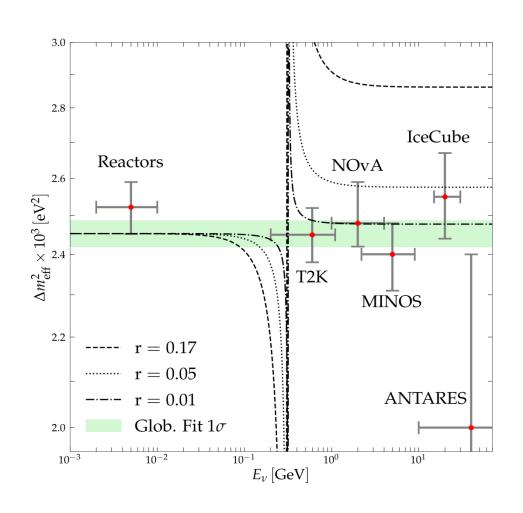
$$\Delta m_{eff}^2$$
 (E)

It is expected that

$$\Delta m_{eff}^2$$
 (E \ll E_R) = Δm^2
 Δm_{eff}^2 (E \gg E_R) = $r \Delta m^2$

MB explanation requires r > 1.6

Data are consistent with Δm_{eff}^2 = const and give bound r < 0.01



Dependence of the

Neutrino oscillations and neutrino mass

Above resonance $E \gg E_R$ (y \gg 1) the potential

C .Lunardini, A.S. Ki-Yong Choi, Eung Jin Chun, Jongkuk Kim, 2012.09474 [hep-ph],

has the same behaviour as the kinetic (mass) term $\Delta m^2/2E$

- general dependence at large E

It is proof of the existence of 1/E term in the Hamiltonian of the evolution equation that allowed to conclude: oscillations imply the mass (coupling of neutrinos with VEV) - MAY IMPLY

The conditions for 1/E dependence from scattering:

Light mediator:

Light target:

$$m_{tar} \ll E$$

Neutrino oscillations without neutrino mass

Effective neutrino mass due to interactions

$$m_{eff}^2 \sim \frac{g^2 n_{\chi}}{4 m_{\chi}}$$

Up to now the condition for 1/E dependence (mass) has been checked down to 0.1 MeV, therefore

Problem?

Due to dependence on energy and number density of scatterers $m_{\rm eff}$ can be different in different space -time points, in contrast to the standard mass due to coupling to VEV (does not depend on z)

$$m_{eff}(z) \sim \sqrt{n(z)}$$
 $n(z) = n_0 (1 + z)^3$

The effective mass increased in the past in contrast to standard generated by VEV.

Dependence of the effective mass on density and energy

$$m_{eff}(z) \sim [\xi (1+z)^3]^{1/2} m_{eff}(loc)$$

 $1/\xi \sim 10^5\,$ - local (near the Earth) over-density of the background

In the epoch of matter-radiation equality, z = 1000, DM should already be formed and structures start to grow.

For
$$m_{eff}$$
 (loc) = 0.05 eV and $1/\xi \sim 10^5$ \implies m_{eff} (1000) ~ 5 eV

- violates cosmological bound on the sum of neutrino masses

For not very small $E_{\rm R}$ one should take into account dependence (decrease) of $m_{\rm eff}$ (loc) with neutrino energy

$$\Delta m_{eff}^{2}(E) \sim \frac{y(y-\epsilon)}{y^2-1} \Delta m^2$$
 $y = E/E_R$

and for relic neutrinos m_{eff} (loc) can be very small

Avoiding cosmological bound

$$|\Delta m_{eff}|^2$$

$$\epsilon \neq 0$$
 existing observations
$$E_R$$

$$E$$

$$E_R = m_\phi^2/2m_\chi$$

For $\varepsilon = 0$ decrease of mass with E is even stronger

Below resonance:
$$m_{eff}^2 (\langle E_R \rangle) = m_{eff}^2 (\rangle E_R) \frac{E}{E_R} = m^2 \frac{E}{E_R}$$

Suppose $E_R = 0.01 \text{ MeV}$

For relic v, E = 10^{-4} eV , m_{eff} < $5~10^{-6}$ eV CMB bound is satisfied

For KATRIN: E = 1 eV: $m_{eff} < 2 \cdot 10^{-4} \text{ eV}$ - not measurable

Neutrino bound states and systems

Generic consequence of long range scalar forces

M. Markov, Phys.Lett. 10,122 (1964)

Neutrino superstars: Massive neutrinos + gravity, used analogy with neutron stars, $m_v = MeV \rightarrow M = 10^6 M_{sun}$, $R = 10^{12}$ cm For $m_v = 0.05$ eV: $M = 4 \times 10^{20}$ M_{sun} , $R = 5 \times 10^{26}$ cm

R. D. Viollier et al, Phys.Lett. B306, 79 (1993) ,....

Gravity, m_v = (10 - 100) keV: M = (10⁸ - 10¹⁰) M_{sun} , R = (10¹⁴ - 10¹⁶) cm essentially, warm DM

G. J. Stephenson et al, Int. J. Mod. Phys. A13, 2765 (1998) ...

Scalar forces, $m_v = 13$ eV, motivated by 3H exp. anomaly, negative m^2

Equations of motion \rightarrow Equations for final configurations \rightarrow Density profiles

Formation of clouds in the Universe - as phase transition.

 $M = (10^8 - 10^{10}) M_{sun}$, $R = 10^{13} cm$, central density: $10^{15} cm^{-3}$

Neutrino clouds revisited

A.Y.S, and Xun-Jie Xu, to appear

The latest bounds on m, and g are used

Final configuration of clouds found

Non-relativistic case: Lane - Emden equations for scalar forces (essentially eq of hydrostatic equilibrium)

Generalized to relativistic case

Formation in analogy to formation of DM halos

Fragmentation \rightarrow Virialization \rightarrow Cooling

Neutrino structure of the Universe: clouds-voids

Applications for detection of relic neutrinos

Characteristics of neutrino clouds

A.Y.S, and Xun-Jie Xu, to appear

Global characteristics for different total numbers of neutrinos N

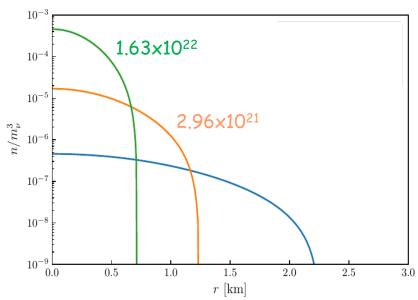
| Ν | 2.96×10 ²¹ | 1.63×10 ²² | 5.96×10 ²² | 9.36×10 ²³ | 2.34×10 ²⁴ |
|-----------------------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| m_{ν}^*/m_{ν} | 0.991 | 0.922 | 0.688 | 0.060 | 0.014 |
| R, km | 1.25 | 0.75 | 0.62 | 1.46 | 2.41 |
| n ⁰ , cm ⁻³ | 2.0×10 ⁶ | 4.9×10^7 | 3.7×10^{8} | 1.5×10^{8} | 6.1×10^7 |

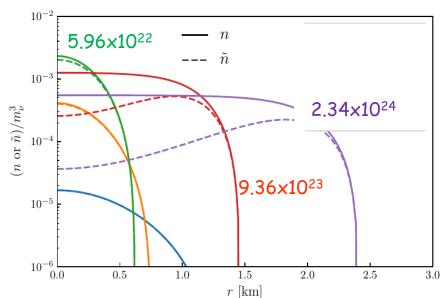
$$y = 10^{-7}$$

 $m_{v}/= 0.1 \text{ eV}$

 $m_v^* = m_v + V$ - the effective neutrino mass in medium, n^0 - central density

Density and effective density distributions in the clouds for different N





Neutrino masses, mixing and flavor symmetries

Modular symmetries as flavor symmetries

F. Feruglio

Gui-Jun Ding, S.F. King, Xiang-Gan Liu, S. Petcov, A V. Titov, M. Tanimoto, T. Nomura, H. Okada, T Kobayashi, O.Popov Y. Shimizu, P Novichkov, J. Penedo, T Osaka, A. Romanino, I. De Medeiros Varzielas, X Wang, S. Zhou ...

Nice introduction by J. Penedo

Motivated by string theory

Symmetry related to (orbifold) compactification of extra dimensions and primary realized on the moduli fields τ which describe geometry of the compactified space.

Hope was to

Reduce number of parameters, more predictive -? Connect masses and mixing -?

Modular symmetries

For single modulus field τ the modular transformation γ

$$\tau \rightarrow \gamma \tau = \frac{a \tau + b}{c \tau + d}$$

The 2x2 matrices of integer numbers $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with det. ad - bc = 1 form the group Γ = SL(2, Z) [special, linear, integer ...]

Modular group Γ_N is finite subgroup of Γ , quotient group of the level N: $\Gamma_N = \Gamma / \Gamma(N)$

Isomorphic to the groups considered previously ..

Yukawa couplings are modular forms

- holomorphic functions of modulus field $\boldsymbol{\tau}$
- Form multiplets of the group Γ_{N} and transform as superfields:

$$Y_i(\tau) \rightarrow Y_i(\gamma \tau) = (c\tau + d)^k \rho(\gamma)_{ij} Y_j(\tau)$$

k is the weight of multiplet $\rho(\gamma)$ is the representation of γ element of the group Γ_N dimension of the multiplet is determined by the level N and weight k

Dependence $Y_i(\tau)$ is determined by transformation properties

Invariant Lagrangians

Invariance of terms of the superpotential

constant $\alpha \ Y \ \varphi_1 \ \varphi_2 \ \varphi_3$ requires

$$\rho_1 \times \rho_2 \times \rho_3 \times \rho_y = I$$

$$\Sigma_1 k_1 + k_2 = 0$$
 for weights

The latter condition acts to some extent as Froggatt-Nielsen symmetry it gives additional restrictions, forbids some term >texture zeros

Comparing with usual approach with flavons

$$Y_1(\langle \tau \rangle)$$
, $Y_2(\langle \tau \rangle)$... $Y_n(\langle \tau \rangle)$

For a given N,k, r - known functions of the same VEV

$$\langle \chi_1 \rangle$$
, $\langle \chi_2 \rangle$, ..., $\langle \chi_n \rangle$

Depend on parameters of potential not controlled by symmetry, independent for different representations

Can do the same

From Model building to Symmetry building

Minimal simpest versions do not reproduce data well.

Generic predictions of masses: weak hierarchy, often quasi-degenerate, Majorana CP-phases

More parameters needed Several moduli, flavons,

Different representations of the same dimension, fine tuning of corresponding couplings

Different ways of construction of finite groups

In the end: # parameters is comparable to # of observables

Use modular symmetry in wrong way?

CKM and the dark sector physics





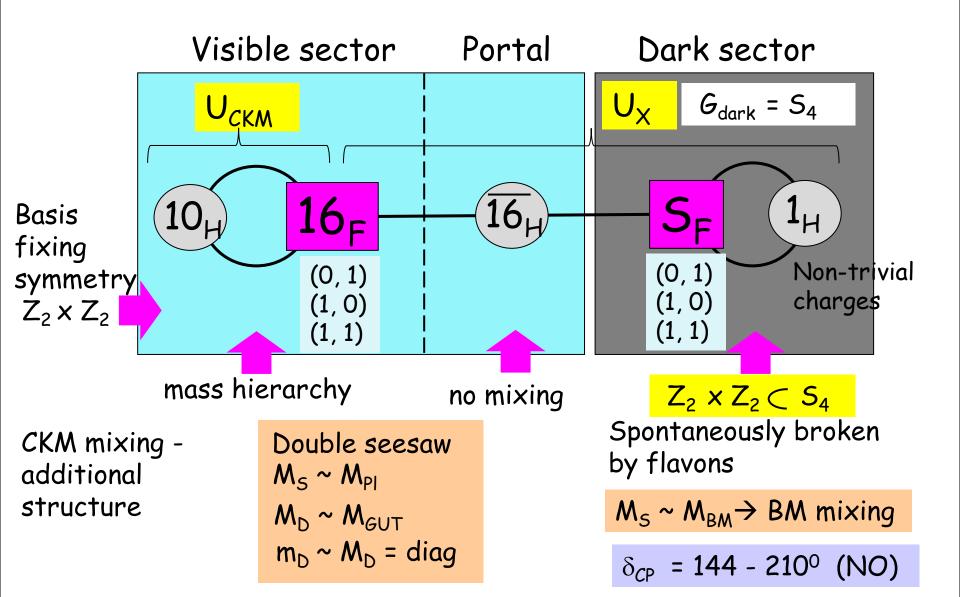
Common sector for quarks and leptons. Implies

$$m_l \sim m_d$$
 $m_v^D \sim m_u$
Q - L unification, GUT

CKM physics: hierarchy of masses and mixings, relations between masses and mixing From the dark sector coupled to neutrinos. Responsible for large neutrino mixing smallness of neutrino mass

may have special symmetries Modular symmetries? which lead to BM or TBM mixing

An SO(10) GUT with $G_{dark} = S_4^{\times un-Jie \times u}$, A.S



Conclusions

 $3 \ v$ - framework works fine, anomalies and tensions found in different oscillation experiments lose sigmas, in particular, the case of sterile neutrino become weaker.

Neutrino interactions with light dark sector -rich phenomenology

- resonance refraction at low energies
- possibility to substitute usual neutrino mass by interactions with medium
 - bound neutrino systems...

Modular symmetries - promising, but in complexity - became comparable to usual symmetries: simplest versions do not work: can fixed by additional parameters, assumptions now via symmetry building instead of model building. Using MS in the wrong way?