Sterile neutrinos with inverse-seesaw and Abelian flavour symmetries

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Based on work in collaboration with **R.G. Felipe** & **F.R. Joaquim**

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Inverse-seesaw (ISS)

$\text{ISS}(n_R, n_s)$ Mohapatra; Mohapatra & Valle '86; Gonzalez-Garcia & Valle '89

- **Sterile neutrino fields:** $\nu_{Ri}\ (i=1,...,n_R),\ s_i\ (i=1,...,n_s)$

$(3+n_R+n_s)\times(3+n_R+n_s)$

$$-\mathcal{L}^{\text{ISS}}_{\text{mass}} = \overline{e_L}\,\mathbf{M}_\ell\, e_R + \overline{\nu_L}\,\mathbf{M}_D \nu_R + \overline{\nu_R}\,\mathbf{M}_R s + \frac{1}{2}\overline{s^c}\,\mathbf{M}_s s + \text{H.c.} \Rightarrow \boldsymbol{\mathcal{M}} = \begin{pmatrix} 0 & \mathbf{M}_D^* & 0 \\ \mathbf{M}_D^\dagger & 0 & \mathbf{M}_R \\ 0 & \mathbf{M}_R^T & \mathbf{M}_s \end{pmatrix}$$

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- **Effective neutrino mass matrix** ($m_D, \mu_s \ll M$):

$$\mathbf{M}_{\text{eff}} = -\mathbf{M}_D^*\,(\mathbf{M}_R^T)^{-1}\,\mathbf{M}_s\mathbf{M}_R^{-1}\mathbf{M}_D^\dagger \Longrightarrow m_\nu \sim \mu_s \frac{m_D^2}{M^2}$$

- **Active-sterile mixing:**

$$\mathbf{U}_{\text{Hl}} \simeq \mathbf{V}_L^\dagger\,(0,\ \mathbf{M}_D(\mathbf{M}_R^\dagger)^{-1})\,\mathbf{U}_s \Longrightarrow U_{\text{Hl}} \sim \frac{m_D}{M} \sim \sqrt{\frac{m_\nu}{\mu_s}}$$

Type-I seesaw:	$m_\nu \sim \dfrac{m_D^2}{M}\,,\ U_{\text{Hl}} \sim \dfrac{m_D}{M} \sim \sqrt{\dfrac{m_\nu}{M}}$

- The ISS is a low-scale neutrino mass generation mechanism
- ISS provides a natural template for (active) neutrino mass suppression with sizeable active-sterile neutrino mixing

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Minimal Inverse Seesaw:

$$\mathrm{ISS}(n_R, n_s) \Longrightarrow \mathrm{ISS}(2,2)$$

Abada & Lucente '14

- One massless neutrino
- Neutrino data can be accommodated
- Still 17 parameters (in the $\mathbf{M}_s$ diagonal basis)

Oscillation data and flavour symmetries

Minimal Inverse Seesaw ISS(2,2):

17 parameters vs 7 observables

$$\theta_{12}\ ,\ \theta_{23}\ ,\ \theta_{13}\ ,\Delta m^2_{21,31}\ ,$$
$$\delta\ ,\ \alpha$$

Parameter	Best Fit $\pm 1\sigma$	3σ range
$\theta_{12}(^\circ)$	34.3 ± 1.0	$31.4 \to 37.4$
$\theta_{23}(^\circ)$[NO]	$48.79^{+0.93}_{-1.25}$	$41.63 \to 51.32$
$\theta_{23}(^\circ)$[IO]	$48.79^{+1.04}_{-1.30}$	$41.88 \to 51.30$
$\theta_{13}(^\circ)$[NO]	$8.58^{+0.11}_{-0.15}$	$8.16 \to 8.94$
$\theta_{13}(^\circ)$[IO]	$8.63^{+0.11}_{-0.15}$	$8.21 \to 8.99$
$\delta(^\circ)$[NO]	216^{+41}_{-25}	$144 \to 360$
$\delta(^\circ)$[IO]	277^{+23}_{-24}	$205 \to 342$
Δm^2_{21} $(\times 10^{-5}$ eV$^2)$	$7.50^{+0.22}_{-0.20}$	$6.94 \to 8.14$
$\lvert\Delta m^2_{31}\rvert$ $(\times 10^{-3}$ eV$^2)$ [NO]	$2.56^{+0.03}_{-0.04}$	$2.46 \to 2.65$
$\lvert\Delta m^2_{31}\rvert$ $(\times 10^{-3}$ eV$^2)$ [IO]	2.46 ± 0.03	$2.37 \to 2.55$

de Salas et. al ‘20; Capozzi et. al ’20; Esteban et. al ’20

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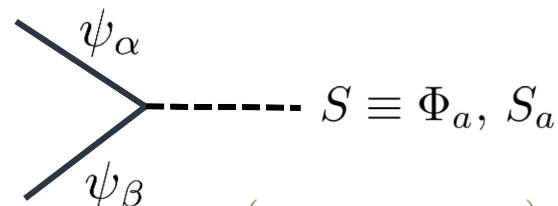
Abelian flavour symmetries:

- All mass terms generated dynamically

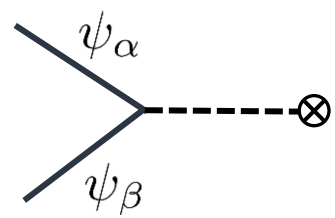
Mass matrices $\mathbf{M}_\ell\ ,\ \mathbf{M}_D$ $\mathbf{M}_R\ ,\ \mathbf{M}_s$



$$(q_\alpha + q_\beta + q_S) = 0$$

- Impose texture zeros in the mass matrices reducing the number of parameters

- CPV from vacuum phases (SCPV)



$$\langle \phi^0_a \rangle = v_a e^{i\theta_a}$$
$$\langle S_a \rangle = u_a e^{i\xi_a}$$

Scalar content and Yukawa Lagrangian

- Need to add a second Higgs doublet to be able to realise the charged-lepton mass matrix textures.
- Add two neutral complex scalar singlets to dynamically generate $\mathbf{M}_S$ and $\mathbf{M}_R$.

$$\Phi_{1,2} = \begin{pmatrix} \phi^+_{1,2} \\ \phi^0_{1,2} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\phi^+_{1,2} \\ v_{1,2} e^{i\theta_{1,2}} + \rho_{1,2} + i\eta_{1,2} \end{pmatrix} \ , \qquad S_{1,2} = \frac{1}{\sqrt{2}} \left(u_{1,2} e^{i\xi_{1,2}} + \rho_{3,4} + i\eta_{3,4} \right)$$

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Yukawa Lagrangian

$$-\mathcal{L}_{\text{Yuk.}} = \overline{\ell_L} \left(\mathbf{Y}^1_\ell \Phi_1 + \mathbf{Y}^2_\ell \Phi_2 \right) e_R + \overline{\ell_L} \left(\mathbf{Y}^1_D \tilde{\Phi}_1 + \mathbf{Y}^2_D \tilde{\Phi}_2 \right) \nu_R$$
$$+ \frac{1}{2} \overline{s^c} \left(\mathbf{Y}^1_s S_1 + \mathbf{Y}^2_s S_1^* \right) s + \overline{\nu_R} \left(\mathbf{Y}^1_R S_2 + \mathbf{Y}^2_R S_2^* \right) s + \text{H.c.}$$

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Scalar potential

$$V(\Phi_a, S_a) = V_{\text{sym.}} + V_{\text{soft}}(\Phi_a, S_a)$$

$$V_{\text{soft}}(\Phi_a, S_a) = \mu_{12}^2 \, \Phi_1^\dagger \Phi_2 + \mu_3^2 \, S_1^2 + \mu_4 \left| S_1 \right|^2 S_1 + \mu_5 \, \left| S_2 \right|^2 S_2 + \text{H.c.}$$

- SCPV is achieved by: $\theta, \xi_2 = 0 \, , \; \xi_1 = \arctan \left(\frac{\sqrt{32\mu_3^4 - \mu_4^2 u_1^2}}{\mu_4 u_1} \right)$

Abelian flavour symmetries

- Maximally-restrictive texture sets compatible with neutrino oscillation data that are realisable by Abelian symmetries:

$$\mathbf{G}_{\mathrm{F}} = \mathrm{U}(1) \times \mathbb{Z}_n \times \mathrm{U}(1)_{\mathrm{F}},\ n = 2, 4$$

Fields	U(1)	$(5^{\ell}_{1,\mathrm{I}}, \mathrm{T}_{45})$ $\mathbb{Z}_2 \times \mathrm{U}(1)_{\mathrm{F}}$	$(4^{\ell}_{3}, \mathrm{T}_{124})$ $\mathbb{Z}_2 \times \mathrm{U}(1)_{\mathrm{F}}$	$(4^{\ell}_{3}, \mathrm{T}_{456})$ $\mathbb{Z}_2 \times \mathrm{U}(1)_{\mathrm{F}}$	$(4^{\ell}_{3}, \mathrm{T}_{136,\mathrm{I}})$ $\mathbb{Z}_4 \times \mathrm{U}(1)_{\mathrm{F}}$	$(4^{\ell}_{3}, \mathrm{T}_{146,\mathrm{I}})$ $\mathbb{Z}_4 \times \mathrm{U}(1)_{\mathrm{F}}$
Φ_1	0	$(1,1)$	$(0,-5)$	$(1,1)$	$(1,2)$	$(0,1)$
Φ_2	0	$(0,-1)$	$(1,-3)$	$(0,-1)$	$(0,1)$	$(3,0)$
S_1	0	$(0,2)$	$(0,-2)$	$(0,-2)$	$(0,-2)$	$(0,-2)$
S_2	1	$(0,0)$	$(0,0)$	$(1,0)$	$(0,0)$	$(0,0)$
ℓ_{e_L}	1	$(1,0)$	$(0,0)$	$(0,0)$	$(2,0)$	$(2,0)$
ℓ_{μ_L}	1	$(0,2)$	$(1,2)$	$(1,-2)$	$(1,-1)$	$(1,-1)$
ℓ_{τ_L}	1	$(0,-2)$	$(0,4)$	$(0,-4)$	$(0,-2)$	$(0,-2)$
e_R	1	$(1,-3)$	$(0,9)$	$(1,-5)$	$(3,-4)$	$(0,-3)$
μ_R	1	$(0,3)$	$(1,7)$	$(0,-3)$	$(0,-3)$	$(1,-2)$
τ_R	1	$(0,-1)$	$(0,5)$	$(1,-1)$	$(1,-2)$	$(2,-1)$
ν_{R_1}	1	$(0,1)$	$(0,-1)$	$(0,-1)$	$(0,-1)$	$(0,-1)$
ν_{R_2}	1	$(1,-1)$	$(1,1)$	$(1,1)$	$(2,1)$	$(2,1)$
s_1	0	$(1,-1)$	$(1,1)$	$(0,1)$	$(2,1)$	$(2,1)$
s_2	0	$(0,1)$	$(0,-1)$	$(1,-1)$	$(0,-1)$	$(0,-1)$

Abelian flavour symmetries

ONLY INTERESTING CASE

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Φ_2	0	$(0,-1)$
S_1	0	$(0,2)$
S_2	1	$(0,0)$
ℓ_{e_L}	1	$(1,0)$
ℓ_{μ_L}	1	$(0,2)$
ℓ_{τ_L}	1	$(0,-2)$
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Mass matrices Yukawa decompositions

$\mathbf{M}_\ell$	$\mathbf{Y}_\ell^1$	$\mathbf{Y}_\ell^2$
$5^{\ell}_{1,\mathrm{I}}$	$\begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$

$\mathbf{M}_R$	$\mathbf{Y}_R$
T_{14}	$\begin{pmatrix} 0 & \times \\ \times & 0 \end{pmatrix}$

$\mathbf{M}_D$	$\mathbf{Y}_D^1$	$\mathbf{Y}_D^2$
T_{45}	$\begin{pmatrix} \times & 0 \\ 0 & 0 \\ 0 & \times \end{pmatrix}$	$\begin{pmatrix} 0 & \times \\ \times & 0 \\ 0 & 0 \end{pmatrix}$

$\mathbf{M}_s$	$\mathbf{Y}_s^1$	$\mathbf{Y}_s^2$
T_{23}	$\begin{pmatrix} \times & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 0 & \times \end{pmatrix}$

Common origin for Leptonic CPV

- Parameterisation of the charged lepton-mass matrix:

$$5_1^{\ell}: \quad \mathbf{M}_\ell = \begin{pmatrix} 0 & 0 & a_1 \\ 0 & m_{\ell_1}^2 & 0 \\ a_2 & 0 & a_4 \end{pmatrix}, \quad \mathbf{H}_\ell = \begin{pmatrix} a_1^2 & 0 & a_1 a_4 \\ 0 & a_3^2 & 0 \\ a_1 a_4 & 0 & a_2^2 + a_4^2 \end{pmatrix}, \quad \mathbf{V}'_L = \begin{pmatrix} c_L & 0 & s_L \\ 0 & 1 & 0 \\ -s_L & 0 & c_L \end{pmatrix} \quad \theta_L$$

$$5_1^e : \mathbf{V}_{L,R} = \mathbf{V}'_{L,R}\mathbf{P}_{12}, \quad 5_1^\mu : \mathbf{V}_{L,R} = \mathbf{V}'_{L,R}, \quad 5_1^\tau : \mathbf{V}_{L,R} = \mathbf{V}'_{L,R}\mathbf{P}_{23},$$

$NO_{e,\mu,\tau}$, $IO_{e,\mu,\tau}$ → 6 distinct cases to be analysed

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NO$_{e,\mu,\tau}$, IO$_{e,\mu,\tau}$ ➡ 6 distinct cases to be analysed

Real Yukawa couplings (CP is conserved at the Lagrangian level)

$$\mathbf{Y}_D^1 = \begin{pmatrix} b_1 & 0 \\ 0 & 0 \\ 0 & b_2 \end{pmatrix}, \quad \mathbf{Y}_D^2 = \begin{pmatrix} 0 & b_3 \\ b_4 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{Y}_R = \begin{pmatrix} 0 & d_2 \\ d_1 & 0 \end{pmatrix}, \quad \mathbf{Y}_s^1 = \begin{pmatrix} f_2 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{Y}_s^2 = \begin{pmatrix} 0 & 0 \\ 0 & f_1 \end{pmatrix}$$

VEV configuration:

$$\langle \phi_1^0 \rangle = v \cos\beta$$
$$\langle \phi_2^0 \rangle = v \sin\beta$$
$$\langle S_1 \rangle = u_1 e^{i\xi}, \quad \langle S_2 \rangle = u_2$$

➡

$$\mathbf{M}_D = \begin{pmatrix} m_{D_1} & m_{D_3} \\ m_{D_4} & 0 \\ 0 & m_{D_2} \end{pmatrix}, \quad \mathbf{M}_R = \begin{pmatrix} 0 & M \\ qM & 0 \end{pmatrix}, \quad \mathbf{M}_s = \begin{pmatrix} p\,\mu_s e^{i\xi} & 0 \\ 0 & \mu_s e^{-i\xi} \end{pmatrix}$$

Correlation between low-energy observables

- Effective neutrino mass matrix: $\mathbf{V}_L^\dagger \mathbf{M}_{\text{eff}} \mathbf{V}_L$

$$\mathbf{M}_{\text{eff}} = e^{-i\xi} \begin{pmatrix} \frac{y^2}{x} + \frac{z^2}{w} e^{2i\xi} & y & z e^{2i\xi} \\ y & x & 0 \\ z e^{2i\xi} & 0 & w e^{2i\xi} \end{pmatrix}, \mathbf{V}_L = \begin{pmatrix} \cos\theta_L & 0 & \sin\theta_L \\ 0 & 1 & 0 \\ -\sin\theta_L & 0 & \cos\theta_L \end{pmatrix}$$

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$$5_1^\mu : \mathbf{V}_{L,R} = \mathbf{V}'_{L,R}$$
$$5_1^\tau : \mathbf{V}_{L,R} = \mathbf{V}'_{L,R} \mathbf{P}_{23}$$

$$z = \mu_s \frac{m_{D_2} m_{D_3}}{M^2} \frac{p}{q^2}, \; w = \mu_s \frac{m_{D_2}^2}{M^2} \frac{p}{q^2}, \; x = \mu_s \frac{m_{D_4}^2}{M^2}, \; y = \mu_s \frac{m_{D_1} m_{D_4}}{M^2}$$

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- The effective light neutrino mass matrix is written solely in terms of 6 effective parameters:

$$(x, y, z, w, \theta_L, \xi) \longrightarrow \mathcal{O}_i \equiv (\Delta m_{21}^2, \Delta m_{31}^2, \theta_{ij}, \delta, \alpha)$$

$$\text{NO} : M_{ij} = \left[\mathbf{U}'^* \text{diag}\left(0, \sqrt{\Delta m_{21}^2}, \sqrt{\Delta m_{31}^2}\right) \mathbf{U}'^\dagger\right]_{ij}$$

$$\text{IO} : M_{ij} = \left[\mathbf{U}'^* \text{diag}\left(\sqrt{\Delta m_{31}^2}, \sqrt{\Delta m_{21}^2 + \Delta m_{31}^2}, 0\right) \mathbf{U}'^\dagger\right]_{ij}$$

$$\text{D}_{ij} = M_{ii} M_{jj} - M_{ij}^2$$

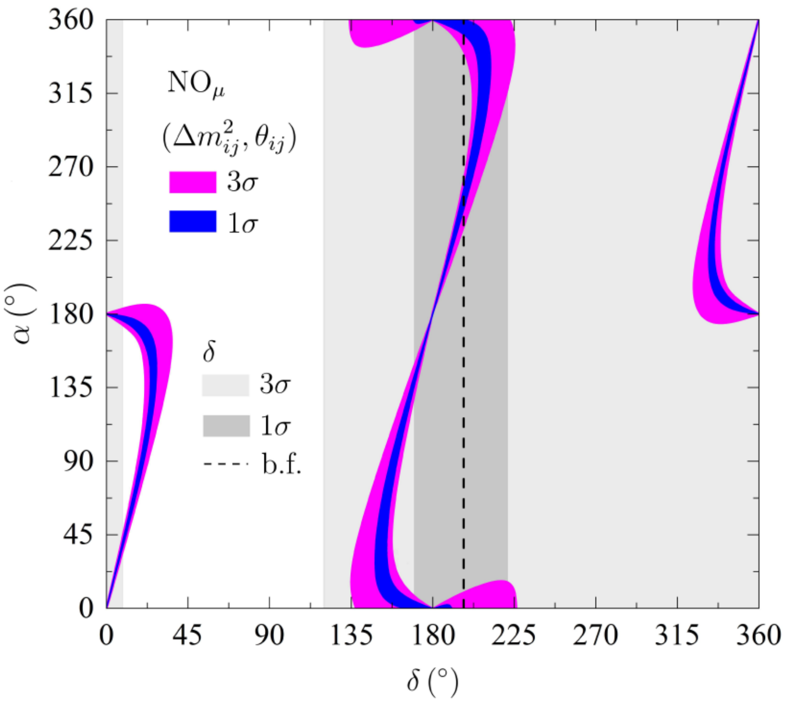
Low-energy relations:

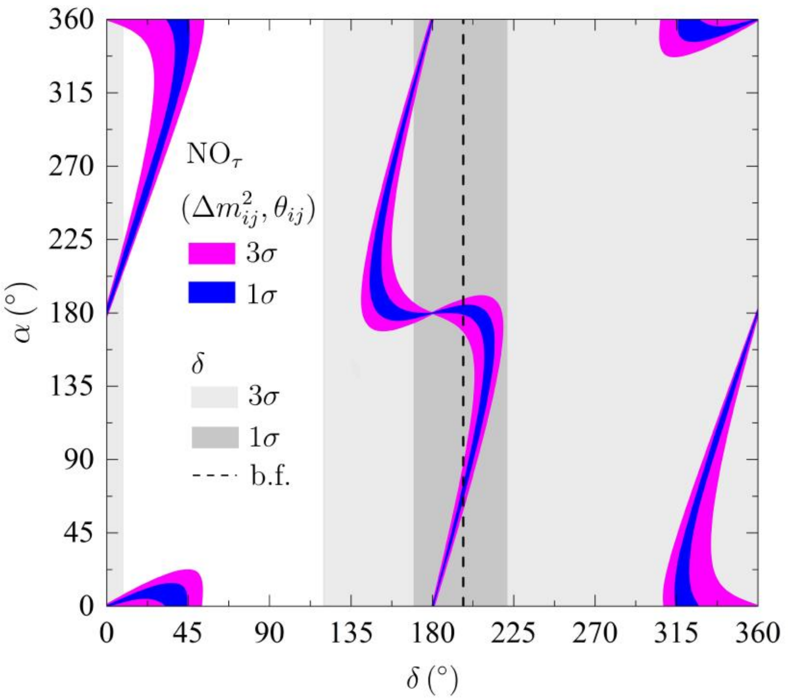
$$5_1^e : \arg\left[M_{11}^{*2} M_{13}^2 \frac{\text{D}_{12}}{\text{D}_{23}}\right] = 0$$

$$5_1^\mu : \arg\left[M_{12}^{*2} M_{23}^2 \frac{\text{D}_{12}}{\text{D}_{23}}\right] = 0$$

$$5_1^\tau : \arg\left[M_{13}^{*2} M_{33}^2 \frac{\text{D}_{12}}{\text{D}_{23}}\right] = 0$$

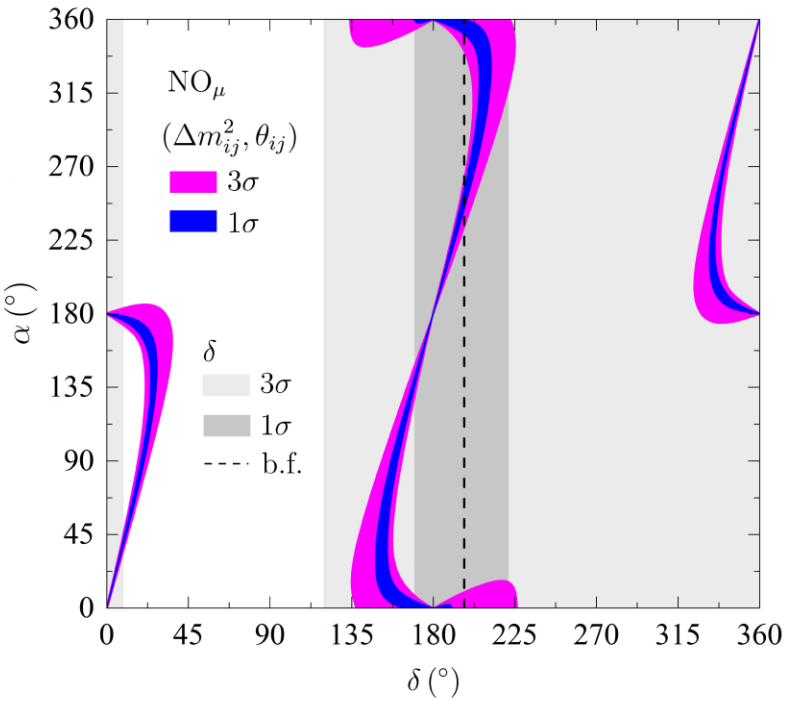
Leptonic CP violation

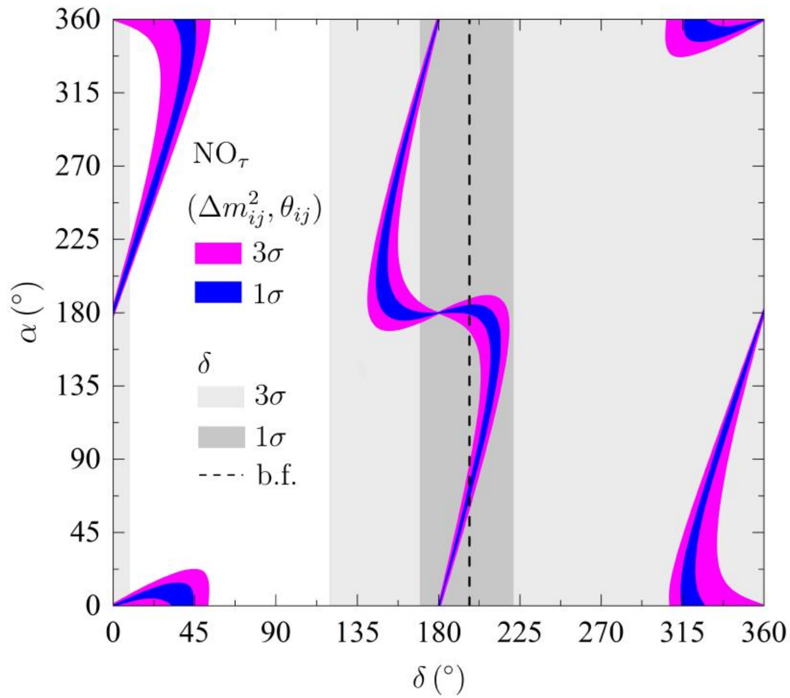




- Strong correlation between α and δ,

Leptonic CP violation





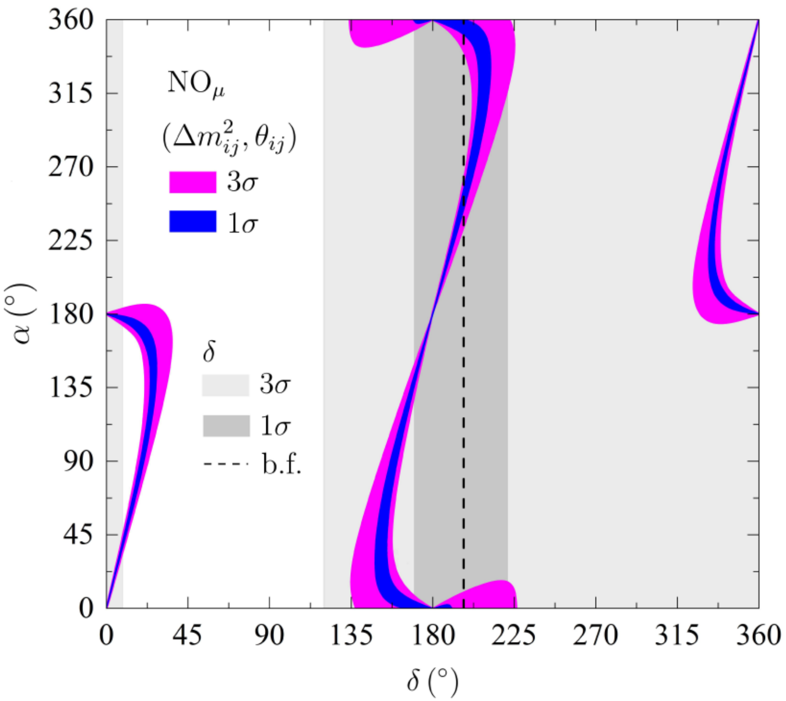
- Strong correlation between α and δ,
- No Dirac CPV implies no Majorana CPV,

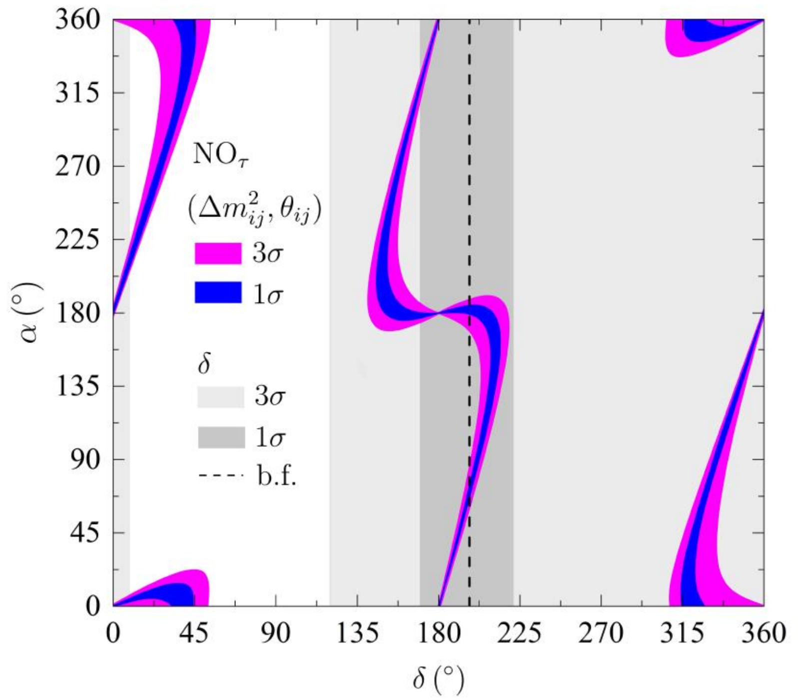
$$\langle S_1 \rangle = u_1 e^{i\xi}$$

$$\mathcal{J}^{\rm CP}_{\rm Dirac}\,,\ \mathcal{J}^{\rm CP}_{\rm Maj} \propto \sin(2\xi)$$

as in Branco, Felipe, Joaquim, Serôdio '12

Leptonic CP violation





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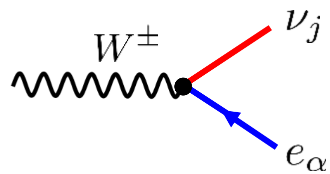
- A measurement of δ in the intervals $[\,45^{\mathrm{o}}, 135^{\mathrm{o}}]$ and $[\,225^{\mathrm{o}}, 315^{\mathrm{o}}]$ would exclude the NO_μ and NO_τ cases,
- $\beta\beta_{0\nu}$ analysis done in the paper (no time to discuss here).

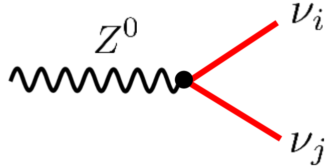
Heavy-light mixing relations

- Charged and neutral current interactions

$$\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}} W_\mu^- \sum_{\alpha=1}^{3} \sum_{j=1}^{n_f} \mathbf{B}_{\alpha j}\, \overline{e_\alpha} \gamma^\mu P_L \nu_j + \text{H.c.}$$



$$\mathcal{L}_Z = \frac{g}{4c_W} Z_\mu \sum_{i,j=1}^{n_f} \overline{\nu_i} \gamma^\mu \left(\mathcal{C}_{ij} P_L - \mathcal{C}_{ij}^* P_R \right) \nu_j \;,\; \mathcal{C}_{ij} = \sum_{\alpha=1}^{3} \mathbf{B}_{\alpha i}^* \mathbf{B}_{\alpha j}$$



$$\frac{\mathbf{B}_{e4}}{\mathbf{B}_{\mu 4}} \simeq \frac{\mathbf{B}_{e5}}{\mathbf{B}_{\mu 5}} \simeq \frac{x}{y c_L} \;,\; \frac{\mathbf{B}_{\tau 4}}{\mathbf{B}_{\mu 4}} \simeq \frac{\mathbf{B}_{\tau 5}}{\mathbf{B}_{\mu 5}} \simeq \tan\theta_L \;,\; \frac{\mathbf{B}_{\mu 6}}{\mathbf{B}_{\tau 6}} \simeq \frac{\mathbf{B}_{\mu 7}}{\mathbf{B}_{\tau 7}} \simeq \frac{z - w \tan\theta_L}{w + z \tan\theta_L} \;,\; \mathbf{B}_{e6} \simeq \mathbf{B}_{e7} \simeq 0$$

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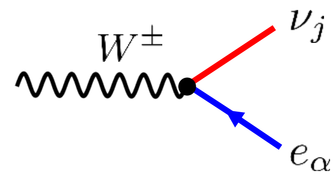

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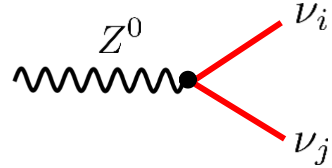

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- Numerical estimates

	NO_e	NO_μ	NO_τ	IO_e	IO_μ	IO_τ
$\mathbf{B}_{e4}/\mathbf{B}_{\mu 4} \simeq \mathbf{B}_{e5}/\mathbf{B}_{\mu 5}$	0.21	0.17	0.17	2.73	0.21	0.41
$\mathbf{B}_{\tau 4}/\mathbf{B}_{\mu 4} \simeq \mathbf{B}_{\tau 5}/\mathbf{B}_{\mu 5}$	0.27	0.88	0.87	0.51	1.09	1.24
$\mathbf{B}_{\tau 4}/\mathbf{B}_{e4} \simeq \mathbf{B}_{\tau 5}/\mathbf{B}_{e5}$	1.27	5.07	5.24	0.19	5.33	5.02
$\mathbf{B}_{e6}/\mathbf{B}_{\mu 6} \simeq \mathbf{B}_{e7}/\mathbf{B}_{\mu 7}$	0	–	0.36	0	–	4.96
$\mathbf{B}_{\tau 6}/\mathbf{B}_{\mu 6} \simeq \mathbf{B}_{\tau 7}/\mathbf{B}_{\mu 7}$	0.61	–	0	1.14	–	0
$\mathbf{B}_{\tau 6}/\mathbf{B}_{e6} \simeq \mathbf{B}_{\tau 7}/\mathbf{B}_{e7}$	–	1.64	0	–	0.23	0

- The $\mathbf{B}_{\alpha i}$ $(\alpha = e, \mu, \tau)$ $(i = 4, \ldots, 7)$ are related to each other;
- The relations are expressed solely in terms of the low-energy neutrino observables;
- Due to the flavour symmetries the heavy-light mixing parameters are not independent.

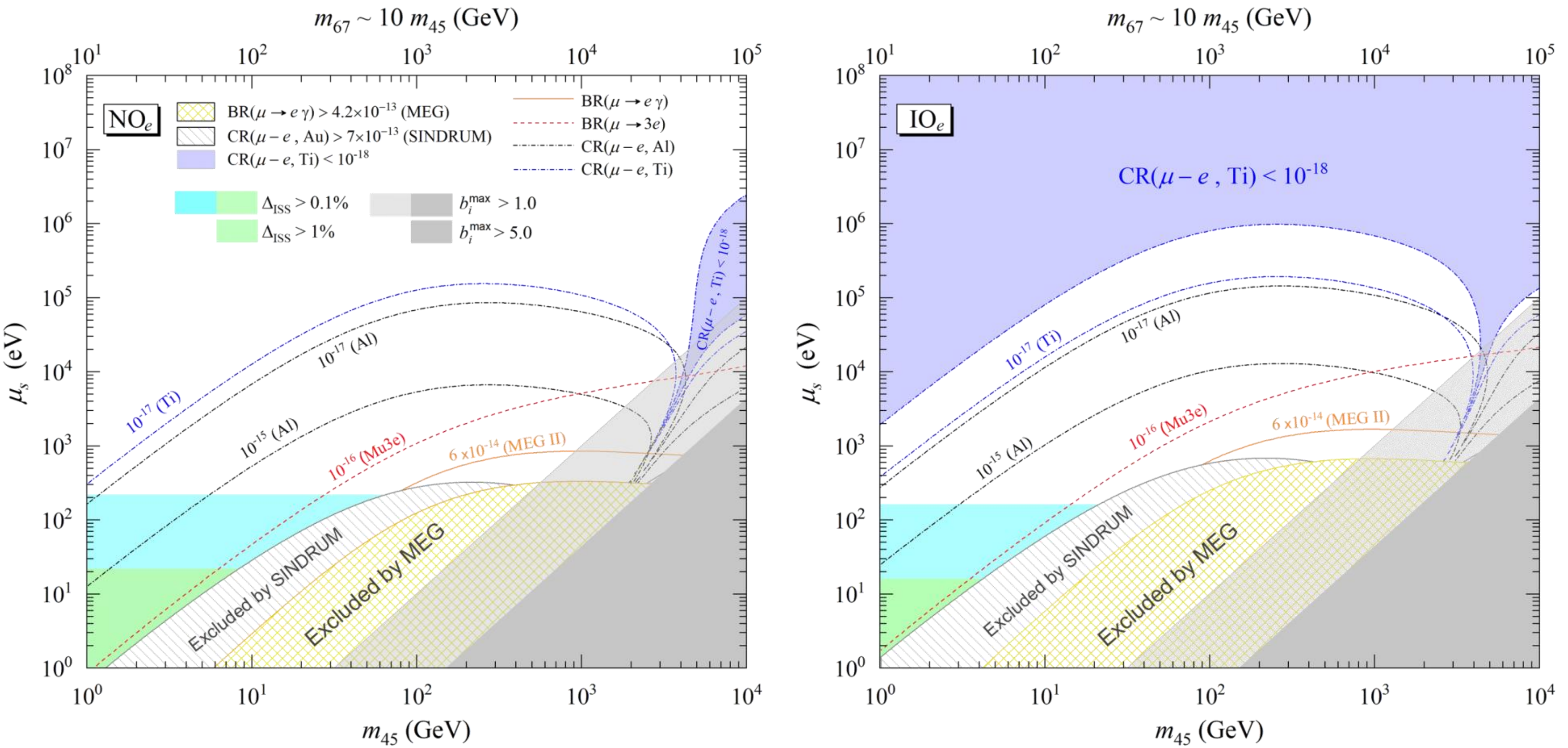
Relations among cLFV processes (no time to discuss here)

Charged lepton flavour violation (cLFV)

cLFV process	Present limit (90% CL)	Future sensitivity
$\mathrm{BR}(\mu \to e\gamma)$	4.2×10^{-13} (MEG)	6×10^{-14} (MEG II)
$\mathrm{BR}(\tau \to e\gamma)$	3.3×10^{-8} (BaBar)	3×10^{-9} (Belle II)
$\mathrm{BR}(\tau \to \mu\gamma)$	4.4×10^{-8} (BaBar)	10^{-9} (Belle II)
$\mathrm{BR}(\mu^- \to e^- e^+ e^-)$	1.0×10^{-12} (SINDRUM)	10^{-16} (Mu3e)
$\mathrm{BR}(\tau^- \to e^- e^+ e^-)$	2.7×10^{-8} (Belle)	5×10^{-10} (Belle II)
$\mathrm{BR}(\tau^- \to e^- \mu^+ \mu^-)$	2.7×10^{-8} (Belle)	5×10^{-10} (Belle II)
$\mathrm{BR}(\tau^- \to e^+ \mu^- \mu^-)$	1.7×10^{-8} (Belle)	3×10^{-10} (Belle II)
$\mathrm{BR}(\tau^- \to \mu^- e^+ e^-)$	1.8×10^{-8} (Belle)	3×10^{-10} (Belle II)
$\mathrm{BR}(\tau^- \to \mu^+ e^- e^-)$	1.5×10^{-8} (Belle)	3×10^{-10} (Belle II)
$\mathrm{BR}(\tau^- \to \mu^- \mu^+ \mu^-)$	2.1×10^{-8} (Belle)	4×10^{-10} (Belle II)
$\mathrm{CR}(\mu - e, \mathrm{Al})$	–	3×10^{-17} (Mu2e)
		$10^{-15} - 10^{-17}$ (COMET I-II)
$\mathrm{CR}(\mu - e, \mathrm{Ti})$	4.3×10^{-12} (SINDRUM II)	10^{-18} (PRISM/PRIME)
$\mathrm{CR}(\mu - e, \mathrm{Au})$	7×10^{-13} (SINDRUM II)	–
$\mathrm{CR}(\mu - e, \mathrm{Pb})$	4.6×10^{-11} (SINDRUM II)	–

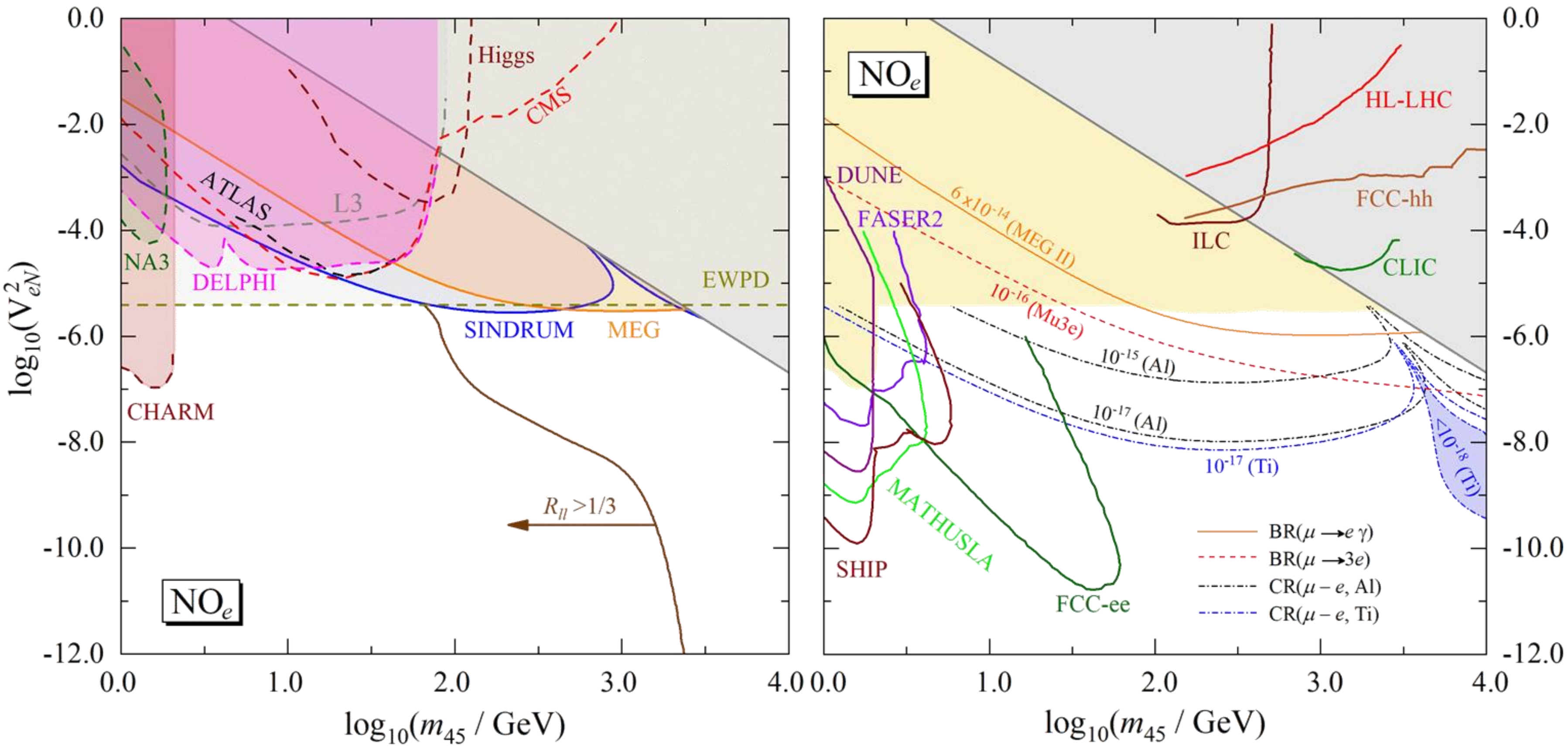
- Muon cLFV: strongest current constraints and future lowest sensitivities

cLFV in the ISS(2,2) with Abelian symmetries



- For NO, almost the whole parameter space will be scrutinized by future $\mu - e$ conversion experiments (Mu2e, COMET, PRISM/PRIME);
- For IO, the prospects are less optimistic.

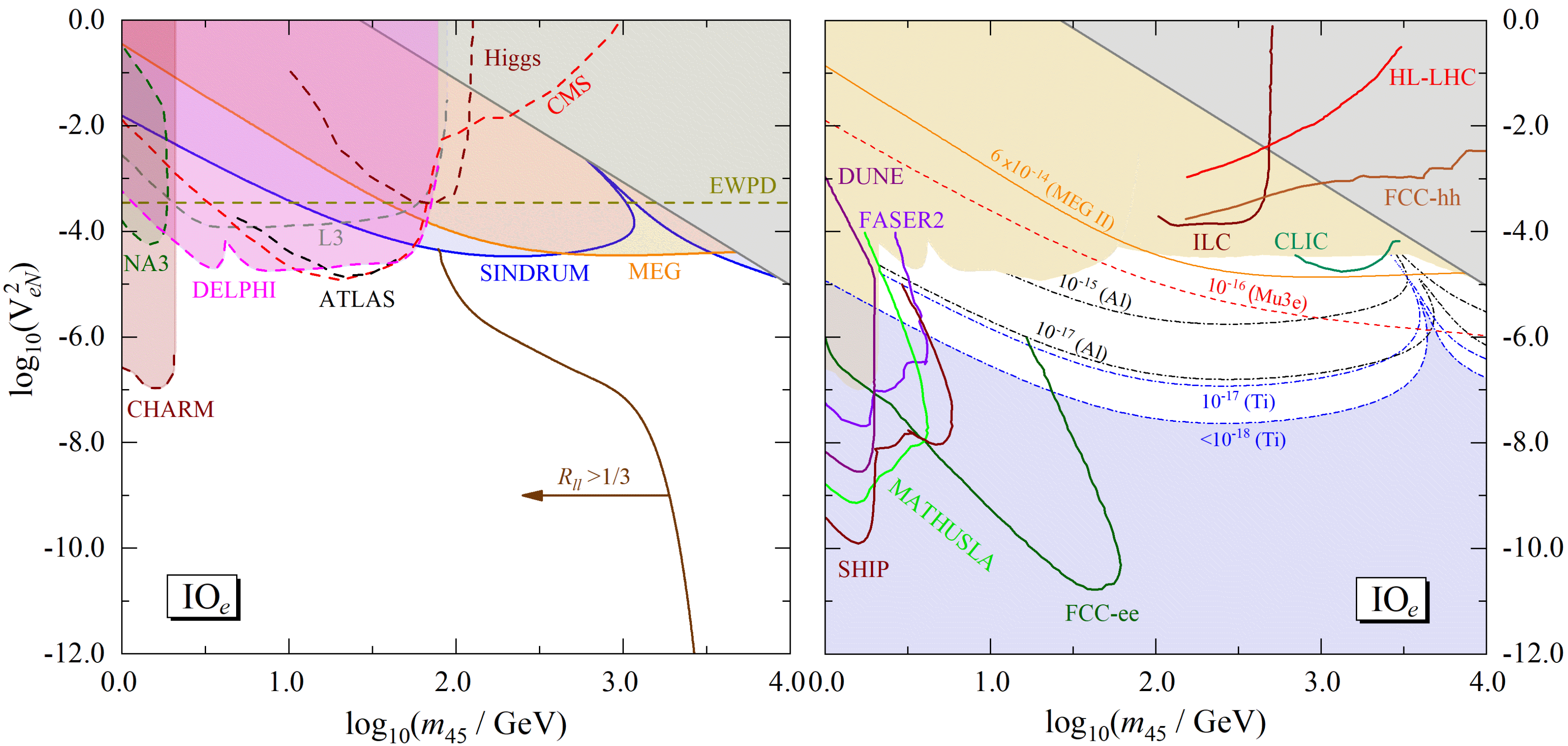
Constraints on heavy sterile neutrinos



- Current data implies an upper bound $V_{eN}^2 \sim 10^{-6} - 10^{-5}$;
- Future probes will be sensitive to much smaller mixings.

  Indirect LFV experiments fully complementary to other direct searches.

Constraints on heavy sterile neutrinos



- EWPD is less constraining in the IO case;
- Future cLFV probes will be sensitive to $V_{eN}^2 \sim 10^{-7}$.

Conclusion

- Comprehensive study of the minimal inverse seesaw model constrained by Abelian flavour symmetries with all mass terms generated via SSB;

- Majorana and Dirac-type CP violation are related;

- Relations among LFV parameters in our framework provide a very constrained setup for phenomenological studies;

- Constraining power of cLFV processes in the model's parameter space;

- Alternative probes such as beam-dump, hadron-collider, linear-collider, displaced-vertex experiments as well as EWPD.

Analysed in paper: Impact of radiative corrections on neutrino masses, neutrinoless double beta decay, relations among tau and muon cLFV decays, …

Thank you!