Conformal symmetry: towards the link between the Fermi and the Planck scales

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References

MS, Andrey Shkerin, and Sebastian Zell:

- Standard Model Meets Gravity: Electroweak Symmetry Breaking and Inflation, Phys. Rev. D 103 (2021) 3, 033006

MS and Andrey Shkerin

- Conformal symmetry: towards the link between the Fermi and the Planck scales, Phys. Lett. B 783 (2018) 253

- Gravity, Scale Invariance and the Hierarchy Problem, *JHEP* 10 (2018) 024

Outline

- Standard Model and conformal symmetry
- Non-perturbative semiclassical relation between the Fermi and Planck scales
- Conclusions

Scales in Nature

We observe several scales in Nature:

- Scale of quantum gravity, related to Newtons constant,
  $G_N = 6.7 \times 10^{-39}$ GeV^{-2} , $M_P = 2.435 \times 10^{18}$ GeV

- Fermi scale, associated with electroweak interactions,
  $G_F = 1.17 \times 10^{-5}$ GeV^{-2}, $M_W = 80.38$ GeV

- Cosmological constant, or vacuum energy, or Dark Energy,
  $\epsilon_{vac} = \left(2.24 \times 10^{-3}\mathrm{eV}\right)^4$

Are all these scales independent?

Origin of scales

Hierarchy: $\left(\epsilon_{vac}\right)^{\frac{1}{4}} \ll M_W \ll M_P$

Could it be that only one of these scales is fundamental, and others are derived from it by some dynamical mechanism?

This talk: M_W from M_P.

Standard Model and scale symmetry

Let's forget about gravity for the moment and ask the question: "Do we get enhanced symmetry if mass of the Higgs boson is put to zero?"

Why this question? t'Hooft naturalness criterion, 1980: the parameter of the theory is small "naturally" if its zero value leads to increased symmetry.

SM with M_H=0: the classical Lagrangian of the SM has a wider symmetry: it is scale invariant. Dilatations "D"- global scale transformations (σ = const) ,

$$\Psi(x) \to \sigma^n \Psi(\sigma x),$$

n = 1 for scalars and vectors, and n = 3/2 for fermions. Space-time Poincaré symmetry with 10 generators, $P = T \rtimes O(1,3)$, is enhanced to a direct product P $\Rightarrow$ P x D (11 generators)

Standard Model and conformal symmetry

In fact, the resulting symmetry is even larger for M_H=0: SM gets conformally invariant: P ⇒ SO(4,2) ! 10 generators ⇒ 15

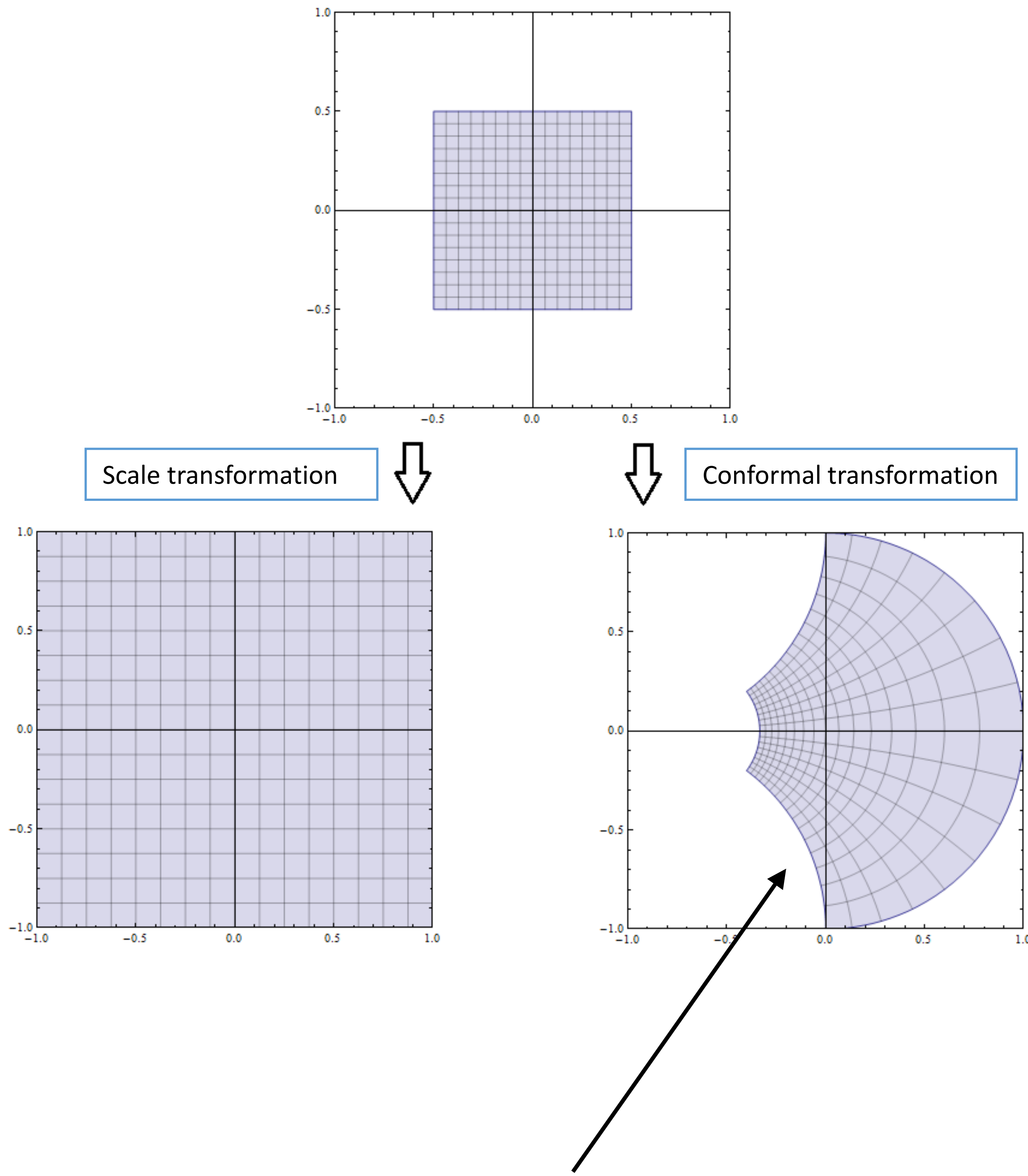
generators = 4 translations + 6 Lorentz transformations + 4 special conformal transformations + 1 scale transformation.

Conformal group: coordinate transformations $x' = F(x)$, which leave the metric $g_{\mu\nu}$ invariant up to a conformal factor $\Omega\,(x')$

$$g_{\mu\nu}(x) = \Omega\,(x')\, g'_{\lambda\sigma}\,(x')\,\frac{\partial F^\lambda}{\partial x^\mu}\frac{\partial F^\sigma}{\partial x^\nu}$$

Conformal symmetry of Maxwell equations: Bateman and Cunningham, 1908.

scale and conformal transformations



Angles do no change

Conformal symmetry and quantum physics

Sidney Coleman, extract from his 1971 Erice Lectures on “Dilatations”, chapter “The death of scale invariance”:

“For scale invariance,...., the situation is hopeless; any cutoff procedure necessarily involves a large mass, and a large mass necessarily breaks scale invariance in a large way.”

No go theorem?

Conformal anomaly

- The statement " any cutoff procedure necessarily involves a large mass" is not true. Counter-example: dimensional regularsation of t'Hooft and Veltman (invented in 1972) does not involve any large mass, just a normalisation point μ which is not send to infinity. Still, the scale invariance is anomalous in realistic renormalisable theories: the renormalisation-group running of the parameters leads to a non-vanishing trace of the energy-momentum tensor, which enters the divergence of the scale current J^μ:

$$\partial_\mu J^\mu \propto \beta(g) G^a_{\alpha\beta} G^{\alpha\beta\, a} \ + \ \ldots$$

Conformal anomaly

The physical quantities depend on the renormalisation scale only logarithmically. Any quadratically divergent contributions to the Higgs boson mass are purely technical and are introduced by artificial explicit breaking of the conformal invariance by regulators (cutoff, Pauli-Villars, etc).

It is possible to make quantum conformal symmetry exact but spontaneously broken (non-linear realisation) in realistic setup. Consequence: existence of the exactly massless dilation (but no fifth source), non-renormalisable interactions relevant at the Planck scale.

Radiative generation of the electroweak scale

- In classically scale invariant/conformal theories the Higgs mass can be predicted :

Radiative Corrections as the Origin of Spontaneous Symmetry Breaking*

Sidney Coleman

and

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(Received 8 November 1972)

We investigate the possibility that radiative corrections may produce spontaneous symmetry breakdown in theories for which the semiclassical (tree) approximation does not indicate such breakdown. The simplest model in which this phenomenon occurs is the electrodynamics of massless scalar mesons. We find (for small coupling constants) that this theory more closely resembles the theory with an imaginary mass (the Abelian Higgs model) than one with a positive mass; spontaneous symmetry breaking occurs, and the theory becomes a theory of a massive vector meson and a massive scalar meson. The scalar-to-vector mass ratio is computable as a power series in e, the electromagnetic coupling constant. We find, to lowest order, $m^2(S)/m^2(V) = (3/2\pi)(e^2/4\pi)$. We extend our analysis to non-Abelian gauge theories, and find qualitatively similar results. Our methods are also applicable to theories in which the tree approximation indicates the occurrence of spontaneous symmetry breakdown, but does not give complete information about its character. (This typically occurs when the scalar-meson part of the Lagrangian admits a greater symmetry group than the total Lagrangian.) We indicate how to use our methods in these cases.

Radiative generation of the electroweak scale

The scale invariance is anomalous due to “dimensional transmutation”: the renormalisation-group running of the parameters leads to a non-vanishing trace of the energy-momentum tensor, which enters the divergence of the scale current. The physical quantities depend on the renormalisation scale only logarithmically. Take a scale-independent renormalisation, e.g. DimReg. No counter-term is needed to renormalise the scalar mass: m_H can be predicted! RG equation has a fixed point at $m_H = 0$:

$$\mu \frac{\partial}{\partial \mu} m_H^2 \propto m_H^2$$

- Procedure: compute the CW effective potential and discover that the U(1) theory is in the Higgs phase. Read off the ratio between the Higgs boson mass and the vector boson mass,

$$\frac{m_H^2}{m_W^2} = \frac{3e^2}{8\pi^2}$$

Radiative generation of the electroweak scale

Does not work for the SM (but may work in its extensions):

If the top quark mass $m_t \lesssim m_t^{crit}$, then the minimum of the effective potential is generated at $\langle H \rangle \simeq 100\ \mathrm{MeV}$ due to chiral symmetry breaking in QCD.

If the top quark mass $m_t \gtrsim m_t^{crit}$, then an extra minimum of the effective potential is generated at $\langle H \rangle \gtrsim M_P$ due to top quark loops.

$m_t^{crit} = 170 - 174$ GeV accounting for uncertainties in the relation between the Monte Carlo and pole masses of the top quark.

Radiative generation of the electroweak scale

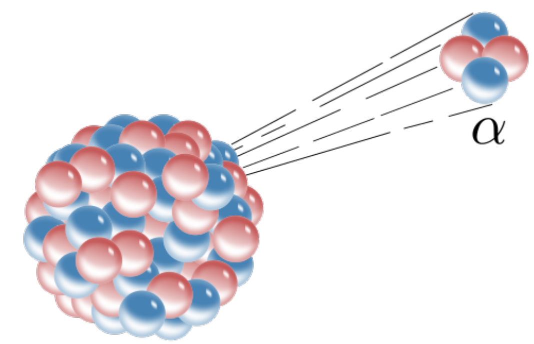
The idea fails. But we do have the breaking of scale invariance! Gravity comes with a dimensionful parameter $M_P \gg m_H$, and this must be taken into account!

- Perturbatively, with mass-independent regularisation (such as DimReg) : no gravity contribution to the Higgs mass: all corrections are suppressed by the Planck mass. The RG equation $\mu \frac{\partial}{\partial \mu} m_H^2 \propto m_H^2$ remains in force!

Gravity + conformally invariant SM is an ideal playground for looking for non-perturbative generation of the weak scale!

Very small numbers in quantum physics: non-perturbative effects

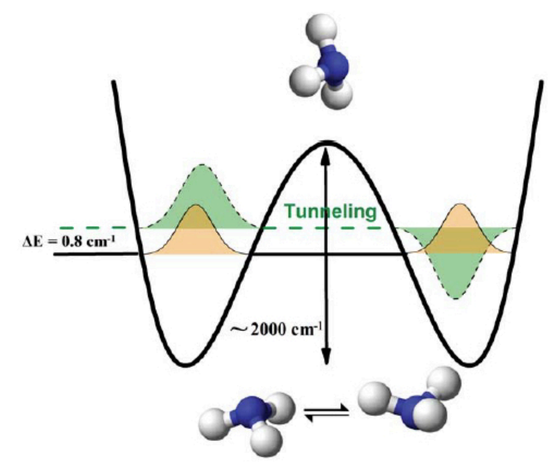
Well known examples:



- 1928, Gamow's theory of α-decay, uranium-238 → thorium-234 + α,

$$\Gamma = E_{\text{bounding}} e^{-S} <<< E_{\text{bounding}}$$

- 1951, Townes, Ammonia Maser,



$$\omega = E_{\text{bounding}} e^{-S} <<< E_{\text{bounding}}$$

- Mass gap in BCS superconductors

$$T_c \sim E_D e^{-1/NV}$$

Proposal : there is only one fundamental scale in Nature - M_P and the electroweak scale is generated from it non-perturbatively. The huge difference between m_H and M_P is due to the non-perturbative phenomena in gravity and the Higgs mass is related to the Planck scale as $m_H^2 = M_P^2 e^{-S}$ with S ~ 80.

Non-perturbative Fermi scale generation

Simplest theory that works

Scalar field with non-minimal coupling to Palatini gravity. Basic structures: metric (distances) and symmetric connection $\Gamma^{\rho}_{\nu\sigma} = \Gamma^{\rho}_{\sigma\nu}$. The action (metric -+++):

$$\frac{\mathscr{L}_{\varphi,g}}{\sqrt{g}} = \frac{1}{2}(M_P^2 + \xi\varphi^2)R - \frac{1}{2}(\partial\varphi)^2 - V(\varphi)$$

The dynamical variables are $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$, variation with respect to $\Gamma^{\rho}_{\nu\sigma}$ gives metricity, $g_{\mu\nu;\alpha} = 0$, i.e. the relation between $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$, the variation with respect to gives Einstein equations. Large ξ - semiclassical parameter which allows for non-perturbative estimate in Palatini gravity 😀. Metric gravity does not provide such a parameter 😢.

"Matter" is scale invariant with $V(\varphi) = \frac{\lambda}{4}\varphi^4$. The cutoff of the theory (onset of perturbation theory breaking) $\Lambda \sim M_P/\sqrt{\xi}$

Non-perturbative Fermi scale generation

We want to compute the Higgs vev:

$$\langle \varphi \rangle \sim \int \mathscr{D}\varphi \mathscr{D} g_{\mu\nu} \varphi e^{-S_E}$$

S_E is the euclidean action of the model.

Remarks:

- Euclidean path integral for gravity may not be well defined due to the problem with the conformal factor of the metric
- We will ignore this problem and follow the crowd: Hawking; Coleman, de Luccia; Veneziano; ..., Isidori, Rychkov, Strumia, Tetradis; ... Branchina, Messina, Sher;..

Fermi scale generation

For small $\varphi \ll M_P$ - gravity is irrelevant – no contribution to the vev of the Higgs from scalar loops.

- Challenge: account for contributions with $\varphi \gg M_P$. Theory for large φ:

$$\mathscr{L} = -\frac{1}{2}\xi\varphi^2 R + \frac{1}{2}(\partial\varphi)^2 + \frac{\lambda}{4}\varphi^4$$

Important properties of this action:

- Scale-invariance
- Planck scale is dynamical, $M_P(\varphi) \propto \sqrt{\xi}\varphi$
- Conjecture: contribution of large Higgs fields $\varphi > M_P/\sqrt{\xi}$ to path integral is better to be found in the Einstein frame
- Redefinition of the Higgs field to make canonical kinetic term

$$\frac{d\chi}{d\varphi} = \frac{1}{\Omega} \quad \Longrightarrow \quad \varphi = \frac{M_P}{\sqrt{\xi}} \sinh\left(\frac{\sqrt{\xi}\chi}{M_P}\right)$$

Fermi scale generation

Resulting action

$$S = \int d^4x\sqrt{-\hat{g}}\left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu\chi\partial^\mu\chi}{2} + \frac{\lambda M_P^4}{4\xi^2}\sinh^4\left(\frac{\sqrt{\xi}\chi}{M_P}\right)\right\}$$

source term

Most important:

$$\langle\varphi(x)\rangle \sim \int \mathscr{D}\mathscr{A}\mathscr{D}\varphi(x)\mathscr{D}g_{\mu\nu}\varphi(x)e^{-S_E} \Longrightarrow \int \mathscr{D}\mathscr{A}\mathscr{D}\chi\mathscr{D}\hat{g}_{\mu\nu}e^{\frac{\sqrt{\xi}\chi(x)}{M_P}-S_E}$$

Modification of the action and equations of motion!

Equations of motion for χ contain a source term $\delta(x) \longrightarrow$ new classical solutions.

Similar to

- computation of $\int dx x^N e^{-x^2}$ for large N ,

- computation of multi-particle production in Khlebnikov, Rubakov, Tinyakov ’91,

- proof of confinement in 3D Georgi-Glashow model, $\langle\exp(\int A_\mu dv^\mu)\rangle$ in Polyakov ’76.

Fermi scale generation

Field equations in the Einstein frame for maximally O(4) symmetric metric

$$ds^2 = f^2(r)dr^2 + r^2 d\Omega_3^2$$

$$\partial_r\left(\frac{r^3\chi'}{f}\right) - r^3 f U'(\chi) = -\frac{\sqrt{\xi}}{2\pi^2 M_P}\delta(r)$$

$$6 - 6f^2 + \frac{2r^2 f^2 U(\chi)}{M_P^2} - \frac{r^2\chi'^2}{M_P^2} = 0$$

Boundary conditions at infinity : $r \to \infty$: $f^2(r) \to 1, \chi(r) \to 0$.

Fermi scale generation

The total action for O(4) symmetric configurations has the form

$$\int_0^\infty dr \left(\frac{\sqrt{\xi}\chi'(r)}{M_P} + 2\pi^2 r^3 \mathscr{L} \right).$$ Rescaling $r \to \xi^{1/6}/M_P, \;\; \chi \to M_P \chi, \;\; f \to \xi^{-1/6} f$

source term

demonstrates the existence of semiclassical parameter $\sqrt{\xi}$, analogue of $1/\hbar$ in WKB approximation.

To regulate singularity at r → 0: add to the action higher dimensional operators. We also require that these operators:

- do not introduce new degrees of freedom
- do not spoil asymptotic scale invariance, when h → ∞

Example (other operators work as well) : $\delta\mathscr{L}_\delta = -\frac{\delta}{M_P^8 \Omega^8}\left(1 + \frac{\delta}{\Omega^2}\right)\left(\partial_\mu h\right)^6$

Fermi scale generation

Natural choice: $\delta \sim \xi^2$: the same cutoff $\Lambda \sim M_P/\sqrt{\xi}$

Constraint from inflation

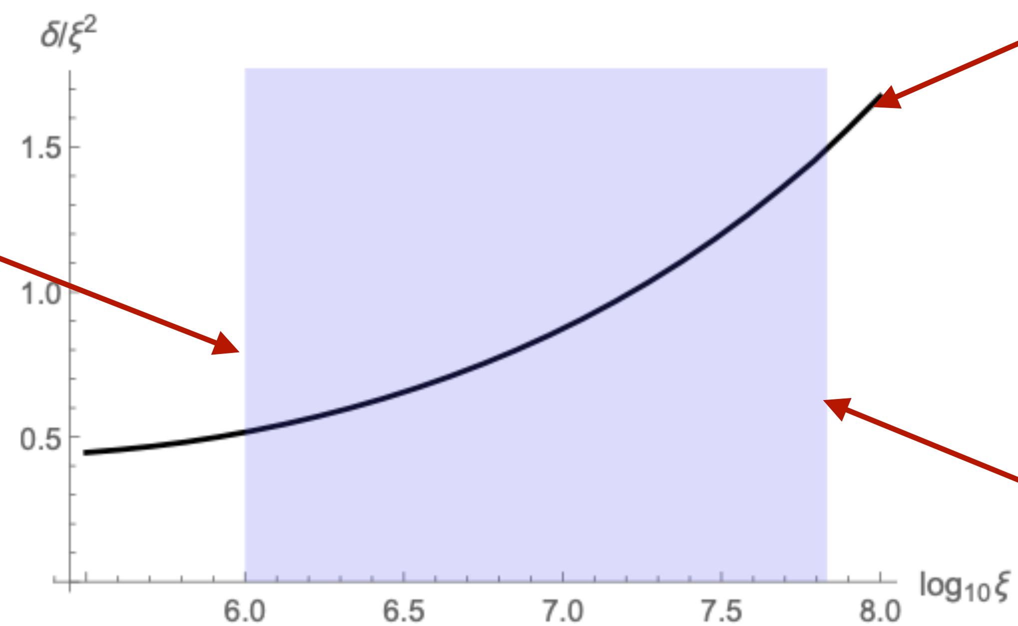


Figure 1. Values of the non-minimal coupling ξ and the coupling δ of the higher-order operator (11), for which $B = \ln(M_P/(\sqrt{\xi}M_F))$. Admissible values of ξ are within the blue area, the left bound coming from inflation and the right bound coming from top quark measurements.

Requirement of the correct hierarchy. Parametrically $S \propto \sqrt{\xi}$

Constraint from t-quark mass: positive λ at the scale of inflation

See also:
Rasanen and Rasanen, 1709.07853;
Rasanen, 1811.09514;
Karananas, Michel and Rubio, 2006.11290

The hierarchy between the Planck and the Fermi scales may be a natural phenomenon when the SM is classically conformal, ξ is large and the gravity is of the Palatini type! In the metric theory the source term $\sqrt{\xi}\delta(r)$ is replaced by $\delta(r)/(6 + 1/\xi)$ and the semiclassical parameter is absent, and the action is too small.

Conclusions

- The conjecture that there is just one scale in Nature - M_P , and that the Fermi scale is generated dynamically from it, may actually work

- The reason why the Fermi scale is much smaller than the Planck scale may be rooted in conformal symmetry and non-perturbative gravity effects

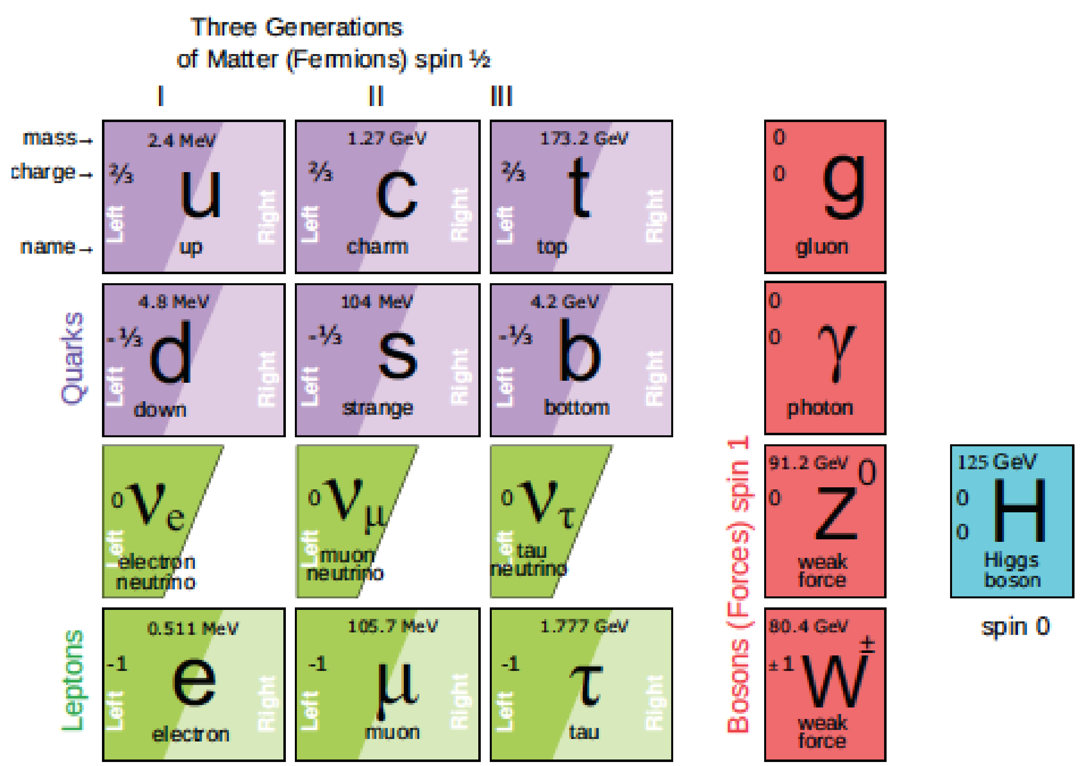
Phenomenology challenge to this picture

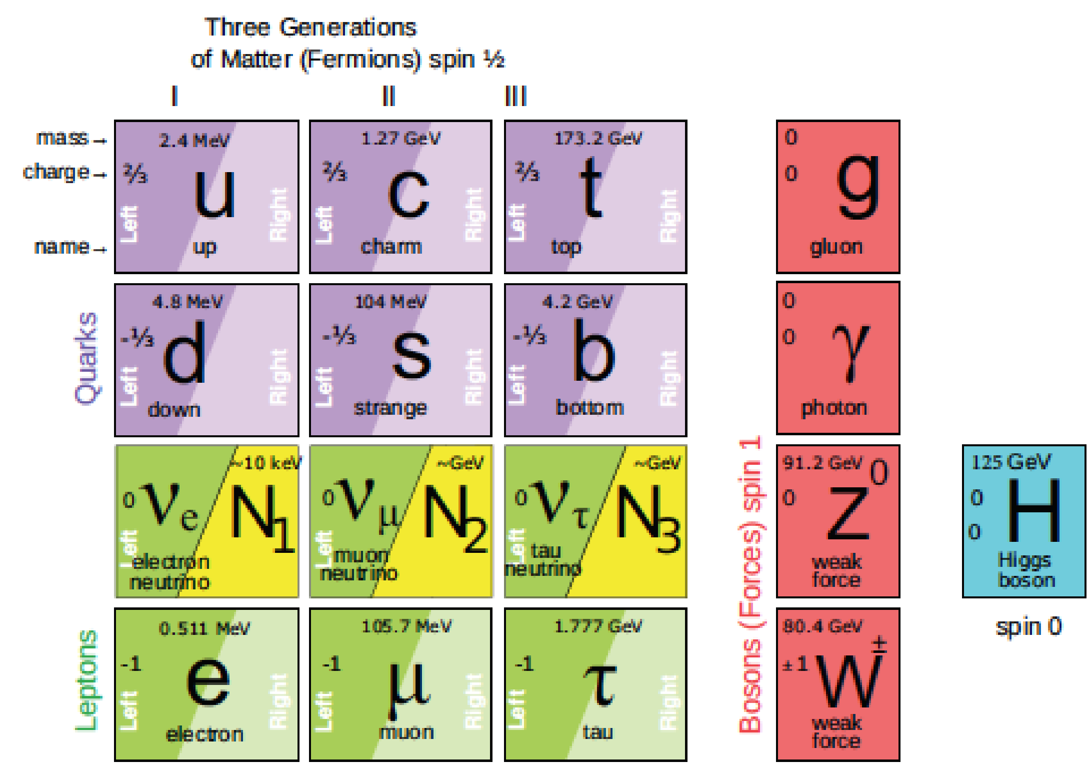
If there are no new particles with masses between the Planck and Fermi scales (such as GUT leptoquarks, superheavy RH neutrinos, SUSY relics,…) what explains:

- origin of neutrino masses
- dark matter
- baryon asymmetry of the Universe
- cosmological inflation

A possibility: new particles which can be relatively light but feebly interacting

Example: SM->νMSM





Role of the Higgs boson: break the symmetry and inflate the Universe

Role of N_1 with mass in keV region: dark matter.

Role of N_2, N_3 with masses > 100 MeV : "give" masses to neutrinos and produce baryon asymmetry of the Universe