Freeze-in produced dark matter at high temperatures

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Workshop on the Standard Model and Beyond - Corfu Summer Institute

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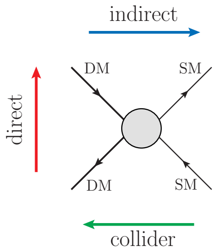
in collaboration with Jacopo Ghiglieri (arXiv 2012.09083, JCAP03(2021)075)

Swiss National Science Foundation

University of Basel

Department of Physics

Freeze-in production mechanism



- DM as a particle: many candidates see review G. Bertone 2016
- non-interacting with photons, absolutely stable or long-lived $\sim \tau_{\rm universe}$
- Any model has to comply with

$$\Omega_{\rm DM} h^2(M_{\rm DM}, M_{\rm DM'}, \alpha_{\rm DM}, \alpha_{\rm SM}) = 0.1200 \pm 0.0012$$

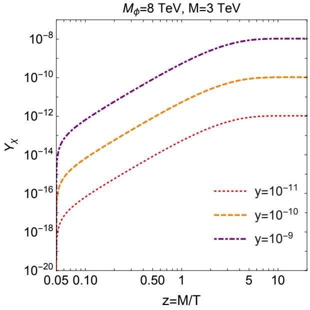
[for **production mechanisms** talks by Kolb, Trócsányi, Cirelli, Mambrini]

Freeze-in mechanism J. McDonald (2002)

- DM never reached thermal equilibrium
- DM from decay and/or annihilations of **equilibrated** species
- for a simple model $\mathcal{L}_{\rm int} = -y\phi\bar{\chi}\chi$, $\phi \to \chi\bar{\chi}$

$$\frac{dn_\chi}{dt} + 3Hn_\chi = \langle \Gamma_{\phi\to\chi\chi} \rangle n_\phi^{\rm eq} \;, \; Y = n_\chi/s$$

- $\Omega_{\rm DM} h^2 = \frac{M}{\rm GeV} \frac{Y_{\rm fin}}{3.645\times 10^{-9}}$



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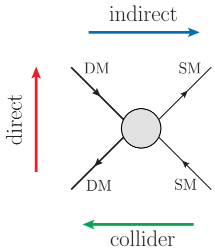

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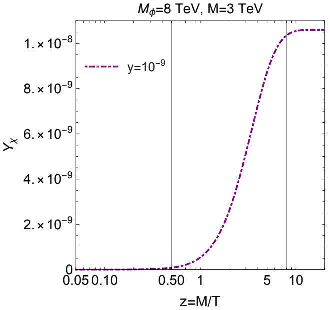

Framing the particle physics model

- Simplified DM models [see also talk by Belyaev]:
  $\Rightarrow$ capture the d.o.f. and parameters needed to study DM phenomenology
- χ Majorana fermion singlet, $\chi \equiv$ DM particle
- η is charged under QCD and $\mathrm{U}(1)_Y$, $\eta \equiv$ mediator with $M_\eta = M + \Delta M$

$$
\begin{aligned}
\mathcal{L} \; &= \; \mathcal{L}_{\mathrm{SM}} + \frac{1}{2}\bar{\chi}\left(i\partial\!\!\!/ - M\right)\chi + (D_\mu \eta)^\dagger D^\mu \eta - M_\eta^2\, \eta^\dagger \eta - \lambda_2 (\eta^\dagger \eta)^2 \\
&- \; \lambda_3\, \eta^\dagger \eta\, \phi^\dagger \phi - y\, \eta^\dagger \bar{\chi} a_R q - y^* \bar{q} a_L \chi\, \eta
\end{aligned}
$$

same model and freeze-in see M. Garny and J. Heisig [1809.10135], similar models see G. Bélanger et al [1811.05478]

$y \lesssim \mathcal{O}(10^{-8})$, $T_{\mathrm{INI.}} \gg M_\eta$ and $T > 150$ GeV

- DM **never** reaches thermal equilibirum $f_\chi(t, \boldsymbol{k}) \ll n_{\mathrm{F}}(k^0)$
- η and q are maintained in equilibrium by SM interactions
- χ accumulates over the thermal history through e.g. $\eta \to \chi q$

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- we shall address the high-temperature dynamics
- multiple soft scatterings and $2 \to 2$ process

$\boxed{T \gg M_\eta \text{ can be very important even for renormalizable interactions}}$

PRODUCTION RATE AND RATE EQUATION

GENERAL APPROACH

- Given a field χ weakly coupled to a an equilibrated bath, with internal couplings g

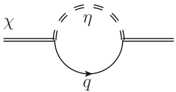
T. Asaka, M. Laine and M. Shaposhnikov [hep-ph/0605209], D. Bödeker, M. Sangel and M. Wörmann [1510.06742]

M. Laine and A. Vuorinen [1701.01554]

- at leading order in y and **all orders in** g one can prove [1510.06742]

$$\dot{f}_\chi(t,\boldsymbol{k}) = \Gamma(k)[n_{\mathrm{F}}(k^0) - f_\chi(t,\boldsymbol{k})], \quad \Gamma(k) = \frac{|y|^2}{2k^0}\int d^4X\, e^{iK\cdot X}\langle[J(X),J(0)]\rangle$$

- $f_\chi(t,\boldsymbol{k})$ is the single-particle phase-space distribution; J made of bath fields



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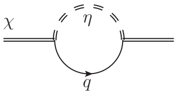
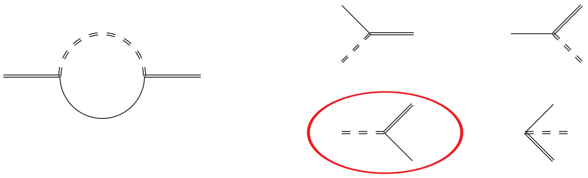


$$\boxed{\Gamma(k) = \frac{|y|^2}{k^0}\mathrm{Im}\Pi_R}$$

- when doing perturbative expansions $\Rightarrow$ **Boltzmann equation is recovered**
- within this approach one can include:

  resummation and NLO computations, non-perturbative and thermal effects

Born term and Boltzmann equation



$\eta \to \chi + q$

For us $M_\eta > M_\chi + M_q \ldots$ (first row: $M_\chi > M_\eta + M_q$, $M_q > M_\eta + M_\chi$)

$$\mathrm{Im}\Pi_R^{\mathrm{Born}} = \frac{N_c(M_\eta^2 - M^2)}{8 n_{\mathrm{F}}(k^0)} \int \frac{d^3\boldsymbol{p}_\eta}{(2\pi)^3} \int \frac{d^3\boldsymbol{p}_q}{(2\pi)^3} \frac{(2\pi)^4 \delta^4(\mathcal{P}_\eta - \mathcal{P}_q - \mathcal{K})}{E_\eta E_q} n_{\mathrm{B}}(E_\eta)(1 - n_{\mathrm{F}}(E_q))$$

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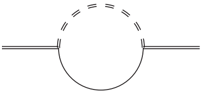

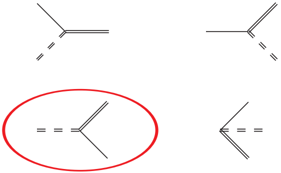

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- $n_{\mathrm{DM}} = 2\int_{\boldsymbol{k}} f_\chi(t,k)$, for $\eta \to \chi + q$

$$\begin{aligned}
\dot{n}_{\mathrm{DM}} + 3Hn_{\mathrm{DM}} &= 2|y|^2 \int_{\boldsymbol{k}} \frac{n_{\mathrm{F}}(k^0)}{k^0} \mathrm{Im}\Pi_R \\
&= 2|y|^2 N_c(M_\eta^2 - M^2) \int_{\boldsymbol{p}_\eta,\boldsymbol{p}_q,\boldsymbol{k}} \frac{(2\pi)^4\delta^4(\mathcal{P}_\eta - \mathcal{P}_q - \mathcal{K})}{8E_\eta E_q\, k^0} n_{\mathrm{B}}(E_\eta)\,[1 - n_{\mathrm{F}}(E_q)]\,[1 - \ldots]
\end{aligned}$$

First improvement: thermal masses

- at high temperatures, $\pi T \gg M_\eta$, repeated interactions within the bath change the dispersion relations $\Rightarrow$ *asymptotic masses* ($p_0 \sim p \sim \pi T$ and $p_0 - p \ll \pi T$)

  see also L. Darmé, A. Hryczuk, D. Karamitros and L. Roszkowski [1908.05685] for thermal masses and freeze-in

- for $T > T_c \simeq 150$ GeV the quarks only have

$$m_q^2 = \frac{T^2}{4}(g_3^2 C_F + Y_q^2 g_1^2 + |h_q|^2)$$

- for the colored scalar

$$m_\eta^2 = T^2 \left(\frac{g_3^2 C_F + Y_q^2 g_1^2}{4} + \frac{\lambda_3}{6} \right)$$

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$$m_B^2 = \left(\frac{n_S}{6} + \frac{5 n_G}{9} + \frac{Y_q^2 N_c}{3} \right) g_1^2 T^2 \,, \quad m_g^2 = \left(\frac{N_c}{3} + \frac{n_G}{3} + \frac{1}{6} \right) g_3^2 T^2$$

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$$\begin{aligned} m_\eta^2(M_\eta, p) &= \frac{Y_q^2 g_1^2 + C_F g_3^2}{2\pi^2} \int_0^\infty dq \left\{ \frac{n_{\mathrm{B}}(E_\eta(q))}{E_\eta(q)} \left[q^2 + \frac{M_\eta^2 q}{2p} \ln \left(\frac{(p+q)^2}{(p-q)^2} \right) \right] + 2\, q\, n_{\mathrm{B}}(q) \right\} \\ &+ \frac{\lambda_3}{\pi^2} \int_0^\infty dq\, q\, n_{\mathrm{B}}(q) \end{aligned}$$

Born rate, thermal masses and IR Freeze-in

- Born rate $\eta \to \chi + q$ with and without thermal masses (recall m_q is purely thermal)

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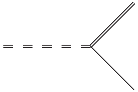
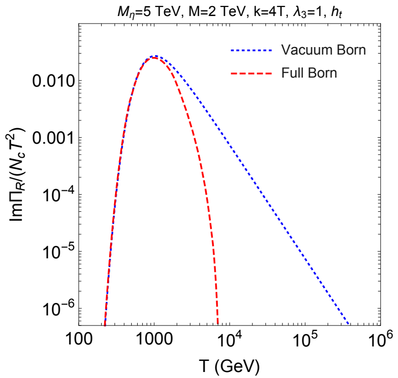
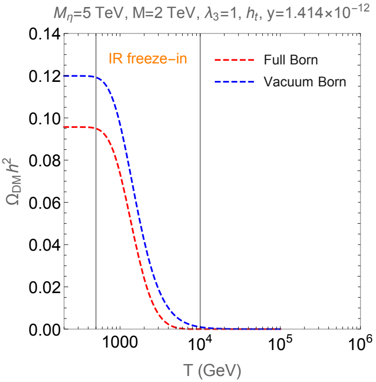


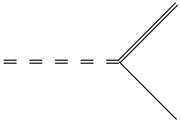




LPM RESUMMATION FOR LIGHT-CONE KINEMATICS

HIGH-TEMPERATURES $\pi T \gg M_\eta$

- all the particles are seen as *massless*,
- momenta of external particles $p \sim \pi T$,
- particles are ultra-relativistic, and gT is a soft scale
  $\Rightarrow$ collinear kinematics $\approx$ high T



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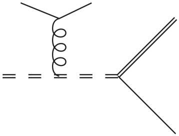

LPM resummation for light-cone kinematics

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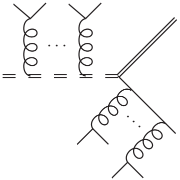

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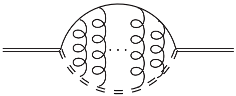

n soft scatterings: LPM resummation

L. Landau and I. Pomeranchuk (1953) and A. B. Migdal (1956)

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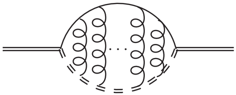

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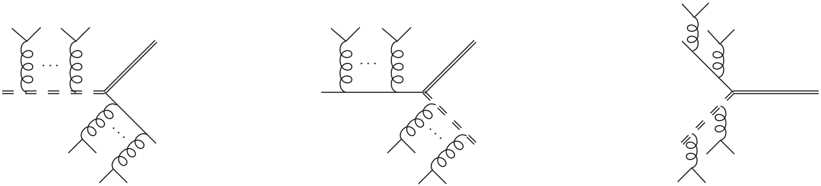
- at $T \gg M_\eta$ three effective processes contribute to the production of χ $[1 + n \leftrightarrow 2 + n]$

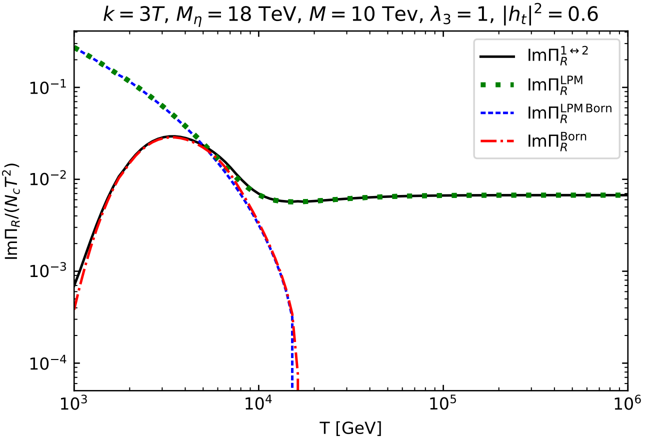
$$\boxed{\eta \to \chi + q, \quad q \to \chi + \eta, \quad q + \eta \to \chi}$$



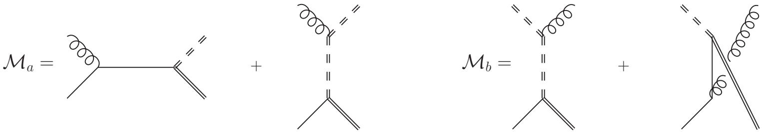
LPM results

- prescription for any temperature I. Ghisoiu and M. Laine [1411.1765]

$$\mathrm{Im}\Pi_R^{1\leftrightarrow 2} = \mathrm{Im}\Pi_R^{\mathrm{LPM}} - \mathrm{Im}\Pi_R^{\mathrm{LPM\ Born}} + \mathrm{Im}\Pi_R^{\mathrm{Born}}$$



$2 \to 2$ scatterings



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- for s/t and u/t contributions from **both hard and soft momentum regions**

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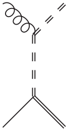
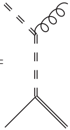
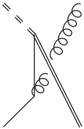
$2 \to 2$ SCATTERINGS

$\mathcal{M}_a =$  $+$ 

$\mathcal{M}_b =$  $+$ 

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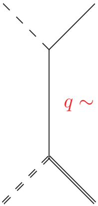

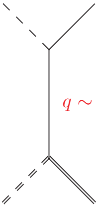

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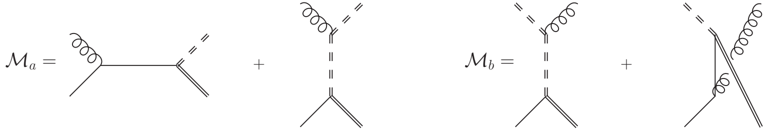

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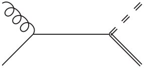
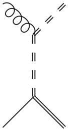
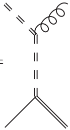
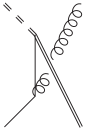
$\mathcal{M}_a =$  $+$ 

$\mathcal{M}_b =$  $+$ 

- Considered by M. Garny and J. Heisig [1809.10135] for $T \leq M$ (possibly with issues about IR of some processes)
- for s/t and u/t contributions from **both hard and soft momentum regions**

D. Besak and D. Bodeker [1202.1288], J. Ghiglieri and M. Laine [1605.07720]

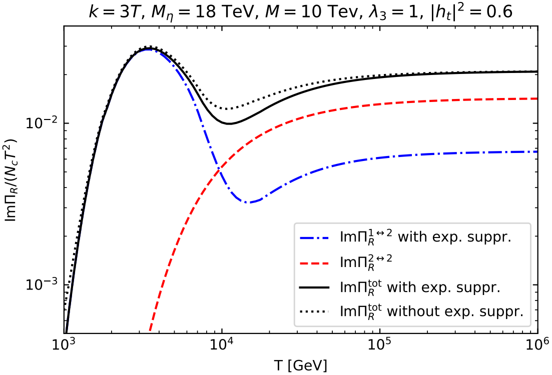
$$
\begin{aligned}
\mathrm{Im}\Pi_R^{2\leftrightarrow 2} \; = \; & \frac{2}{(4\pi)^3 k} \int_k^\infty \mathrm{d}q_+ \int_0^k \mathrm{d}q_- \Big\{ \big[n_{\mathrm{F}}(q_0) + n_{\mathrm{B}}(q_0 - k) \big] N_c \left(Y_q^2 g_1^2 + C_F g_3^2 + |h_q|^2 \right) \Phi_{s2} \Big\} \\
+ \; & \frac{2}{(4\pi)^3 k} \int_0^k \mathrm{d}q_+ \int_{-\infty}^0 \mathrm{d}q_- \Big\{ \big[1 - n_{\mathrm{F}}(q_0) + n_{\mathrm{B}}(k - q_0) \big] N_c \left(Y_q^2 g_1^2 + C_F g_3^2 + |h_q|^2 \right) \Phi_{t2} \\
- \; & \left[n_{\mathrm{B}}(k) + \frac{1}{2} \right] N_c \left(Y_q^2 g_1^2 + C_F g_3^2 + |h_q|^2 \right) \frac{k\pi^2 T^2}{q^2} \Big\} + N_c \frac{m_q^2}{16\pi} \left[n_{\mathrm{B}}(k) + \frac{1}{2} \right] \ln\left(1 + \frac{4k^2}{m_q^2} \right)
\end{aligned}
$$

SUMMARY OF THE RATES

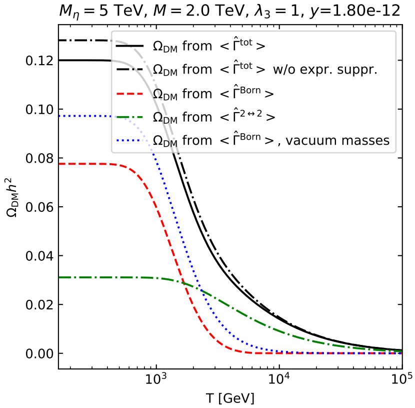
- Phenomenological switch off the high-temperature processes (lack of NLO rates)

$$\kappa(M_\eta) = \frac{3}{\pi^2 T^3} \int_0^\infty dp\, p^2\, n_{\rm B}(E_\eta)[1 + n_{\rm B}(E_\eta)]\,, \qquad \boxed{\Pi_R^{\rm tot} = {\rm Im}\Pi_R^{1\leftrightarrow 2} + {\rm Im}\Pi_R^{2\leftrightarrow 2}}$$

- ${\rm Im}\Pi_R^{1\leftrightarrow 2} = ({\rm Im}\Pi_R^{\rm LPM} - {\rm Im}\Pi_R^{\rm LPM\ Born})\kappa(M_\eta) + {\rm Im}\Pi_R^{\rm Born}$



First example for high-T effects



- integrate over $\boldsymbol{k}$ the rates $(x \equiv \ln(T_{\rm max}/T))$

$$\langle \hat{\Gamma} \rangle = \int_{\boldsymbol{k}} \hat{\Gamma}\, n_{\rm F}(k^0)\,, \quad \hat{\Gamma} \equiv \frac{\Gamma}{3c_s^2 H}\,, \quad \frac{dY}{dx} = 2\frac{\langle \hat{\Gamma} \rangle}{s}$$

- at $T \gg M_\eta, M$

  $\rightarrow$ **non-negligible production of χ!**

- important high-temperature window

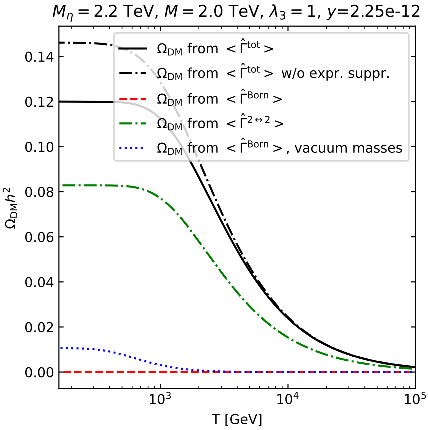
  $2 \to 2$ and effective $1 \leftrightarrow 2$

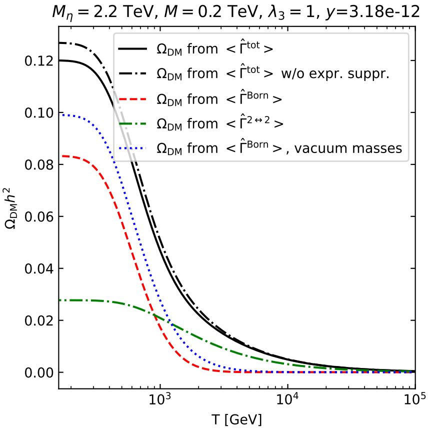
- Born rate with vacuum masses $\Rightarrow$ 20% reduction of $\Omega_{\rm DM} h^2$ with respect to $\Pi_R^{\rm tot}$
- 30% when including thermal masses but excluding $2 \to 2$ and effective $1 \leftrightarrow 2$
- estimation of theoretical error: LPM with and without $\kappa(M_\eta)$, here $\sim 10\%$ effect

LARGE AND SMALL MASS SPLITTINGS

- the smaller $\Delta M/M$ the larger the effect of thermal masses, LPM and $2 \to 2$

  Left plot: $\Delta M/M = 0.1$; Right plot: $\Delta M/M = 10$
- also other parameters of the model are relevant (h_q, λ_3)

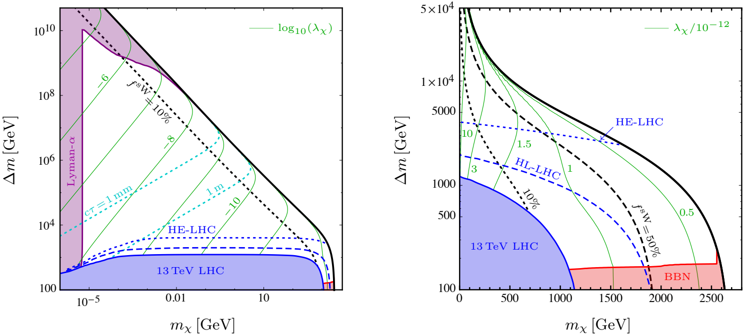




IMPACT ON PHENO AND EXPERIMENTAL SEARCHES

- the relic density within freeze-in implies long-lived particles, $\Gamma_\eta \approx \frac{M_\eta |y|^2}{8\pi}\left(1 - \frac{M^2}{M_\eta^2}\right)^2$
- the colored η together with a SM coloured states can form **R-hadrons**

  A. C. Kraan, J. B. Hansen and P. Nevski hep-ex/0511014, G. Aad *et al.* [ATLAS] 1211.1597, S. Chatrchyan *et al.* 1305.0491, G. Aad *et al.* [ATLAS] 1604.04520
- for decay lengths $c\tau \gtrsim$ *detector size* $\Rightarrow$ stable, otherwise some fraction lost in the detector



[1809.10135]

Summary

- we studied the impact of the ultra-relativistic regime on the production of a feebly interacting DM particle
- Before: in renormalizable models bulk DM population produced at $T \sim M$
- Our work: this is not always the case
  high-temperature $1 \leftrightarrow 2$, $2 \to 2$ can give $\mathcal{O}(1)$ contribution

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- simplified dark matter model:
  χ Majorana fermion DM and η mediator charged under $\mathrm{SU}(3) \otimes \mathrm{U}(1)_Y$
- important effects on freeze-in from $1 \leftrightarrow 2$, $2 \to 2$

$$M = 2\ \mathrm{TeV}, \Delta M = 0.2\ \mathrm{TeV} \Rightarrow \frac{(\Omega h^2)_{\mathrm{full}}}{(\Omega h^2)_{\mathrm{Born}}} \simeq 10$$

$$M = 0.2\ \mathrm{TeV}, \Delta M = 2\ \mathrm{TeV} \Rightarrow \frac{(\Omega h^2)_{\mathrm{full}}}{(\Omega h^2)_{\mathrm{Born}}} \simeq 1.2$$

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- high-T effects can affect other models with similar portals [see back up slides]
- Our main uncertainty comes from the lack of NLO rates (state-of-art M.Laine (2013))

THE BORN RATE WITH VANISHING THERMAL MASSES

- Let us look at the model at hand

$$
\left(\frac{\partial}{\partial t} - Hk_i \frac{\partial}{\partial k_i}\right) f_\chi(t, \boldsymbol{k}) = \Gamma(k)[n_{\mathrm{F}}(k^0) - f_\chi(t, \boldsymbol{k})],
$$

$$
\Gamma(k) = \frac{|y|^2}{k^0} \mathrm{Im}\Pi_R = \frac{|y|^2}{2k^0} \mathrm{Tr}\left\{ \not{\mathcal{K}}\, a_R\, [\rho(\mathcal{K}) + \rho(-\mathcal{K})]\, a_L \right\}\,,
$$

- Retarded correlator, Euclidean correlator and spectral function are connected

The born rate with vanishing thermal masses

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- Retarded correlator, Euclidean correlator and spectral function are connected

$$\Pi^E(K) \equiv \mathrm{Tr}\left\{ i\not{K}\left[\int_X e^{iK\cdot X} a_R \langle(\eta^\dagger q)(X)(\bar{q}\eta)(0)\rangle a_L\right]\right\}$$

$$= N_c \int_{\boldsymbol{p}} T \sum_n \frac{-i\not{P} a_L}{p_n^2 + E_q^2} \frac{1}{(p_n + k_n)^2 + E_\eta^2}$$

- with $E_q = |\boldsymbol{p}| = p$ and $E_\eta = \sqrt{(\boldsymbol{p}+\boldsymbol{k})^2 + M_\eta^2}$

see M. Laine and A. Vuorinen (2017)

Finite-T quark propagator

- the quark thermal mass is momentum-independent only for $p \gg gT$
- if p becomes soft $\Rightarrow$ the full momentum dependence of the quark HTL dispersion relation becomes necessary
- we use the UV limit of the quark HTL propagator

$$S_R(p^0, p \gg gT) = \frac{ih_{\mathrm{p}}^+}{p^0 - p - m_q^2/(2p) + i\epsilon} + \frac{ih_{\mathrm{p}}^-}{p^0 + p + m_q^2/(2p) + i\epsilon},$$

- $h_{\mathrm{p}}^{\pm} = (\gamma^0 \mp \boldsymbol{\gamma} \cdot \boldsymbol{p})/2$, and for $p \gg gT$ we can rewrite it as follows

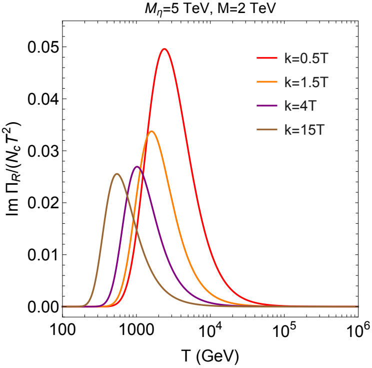
$$S_R(p^0, p \gg gT) = \frac{ih_{\mathrm{p}}^+}{p^0 - \sqrt{p^2 + m_q^2} + i\epsilon} + \frac{ih_{\mathrm{p}}^-}{p^0 + \sqrt{p^2 + m_q^2} + i\epsilon},$$

In-vacuum Born

$$\mathrm{Im}\Pi_R^{\mathrm{Born}} = \frac{N_c(M_\eta^2 - M^2)}{8 n_{\mathrm{F}}(k^0)} \int \frac{d^3\boldsymbol{p}_\eta}{(2\pi)^3} \int \frac{d^3\boldsymbol{p}_q}{(2\pi)^3} \frac{(2\pi)^4 \delta^4(\mathcal{P}_\eta - \mathcal{P}_q - \mathcal{K})}{E_\eta E_q} n_{\mathrm{B}}(E_\eta)(1 - n_{\mathrm{F}}(E_q))$$

$$= \frac{N_c T(M_\eta^2 - M^2)}{16\pi k} \left[\ln\left(\frac{\sinh(\beta(k^0 + p_{\mathrm{max}})/2)}{\sinh(\beta(k^0 + p_{\mathrm{min}})/2)} \right) - \ln\left(\frac{\cosh(\beta p_{\mathrm{max}}/2)}{\cosh(\beta p_{\mathrm{min}}/2)} \right) \right]$$

- $p_{\mathrm{min}} = \frac{M_\eta^2 - M^2}{2(k^0 + k)}$ and $p_{\mathrm{max}} = \frac{M_\eta^2 - M^2}{2(k^0 - k)}$



In-vacuum versus finite thermal masses

- Scalar mass: $\mathcal{M}_\eta^2 = M_\eta^2 + m_\eta^2$, for $\boldsymbol{\eta} \to \boldsymbol{\chi} + \boldsymbol{q}$ and $E_p = \sqrt{p^2 + m_q^2}$

$$\mathrm{Im}\Pi^{\mathrm{Born}}_{\mathrm{R},\eta\to\chi q} = \frac{N_c}{16\pi k}\int_{p_{\min}}^{p_{\max}} dp[\mathcal{M}_\eta^2 - M^2 - m_q^2 - 2k^0(E_p - p)][n_{\mathrm{B}}(k^0 + E_p) + n_{\mathrm{F}}(E_p)]$$

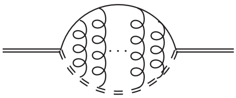
$$p_{\min,\,\max} = \frac{\mathcal{M}_\eta^2 - M^2 - m_q^2}{2M^2}\left| k^0\sqrt{1 - \frac{4M^2 m_q^2}{(\mathcal{M}_\eta^2 - M^2 - m_q^2)^2}} \mp k \right| .$$

- for $k_0 \gg k$ (small temperatures mostly) the phase space vanishes

LPM resummation for light-cone kinematics

$\pi T \gg M_\eta$

- all the particles are seen as *massless*,
- all the momenta of extarnal particles are $p \sim \pi T$,
- particles are ultra-relativistic, and gT is a soft scale
  $\Rightarrow$ collinear kinematics $\approx$ high T



- connection between LPM resummation and $\mathrm{Im}\Pi_R^{\mathrm{LPM}} \Rightarrow \chi$ self-energy
- notation and computational setting

$$\hat{H} \equiv -\frac{M^2}{2k_0} + \frac{m_q^2 - \nabla_\perp^2}{2E_q} + \frac{\mathcal{M}_\eta^2 - \nabla_\perp^2}{2E_\eta} + i\Gamma(y)\,, \quad y \equiv |\boldsymbol{y}_\perp|$$

$$\Gamma(y) = \frac{T}{2\pi} g_1^2 Y_q^2 \left[\ln\left(\frac{m_{\mathrm{B}} y}{2}\right) + \gamma_E + K_0(m_{\mathrm{B}} y)\right] + \frac{T}{2\pi} g_3^2 C_F \left[\ln\left(\frac{m_g y}{2}\right) + \gamma_E + K_0(m_g y)\right]$$

Numerical strategy and LPM Born

- $\hat{H}$ enters the inhomogeneous equations for the functions $g(\boldsymbol{y})$ and $\boldsymbol{f}(\boldsymbol{y})$

$$(\hat{H}+i0^+)g(\boldsymbol{y}) = \delta^{(2)}(\boldsymbol{y})\,, \quad (\hat{H}+i0^+)\boldsymbol{f}(\boldsymbol{y}) = -\nabla_\perp \delta^{(2)}(\boldsymbol{y})$$

$$\begin{aligned}\mathrm{Im}\Pi_R^{\rm LPM} &= -\frac{N_c}{8\pi}\int_{-\infty}^{+\infty} dE_q \int_{-\infty}^{+\infty} dE_\eta\, \delta(k_0-E_q-E_\eta)[1-n_{\rm F}(E_q)+n_{\rm B}(E_\eta)] \\ &\quad \frac{k_0}{E_\eta}\lim_{\boldsymbol{y}\to 0}\left\{\frac{M^2}{k_0^2}\mathrm{Im}[g(\boldsymbol{y})] + \frac{1}{E_q^2}\mathrm{Im}[\nabla_\perp\cdot \boldsymbol{f}(\boldsymbol{y})]\right\}\end{aligned}$$

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$$\begin{aligned}\mathrm{Im}\Pi_{\mathrm{R}}^{\mathrm{LPM}} \quad = \quad & -\frac{N_c}{8\pi}\int_{-\infty}^{+\infty} dE_q \int_{-\infty}^{+\infty} dE_\eta\, \delta(k_0 - E_q - E_\eta)[1 - n_{\mathrm{F}}(E_q) + n_{\mathrm{B}}(E_\eta)] \\ & \frac{k_0}{E_\eta} \lim_{\boldsymbol{y}\to 0} \left\{ \frac{M^2}{k_0^2}\mathrm{Im}[g(\boldsymbol{y})] + \frac{1}{E_q^2}\mathrm{Im}[\nabla_\perp \cdot \boldsymbol{f}(\boldsymbol{y})] \right\}\end{aligned}$$

Once $E_\eta = k^0 - E_q$ by the δ

1. $k^0 > E_q > 0$: this corresponds to the effective $2 \to 1$ process $\eta, q \to \chi$
2. $E_q < 0$: this corresponds to the effective $1 \to 2$ process $\eta \to q\chi$

   $\Rightarrow$ LPM-Born with thermal masses is the $n = 0$ limit (no scatterings)
3. $E_q > k^0$: this corresponds to the effective $1 \to 2$ process $q \to \eta\chi$.

Collinear Born limit

$$\mathrm{Im}\Pi_{\mathrm{R}}^{\mathrm{LPM\,Born}} = \frac{N_c}{16\pi k_0}\int_{E_{\mathrm{min}}}^{E_{\mathrm{max}}} dE_q \frac{E_q(\mathcal{M}_\eta^2 - M^2 - m_q^2) - k_0 m_q^2}{E_q}[n_{\mathrm{B}}(k_0 + E_q) + n_{\mathrm{F}}(E_q)]$$

$$E_{\mathrm{min}}, E_{\mathrm{max}} = \frac{k_0}{2M^2}\left(\mathcal{M}_\eta^2 - M^2 - m_q^2 \mp \sqrt{(\mathcal{M}_\eta^2 - m_q^2)^2 + M^4 - 2M^2(\mathcal{M}_\eta^2 + m_q^2)}\right)$$

- In the $m_q \to 0$ and $\mathcal{M}_\eta \to M_\eta$ limit, we obtain

$$\mathrm{Im}\Pi_{\mathrm{R}}^{\mathrm{LPM\,Born}}\Big|_{m_q=0} = \frac{N_c T(M_\eta^2 - M^2)}{16\pi k}\ln\frac{2n_{\mathrm{B}}(k_0)(e^{k_0 M_\eta^2/(M^2 T)} - 1)}{e^{k_0 M_\eta^2/(M^2 T) - k_0/T} + 1}$$

- it agrees with the collinear limit of in-vacuum Born, i.e. taking $k^0 - k \ll k^0 + k$

DM energy density and super-WIMP contribution

$$(\Omega_{\mathrm{DM}} h^2)_{\text{obs.}} = (\Omega_{\mathrm{DM}} h^2)_{\text{freeze-in}} + (\Omega_{\mathrm{DM}} h^2)_{\text{super-WIMP}}$$

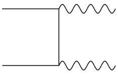
- η particles stays in chemical equilibrium till late times $T \sim M_\eta/25$
- there is a populations of η, as governed by freeze-out, which decays into χ

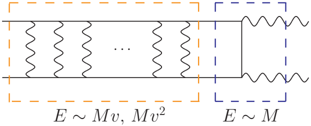
M. Garny, J. Heisig (2018)

$$(\Omega_{\mathrm{DM}} h^2)_{\text{super-WIMP}} = \frac{M}{M_\eta} (\Omega h^2)_\eta \, .$$

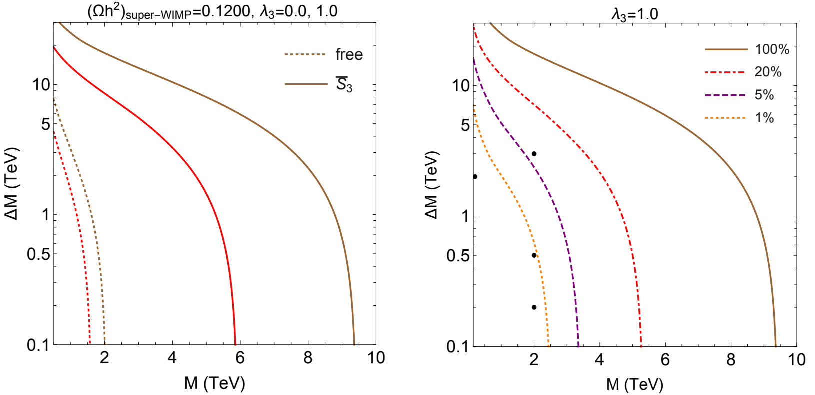
$$\frac{dn_\eta}{dt} + 3Hn_\eta = -\langle \sigma_{\text{eff}} v \rangle (n_\eta^2 - n_{\eta,\text{eq}}^2) \, , \quad \langle \sigma_{\text{eff}} v \rangle = \frac{c_3 \bar{S}_3 + c_4 C_{\mathrm{F}} \bar{S}_4}{N_c}$$

A. Mitridate et al (2017), S. B. and M. Laine (2018), S. B. and S. Vogl (2019), J. Hartz and K. Petraki (2018)





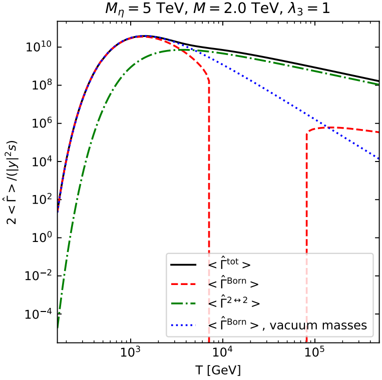
Benchmark values with bound-state effects

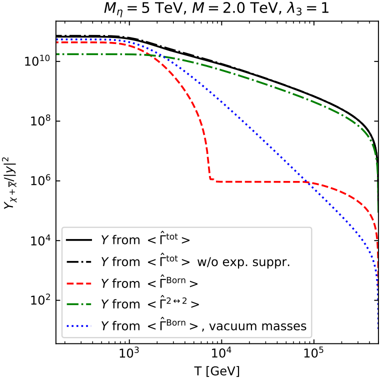


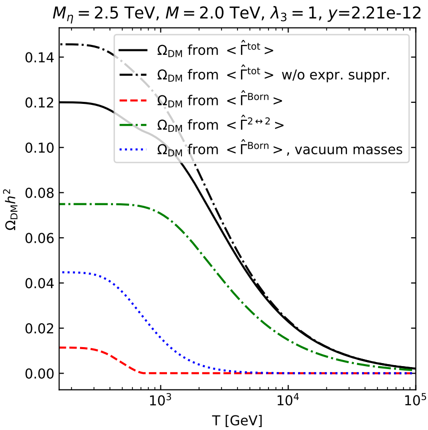
- P0 ($M = 2.0$ Tev, $M_\eta = 5.0$ TeV); P1 ($M = 0.2$ Tev, $M_\eta = 2.2$ TeV); P2 ($M = 2.0$ Tev, $M_\eta = 2.5$ TeV); P3 ($M = 2.0$ Tev, $M_\eta = 2.2$ TeV);

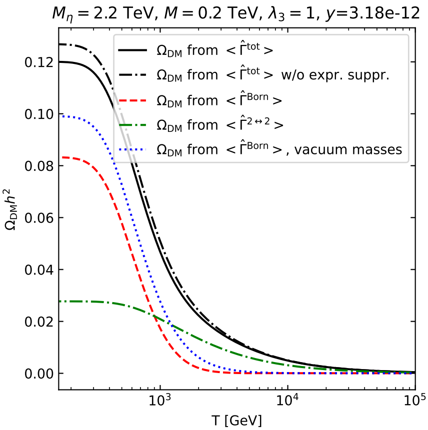
- $n_{\rm DM} = 2\int_k f_\chi(k)$, and defining $Y = n_{\rm DM}/s$

$$\frac{d\,Y}{d\,x} = 2\frac{\langle\hat\Gamma\rangle}{s}\,, \qquad x \equiv \ln(T_{\rm max}/T)$$









- for $M_\eta = 2.2$ TeV the freeze-in production stops fairly close to $T_c \simeq 150$ GeV
  $\Rightarrow$ follow DM production in the SM broken phase
- CMS analysis provides us with $M_\eta > 1250$ GeV CMS-PAS-EXO-16-036

NLO in the relativistic regime

- NLO corrections come both in the form of $2 \leftrightarrow 2$ and $1 \leftrightarrow 3$ real processes with full account of M_η and M
- virtual processes, that is, the interference of the Born amplitude with thermal one-loop corrections thereto
- real and virtual processes are separately IR divergent and need to be consistently regulated
- See refs. on Majorana neturino production by M.Laine [1304.0202,1307.4909,1310.0164]

- M. Garny, A. Ibarra and S. Vogl [1503.01500], Majorana DM fermion with SM charged mediators

1) χ couples to a RH quark or (lepton) with hypercharge Y, then η is a color triplet (singlet), a singlet under SU(2) and $Y_\eta = -Y$

$$\mathcal{L}_{\rm int} = -y\bar{\chi}P_R f \eta + \text{h.c.} - \lambda_3 \phi^\dagger \phi \eta^\dagger \eta$$

2) χ couples to a LH quark or (lepton) doublet with hypercharge Y, then η is a color triplet (singlet), a doublet under SU(2) and $Y_\eta = -Y$

$$\mathcal{L}_{\rm int} = -y\bar{\chi}P_L f\, i\sigma^2 \eta + \text{h.c.} - \lambda_3 \phi^\dagger \phi \eta^\dagger \eta - \lambda_4 \phi^\dagger \eta \eta^\dagger \phi$$

- G. Bélanger et *al* [1811.05478], the mediator (F) is a fermion rather than a scalar particle, the DM is a real scalar (S) rather than a Majorana fermion

$$\mathcal{L}_{\rm int} = -y\, S\bar{F}P_R f + \text{h.c.}$$