Axion Gravitodynamics

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Workshop on the Standard Model and Beyond

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Axion Electrodynamics

The θ-term in electrodynamics

Tong (2018)
Litvinov (2020)

- Maxwell action

$$\mathcal{S}_{em} = -\frac{1}{4}\int \mathrm{d}^4x F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\int \mathrm{d}^4x \left(\mathbf{E}^2 - \mathbf{B}^2\right)$$

  – $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,,\; E_i = F_{0i}\,,\; F_{ij} = -\epsilon_{ijk}B_k$

- Extra term seemingly of equal importance (Chern-Simons):

$$\mathcal{S}_\theta = \kappa\theta \int \mathrm{d}^4x \tilde{F}^{\mu\nu}F_{\mu\nu} = \kappa\theta\int \mathrm{d}^4x \mathbf{E}\cdot\mathbf{B}$$

  – Two cases of κ
    - classical case $\kappa = 1$
    - quantum case $\kappa = \frac{e^2}{16\pi^2\hbar}$
  – θ dimensionless constant
  – $\tilde{F}^{\mu\nu}$ the dual tensor: $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$
  – Same as $F_{\mu\nu}$ with $\mathbf{E}\to\mathbf{B}$ and $\mathbf{B}\to-\mathbf{E}$

- θ term can be written as total derivative - topological

Axion Electrodynamics

Qi, Hughes, Zhang (2008)
Qi, Zhang (2010)
Qi, Zhang, Witten (2012)

- Let us consider $\theta = \theta(\vec{x}, t)$
- Total action would be

$$\mathcal{S} = \int \mathrm{d}^4x \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{e^2}{16\pi^2\hbar} \theta(\vec{x}, t) \tilde{F}^{\mu\nu} F_{\mu\nu} \right)$$

- Lorentz and gauge invariant
- Eom - modified Maxwell eqns - vacuum axion electrodynamics[1]

$$\vec{\nabla} \cdot \vec{E} = -\frac{\alpha c}{\pi} \vec{\nabla}\theta \cdot \vec{B} \,, \quad -\frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} + \vec{\nabla} \times \vec{B} = \frac{\alpha}{\pi c} \left(\dot{\theta} \vec{B} + \vec{\nabla}\theta \times \vec{E} \right)$$

- The rest (Bianchi identities) remain the same
- Magnetic field acts like charge density
- Combination acts like current density

[1] A bit of overstatement - not a dynamical field

Time-Reversal Invariant Topological Insulators

- With θ constant - CP-violating term (like in QCD)
  CP : $E \to E$, CP : $B \to -B$, CP : $EB \to -EB$
- Extra term $(E \cdot B)$ is pseudoscalar
- Spacetime-dependent θ also CP-violating
- Two cases for θ to absorb are:
  - $\theta = 0$ (trivial case)
  - CP : $\theta \to -\theta$, $\theta = \pi$ (θ periodic[2]) $\to$ symmetry reinstated
- In Condensed Matter context use of time reversal instead of CP (they transform the fields in the same way)
- Topological Insulators whose dynamics are characterized by $\theta = \pi$
- Special about them: Time-Reversal Invariant (CP-preserving)

[2] Due to arguments involving the partition function

Topological Magnetoelectric Effect

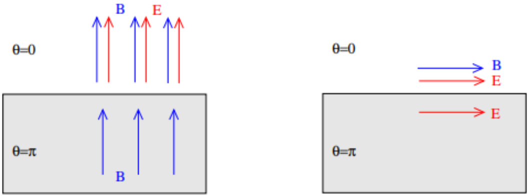

Figure: TI Magnetoelectric effect

- Magnetic field perpendicular to the interface
- Modified Gauss law: effective accumulation of surface charge density $\rightarrow$ electric field outside the TI
- Electric field tangential to the interface
- Modified Ampere law: Electric field acts as surface current $((\partial_z\theta) \neq 0) \rightarrow$ magnetic field outside the TI

Image from Tong (2018)

GravitoElectroMagnetism

Basic GravitoElectroMagnetism (GEM)

Medina (2006)
Mashhoon (2008)

- Basic Idea
  - Mass produces gravitoelectric field
  - Mass current produces gravitomagnetic field
- Due to the fact that General Relativity is compatible to Lorentz invariance (at least locally) - like e/m
- No analogue of magnetic field in Newton's gravity - not a relativistically invariant theory

→ Static (Newton's) Gravity + Special Relativity =
Gravitomagnetism

- GEM: subject dealing with effects caused by mass-energy currents

No Physical Electric or Magnetic fields are predicted this way - Analogy is only mathematical

Frame dragging - Lense-Thirring effect

- Frame-dragging (GR prediction): Effect on spacetime due to non-static stationary distributions of mass-energy

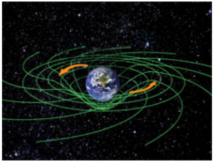

Figure: Spacetime dragged

Lense, Thirring (1918)

- Lense-Thirring effect (Rotational frame dragging): Massive rotating object distorting spacetime metric

→ Precession of the orbit of nearby test particles

- Effect not happening in Newtonian mechanics - gravitational field depends only on mass (not rotation)
- Small effect: Either very massive rotating object or very sensitive instruments
- Frame dragging caught recently in action (GP-B)

Image from https://einstein.stanford.edu/MISSION/mission1.html

From Einstein to Maxwell

- Einstein Field Equation (Curvature due to matter-energy):

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

- For a Maxwell-like form → Weak Field Approximation to GR
- Spacetime metric considered as a linear perturbation of the flat spacetime

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,, \;\; |h_{\mu\nu}| \ll 1$$

- Define the trace-reversed quantity

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\,, \text{where } \; h = \eta^{\mu\nu}h_{\mu\nu}$$

- Keeping only linear terms and applying the De Donder gauge

$$\partial_\mu \bar{h}^{\mu\nu} = 0$$

Fundamental Equations of the Linearized theory of Gravity

Analogy to the corresponding Maxwell's equations is evident $\Box A^{\mu} = 4\pi j^{\mu}$

- The EFE become

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

- Let us assume the source of this weak field to be a rotating astronomical source consisting of slowly ($|\vec{v}| \ll c$) moving matter

$$T_{\mu\nu} = \begin{pmatrix} \rho_g c^2 & c j_g^i \\ c j_g^i & T_{ij} \end{pmatrix}, \; T^{00} = \rho_g c^2 \, , \; T^{0i} = j_g^i = \rho_g v^i$$

  - Almost no pressure
  - almost no shear
  - small momentum density (slowly moving matter)
  - main contribution the mass density

$$|T^{00}| \gg |T^{0i}| \gg |T^{ij}|$$

- Define the potentials
  - Gravitoelectric: $\bar{h}^{00}(\vec{x},t) = -\frac{4\Phi_g(\vec{x},t)}{c^2}$
  - Gravitomagnetic: $\bar{h}^{0i}(\vec{x},t) = \frac{2A_g^i(\vec{x},t)}{c^2}$
  - $\bar{h}^{ij} \sim 1/c^4$ and is omitted - no relevant potential involved
- Define the physical fields (in e/m analogy):
  - Gravitoelectric: $\vec{E}_g = -\vec{\nabla}\Phi_g - \frac{1}{c}\frac{\partial}{\partial t}(\frac{1}{2}A_g)$
  - Gravitomagnetic: $\vec{B}_g = \vec{\nabla} \times \vec{A}_g$
- De Donder gauge in terms of e/m-like potentials

$$\partial_\mu \bar{h}^{\mu\nu} = 0 \Rightarrow \frac{2}{c}\frac{\partial}{\partial t}\Phi_g + \vec{\nabla}\vec{A}_g = 0 \text{ and } \dot{\vec{A}} = 0$$

- Combining:
  1. Linearized EFE
  2. De Donder gauge
  3. Potential and Field definitions

  $\Rightarrow$ GEM Field Equations

GEM Field Equations VS Maxwell Equations

GEM equations

$$\vec{\nabla} \cdot \vec{E}_g = -4\pi G \rho_g$$

$$\vec{\nabla} \times (\frac{1}{2}\vec{B}_g) - \frac{1}{c}\frac{\partial \vec{E}_g}{\partial t} = -\frac{4\pi G}{c}\vec{j}_g$$

$$\vec{\nabla} \cdot (\frac{1}{2}\vec{B}_g) = 0$$

$$\vec{\nabla} \times \vec{E}_g = -\frac{1}{c}\frac{\partial}{\partial t}(\frac{1}{2}\vec{B}_g)$$

Maxwell equations

$$\vec{\nabla} \cdot \vec{E} = 4\pi \rho$$

$$\vec{\nabla} \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\vec{j}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

Therefore we understand:

- GR in the WFA admits a Maxwell description
- Prediction of the physical field analogous to the magnetic
- E/M analogy: A rotating charged ball producing magnetic field

GEM Example

- Solving the GEM Ampere's law:

$$\nabla^2 \vec{A} = -4\pi G \vec{j}$$

for a massive (uniformly distributed) spinning spherical body of radius R (stationary source):

$$\vec{J} = \rho\, \vec{\omega} \times \vec{r}\, \Theta(R - r)$$

- Find the magnetic field everywhere

$$\vec{B}_{GR} = \frac{4\pi G \rho R^2}{15} \times \begin{cases} \left(5 - 3\frac{r^2}{R^2}\right)\vec{\omega} + 3\frac{r^2}{R^2}\hat{r} \times (\hat{r} \times \vec{\omega})\,, r \leq R\,, \\ \frac{R^3}{r^3}\left((2\vec{\omega} + 3\hat{r} \times (\hat{r} \times \vec{\omega})\right)\,, r \geq R \end{cases}$$

- Poloidal contribution - No toroidal

Axion Electrodynamics i.e. modifications on the Maxwell action led to observable effects in Condensed Matter Physics

$$\Downarrow$$

Modify Gravitodynamics along the same lines to examine if there are corresponding (observable) effects

Chern-Simons Gravitodynamics

Chern-Simons θ-term

Smith, Erickcek, Caldwell, Kamionkowski (2008)

- GEM analogue of the previous θ term
- BUT not axion gravitodynamics...yet!
- Addition to GR of a Chern-Simons term with θ dynamical
- The total action is:

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2\kappa^2}R + \frac{\ell}{12}\theta\mathbf{R}\tilde{\mathbf{R}} - \frac{1}{2}(\partial\theta)^2 - V(\theta) + \mathcal{L}_m\right]$$

- $\mathbf{R}\tilde{\mathbf{R}} = R^{\beta}{}_{\alpha}{}^{\gamma\delta}\tilde{R}^{\alpha}{}_{\beta\gamma\delta} \quad \tilde{R}^{\mu}{}_{\nu\alpha\beta} = \frac{1}{2}\epsilon_{\sigma\tau\alpha\beta}R^{\mu}{}_{\nu}{}^{\sigma\tau}$
- EOM for θ: $\Box\theta = \frac{\mathrm{d}V}{d\theta} - \frac{\ell}{12}\mathbf{R}\tilde{\mathbf{R}}$
- Gravitational field equations:

$$G_{\mu\nu} - \frac{2}{3}\ell\kappa^2 C_{\mu\nu} = -\kappa^2 T_{\mu\nu}$$

Chern-Simons GEM equations

Carroll (1998)

Tsujikawa (2003)

- Assume only temporal dependence $\theta(\vec{x},t) = \theta(t)$ - quintessence (spatially homogeneous) *Jackiw, Pi (2003)*

Carroll, Jackiw, Field (1989)

- Not Lorentz Invariant: $\partial\theta$ points at the cosmic time direction
- Assume perturbation around the flat metric: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- Work with the trace-reversed perturbation: $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$
- Assume de Donder condition: $\partial^\mu \bar{h}_{\mu\nu} = 0$
- Define the 4-vector potential: $A_\mu = -\frac{1}{4}\bar{h}_{\mu 0}$ and $T_{0i} = J_i$
- Linearized temporal-spatial field equations:

$$G_{0i}^{lin} - \frac{2}{3}\ell\kappa^2 C_{0i}^{lin} = -\kappa^2 T_{0i}$$

- Modified GEM Ampere law:

$$\vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} - \frac{1}{m_{cs}}\Box\vec{B} = 4\pi G\vec{J}, \quad m_{cs} = \frac{-3}{\ell\kappa^2\dot{\theta}}$$

- Temporal field equations - Gauss law - remains the same

Chern-Simons modification on the Field equations

- Assuming homogeneous rotating sphere
- The PDE for the vector potential:

$$\nabla^2 \vec{A} + m_{cs}^2 \vec{A} = m_{cs}^2 G \left(\mathbf{I} - \frac{1}{m_{cs}} \vec{\nabla} \times \right) \int \frac{\vec{J}}{|\vec{r} - \vec{r}'|} \mathrm{d}^3 r'$$

- Find $\vec{A}$ using the Green's function for the inhomogeneous Helmholtz equation
- The modification to magnetic field is:

$$\vec{B}_{cs} = 4\pi G \rho R^2 \left(D_1(\vec{r})\vec{\omega} + D_2(\vec{r})\hat{r} \times \vec{\omega} + D_3(\vec{r})\hat{r} \times (\hat{r} \times \vec{\omega}) \right)$$

- For $r \leq R$: $$\begin{aligned} D_1(\vec{r}) &= \tfrac{2}{(m_{cs}R)^2} + \tfrac{2R}{r} y_2(m_{cs}R) j_1((m_{cs}r) \\ D_2(\vec{r}) &= \tfrac{m_{cs}r}{(m_{cs}R)^2} + m_{cs}R y_2(m_{cs}R) j_1(m_{cs}r) \\ D_3(\vec{r}) &= m_{cs}R y_2(m_{cs}R) j_2(m_{cs}r) \end{aligned}$$
- For $r \geq R$: $$\begin{aligned} D_1(\vec{r}) &= \tfrac{2R}{r} j_2(m_{cs}R) y_1((m_{cs}r) \\ D_2(\vec{r}) &= m_{cs}R j_2(m_{cs}R) y_1(m_{cs}r) \\ D_3(\vec{r}) &= m_{cs}R j_2(m_{cs}R) y_2(m_{cs}r) \end{aligned}$$

- Quantitative: Change the existing component along the $\vec{\omega}$ and $\hat{r} \times (\hat{r} \times \vec{\omega})$
- Qualitative: Introduce an extra component perpendicular to the $\vec{\omega} - \hat{r}$ plane
  - GR: Toroidal mass current $\rightarrow$ poloidal gravitomagnetic field
  - CS: Addition of toroidal component to the gravitomagnetic field

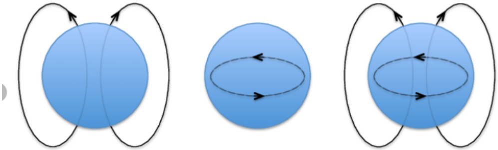

Figure: Poloidal and toroidal gravitomagnetic components

- Notes:
  - Parity-violating extra $\theta(x)$ term led to the emergence of the toroidal component
  - For $m_{cs} \rightarrow \infty$: GR

Image from A. Ohnisi in Researchgate

Gyroscopic precession - Boundary on m_{cs}

- Gravity Probe b satellite (2004-2005) verified GR predictions on frame dragging up to 19%[3]
- Telescope pointing on a guide star and four gyroscopes aligned
- Executing polar orbits around earth
- Rate of change of gyroscopes: $\dot{\vec{S}} = 2\vec{B} \times \vec{S}$, $\vec{S}$ the angular momentum of the gyroscope
- Relative to GR they found:

$$\frac{\dot{\Phi}_{cs}}{\dot{\Phi}_{GR}} = 15\frac{a^2}{R^2} j_2(m_{cs}R)[y_1(m_{cs}a) + m_{cs}ay_0(m_{cs}a)]$$

- $\dot{\Phi} = \frac{|\dot{\vec{S}}|}{|\vec{S}|}$: rate of change of the axis of Φ due to gravitomagnetic field
- Lower limit on the Chern-Simons mass: $|m_{cs}| \gtrsim 0.01\text{km}^{-1}$

[3]The time the paper was published the data was not published and the authors considered a 10% verification

Why not axion gravitodynamics yet?

- In terms of gravitoelectric and gravitomagnetic fields the Chern-Simons term $\mathbf{R}\tilde{\mathbf{R}}$ can be written as:

$$\mathbf{R}\tilde{\mathbf{R}} = -16(\partial_i E_j)(\partial_k B_\ell)(\eta^{ik}\eta^{j\ell} + \eta^{i\ell}\eta^{jk})$$

- Extra derivatives from the EM case

- Can we find a term in the initial (non-linear) theory and result with the GEM analogue of the axion electrodynamics term $\vec{E} \cdot \vec{B}$?

→ Axion gravitodynamics

Axion Gravitodynamics

Torsion-induced gravitational θ-term

Chatzistavrakidis, Karagiannis, Schupp (2020)

- Write down GR action in a torsion inclusive form (anholonomy coeffs, torsion, contorsion)

Nieh, Yan (1981)

- In Weitzenböck connection, the topological Nieh-Yan term gives a nice candidate for a θ-term

$$\mathcal{S}_\theta = \frac{c^4}{64\pi G}\int \theta \eta_{ab}(\mathrm{d}e^a \wedge \mathrm{d}e^b)$$

- More generally:

$$\mathcal{S} = \mathcal{S}_G + \mathcal{S}_\theta + \mathcal{S}_m$$

  - $\mathcal{S}_G$: GR written in a form including e
  - $\mathcal{S}_\theta$: general form of Nieh-Yan $(R \neq 0) \to \theta \mathrm{d}(e^a \wedge T_a)$

Chabdia, Zanelli (1997)

- For constant θ - no contribution $\to \theta = \theta(\vec{x}, t)$

- Trace-reversed metric
- De donder gauge
- Linearized limit

→ Field equations:

$$\Box \bar{h}_{\mu\nu} - \partial^\alpha \theta \epsilon_{\alpha\beta\gamma(\mu} \partial^\beta \bar{h}^\gamma_{\nu)} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

- After potential and field definition → modified GEM field equations:

$$\vec{\nabla} \cdot \vec{E} = -4\pi G \rho - \frac{1}{2} \vec{\nabla}\theta \cdot \vec{B}$$

$$\vec{\nabla} \times \vec{B} - \frac{2}{c} \frac{\partial \vec{E}}{\partial t} = -\frac{8\pi G}{c} \rho \vec{v} + \vec{\nabla}\theta \times \vec{E} + \frac{1}{2c} \frac{\partial \theta}{\partial t} \vec{B}$$

- Bianchi ids remain the same
- The θ-term reduces to the $\vec{E} \cdot \vec{B}$ → (almost) axion gravitodynamics

To true axion gravitodynamics

- Upgrade the background field to a dynamical one
- Action (linearized)

$$\mathcal{S} = \frac{1}{\kappa} \int \mathrm{d}^4 x \left(E^2 - B^2 - 2\ell\theta \vec{E}\vec{B} - \frac{\kappa}{2}(\partial\theta)^2 - \kappa V(\theta) \right) + S_m$$

- ℓ a length scale - for $\ell \to 0$ Linearized EH
- Field equation for the θ field

$$\Box\theta = \frac{2\ell}{\kappa}\vec{E}\cdot\vec{B} + V'(\theta)$$

- Field equations rewritten in terms of mass scale parameters $m = \frac{\ell\dot{\theta}}{2}$ and $\vec{p} = \frac{\ell\vec{\nabla}\theta}{2}$

$$\vec{\nabla}\cdot\vec{E} = -\frac{\kappa}{2}\rho - \vec{p}\cdot\vec{B}$$

$$\vec{\nabla}\times\vec{B} - 2\dot{\vec{E}} = -\kappa\vec{J} + 2\vec{p}\times\vec{E} + m\vec{B}$$

- Opposite GR limit from CS case

- Assuming quintessence-like θ field and the same kind of source $\vec{J} = \rho r \Theta(R-r)\vec{\omega} \times \hat{r}$
- Ampere law takes the form

$$(\nabla^2 + m^2)\vec{B} = \kappa(m\vec{J} + \vec{\nabla} \times \vec{J})$$

- Gravitomagnetic field is found to be

$$\vec{B}_{NY} = \kappa\rho R^2(f_1(r)\vec{\omega} + f_2(r)\hat{r} \times \vec{\omega} + f_3(r)\hat{r} \times (\hat{r} \times \vec{\omega}))$$

- For $r \leq R$:

$$f_1(r) = \frac{2}{m^2 R^2} + \frac{2R}{r} y_2(mR) j_1(mr)$$
$$f_2(r) = -\frac{r}{mR^2} - mR y_2(mR) j_1(mr)$$
$$f_3(r) = mR y_2(mR) j_2(mr)$$

- For $r \geq R$:

$$f_1(r) = \frac{2R}{r} j_2(mR) y_1(mr)$$
$$f_2(r) = -mR j_2(mR) y_1(mr)$$
$$f_3(r) = mR j_2(mR) y_2(mr)$$

- GP B result in upper bound: $m_{ny} \lesssim 5 \times 10^{-5} \mathrm{km}^{-1}$

Conclusions

- Motivated by axion electrodynamics and Chern-Simons gravitodynamics found an axion gravitodynamics action (Nieh-Yan)
- Assumed time-dependence on the axion field
- Solved the problem for a massive spherical spinning object
- Calculated an upper bound on the m parameter of the problem

More to examine

- Find the corresponding Cotton-York tensor corresponding to the Nieh-Yan term
- Obtain the dispersion relation and examine the modifications on the GR gravitational waves predictions
- Try the static case of the θ-term

Thank you for your attention