Sky Meets Laboratory via Renormalization Group Equations:: Higgs Inflation and Light Dark Sector

Anish Ghoshal

Istituto Nazionale di Fisica Nucleare
Tor Vergata, Rome, Italy

anish.ghoshal1@protonmail.ch
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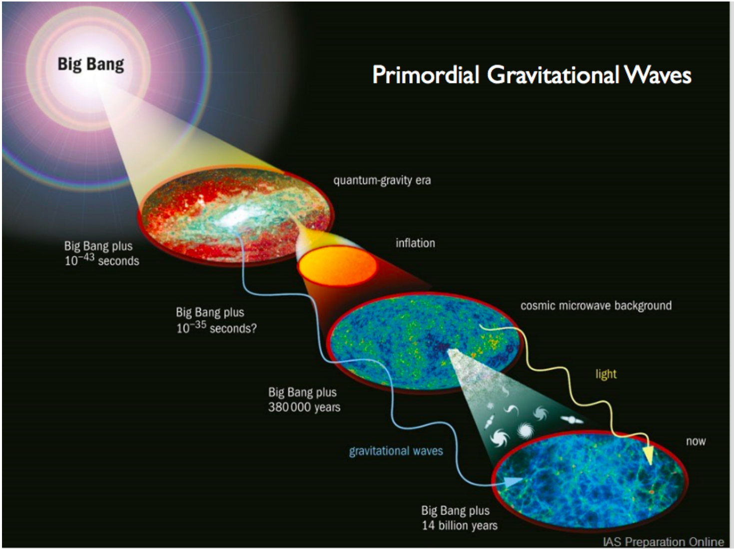
Outline of the talk:

- Ultraviolat (UV) and Infrared (IR)physics is connected via RGE flow.
- Complementarity between Lab and Cosmic observables.
- Inflation and Scalar Field Models of Inflation.
- Particle Models: Higgs Inflation
- Particle Model: Creating an Inflection-point
  - Creating an Inflection-point via RGE.
  - Complementary Probes via CMB & Light Dark Sector Experiments.
- Various model-building: dark matter, neutrinos & conformal models.
- Conclusion

History of the Universe



Inflation: Motivations

- Cosmic Inflation, characterised as quasi-de Sitter expansion is invoked to solve the problems of Big Bang Cosmology:
  - Horizon Problem.
  - Flatness Problem.
  - Origin of Primordial density fluctuations seen in CMB.
  - Monopole problem.
  - Others...
- Slow-roll Inflation:
  - A scalar field inflaton rolling down a potential.
  - This potential needs to be flat from CMB constraints.

Planck 2018 and Constraints

Constraints on scalar and tensor perturbations from the PLANCK satellite

Observational constraints :

$$\begin{cases} \Delta_\zeta(k_0) = 2.137^{+0.063}_{-0.061} \times 10^{-9}, \\ n_s = 0.968 \pm 0.006, \\ r < 0.11, \\ k_0 = 0.002\text{Mpc}^{-1}. \end{cases}$$

(TT+lowP+lensing)

Theoretical predictions :

$$\begin{cases} \Delta_\zeta(k) \simeq \frac{1}{8\pi^2\epsilon}\left(\frac{H}{M_G}\right)^2, \\ n_s - 1 = \frac{d\ln\Delta_\zeta(k)}{d\ln k} \simeq -2\epsilon - 2\eta, \\ \Delta_h(k) \simeq \frac{2}{\pi^2}\left(\frac{H}{M_G}\right)^2, \quad n_T = \frac{d\ln\Delta_h(k)}{d\ln k} \simeq -2\epsilon, \\ r \equiv \frac{\Delta_h(k)}{\Delta_\zeta(k)} \simeq 16\epsilon (= -8n_T). \end{cases}$$

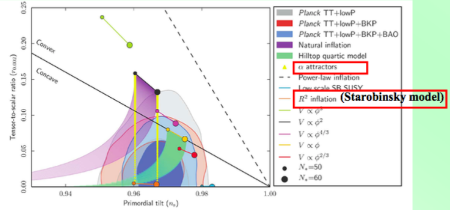


Fig. 54. Marginalized joint 68 % and 95 % CL regions for n_s and $r_{0.002}$ from *Planck* alone and in combination with its cross-correlation with BICEP2/Keck Array and/or BAO data compared with the theoretical predictions of selected inflationary models.

Attractor models like Starobinsky model fit the data well.

Planck 2015 results. XX

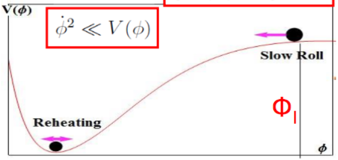
Scalar Field with Slow-roll

Single Scalar Field: Slow Roll Inflation Scenario

- **Slow Roll Inflation**

$$\epsilon(\phi) = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2$$
$$\eta(\phi) = M_P^2\left(\frac{V''}{V}\right)$$
$$\zeta^2(\phi) = M_P^4\left(\frac{V'V'''}{V^2}\right)$$

Slow Roll $\{\epsilon,\ |\eta|,\ \zeta^2\} \ll 1$



- **e-folds:** $N = \frac{1}{M_P^2}\int_{\phi_E}^{\phi_I}\left(\frac{V}{V'}\right)d\phi$

N ≃ 60

To solve the horizon problem

- **Observables**

$$\begin{aligned} r &= 16\epsilon \\ n_s &= 1 - 6\epsilon + 2\eta \\ \alpha &= \frac{dn_s}{d\ln k} = 16\epsilon - 24\epsilon^2 - 2\zeta \\ \Delta_{\mathcal{R}}^2 &= \frac{1}{24\pi^2}\frac{1}{M_P^4}\frac{V}{\epsilon}\Big|_{k_0=0.002\ \mathrm{Mpc}^{-1}} \end{aligned}$$

Planck 2015 Measurements

$$\begin{aligned} r &\leq 0.11 \\ n_s &\simeq 0.9655 \pm 0.0062 \\ \alpha &= -0.0057 \pm 0.0071 \\ \Delta_{\mathcal{R}}^2 &= 2.195 \times 10^{-9} \end{aligned}$$

Non-minimally Coupled Inflaton

Non-minimal Quartic Inflation: simple & successful scenario

Action in Jordan Frame

See, for example,
NO, Rehman & Shafi, PRD 82 (2010) 04352

$$\mathcal{S}_J = \int d^4x\sqrt{-g}\left[-\frac{1}{2}f(\phi)\mathcal{R} + \frac{1}{2}g^{\mu\nu}\left(\partial_\mu\phi\right)\left(\partial_\nu\phi\right) - V_J(\phi)\right],$$

- Non-minimal gravitational coupling

$$f(\phi) = (1 + \xi\phi^2)$$ with a real parameter $\xi > 0$,

- Quartic coupling dominates during inflation

$$V_J(\phi) = \frac{1}{4}\lambda\phi^4$$

3

ϕ can be the Standard Model Higgs field or any other scalar field.
Slides (N Okada).

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Quartic Higgs Inflation

Inflationary Predictions VS Planck 2018 results

ξ	ϕ_0/M_p	ϕ_e/M_p	n_s	r	$\alpha(10^{-4})$	λ
0	22.1	2.83	0.951	0.262	-8.06	1.43×10^{-13}
0.00333	22.00	2.79	0.961	0.1	-7.03	3.79×10^{-13}
0.00642	21.85	2.76	0.963	0.064	-7.50	3.79×10^{-13}
0.0689	18.9	2.30	0.967	0.01	-5.44	6.69×10^{-12}
1	8.52	1.00	0.968	0.00346	-5.25	4.62×10^{-10}
10	2.89	0.337	0.968	0.00301	-5.24	4.01×10^{-8}

- Only one free parameter ξ decides the scenario.
- CMB can be satisfied as long as $\xi \geq O(10^{-2})$.
- No direct sensitivity to particle model-building and laboratory observables as any scalar with such a potential can be the inflaton.

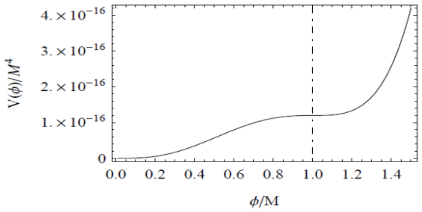
Q: Can we have a scenario be one-to-correspondence between particle properties like coupling & mass and CMB values ? Or else, lots of degeneracies in cosmology. In the similar spirit as DM relic density, or the baryon asymmetry particle physics models ?

Inflection-point Inflation

- Inflection-point Inflation is a small-field ($\phi_I \leq M_{pl}$) inflationary scenario where the scalar field potential is expanded around a point-of-ínflection M in its plane.
- Conditions for inflection-point: $V'(\phi_I) \simeq 0$.. $V''(\phi_I) \simeq 0$..

- **Potential expansion around the inflection-point**

$$V(\phi) \simeq V_0 + V_1(\phi - M) + \frac{V_2}{2}(\phi - M)^2 + \frac{V_3}{6}(\phi - M)^3$$



$M = \phi_I$

Idea is to make the cubic term dominate in the potential !

Inflection-point Analysis for PLANCK Data

- **Summary** of Inflection-point inflation analysis:

Constraint on Potential to satisfy Planck 2015 inflationary measurements

$$\frac{V_1}{M^3} \simeq 1961 \left(\frac{M}{M_P}\right)^3 \left(\frac{V_0}{M^4}\right)^{3/2},$$
$$\frac{V_2}{M^2} \simeq -1.725 \times 10^{-2} \left(\frac{M}{M_P}\right)^2 \left(\frac{V0}{M^4}\right)$$
$$\frac{V_3}{M} \simeq 6.989 \times 10^{-7} \left(\frac{M}{M_P}\right) V_0^{1/2}$$

$M = \phi_I$

$N = 60$; $n_s = 0.9655$
$r = 0.11$; $\Delta_R^2 = 2.195 * 10^{-9}$

Free Parameters:
V_0, M

- **Model-independent Prediction** for the Running of the Spectral Index

$$\alpha \simeq -2\zeta^2(M) = -\, 2.742 \times 10^{-3} \left(\frac{60}{N}\right)^2$$

Planck 2015

$$\alpha = -0.0057 \pm 0.0071$$

- The future experiments can reduce the error to ±0.002.

(*Abazajian et. al. , arXiv:1309.5381*)

- Hence this prediction can be tested in the future.

BSM Model

With such an analysis in hand, let us now ask the following:

- We stick to quartic potential (only re-normalizable term in QFT sense).
- Can the SM Higgs play such a role ?
- Can in any BSM Higgs motivated from neutrino, dark matter, axion or flavor models play such a role ?
- Will there be complementarity between CMB & Laboratory observables ?

We start with a very generic $U(1)_X$ quartic Higgs potenial.

Inflection-point Analysis for PLANCK Data

U(1)$_{B\text{-}L}$ Model

- Minimal Gauged B-L(Baryon-Lepton) Extension of Standard Model
  - Gauge Anomaly Free: 3 generation of right handed Neutrinos (N_i).
  - B-L Higgs Field : Breaks B-L gauge symmetry.
  - B-L Symmetry Breaking: Generates Z' boson mass and Majorana mass for N_i.

$$\mathcal{L} \supset -\frac{1}{2}\sum_{i=1}^{3} Y\, \varphi\, \overline{N^c} N + \text{h.c.}$$

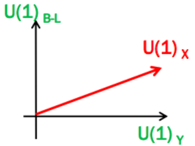
	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$
q_L^i	3	2	$+1/6$	$+1/3$
u_R^i	3	1	$+2/3$	$+1/3$
d_R^i	3	1	$-1/3$	$+1/3$
ℓ_L^i	1	2	$-1/2$	-1
NR^i	1	1	0	-1
e_R^i	1	1	-1	-1
H	1	2	$-1/2$	0
φ	1	1	0	$+2$

  - See-Saw Mechanism
  - Mass Spectrum : $m_{NR} = \frac{1}{\sqrt{2}} Y_N v_{BL},\ m_{Z'} = 2 g v_{BL},\ m_\phi^2 = 2\lambda v_{BL}^2$

Inflection-point Analysis for PLANCK Data

$U(1)_X$ Model $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$

- Generalization of the minimal B-L model.
- $U(1)_X$ is defined as a linear combination of $U(1)_Y$ and $U(1)_{B\text{-}L}$.



	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_X = Q_Y x_H + Q_{B-L} x_\Phi$
q_L^i	3	2	$1/6$	$(1/6)x_H + (1/3)x_\Phi$
u_R^i	3	1	$2/3$	$(2/3)x_H + (1/3)x_\Phi$
d_R^i	3	1	$-1/3$	$(-1/3)x_H + (1/3)x_\Phi$
ℓ_L^i	1	2	$-1/2$	$(-1/2)x_H - x_\Phi$
e_R^i	1	1	-1	$(-1)x_H - x_\Phi$
H	1	2	$-1/2$	$(-1/2)x_H$
N_R^i	1	1	0	$-x_\Phi$
Φ	1	1	0	$+2x_\Phi$

Parameterization

$x_\Phi = 1$

x_H (Free)

B-L limit: $x_H = 0$

$U(1)_Y$ limit: $x_H \to \infty$

Inflection-point Analysis for PLANCK Data

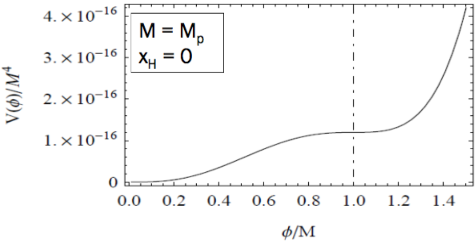
U(1)$_X$ Higgs/Inflaton Potential

- RG improved U(1)$_X$ Higgs/Inflaton potential: $V(\phi) = \frac{1}{4}\,\lambda_\Phi(\phi)\,\phi^4$

Constraint on U(1)$_X$ couplings for a successful inflection-point inflation

$$\lambda_\Phi(M) \simeq 4.770 \times 10^{-16} \left(\frac{M}{M_P}\right)^2, \qquad \boxed{Y(M) \simeq 32^{1/4} g_X(M)}$$

$$g_X(M, x_H) \simeq \frac{1.511 \times 10^{-2}}{(93 + 256 x_H + 164 x_H{}^2)^{1/6}} \left(\frac{M}{M_P}\right)^{1/3}.$$



Free Parameters:
x_H , M

Expanding the potential:

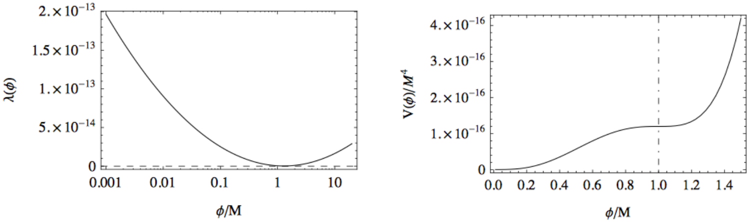
$$\frac{V_1}{M^3} = \frac{1}{4}(4\lambda_\phi + \beta_{\lambda_\phi}),$$
$$\frac{V_2}{M^2} = \frac{1}{4}(12\lambda_\phi + 7\beta_{\lambda_\phi} + M\beta'_{\lambda_\phi}),$$
$$\frac{V_3}{M} = \frac{1}{4}(24\lambda_\phi + 26\beta_{\lambda_\phi} + 10M\beta'_{\lambda_\phi} + M^2\beta''_{\lambda_\phi}),$$

$$\beta_\lambda = \frac{1}{16\pi^2}\left(20\lambda^2 - (48g^2 - 6Y^2)\lambda + 96g^4 - 3Y^4\right)$$

- Simplified assumptions taken: Yukawa degenracy.
- Gauge coupling and Yukawa coupling cancel each other and creates the inflection-point.
- Gauge and Yukawa couplings themselves do not run so much.
- Logarithimic-corrected RGE-improved Higgs potential responsible for cosmic inflation.

Inflection-point Analysis for PLANCK Data

As the quartic reaches the very small value to satisfy the CMB constraint, due the inflection-point conditions imposed from the Yukawa and gauge coupling cancelling each other, the flattened inflationary potential is generated.



- Inflation basically imposes boundary conditions on particle physics model.
- Similar to the MPP principle (Nielsen & Froggart) for the Higgs.
- Small gauge coupling required. Cannot be done with the SM Higgs. But any dark sector works.
- Laboratory phenomenology for the particle model becomes very predictive.

Ghoshal (2021).

Inflection-point Analysis for PLANCK Data

Constraint on Low Energy Observables

- Low Energy Observables evaluated at VEV

Inflection-point condition leads to a relation between low energy observables!

$$Y \equiv Y_1 = Y_2 = Y_3$$

$$\frac{m_N}{m_{Z'}} \simeq 0.84, \qquad (m_{Z'} > m_N)$$

$$\frac{m_\phi}{m_{Z'}} \simeq 2.911 \times 10^{-6} \left(\frac{M}{M_P}\right)^{2/3} \left(87 + 256 x_H + 164 x_H^2\right)^{1/6} \ln\left[2 g_X \frac{M}{m_{Z'}}\right].$$

$$g_X(M, x_H)$$

- Free Parameters

  x_H, M, $m_{Z'}$

Raut slides (2016)

Inflection-point Analysis for PLANCK Data

Decay of Inflaton and Reheating

- Φ decays into the SM particle
- Themalization of decay products recreates Standard Big Bang Scenario.

Reheating Temperature

$$T_R \simeq 0.55 \left(\frac{100}{g_*}\right)^{1/4} \sqrt{\Gamma_\phi M_P}$$

BBN Constraint
T_R > 1 MeV

- Inflaton being very light, it can only decay through SM Higgs coupling

$$V = \lambda_H \left(H^\dagger H - \frac{v_h^2}{2}\right)^2 + \lambda_\Phi \left(\Phi^\dagger \Phi - \frac{v_X^2}{2}\right)^2 + \lambda_{\rm mix} \left(H^\dagger H - \frac{v_h^2}{2}\right)\left(\Phi^\dagger \Phi - \frac{v_X^2}{2}\right)$$

$$\Gamma_\phi(m_\phi, \xi) \simeq \theta^2 \, \Gamma_h(m_\phi)$$

$$m_\phi(x_H, M, m_{Z'})$$

$$\lambda_{mix} = \left(\frac{m_H^2}{v_H \; v_X}\right)\theta$$

Additional Constraint

$$\theta^2 = \left(\frac{m_\phi}{m_H}\right)^2 \xi$$

$$\xi < 1$$

- Free Parameters: ξ , X_H, M , $m_{Z'}$

Raut slides (2016)

Inflection-point Analysis for PLANCK Data

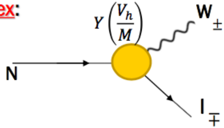
Collider Z' Phenomenology

- Z' boson direct search : $pp \to Z' + X \to \ell^+ \ell^- + X$

- Heavy Neutrino search via displaced vertex:

$Z' \to N\ N$

$N \to W^{\pm} + l^{\mp}$



The partial decay width of heavy neutrinos is suppressed by See-Saw mechanism.

$$\Gamma_N \sim Y^2 \left(\frac{V_h}{M}\right)^2 M \sim \frac{m_D{}^2}{M} \sim m_\nu$$

- Kinematic Constraint:

$m_{Z'}{}^2 > 4\, m_N{}^2$

Non-degenerate Yukawa

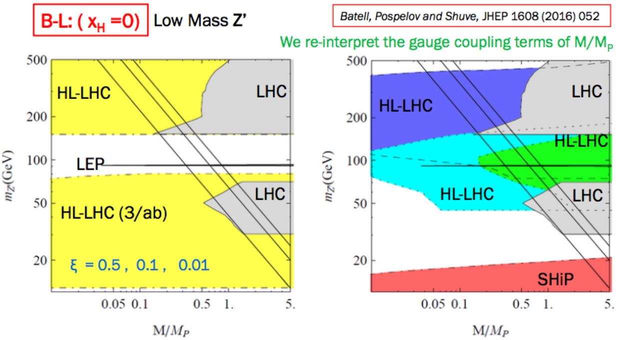
$Y_2 = Y_3$

$m_{Z'}/m_{N^1} = 3 \quad \rightarrow \quad \frac{m_{N^{2,3}}}{m_{Z'}} \simeq 0.929$

Batell, Pospelov and Shuve, JHEP 1608 (2016) 052

Raut slides [Pheno (2017)]

Inflection-point Analysis for PLANCK Data



- Constraints on the parameter space from current and future colliders.
- Diagonal lines are for re-heating temperatures 1 MeV for mixing various angles ξ. The region on the right is ruled out due to BBN constraints.
- Inflection-point scale M and Higgs vev are the free parameter of the model; rest are all related via RGE running.

Dark Matter

What about Dark Matter candidate ?

- Condition for inflection-point dictates gauge coupling to be very very small.
- Freeze-in Z'-portal dark matter.

Model Content:

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$
q_L^i	**3**	**2**	$+1/6$	$+1/3$
u_R^i	**3**	**1**	$+2/3$	$+1/3$
d_R^i	**3**	**1**	$-1/3$	$+1/3$
ℓ_L^i	**1**	**2**	$-1/2$	-1
N_R^i	**1**	**1**	0	-1
e_R^i	**1**	**1**	-1	-1
H	**1**	**2**	$-1/2$	0
φ	**1**	**1**	0	$+2$
ζ	**1**	**1**	0	Q

ζ is a dark vector-like fermion is the dark matter candidate.

$$\mathcal{L}_{Z_{BL}} = y_l \bar{L}\tilde{H}N + g_{BL}\,(Z_{BL})_\mu \left[\sum_f (B-L)_f \bar{f}\gamma^\mu f + Q_\zeta \bar{\zeta}\gamma^\mu \zeta\right]$$

Ghoshal (2021)

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ℓ_L^i	**1**	**2**	$-1/2$	-1
N_R^i	**1**	**1**	0	-1
e_R^i	**1**	**1**	-1	-1
H	**1**	**2**	$-1/2$	0
φ	**1**	**1**	0	$+2$
ζ	**1**	**1**	0	Q

$$\mathcal{L}_{Z_{BL}} = y_l \bar{L}\tilde{H}N + g_{BL}\,(Z_{BL})_\mu \left[\sum_f (B-L)_f \bar{f}\gamma^\mu f + Q_\zeta \bar{\zeta}\gamma^\mu \zeta\right]$$

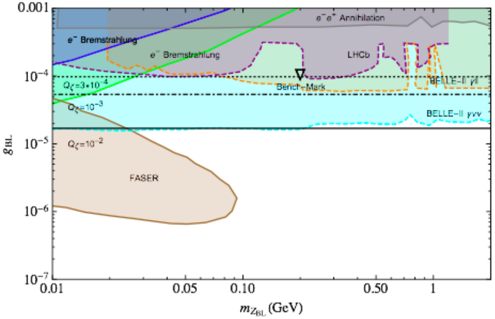
Dark Matter

Dark Matter Relic Density:

$$\sigma(\bar{\zeta}\zeta \to f\bar{f})\,v \simeq \frac{37}{36\pi s}(Q_\zeta g_{BL})^2 g_{BL}^2,$$

$$\sigma(\bar{\zeta}\zeta \to Z_{BL}Z_{BL})\,v \simeq \frac{(Q_\zeta g_{BL})^4}{4\pi s}\left(\ln\left[\frac{s}{m_\zeta^2}\right] - 1\right),$$

Collider Searches:



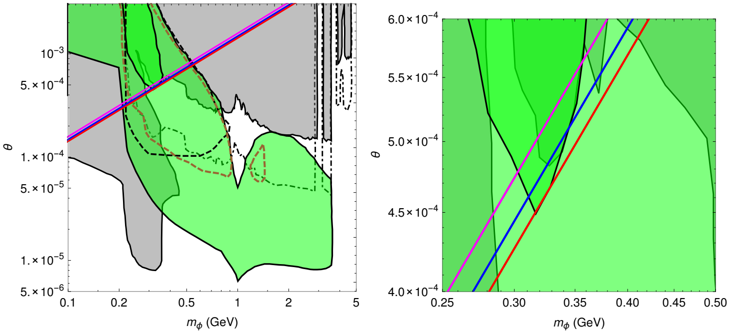
Inflaton Hunt

Inflaton Hunt (no gauge extension: SM + sterile neutrinos)

$$\beta_{\lambda_\phi} = \frac{1}{16\pi^2}\left(12\lambda_\phi y^2 - 6y^4 + 8\lambda_{H\phi}^2 + 20\lambda_\phi^2\right).$$

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\lambda_{H\phi} m_\phi v_h}{\sqrt{2\lambda_\phi}\left(m_h^2 - m_\phi^2\right)}\right)$$

Long Lived Particle Searches:



Conformal Model

We want to construct a conformal model where no scales are present at the tree-level. In the IR, Seesaw Scale and EW Scale is generated via Coleman-Weinberg. Inflection-point Inflation happens in the UV. All determined by the RGE.

$$\mathcal{V}\left(H,\phi\right) = \lambda_H |H|^4 - \lambda_{H\phi}\,|H|^2|\phi|^2 + \lambda_\phi|\phi|^4.$$

Conformal Model

The Model Content:

The B-L Extended Model

Field	Group	Coupling
Z_{BL}	$U(1)_{B-L}$	g_{BL}

TABLE I. New gauge sector of the model

Field	Spin	$U(1)_{B-L}$
ϕ	0	2
$\psi_{L,R}$	$\frac{1}{2}$	-1
N_R^i	$\frac{1}{2}$	-1

TABLE II. New scalars and fermions in the model.

$$\mathcal{V}(H,\phi) = \lambda_H |H|^4 - \lambda_{H\phi} |H|^2 |\phi|^2 + \lambda_\phi |\phi|^4.$$

Conformal Model: RGE

RGE:

$$\phi\frac{d\lambda_\phi}{d\phi}=\frac{1}{16\pi^2}\left(20\lambda_\phi^2+96g_{BL}^4-\sum_j Y_j^{low\ 4}+\lambda_\phi\left(2\sum_j Y_j^{low\ 2}-48g_{BL}^2\right)\right),$$

$$\phi\frac{dg_{BL}}{d\phi}=\frac{1}{16\pi^2}\left(12+\frac{4}{3}\right)g_{BL}^3,$$

$$\phi\frac{dY_i^{low}}{d\phi}=\frac{1}{16\pi^2}\left(6g_{BL}^2Y_i^{low}+Y_i^{low}\left(\frac{1}{2}\left(\sum_j Y_j^{low\ 2}+y_L^2+y_R^2\right)-12g_{BL}^2+Y_i^{low\ 2}\right)\right),$$

$$\phi\frac{dy_L}{d\phi}=\frac{1}{16\pi^2}\left(6g_{BL}^2y_L+y_L\left(\frac{1}{2}\left(\sum_j Y_j^{low\ 2}+y_L^2+y_R^2\right)-12g_{BL}^2+y_L^2\right)\right),$$

$$\phi\frac{dy_R}{d\phi}=\frac{1}{16\pi^2}\left(6g_{BL}^2y_R+y_R\left(\frac{1}{2}\left(\sum_j Y_j^{low\ 2}+y_L^2+y_R^2\right)-12g_{BL}^2+y_R^2\right)\right),$$

$$\phi\frac{d\lambda_\phi}{d\phi}=\frac{1}{16\pi^2}\left(20\lambda_\phi^2+96g_{BL}^4-\left(\sum_j Y_j^{low\ 4}+y_L^4+y_R^4\right)+\lambda_\phi\left(2\sum_j Y_j^{low\ 2}+2y_L^2+2y_R^2-48g_{BL}^2\right)\right) \quad (14)$$

Conformal Model

RGE:

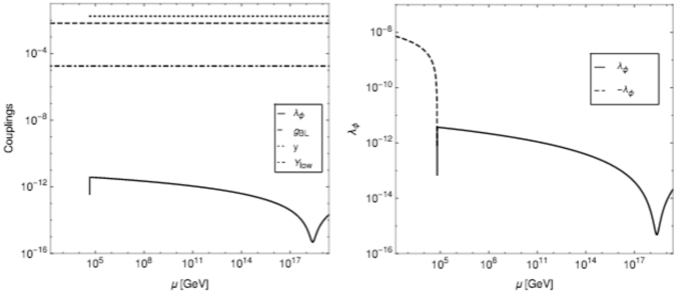


FIG. 2. ***Left Panel:*** *RG running of all the couplings for the benchmark point ($M = 1\ M_P$, $\mu_T = 44.85$ TeV) against μ.* ***Right Panel:*** *RG running of λ_ϕ against μ. Note the abrupt drop of λ_ϕ to negative value at the threshold. We have chosen negligible $Y^{low} = 10^{-3}y$ for this work.*

Conformal Model

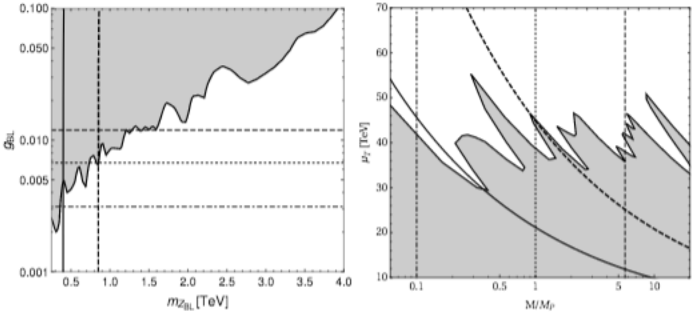


FIG. 4. ***Left Panel:*** *The diagonal jagged solid line is the upper bound on the B-L gauge coupling as a function of Z_{BL} mass, from the ATLAS final result [84] (ATLAS-CONF-2019-001). The horizontal lines correspond to the inflection-point scale $M = 5.67\ M_P$ (dashed), M_P (dotted) and $0.1\ M_P$ (dot-dashed) respectively* [6]*. This corresponds to $m_{Z_{BL}}$ lower bounds to be 1.64 TeV, 850 GeV and 360 GeV, respectively. The vertical solid line and the vertical thick dashed line correspond to $m_{Z_{BL}} = 3 \times 133$ and 850 GeV respectively, the lower limit for theoretical consistency*

Conclusions:

- UV is connected to the IR via the RGE.
- Complementary probes of BSM models via light dark sector experimental searches and CMB.
- SM Higgs cannot be play such a role as the gauge coupling is too high. But can be done in any dark BSM $U(1)_X$ or $SU(N)_X$ sector.
- For Type-I seesaw neutrino models we showed the collider and CMB complementarity, for freeze-in dark matter.
- We showed conformal models where radiative symmetry-breaking generates EW scale and Seesaw scale in the IR via Coleman-Weinberg and achieve inflection-point inflation in the UV via RGE.
- We showed how to construct gauged-free extensions where inflection point is achieved. In this case actual light inflaton hunt is possible via light scalar decay searches. *Old idea by Berzukov & Gorbunov*.
- Plethora of particle physics model-building directions possible now involving dark sector and CMB, now that we have one-to-one correspondence between particle property and CMB.

Generic Inflection-point Condition for SU(N) Theory:

Gauge-Yukawa-Higgs Theory:

$$\beta_g = -\kappa\, g^3 \left(\frac{11}{3} N_c - \frac{1}{6} - \frac{2n_f}{3} \right) ,$$

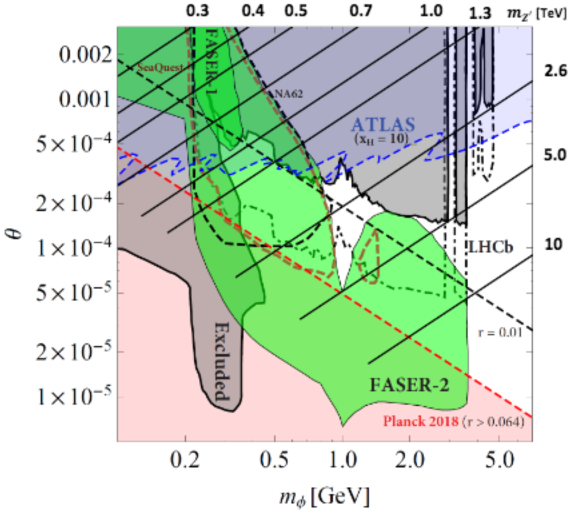
$$\beta_{\mathbf{Y}} = \kappa \left(\frac{3}{2} \mathbf{Y}\mathbf{Y}^\dagger \mathbf{Y} + \mathbf{Y}\mathrm{tr}(\mathbf{Y}^\dagger \mathbf{Y}) - 3 \frac{N_c^2 - 1}{2N_c} g^2\, \mathbf{Y} \right) ,$$

$$\beta_\lambda = \kappa \left(\frac{3(N_c - 1)(N_c^2 + 2N_c - 2)}{4N_c^2} g^4 - 2\,\mathrm{tr}(\mathbf{Y}^\dagger \mathbf{Y}\,\mathbf{Y}^\dagger \mathbf{Y}) - \frac{6(N_c^2 - 1)}{N_c} \lambda\, g^2 + 4\,\lambda\,\mathrm{tr}(\mathbf{Y}^\dagger \mathbf{Y}) + 4\,(N_c + 4)\,\lambda^2 \right) ,$$

$$Y^4 = \frac{3(N_c - 1)(N_c^2 + 2N_c - 2)}{8N_c^2} g^4 \,.$$

Names of experiments

Dark Matter Relic Density:



Thank You

Sky Meets Laboratory via Renormalization Group Equations: Gravitational Waves from Peccei-Quinn Phase Transition

Anish Ghoshal

Istituto Nazionale di Fisica Nucleare
Tor Vergata, Rome, Italy

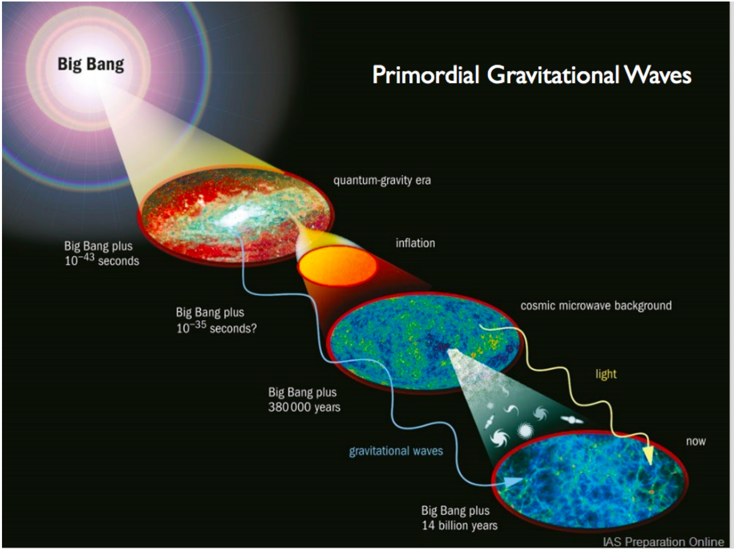
anish.ghoshal1@protonmail.ch

September 2021, CORFU, Greece

Outline of talk:

- UV & IR is connected via RGE.
- Complementarity between Lab versus Cosmic Observables.
- Stochastic Gravitational Waves from Cosmological Phase Transitions
  - Peccei-Quinn Phase Transition & Gravitational Waves.
  - Conformal Invariance & TAF as a direction of UV-completion.
  - Predictions in UV-complete Axion model.
  - Predictions with Conformal Symmetry Breaking.
- Predictions on the GW detectors sensitivity map.
- Recent NanoGrav GW detection.
- Conclusion

History of the Universe



Gravitational Waves

- Gravitational Waves (GW) first detected in 2016.
- New Window into the Early Universe.
- New Probes of Particle Phenomenology beyond TeV (LHC scale).
- Robust predictions of GW signatures from UV-completion conditions.
- Sources of GW of cosmological origin & corresponding GW spectrum:
  - Inflation: Primordial GW.
  - Inflation: Secondary GW.
  - Strong First-order Phase Transition.
  - Re-heating.
  - Graviton bremsstrahlung.
  - Topological Defects.
  - Oscillon.
  - Primordial BH-induced GW.
- Strong CP Problem dictates $U(1)_{PQ}$ symmetry breaking.
  Peccei-Quinn Phase Transition.

GW - - A Primer

perturbations of the background metric: $ds^2 = a^2(\tau)(\eta_{\mu\nu} + h_{\mu\nu}(\mathbf{x},\tau))dx^\mu dx^\nu$

scale factor: cosmological expansion ↗ ; background metric ↑ ; GW ↖

governed by linearized Einstein equation $(\tilde{h}_{ij} = a h_{ij}, \text{TT - gauge})$

$$\tilde{h}''_{ij}(\boldsymbol{k},\tau) + \underbrace{\left(k^2 - \frac{a''}{a}\right)}_{\sim a^2H^2} \tilde{h}_{ij}(\boldsymbol{k},\tau) = \underbrace{16\pi G\, a\, \Pi_{ij}(\boldsymbol{k},\tau)}_{\text{source term from } \delta T_{\mu\nu}}$$

source: anisotropic stress-energy tensor

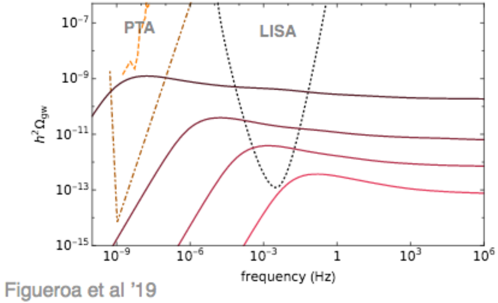
$k \gg aH : h_{ij} \sim \cos(\omega\tau)/a\,, \quad k \ll aH : h_{ij} \sim \text{const.}$

a useful plane wave expansion: $$h_{ij}(\boldsymbol{x},\tau) = \sum_{P=+,\times} \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \int d^2\hat{\boldsymbol{k}}\; h_P(\boldsymbol{k}) \underbrace{T_k(\tau)}_{\sim a(\tau_i)/a(\tau)} e^P_{ij}(\hat{k})\, e^{-ik(\tau - \hat{\boldsymbol{k}}\boldsymbol{x})}$$

transfer function , expansion coefficients , polarization tensor $P = +, \times$

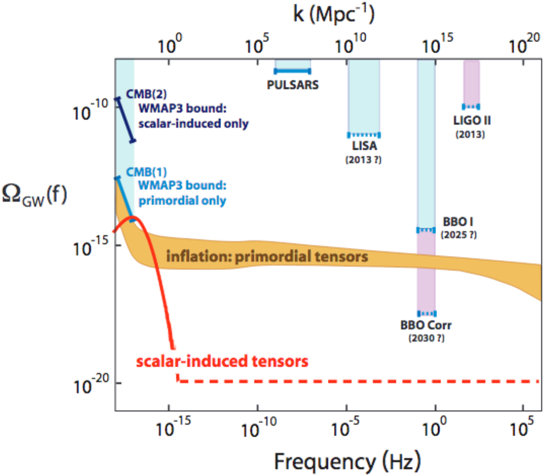
GW - - Cosmic String

Topological defects like cosmic strings give rise to scale invariant GW spectrum.



GW - - Primordial and Scalar Induced Secondary GW

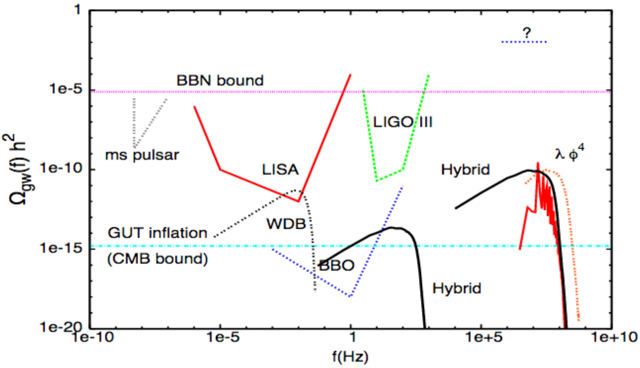
Secondary Tensor Spectrum induced by first-order scalar perturbation via mixing.
Can be tuned to generate high amplitude in high frequency regions.



Baumann (2007)

GW - - (P)-reheating

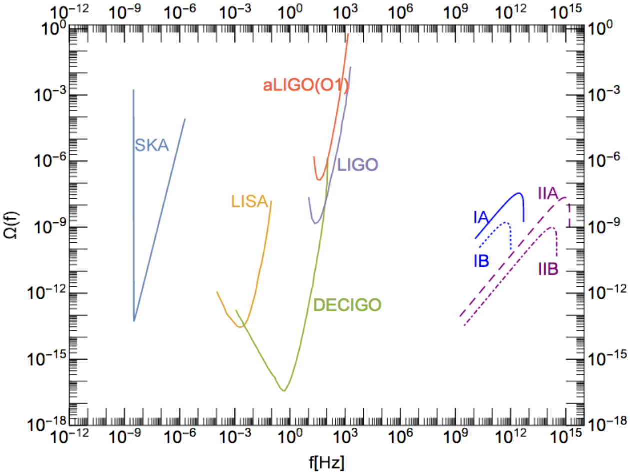
Production during inflaton oscillating in FRW background.



Figuera (2007)

GW - - Graviton Bremmstrahlung

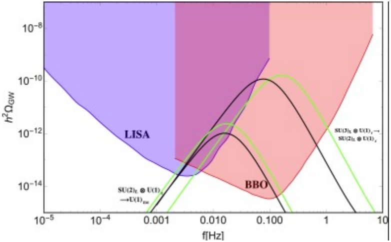
Inflaton radiating away gravitons forming Stochastic GW background.



Nakayama (2018)

GW - - (P)-reheating

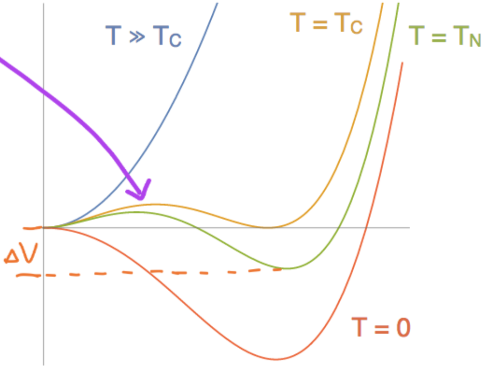
Typical GW spectrum from thermal first-order phase transition:



Huang (2018)

Phase Transition

- QFT at finite temperature → symmetry restoration
- For first order PT
  - Need barrier here
- PT occurs at T_N
- Potential energy ΔV ↓ GWs
- Not in SM! Possible in BSM scenarios

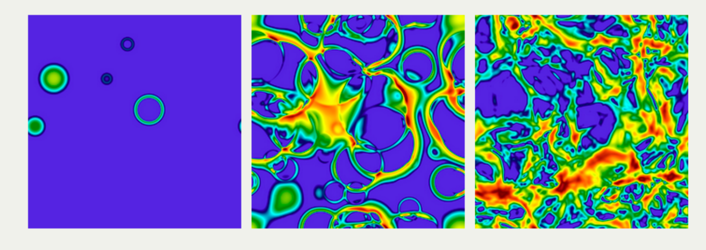


Schwaller (Amsterdam (2019))

Phase Transition

Phase Transitions:

- Bubbles nucleate and grow.
- Expand in plasma.
- Bubbles and fronts collide - - violent process.
- Sound Waves left behind in thermal plasma.
- Turbulence, damping.



Phase Transition GW - Parameter Dependence

- Total GW energy budget from 3 sources

$$h^2\Omega_{\mathrm{GW}} = h^2\Omega_\phi + h^2\Omega_{\mathrm{SW}} + h^2\Omega_{\mathrm{MHD}}$$

Depends on two important parameters:

- Vacuum energy density: $\alpha = \frac{\rho_{\rm vac}}{\rho^*_{\rm rad}}$ with $\rho^*_{\rm rad} = g_*\pi^2 \frac{T_*^4}{30}$
- (Inverse) Bubble nucleation rate: $\beta/H_* = T\sqrt{\frac{d^2 S_E(T)}{dT^2}}\Big|_{T=T_*}$

- $$h^2\Omega_\phi \propto \left(\frac{\beta}{H_*}\right)^{-2},\quad h^2\Omega_{\rm SW} \propto \left(\frac{\beta}{H_*}\right)^{-1},\quad h^2\Omega_{\rm MHD} \propto \left(\frac{\beta}{H_*}\right)^{-1}$$

- The bubble nucleation rate per unit volume at a finite temperature is given by

$$\Gamma(T) = \Gamma_0 e^{-S(T)} \simeq \Gamma_0 e^{-S_E^3(T)/T},$$

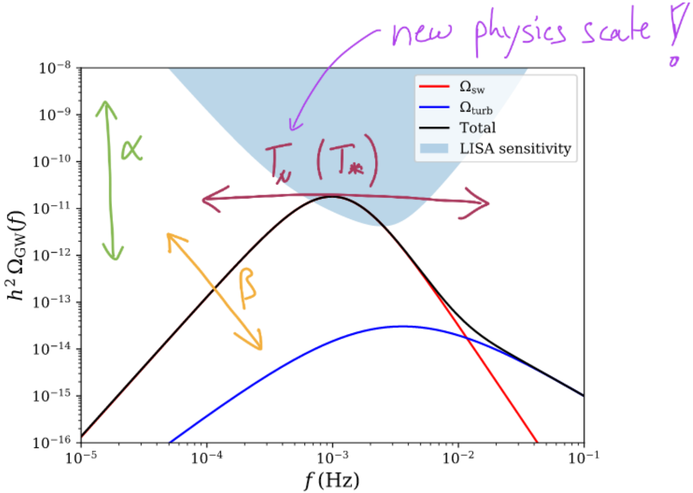
- Other important parameter: bubble wall speed v_w, efficiency factors.

Bounce Action S_3:

$$\partial^2 \phi + V'_{\text{eff}}(\phi, T) + \sum_i \frac{dm_i^2}{d\phi} \int \frac{\mathrm{d}^3 k}{(2\pi)^3\, 2E_i} \delta f_i(\mathbf{k}, \mathbf{x}) = 0$$

- $V'_{\text{eff}}(\phi)$: gradient of finite-T effective potential
- $\delta f_i(\mathbf{k}, \mathbf{x})$: deviation from equilibrium phase space density of ith species
- m_i: effective mass of ith species

Phase Transition GW - Parameter Dependence



Schwaller (Amsterdam, 2019)

Axion

Strong CP Problem:

$$\frac{\theta}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \qquad \theta + \mathrm{Arg}[\mathrm{Det}(y_u y_d)] < 10^{-10}$$

Axion solution:

$$\theta \to \frac{a(x)}{f} \qquad \mathcal{L} = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G}$$

[Peccei-Quinn '77
Weinberg-Wilczek '78]

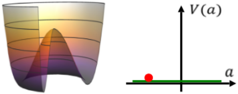
- PQWW axion:
  Axion identified with the phase of the Higgs in a 2HDM
  ($f_a \sim V_{EW}$ was quickly ruled out long ago) [Peccei, Quinn (1977), Weinberg (1978), Wilczek (1978)]

The need to require $f_a \gg V_{EW}$: "invisible axion"

- DSFZ Axion: SM quarks and Higgs charged under PQ.
  Requires 2HDM + 1 scalar singlet. SM leptons can also be charged.
  [Dine, Fischler, Srednicki (1981), Zhitnitsky (1980)]
- KSVZ axion (or QCD axion, or hadronic axion):
  All SM fields are neutral under PQ. QCD anomaly is induced by new quarks, vectorlike under the SM, chiral under PQ.
  [Kim (1979), Shifman, Vainshtein, Sakharov (1980)]

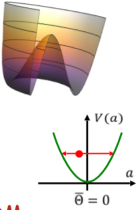
Axion as Dark Matter

- **As long as $\Lambda_{QCD} < T < f_a$:**
  $U(1)_{PQ}$ broken only spontaneously,
  $m_a = 0$, $\langle a_0 \rangle = \theta_0 f_a \sim f_a$



- **As soon as $T \sim \Lambda_{QCD}$:**
  $U(1)_{PQ}$ explicit breaking (instanton effects)
  $m_a(T)$ turns on. When $m_a(T) > H \sim 10^{-9}$ eV,
  $\langle a_0 \rangle \longrightarrow 0$ and starts oscillating undamped

$$\ddot{a} + 3H\dot{a} + m_a^2(T) f_a \sin\left(\frac{a}{f_a}\right) = 0$$



- **Energy stored in oscillations behaves as CDM**

Axion Pheno

- Axion or ALP couplings to SM particles are always suppressed by inverse powers of $U(1)_{PQ}$ symmetry breaking scale f_a.
- Phenomenological scalar with complex singlet scalar Φ:

$$\Phi(x) = \frac{1}{\sqrt{2}}(f_a + \phi(x))e^{ia(x)/f_a} \tag{1}$$

- Spontaneous breaking of U(1) may lead to strong first-order phase transition at the f_a scale & generate GW signals to be detected at the current and future detectors.

Phase Transition GW - Finite Temperature

$$\mathcal{V}(\phi,T) = \mathcal{V}_0(\phi)+\mathcal{V}_{\rm CW}(\phi)+\mathcal{V}_T(\phi,T)\,,$$

- Tree-level: $$\mathcal{V}_0 = -\mu^2|H|^2+\lambda|H|^4+\kappa|\Phi|^2|H|^2+\lambda_a\left(|\Phi|^2-\frac{1}{2}f_a^2\right)^2 .$$
$$= \frac{\lambda_a}{4}\left(\phi^2-f_a^2\right)^2+\left[\frac{\kappa}{2}\phi^2-\mu^2\right]\left(\frac{1}{2}h^2+\frac{1}{2}G_0^2+G_+G_-\right) + \lambda\left[\frac{1}{2}h^2+\frac{1}{2}G_0^2+G_+G_-\right]^2 .$$
- One-loop: $$\mathcal{V}_{\rm CW}(\phi) = \sum_i(-1)^F n_i\frac{m_i^4(\phi)}{64\pi^2}\left[\log\frac{m_i^2(\phi)}{\Lambda^2}-C_i\right].$$
- Finite-temperature: $$\mathcal{V}_T(\phi,T) = \sum_i(-1)^F n_i\frac{T^4}{2\pi^2}J_{B/F}\left(\frac{m_i^2(\phi)}{T^2}\right),$$
- Temperature-dependent mass terms:
$$\Pi_h(T) = \Pi_{G_{0,\pm}}(T) = \frac{1}{48}\left(9g_2^2+3g_1^2+12y_t^2+24\lambda+4\kappa\right)T^2\,,$$
$$\Pi_\phi(T) = \frac{1}{3}(\kappa+2\lambda_a)T^2\,.$$

[Dolan, Jackiw (PRD '74); Arnold, Espinosa (PRD '93); Curtin, Meade, Ramani (EPJC '18)]

Phase Transition GW - sensitivity

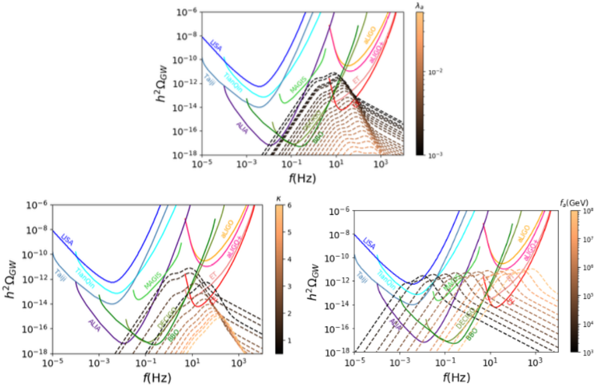
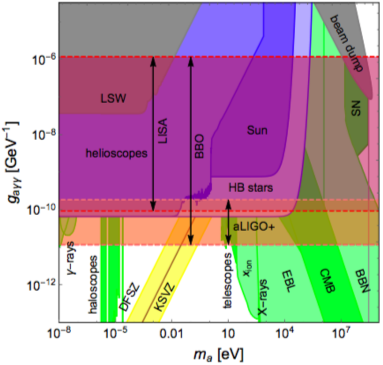
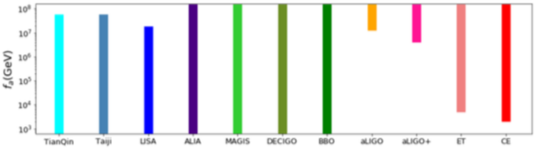


Figure 4. The detection prospects for the GW experiments TianQin [27], Taiji [28], LISA [29, 30], ALIA [31], MAGIS [32], DECIGO [33], BBO [34], aLIGO [37], aLIGO+ [38], ET [36] and CE [35], and the curves of GW strength $h^2\Omega_{\rm GW}(f)$ as functions of the three parameters f_a, κ and λ_a in the ALP model. In the upper panel, we have fixed $f_a = 10^6$ GeV and $\kappa = 1.0$ and varied λ_a from 0.001 to 0.2; in the lower left panel $f_a = 10^6$ GeV and $\lambda_a = 0.001$, with κ varying from 1.0 to 6.00; in the lower right panel $\kappa = 1.0$ and $\lambda_a = 0.001$, with f_a between 10^3 GeV and 10^8 GeV.

Dev et. al. (2019)

Phase Transition GW - Parameter Dependence





Dev et. al. (2019)

KSVZ Axion

– KSVZ axion:

$$U(1)_{\rm PQ}: \qquad X \to e^{i\alpha} X$$

$$\lambda_X(|X|^2 - f^2/2)^2 + (yXQQ^c + h.c.)$$

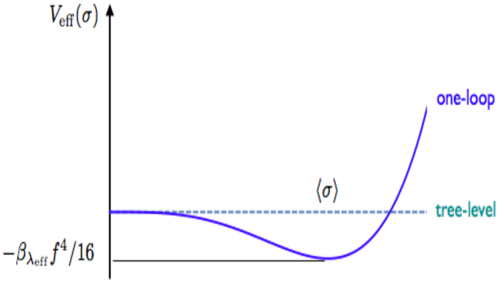
- No massless bosons coupling to X while Peccei-Quinn symmetry is restored.
- Fermion contribution to V_{eff} contributes is negatively.
- Finite temperature corrected potential is of the form $m(T)^{|}X|^2 + \lambda(T)|X|^4$.
- PQ phase transition is of second-order in the minimal case.
- In order of make strong first-order phase transition (PT), and thus enhanced GW, we go to supercooling regime. This requires PT to last long enough.
- This means $\frac{S_3}{T} \sim$ constant $\to$ scale invariant.
- Break PQ symmetry radiatively.
- Or, break non-minimally like strong coupling regime, non-perturbative, extra-dimension etc. (See Delle Rosse (2019) & Von Harling (2019).)

Conformal Symmetry Breaking

Due to conformal symmetry-breaking, the flat direction is lifted at 1-loop when

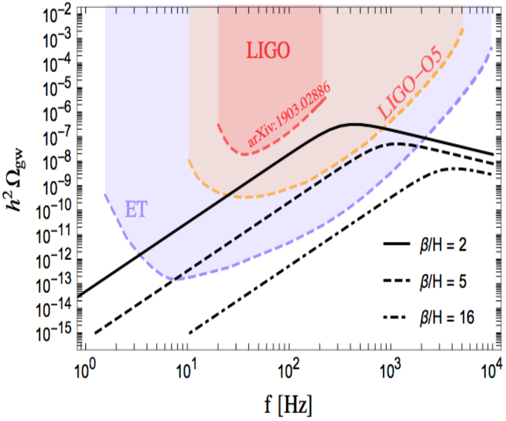
$$V_{eff} = \frac{\beta_\lambda}{4}\sigma^4(log(\frac{\sigma}{f_a} - \frac{1}{4})),$$

where $< \sigma >= f_a$.



Conformal Symmetry Breaking

Strong super-cooling enhances GW signals:



delle Rosse et. al. (2019), von Harling et. al. (2019)

Total Asymptotic Freedom Principle

- No scales are fundamental are nature, all scales that we observe are generated dynamically: 1-loop or via non-perturbative physics.
- Gravitational Corrections not included. For re-normalizable theories of gravity like Quadratic Gravity or non-local gravity, all corrections are softened in the UV.
- Still suffers from Landau poles.
- Total Asymptotic Freedom (TAF) as a direction for UV completion of particle physics. All couplings flow to zero in the UV.
- No Landau poles in theory.
- Theory valid and perturbative upto infinite energy scales.
- For $U(1)_{PQ}$, simplest possibility to replace by $SU(2)_a$.
- Generic conditions for TAF already studied in several places [Giudice (2014), Holdom (2015), Pelaggi (2015)].

Low energy spectrum of the theory contains extra dark photon on top of the SM.
All masses of extra quarks and scalars are expressed in terms of the free parameter f_a.

Phase Transition GW - Parameter Dependence

Renormalization Group Equations of the parameters:

$$\frac{dg^2}{dt} = -bg^4, \qquad b \equiv \frac{11}{3}C_2(G) - \frac{4}{3}S_2(F) - \frac{1}{6}S_2(S),$$

$$b_s = \frac{29}{3} - \Delta, \quad \frac{dy^2}{dt} = y^2\left(\frac{9y^2}{2} - 8g_s^2 - \frac{9g_a^2}{2}\right)$$

and $b_a = \frac{14}{3}$, $t = \frac{ln(\mu^2/\mu_0^2)}{(4\pi)^2}$, where μ_0 is arbitrary energy scale.
Δ is the extra contributions from scalars and fermions in the theory.

$$g_s^2(t) = \frac{\tilde{g}_s^2}{t}, \quad g_a^2(t) = \frac{\tilde{g}_a^2}{t}, \quad y^2(t) = \frac{\tilde{y}^2}{t}, \quad \lambda_i(t) = \frac{\tilde{\lambda}_i}{t},$$

Phase Transition GW - Parameter Dependence

Axion potential:

$$V_A = -m^2 \mathrm{Tr}(A^\dagger A) + \lambda_1 \mathrm{Tr}^2(A^\dagger A) + \lambda_2 |\mathrm{Tr}(AA)|^2,$$

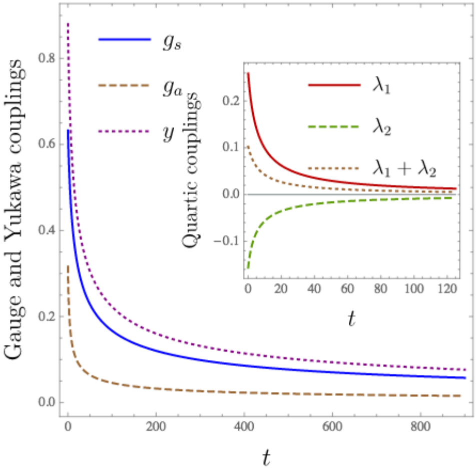
RGEs of λ_1 and λ_2 are $\frac{d\lambda_1}{dt} = \beta_1$, and $\frac{d\lambda_2}{dt} = \beta_2$, where

$$\beta_1(g, y, \lambda) = \frac{9}{2} g_a^4 + \lambda_1 \left(8\lambda_2 + 6y^2 - 12 g_a^2\right) + 14\lambda_1^2 + 8\lambda_2^2 - 3y^4$$

$$\beta_2(g, y, \lambda) = \frac{3}{2} g_a^4 + \lambda_2 \left(12\lambda_1 + 6y^2 - 12 g_a^2\right) + 6\lambda_2^2 + \frac{3}{2} y^4.$$

X and A are used inter-changeably for denoting the radial part of the axion field.

Phase Transition GW - Parameter Dependence



$t = 100 \equiv \mu = 10^{100}$ GeV.

Phase Transition GW - Parameter Dependence

Some values for satisfying TAF principle.

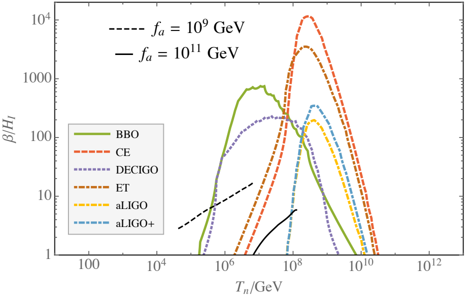
Δ	n_e	unstable vacuum	stable vacuum
28/3	1	$(0.219, -3.25)$	$(1.70, -0.965)$
//	2	$(0.268, -3.27)$	$(1.73, -0.986)$
//	3	$(0.344, -3.30)$	$(1.77, -1.02)$
//	4	$(0.469, -3.34)$	$(1.84, -1.08)$
//	5	$(0.722, -3.42)$	$(1.97, -1.20)$
//	6	$(1.50, -3.49)$	$(2.34, -1.70)$
26/3	1	$(0.185, -1.06)$	$(0.593, -0.362)$
//	2	$(0.237, -1.07)$	$(0.619, -0.389)$
//	3	$(0.314, -1.08)$	$(0.656, -0.435)$
//	4	$(0.447, -1.08)$	$(0.712, -0.528)$
8	1	$(0.182, -0.601)$	$(0.365, -0.255)$
//	2	$(0.236, -0.599)$	$(0.387, -0.294)$
//	3	$(0.324, -0.570)$	$(0.411, -0.376)$

Figure: Values of $(\tilde{\lambda_1}, \tilde{\lambda_2})$ satisfying TAF condition. n_e is the number of vector-like Dirac fermions in the adjoint of $SU(2)_a$.

Salvio et. al. (2020)

Phase Transition GW - Parameter Dependence

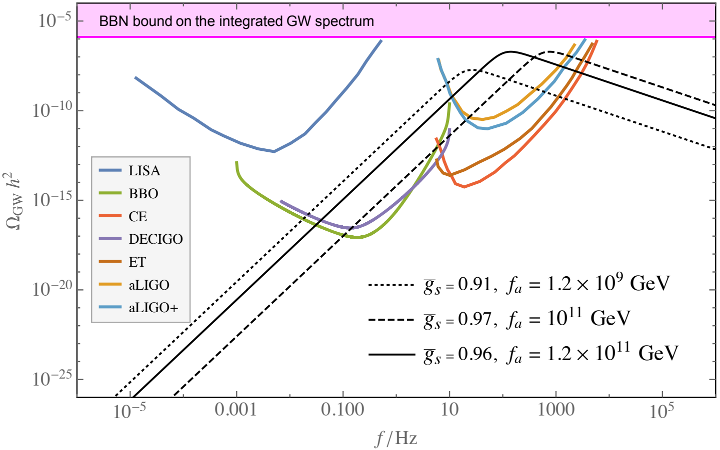
Predictions for some benchmark values.



Imposing conformal symmetry on the axion potential leaves us with only 2 free parameters, thereby very predictive.
Ghoshal et. al. (2020)

Phase Transition GW - Parameter Dependence

Predictions on the GW spectrum



Ghoshal et. al. (2020)

Conclusion: PQ Phase Transition & Gravitational Waves

- Complementarity between the Sky and the Lab via RGE.
- GW detectors will be probing the pre-BBN era.
- UV completion of axion (or any BSM) particle models is insensitive to laboratory or astrophysics searches but predictable in early universe dynamics.
- GW from strong first-order Peccei-Quinn phase transitions will be testable in near future.
- Conformal symmetry breaking makes PQ phase transition very very strong due to supercooling.
- TAF Principle predicts very characteristic & verifiable GW spectrum.
- Gravitational Wave era invites us to dare to imagine, propose and test UV completions of Quantum Field Theory and Gravity !

Percolation Criterion for PT to end

$$P(T) \equiv e^{-I(T)} \lesssim 1/e \Rightarrow I(T) \gtrsim 1, \tag{D.1}$$

where

$$I(T) = \frac{4\pi}{3} \int_T^{T_c} dT' \frac{\Gamma(T')}{(T' H(T'))^4} \left(\int_T^{T'} \frac{d\tilde{T}}{H(\tilde{T})} \right)^3 . \tag{D.2}$$

One also requires that the physical volume of the false vacuum be decreasing significantly inside of one Hubble time [91, 94–97]

$$\frac{1}{H\mathcal{V}_{\rm false}} \frac{d\mathcal{V}_{\rm false}}{dt} = 3 + T\frac{dI}{dT} \lesssim -1. \tag{D.3}$$

NanoGrav GW Detection

NanoGrav recently detected GW events. Many cosmic sources have been proposed. The GW spectrum nicely fits cosmic strings origin hypothesis.

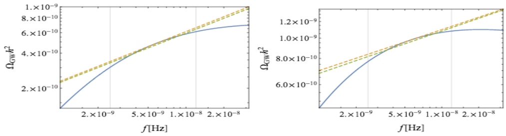
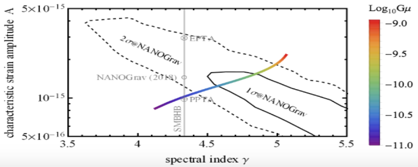


Figure 1. *Cosmic string spectra (solid blue curves) together with our fitted power laws for* $G\mu = 4 \times 10^{-11}$, *and* $G\mu = 10^{-10}$. *The green dashed lines show the results of numerically fitting the curves, while the orange lines result from the simple logarithmic derivative in Eq. (3.3). The thin grey lines indicate the frequency range of interest that was used in the NANOGrav linear fit.*



Ellis (2020)

Thank You