Can we localise asymptotic-AdS Black Holes in brane-world models?

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in collaboration with
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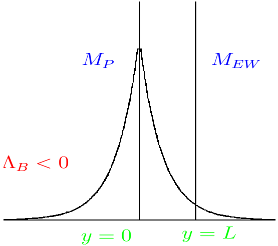
Black Holes in Brane-world models

• In 1999, Randall & Sundrum proposed a model where our universe is a flat 3-brane embedded in a 5-dimensional bulk filled with a negative cosmological constant Λ_B

The line-element of this spacetime had the form

$$ds^2 = e^{-2k|y|}\left(-dt^2 + d\vec{x}^2\right) + dy^2$$

where $k \sim \sqrt{|\Lambda_B|}$ is the AdS curvature scale



In the Randall-Sundrum I model, two branes were introduced, the Planck and the electroweak brane, in order to resolve the hierarchy problem:

$$\boxed{M_{EW} = e^{-kL} M_P}$$

for $kL \simeq 10$, and brane self-energies satisfying $\sigma_P = 24 M_P^3 k = -\sigma_{EW}$

Black Holes in Brane-world models

In the Randall-Sundrum II model, only one brane with a positive tension – our own brane – was introduced

Despite the infinite size of the 5th dimension, the zero-mode graviton is localised near the brane thus restoring 4-dimensional gravity. It holds

$$\boxed{M_P^2 = (M_*^3/k)\,(1 - e^{-2kL})}$$

where M_* and M_P are the fundamental and effective gravitational scales

• **Black-Hole Creation :** When matter localised on our brane undergoes complete gravitational collapse, a black hole should be formed

The BH will span all spacelike dimensions – its singularity should be localised on the brane and its horizon should have a 'pancake' shape

If indeed localised on the brane, the BH should have a regular asymptotic form: Minkowski on the brane and Anti de Sitter in the bulk

Black Holes in Brane-world models

This task has proven to be much more difficult than we thought...

- **The brane perspective:** By Chamblin, Hawking & Reall (2000):

$$ds^2 = e^{-2k|y|}\left[-\left(1-\frac{2M}{r}\right)dt^2 + \frac{dr^2}{\left(1-\frac{2M}{r}\right)} + r^2\, d\Omega_2^2\right] + dy^2$$

It satisfies the RS field equations without the need for any extra $T_{\mu\nu}$, and the $y=0$ projection is a 4D Schwarzschild BH solution

Is it really a localised BH solution? Unfortunately no, since we find:

$$R_{MNRS}\,R^{MNRS} \sim \frac{48M^2 e^{4k|y|}}{r^6}$$

The singularity at $r=0$ that extends from $y=0$ to $y=\infty$ thus the solution is a black string rather than a black hole. In addition, a singularity emerges at the AdS boundary

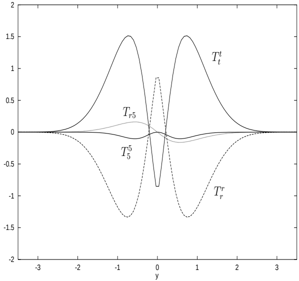
Black Holes in Brane-world models

• **Brane perspective with a bulk profile :** We may write instead the following non-factorised line-element (Kanti & Tamvakis, 2002)

$$ds^2 = e^{-2k|y|}\left[-\left(1-\frac{w(y)}{r}\right)dt^2 + \frac{dr^2}{\left(1-\frac{w(y)}{r}\right)} + r^2\,d\Omega_2^2\right] + dy^2$$

where, for instance,

$$w(y) \simeq 2M\,e^{-\lambda|y|^n}$$



It demands a non-trivial bulk T_{MN} which satisfies all energy conditions on the brane: $\rho > 0\,, \quad \rho + p_i > 0$

(Un)conventional scalar and gauge field configurations do not seem to work

(Kanti, Pappas & Zuleta Estrugo, 2013; Kanti, Pappas & Pappas, 2015)

Black Holes in Brane-world models

• **The bulk perspective :** We start with a regular 5D black hole solution (Creek, Gregory, Kanti & Mistry, 2006)

$$ds^2 = -U(r)\,dt^2 + \frac{dr^2}{U(r)} + r^2\,(d\chi^2 + \sin^2\chi\,d\Omega_2^2)$$

in AdS spacetime with

$$U(r) = 1 + k^2 r^2 - \frac{\mu}{r^2}$$

Then, a 3-brane is introduced through the junction conditions

$$[K_{\mu\nu} - K\,h_{\mu\nu}]^+_- = \kappa_5\,T_{\mu\nu}$$

where $h_{\mu\nu}$ is the induced metric on the brane, $K_{\mu\nu}$ the extrinsic curvature and $T_{\mu\nu}$ the brane energy-momentum tensor of a perfect fluid

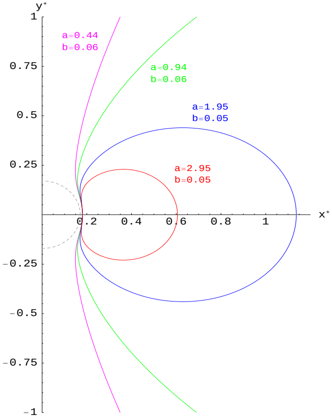
Black Holes in Brane-world models

Since $U(r)$ is fixed through the 5D field equations, the junction conditions form a system of coupled differential equations for:

- $\chi(t,r)$: the trajectory of the brane
- $\rho(t,r)$: the brane energy density
- $p(t,r)$: the brane pressure

We studied in detail the static case, and:

- all regular trajectories stay clear of the BH horizon
- no regular BH solution with the singularity on the brane was found
- the ρ and p resemble those of a star



Black Holes in Brane-world models

• **The brane-only prespective :** We do not know the 5D solution but we can find BH solutions on the brane as solutions to the 4D effective field equations (Shiromizu, Maeda & Sasaki, 2000)

$$^{(4)}G_{\mu\nu} = 8\pi G_4 T_{\mu\nu} + (8\pi G_5)^2 \pi_{\mu\nu} + E_{\mu\nu}$$

where $\pi_{\mu\nu}$ is a tensor quadratic to $T_{\mu\nu}$, and $E_{\mu\nu} =^{(5)} C_{y\mu y\nu}$ is a projected-on-the-brane 5D Weyl tensor - certain assumptions can be made but it may be exactly determined only by solving the bulk equations (Dadhich, Maartens, Papadopoulos & Rezania, 2000; Casadio, Fabri & Mazzacurati, 2002; Kofinas, Papantonopoulos & Zamarias, 2002; ...)

Note : There is a huge literature on black holes, black strings and black branes and their properties in both flat and warped extra dimensions... (Gubser, 2002; Charmousis & Gregory, 2004; Galfard, Germani & Ishibashi, 2005; Kudoh & Wiseman, 2005; Sorkin, 2006; Anber & Sorbo, 2008; Cuadros-Melgar et al, 2008; Bogdanos et al, 2010; ...)

Black Holes in Brane-world models

• **Numerical Solutions :** The following ansatz for the BH line-element was assumed (in e.g. Kudoh, Tanaka, & Nakamura, 2003)

$$ds^2 = \frac{\ell_{AdS}^2}{z^2}\left[-T^2 dt^2 + e^{2R}\left(dr^2 + dz^2\right) + r^2\, e^{2C}\, d\Omega_2^2\right]$$

Small, static, localised and asymptotically AdS BH were indeed found

Large, localised, asymptotically AdS and Schwarzschild on the boundary BHs were also found (Figueras and Wiseman, 2011; Abdolrahimi, Cattoen, Page & Yaghoobpour-Tari, 2012)

$$ds^2 = \frac{\ell_{AdS}^2}{\Delta^2(r,x)}\left[-r^2\, T(r,x) dt^2 + \frac{4A(r,x)}{f^4(r)}\, dr^2 + \frac{4B(r,x)}{f^2(r) g(x)}\, dx^2 + \frac{2rxF(r,x)}{f^3(r)}\, dr\, dx + \frac{x^2\, g(x) S(r,x)}{f^2(r)}\, d\Omega_2^2\right]$$

Our Brane/Bulk Perspective

Our goal: To analytically construct a 5D solution describing a BH with its singularity localised on the brane, a pancake-shaped horizon and an AdS asymptotic behaviour - and, if successful, ask what the price is....

We will start from the line-element of the Randall-Sundrum model

$$ds^2 = e^{-2k|y|}\left(-dt^2 + d\vec{x}^2\right) + dy^2$$

and apply the coordinate transformation $z = sgn(y)\,(e^{k|y|} - 1)/k$, to rewrite it as

$$ds^2 = \frac{1}{(k|z|+1)^2}\left(-dt^2 + dr^2 + r^2\,d\Omega_2^2 + dz^2\right)$$

in terms of spherical coordinates for the spatial directions on the brane

The location of our brane, that was at $y = 0$, is also at $z = 0$; there, the warp/conformal factor is unity and we obtain a flat, Minkowski spacetime. The AdS asymptotic boundary corresponds to $|y| \to \infty$ or $|z| \to \infty$

Our Brane/Bulk Perspective

We will now introduce five-dimensional spherical symmetry by using

$$\{r = \rho \sin\chi\,, \quad z = \rho \cos\chi\}$$

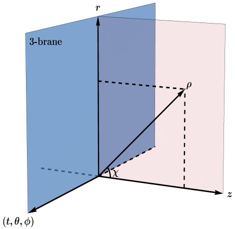
where $\chi \in [0, \pi]$. Then,

$$ds^2 = \frac{1}{(1 + k\rho|\cos\chi|)^2}\left(-dt^2 + d\rho^2 + \rho^2\, d\Omega_3^2\right)\,,$$

where now $d\Omega_3^2 = d\chi^2 + \sin^2\chi\, d\theta^2 + \sin^2\chi\, \sin^2\theta\, d\phi^2$. The inverse transformation reads

$$\left\{\rho = \sqrt{r^2 + z^2}\,, \quad \tan\chi = r/z\right\}$$

The coordinate ρ is positive since $\sin\chi \geq 0$ for $\chi \in [0, \pi]$. The brane is at $\cos\chi = 0$, or $\chi = \frac{\pi}{2}$ The two sides of the brane correspond to $\{z > 0 : \chi \in \left[0, \frac{\pi}{2}\right)\}$ and $\{z < 0 : \chi \in \left(\frac{\pi}{2}, \pi\right]\}$ - identical due to the symmetry $\chi \to \pi - \chi$



Our Brane/Bulk Perspective

Since $\rho = \sqrt{r^2 + z^2}$, on the brane where $z = 0$, $\rho \equiv r$. Also, the 5D radial infinity, $\rho \to \infty$ describes both the asymptotic AdS boundary ($|z| \to \infty$) and the radial infinity on the brane ($r \to \infty$)

Inspired by Chamblin et al (2000), we replace the flat part $(-dt^2 + d\rho^2)$ with the corresponding part of the Schwarzschild solution:

$$ds^2 = \frac{1}{(1 + k\rho\cos\chi)^2}\left[-f(\rho)\,dt^2 + \frac{d\rho^2}{f(\rho)} + \rho^2\,d\Omega_3^2\right],$$

where

$$f(\rho) = 1 - \frac{2M}{\rho}$$

At the location of the brane, $\chi = \pi/2$ and $\rho = r$, and the line-element on the brane becomes Schwarzschild solution with the horizon located at $r = 2M = r_h$.

But what kind of solution do we have in the bulk? We need to evaluate the 5D scalar invariant quantities...

The Geometry

For instance, the 5D Ricci scalar is found to have the form

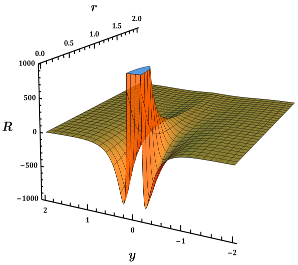
$$R = -20k^2 + \frac{12k^2 M \cos^2\chi}{\rho} - \frac{24kM\cos\chi}{\rho^2} + \frac{4M}{\rho^3}$$

The above contains a constant contribution $-20k^2$, supporting an AdS spacetime, plus additional terms sourced by the mass M

These terms diverge when $\rho \to 0$, i.e. when $z = 0$ (i.e. on the brane) ***and*** $r = 0$ (i.e. at the location of the mass M)

No other singular point exists either in the bulk or on the brane, since these would have either $z \neq 0$ or $r \neq 0$. Thus the singularity remains strictly on the brane

In the limit $\rho \to \infty$, all singular terms vanish leaving behind a pure AdS spacetime



The Geometry

In terms of the original non-spherical coordinates $\{r, y\}$, the 5D line-element has the form

$$ds^2 = e^{-2k|y|} \left\{ -f(r,y)dt^2 + \frac{dr^2}{r^2+z^2(y)} \left[\frac{r^2}{f(r,y)} + z^2(y) \right] + r^2 d\Omega_2^2 \right.$$
$$\left. + \frac{2rz(y)\, e^{k|y|}}{r^2+z^2(y)} \left[\frac{1}{f(r,y)} - 1 \right] drdy \right\} + \frac{dy^2}{r^2+z^2(y)} \left[r^2 + \frac{z^2(y)}{f(r,y)} \right],$$

where

$$z(y) = sgn(y)\,(e^{k|y|} - 1)/k\,, \quad f(r,y) = 1 - \frac{2M}{\sqrt{r^2+z^2(y)}}\,.$$

The aforementioned line-element differs significantly from the ones employed in the brane or bulk perspective and comes closer to the ones employed in numerical works – in fact, it has the same general structure of the ansatz used in the work by Figueras & Wiseman (2011), but here all metric functions are analytically known.

The Geometry

Although the singularity is strictly restricted on the brane, the horizon of the BH is expected to extend into the bulk

The causal structure of the bulk spacetime is reflected in the radial null trajectories in the 5D background. For a fixed value $y = y_0$ and with θ and ϕ kept constant, the condition $ds^2 = 0$ reads

$$\frac{dt}{dr} = \pm \frac{1}{f(r, y_0)} \left[\frac{r^2 k^2 + f(r, y_0) \left(e^{k|y_0|} - 1 \right)^2}{r^2 k^2 + \left(e^{k|y_0|} - 1 \right)^2} \right]^{1/2},$$

where

$$f(r, y_0) = 1 - 2M \left[r^2 + \frac{\left(e^{k|y_0|} - 1 \right)^2}{k^2} \right]^{-1/2}.$$

As $r \to \infty$ along this slice of AdS spacetime, one gets $f(r, y_0) \to 1$, and the light-cone slope dt/dr goes to ± 1 as in Minkowski. But, at a point $r = r_h(y_0)$, where $f(r_h, y_0) = 0$, we obtain $dt/dr = \pm\infty$

The Geometry

This is the point where we meet the horizon of the black hole as it extends into the bulk. Its exact location is

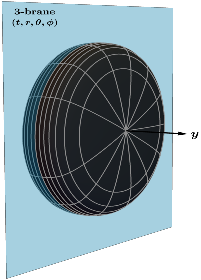
$$r_h^2 = 4M^2 - \frac{\left(e^{k|y_0|} - 1\right)^2}{k^2}$$

It is a function of y_0, i.e. at different locations in the bulk, r_h has a different value: on the brane ($y_0 = 0$), $r_h = 2M$ but for any $y_0 \neq 0$, r_h is exponentially suppressed with y_0

In fact, the horizon shrinks to zero at

$$|y_0| = \frac{1}{k} \ln\left(2Mk + 1\right)$$

The BH horizon has the shape of a "pancake" : its long side lies along the brane and its short side extends over an exponentially small distance in the bulk



The Matter Content

The complete 5D gravitational theory has the form

$$S_B = \int d^5x \sqrt{-g} \left(\frac{R}{2\kappa_5^2} + \mathcal{L}_m^{(B)} \right)$$

The gravitational field equations in the bulk read

$$G_{MN} = R_{MN} - \frac{1}{2} g_{MN} R = \kappa_5^2 \, T_{MN}^{(B)}$$

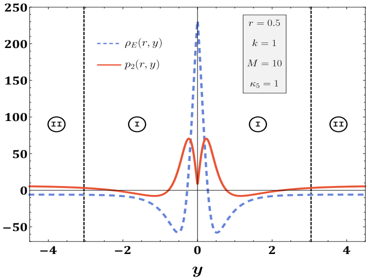
Employing the form of the gravitational background in ‘spherical coordinates’, we obtain

$$T^{(B)t}{}_t = T^{(B)\rho}{}_\rho = \frac{1}{\kappa_5^2} \left(6k^2 + \frac{9kM \cos\chi}{\rho^2} - \frac{3M}{\rho^3} \right)$$

$$T^{(B)\chi}{}_\chi = T^{(B)\theta}{}_\theta = T^{(B)\phi}{}_\phi = \frac{1}{\kappa_5^2} \left(6k^2 - \frac{6k^2 M \cos^2\chi}{\rho} + \frac{6kM \cos\chi}{\rho^2} \right)$$

Thus, T_{MN}^B is diagonal and has two independent components: (ρ_E, p_2).

The Matter Content



The bulk spacetime is divided in regions I (region between the brane and the BH horizon) and II (between the BH horizon and AdS boundary)

The quantities ρ_E and p_2 satisfy all energy conditions on the brane and reduce to the AdS values right outside the BH Horizon

Inside region I though there is a y-interval where the energy conditions are violated - the price to pay for supporting the desired geometry...

The Matter Content

In fact, the complete theory should also accommodate the brane, i.e.

$$T_{MN} = T^{(B)}_{MN} + \delta^{\mu}_{M}\delta^{\nu}_{N}\, T^{(br)}_{\mu\nu}\delta(y)$$

where $T^{(br)}_{\mu\nu}$ is the brane energy-momentum tensor written also as

$$T^{(br)}_{\mu\nu} = -\sigma\, h_{\mu\nu} + \tau_{\mu\nu}$$

σ is the tension of the brane, and $\tau_{\mu\nu}$ encodes all the other possible sources of matter on the brane. Do we need a $\tau_{\mu\nu}$ for the embedding?

We use Israel's (1967) junction conditions at $y = 0$, which are

$$[K_{\mu\nu}] = -\kappa_5^2 \left(T^{(br)}_{\mu\nu} - \frac{1}{3} h_{\mu\nu}\, T^{(br)} \right)$$

In the above, $K_{MN} \equiv h^A_M h^B_N \nabla_A n_B$ is the extrinsic curvature of the brane, h_{MN} the induced 4D metric and n^M the tangent vector on the brane

The Matter Content

The induced line-element on the brane is found by setting $y = 0$:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2$$

Using the above and that $n^M = \delta^M{}_y$, the extrinsic curvature close to the 3-brane is found to be

$$K_{MN} = -k \frac{d|y|}{dy} \, \delta_M^\mu \delta_N^\nu \, h_{\mu\nu},$$

After a little bit of algebra, we find that

$$T_{\mu\nu}^{(br)} = -\frac{6k}{\kappa_5^2} \, h_{\mu\nu}$$

Comparing with the general form of $T_{\mu\nu}^{(br)}$, we obtain $\sigma = 6k/\kappa_5^2 > 0$, while $\tau_{\mu\nu} = 0$. Therefore, no additional distribution of matter is necessary to be introduced on the brane for its consistent embedding in the bulk.

The Matter Content

Let us also derive the effective field equations on the brane (Shiromizu, Maeda & Sasaki, 2000). Decomposing the 5D Riemann and Ricci tensors in bulk and brane contributions, one obtains

$$\bar{G}_{\mu\nu} = \frac{2\kappa_5^2}{3}\,\mathcal{T}^{(B)}_{\mu\nu} - 3k^2\,h_{\mu\nu} + 8\pi G_4\,\tau_{\mu\nu} + \kappa_5^4\,\pi_{\mu\nu} - E_{\mu\nu}\Big|_{y\to 0},$$

where $G_4 = \kappa_5^4\,\sigma/48\pi > 0$ is the 4D Newton's constant, and

$$\mathcal{T}^{(B)}_{\mu\nu} \equiv \left[T^{(B)}_{\mu\nu} + \left(T^{(B)}_{yy} - \frac{T^{(B)}}{4}\right)h_{\mu\nu}\right]_{y\to 0} = \frac{9k^2}{2\kappa_5^2}\,h_{\mu\nu} + \frac{3M}{2\kappa_5^2}\,\frac{1}{r^3}\begin{pmatrix} -h_{tt} & 0 & 0 & 0 \\ 0 & -h_{rr} & 0 & 0 \\ 0 & 0 & h_{\theta\theta} & 0 \\ 0 & 0 & 0 & h_{\varphi\varphi} \end{pmatrix}$$

Also, by direct calculation, we find that

$$(E_{\mu\nu})\Big|_{y\to 0} \equiv C^A{}_{BCD}\,n_A\,n^C\,h^B{}_\mu\,h^D{}_\nu = \frac{M}{r^3}\begin{pmatrix} -h_{tt} & 0 & 0 & 0 \\ 0 & -h_{rr} & 0 & 0 \\ 0 & 0 & h_{\theta\theta} & 0 \\ 0 & 0 & 0 & h_{\varphi\varphi} \end{pmatrix}$$

Substituting all the above in the effective field eqs, we obtain $\bar{G}_{\mu\nu} = 0$. This was expected since the Schwarzschild solution is a vacuum solution!

The (A)dS-Reissner-Nördstrom solution (arXiv: 2105.06915)

Once again, we build the 5D spherically-symmetric line-element

$$ds^2 = \frac{1}{(1 + k\rho\cos\chi)^2}\left[-f(\rho)\,dt^2 + \frac{d\rho^2}{f(\rho)} + \rho^2\,d\Omega_3^2\right], \qquad \chi \in [0, \pi/2]$$

but assume that

$$f(\rho) = 1 - \frac{2M}{\rho} + \frac{Q^2}{\rho^2} - \frac{\Lambda}{3}\,\rho^2$$

Now, on the brane, where $\cos\chi = 0$ and $\rho = r$, the above reduces to an (A)dS-Reissner-Nordström BH, with M the BH mass, Q its charge and Λ the effective cosmological constant on the brane

The analysis follows along the same lines: the singularity is localised on the brane, the horizon(s) exponentially suppressed with y, given by

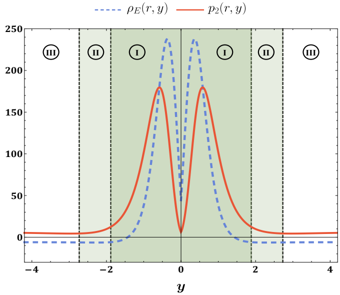
$$r_h^2 = \rho_h^2 - \frac{\left(e^{k|y_0|} - 1\right)^2}{k^2},$$

and the 5D spacetime everywhere regular and asymptotically AdS_5

The (A)dS-Reissner-Nörstrom solution

The $T^{(B)}_{MN}$ in the bulk is again diagonal and described only by (ρ_E, p_2)

The bulk spacetime is now divided in regions: I (between the brane and the Cauchy horizon), II (between the CH and the BH horizon), III (between the BH horizon and the Cos-H) and IV (between the Cos-H and the AdS boundary - not shown)



ρ_E and p_2 satisfy all energy conditions on the brane and reduce to the AdS values even before the BH Horizon

The energy conditions are again violated inside region I - can we find a sensible field theory that would justify this?

The (A)dS-Reissner-Nörstrom solution

We considered the following 5D scalar-vector theory

$$\mathcal{L}_m^{(B)} = -\frac{1}{4}\,F^{MN}F_{MN} - f_1(\xi,\psi)(\partial\xi)^2 - f_2(\xi,\psi)(\partial\psi)^2 - V(\xi,\psi)$$

Above, $F_{MN} = \nabla_M A_N - \nabla_N A_M$ is the field-strength tensor of an Abelian gauge field A_M and $\{\xi(\rho,\chi),\psi(\rho,\chi)\}$ are two scalar fields

The gauge field confinguration was chosen to be

$$\left(F^{MN}\right) = \begin{pmatrix} 0 & E_1(\rho,\chi) & E_2(\rho,\chi) & 0 & 0 \\ -E_1(\rho,\chi) & 0 & 0 & 0 & 0 \\ -E_2(\rho,\chi) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & B(\rho,\chi,\theta) \\ 0 & 0 & 0 & -B(\rho,\chi,\theta) & 0 \end{pmatrix},$$

where E_1, E_2 and B stand for two components of the “electric” bulk gauge field and a sole component of the “magnetic” field

The (A)dS-Reissner-Nörstrom solution

These lead to the following components of the bulk $T^{(B)}_{MN}$

$$T^{(B)t}{}_{t} = \tfrac{1}{2}(-bB^2 + a_1 E_1^2 + a_2 E_2^2) - f_1(\partial\xi)^2 - f_2(\partial\psi)^2 - V,$$

$$T^{(B)\rho}{}_{\rho} = \tfrac{1}{2}(-bB^2 + a_1 E_1^2 - a_2 E_2^2) + f_1\left(\partial^\rho\xi\partial_\rho\xi - \partial^\chi\xi\partial_\chi\xi\right) + f_2\left(\partial^\rho\psi\partial_\rho\psi - \partial^\chi\psi\partial_\chi\psi\right) - V,$$

$$T^{(B)\rho}{}_{\chi} = a_2 E_1 E_2 + 2\left(f_1\,\partial^\rho\xi\partial_\chi\xi + f_2\,\partial^\rho\psi\partial_\chi\psi\right),$$

$$T^{(B)\chi}{}_{\chi} = \tfrac{1}{2}(-bB^2 - a_1 E_1^2 + a_2 E_2^2) - f_1\left(\partial^\rho\xi\partial_\rho\xi - \partial^\chi\xi\partial_\chi\xi\right) - f_2\left(\partial^\rho\psi\partial_\rho\psi - \partial^\chi\psi\partial_\chi\psi\right) - V,$$

$$T^{(B)\theta}{}_{\theta} = T^{(B)\phi}{}_{\phi} = \tfrac{1}{2}(bB^2 - a_1 E_1^2 - a_2 E_2^2) - f_1(\partial\xi)^2 - f_2(\partial\psi)^2 - V.$$

Demanding that $T^{(B)t}{}_{t} = T^{(B)\rho}{}_{\rho}$, $T^{(B)\rho}{}_{\chi} = 0$, and $T^{(B)\chi}{}_{\chi} = T^{(B)\theta}{}_{\theta}$, we arrive at novel constraints of the form

$$E_1^2 = \frac{1}{g_{tt}g_{\chi\chi}}\,\frac{2\left(f_1\,\partial^\rho\xi\partial_\chi\xi + f_2\,\partial^\rho\psi\partial_\chi\psi\right)^2}{f_1\,\partial^\rho\xi\partial_\rho\xi + f_2\,\partial^\rho\psi\partial_\rho\psi}$$

In conjunction with the field eqs and metric signature, it follows that the gauge field and/or scalar field(s) should be phantom-like in the bulk

Conclusions

- We constructed a 5D gravitational solution describing a BH with its singularity localised on the brane and its horizon having a pancake shape
- No matter needs to be introduced on the brane for the embedding of the brane in the bulk, and the induced-on-the-brane Schwarzschild metric satisfies the effective field equations
- Other variants, such as the Reissner-Nordström-(A)dS solution, may be constructed - Q is a tidal charge which appears in many 'brane' solutions (e.g. in Dadhich, R. Maartens, P. Papadopoulos, and V. Rezania, 2000)
- The field-theory interpretation is still lacking - the violation of the energy conditions (similar to that in the case of wormholes) points to either some exotic form of matter (phantom fields, K-essence) or perhaps gravitational effects (Gauss-Bonnet term)