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SLOW AND SAFE GRAVITINOS

Based on :

E.D., M.A.G.Garcia, Y.Mambrini, K.A.Olive, M.Peloso and S.Verner,
Phys. Rev. **D103** (2021), 123519 [arXiv:2104.03749 [hep-th]]



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Outline

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- 2) Complete eq. for the longitudinal gravitino
- 3) Specific models
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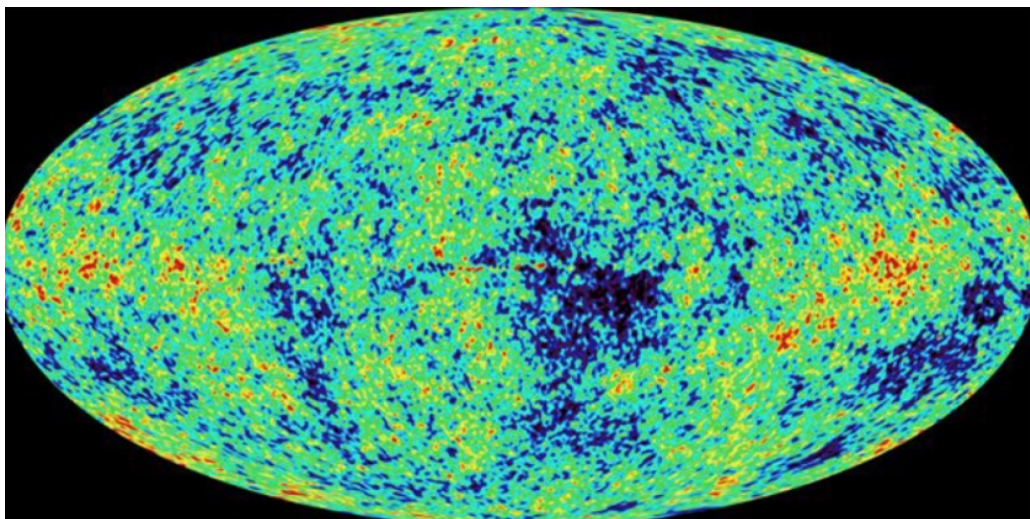
1) Gravitino sound speed in supergravity

Supergravity = Supersymmetry + Gravity

It contains :

- gravity multiplet: Graviton $g_{\mu\nu}$, gravitino ψ_μ
- « matter » fields: (complex) Scalars ϕ_i , Weyl Fermions ψ_i
chiral superfields Φ_i

Why Supergravity for early cosmology ?



- Inflation with super-Planckian field variations needs an UV completion \longrightarrow **String Theory**
- **Supersymmetry** crucial ingredient in String Theory, **supergravity** its low-energy effective action

- In supergravity, the gravitino Ψ_μ becomes **massive** by absorbing the **goldstino** G

$$\Psi_\mu \begin{pmatrix} 3/2 \\ - \\ - \\ -3/2 \end{pmatrix} + G \begin{pmatrix} - \\ 1/2 \\ -1/2 \\ - \end{pmatrix} = \Psi_\mu \begin{pmatrix} 3/2 \\ 1/2 \\ -1/2 \\ -3/2 \end{pmatrix}$$

and its mass is $m_{3/2} = e^{\frac{K}{2}} |W|$

The **sound speed** c_s is defined from the dispersion relation

$$\omega^2 = c_s^2 \mathbf{k}^2 + a^2 m^2$$

The transverse spin 3/2 component in a FRW background has a standard dispersion relation with $c_s = 1$

$$(\gamma^0 \partial_0 + i \gamma^i k_i + a m_{3/2}) \Psi_{3/2, \mathbf{k}} = 0$$

scale factor

Obs: **Supergravity** solves potential **pathological behaviour** of Rarita-Schwinger spin 3/2 fields pointed out by Velo-Zwanziger in 1969.

The longitudinal (goldstino) component satisfies a more involved equation

$$(\gamma^0 \partial_0 - i\gamma^i k_i \frac{\alpha_1 + \gamma^0 \alpha_2}{\alpha} + am_{3/2}) \Psi_{1/2, \mathbf{k}} = 0$$

with $\alpha_1, \alpha_2, \alpha$ specific functions of scalar fields in SUGRA, with the sound speed depending generically on time

$$c_s^2 = \frac{|\alpha_1|^2 + |\alpha_2|^2}{\alpha^2}$$

$$c_s < 1 \quad \longrightarrow \quad \text{Slow gravitino} \quad (\text{Benakli, Darmé, Oz, 2014})$$

A general expression for longitudinal gravitino speed sound is

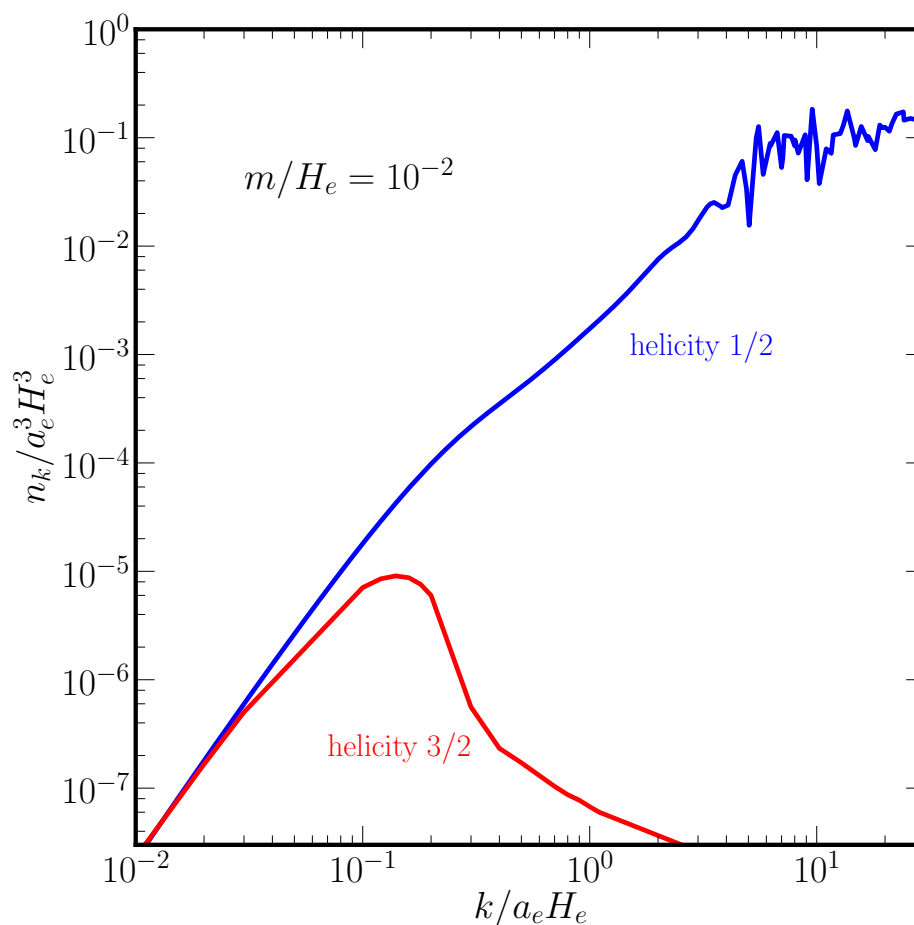
$$c_s^2 = \frac{\overset{\text{pressure}}{\downarrow} \left(p - 3m_{3/2}^2 \right)^2}{\left(\underset{\text{energy density}}{\uparrow} \rho + 3m_{3/2}^2 \right)^2} + \frac{\overset{\text{time-derivative}}{\downarrow} 4\dot{m}_{3/2}^2}{\left(\rho + 3m_{3/2}^2 \right)^2}$$

$c_s = 0$ is possible if $m_{3/2}$ is const. and $p = 3m_{3/2}^2$

In this case, there would be a **catastrophic production** of gravitinos during inflation

(Hasegawa et al, 2017; Kolb, Long, McDonough, 2021).

The problem was argued to arise for $m_{3/2} < H$. If the problem is generic \longrightarrow **potential issue** for low-energy SUSY models.



(taken from
Kolb et al, 2021)



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The explicit formula in SUGRA is (Kolb et al, 2021)

$$c_s^2 = 1 - \frac{4}{(|\dot{\varphi}|^2 + |F|^2)^2} \left\{ |\dot{\varphi}|^2 |F|^2 - |\dot{\varphi} \cdot F^*|^2 \right\}$$

where $F^i \equiv e^{K/2} K^{ij*} D_{j*} W^*$ in standard SUGRA,

$$D_i W \equiv \frac{\partial W}{\partial \varphi^i} + \frac{\partial K}{\partial \varphi^i} W$$

and we used the compact notation $|\dot{\varphi}|^2 = \dot{\varphi}^i K_{ij*} \dot{\varphi}^{j*}$, etc

Obs: Cauchy-Schwarz inequality \longrightarrow causality $c_s \leq 1$
respected in all **standard SUGRA's**

2) Complete eqs. for the longitudinal gravitino

In an expanding background, the longitudinal gravitino θ is coupled to another fermion, the **inflatino**

$$\Upsilon = K_{ij^*} \left(\chi^i \partial_0 \varphi^{j^*} + \chi^{j^*} \partial_0 \varphi^i \right)$$

(Kallosh, Kofman, Linde, Van Proeyen, 2000; Nilles, Peloso, Sorbo, 2001)

θ and Υ are coupled via

$$(\gamma^0 \partial_0 + i \gamma^i k_i N + M) X = 0 \quad , \quad X = \begin{pmatrix} \tilde{\theta} \\ \tilde{\Upsilon} \end{pmatrix}$$



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where the « sound speed matrix »

$$N = \begin{pmatrix} -\frac{\alpha_1}{\alpha} - \gamma^0 \frac{\alpha_2}{\alpha} & -\gamma^0 \Delta \\ -\gamma^0 \Delta & -\frac{\alpha_1}{\alpha} + \gamma^0 \frac{\alpha_2}{\alpha} \end{pmatrix}$$

with $\Delta = \sqrt{1 - c_s^2}$, is now the key to the « slow gravitino » problem.

When $c_s = 0$, then $N = \begin{pmatrix} 0 & -\gamma^0 \\ -\gamma^0 & 0 \end{pmatrix}$

is nonsingular, leading to a nonvanishing sound speed for the physical eigenstates.

(DGMOPV,2021; see also Antoniadis,Benakli, Ke, 2021 for a more general analysis)



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3) Specific models

For the (large) majority of SUGRA models we investigated , we found no problems, $c_s \neq 0$:

- standard SUGRA models with two chiral superfields (inflaton+SUSY breaking): general statement
- SUGRA models with nilpotent SUSY breaking field

$$S^2 = 0$$



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The only models with problems we found is with the
« **orthogonal constraint** » for the inflaton multiplet Φ

$$S(\Phi - \bar{\Phi}) = 0$$

Only $Re \phi$ is a **dynamical** degree of freedom.

$Im \phi$, the inflatino ψ_ϕ and the auxiliary field F_ϕ
are **determined by the constraint**.

In particular F_ϕ is a bilinear in fermions and **does not appear**
in the scalar potential : $F^\Phi \neq e^{K/2} K^{\Phi\bar{i}} D_{i^*} W^*$



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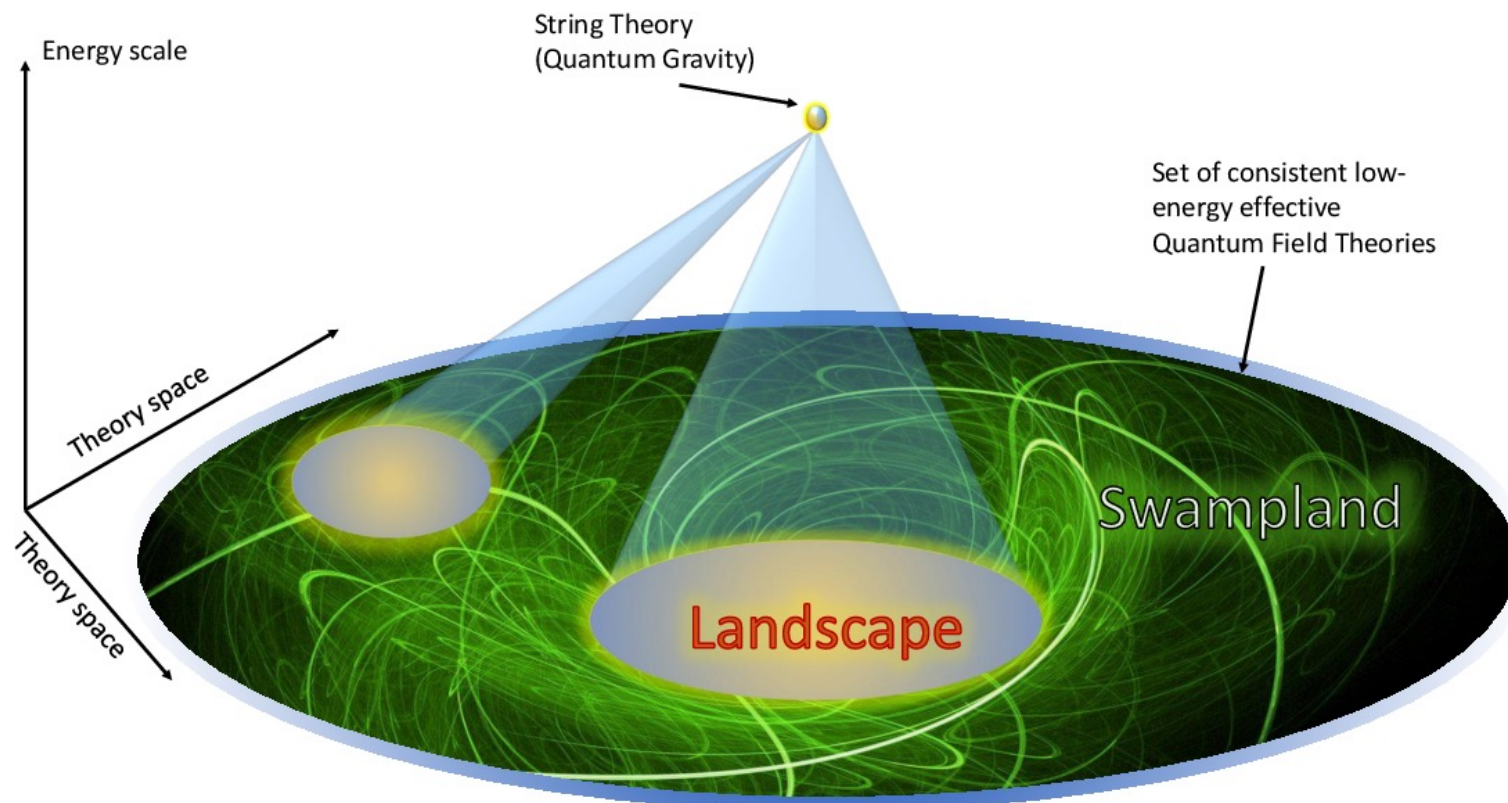


Consequences:

- There is **no inflatino** $\Rightarrow \Upsilon = 0$, the gravitino sound speed problem can reappear (model-dependent)
- The Cauchy-Schwarz argument for $c_s \leq 1$ **not valid** ! We found examples with $c_s > 1$!

On the other hand, the **UV origin** of the orthogonal constraint is not clear (dall'Agata, E.D., Farakos, 2006)

\Rightarrow **Pathological behaviour** reminiscent of the **swampland program** ! (Vafa 2005, Ooguri & Vafa...)



(taken from E. Palti, « The Swampland: Introduction and Review », Fortsch. Phys. **67**, no.6, 1900037 (2019) [arXiv:1903.06239 [hep-th]])



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Interesting to contemplate a « **gravitino swampland conjecture** »

« In all 4d effective field theories that are low-energy limits of quantum gravity, at all points in moduli space and for all initial conditions, the sound speed of the gravitino(s) must be non-vanishing $c_s > 0$ »
(Kolb, Long, McDonough)



our refined version

« In all 4d effective field theories that are low- energy limits of quantum gravity, at all points in moduli space and for all initial conditions, all eigenvalues of the sound speed **matrix** for fermions must be non-vanishing and **subluminal** $0 < \underline{c_s^i} \leq 1$ »

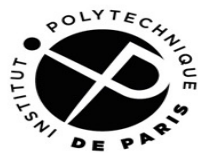


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Conclusions

- Gravitino production constraints important for viability of SUGRA models.
- Very often, the inflatino is produced, alleviating the gravitino problem.
- Important to check and impose sound speed
 $0 < c_s \leq 1 \longrightarrow$ gravitino swampland conjecture
- We believe that most SUGRA models do satisfy it, except peculiar models with nonlinear supersymmetry (orthogonal constraint). Interesting to test other models.



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