Gauge Field Theory Vacuum and Cosmological Inflation without Scalar Field

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1. Heisenberg-Euler Effective Lagrangian

2. Effective Lagrangian in YM theory

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5. Solution of Freidmann Equation in Quantum Vacuum

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Contribution of Vacuum Fluctuations to the Cosmological Constant

The calculation of the effective Lagrangian in QED by Heisenberg and Euler was the first example of a well-defined physically motivated prescription allowing to obtain a finite, gauge and renormalisation group-invariant result when investigating the vacuum fluctuations of quantised fields [29]. It appears that only the difference between vacuum energy in the presence and in the absence of external sources has a well-defined physical meaning [29, 30, 31, 32, 33, 34, 35, 36, 1, 2, 3, 4, 5]. Here we will follow this prescription and will derive the quantum equation of state for non-Abelian gauge fields by using the effective Lagrangian approach [37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54,

Heisenberg-Euler Effective Lagrangian

$$\mathcal{L}_{eff} = \frac{\mathcal{E}^2 - \mathcal{H}^2}{2} - \pi mc^2 (\frac{mc}{h})^3 \int_0^\infty \frac{ds}{s^3} e^{-s} \{ \frac{as\cos(as)}{\sin(as)} \; \frac{bs\cosh(bs)}{\sinh(bs)} - 1 + \frac{a^2 - b^2}{3} s^2 \}$$

where dimensionless fields are

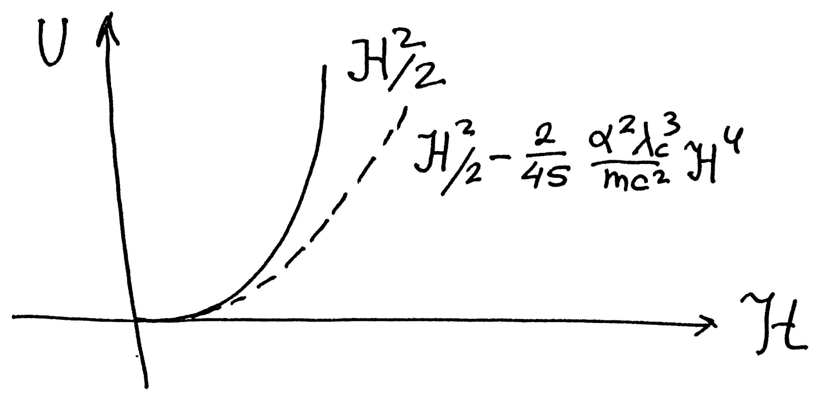
$$a = \frac{e\hbar\mathcal{E}}{m^2c^3}, \qquad b = \frac{e\hbar\mathcal{H}}{m^2c^3}$$

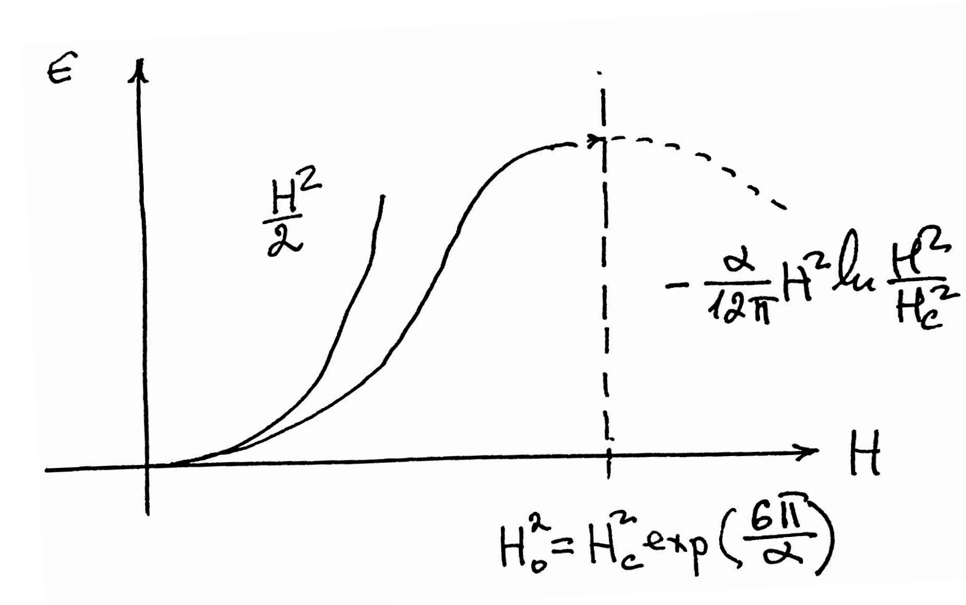
$$mc^2 = 8.2 \cdot 10^{-7} \; \frac{g\; cm^2}{s^2} \qquad \lambda_c = \frac{\hbar}{mc} = 3.86 \cdot 10^{-11} cm \qquad \frac{mc^2}{(\frac{\hbar}{mc})^3} = 1.43 \cdot 10^{25} \frac{g}{cm\; s^2}$$

$$\mathcal{E}_c = \frac{m^2c^3}{e\hbar} \sim 10^{16} \; Volt/cm \qquad \mathcal{H}_c = \frac{m^2c^3}{e\hbar} \sim 4.4 \cdot 10^{13} \; Gauss$$

Heisenberg-Euler Effective Lagrangian

Behaviour of the quantum correction at week and strong magnetic fields





$$\mathcal{L}_{eff} \approx -\frac{\mathcal{H}^2}{2} + \frac{2}{45}(\frac{e^2}{4\pi\hbar c})^2(\frac{\hbar}{mc})^3\frac{1}{mc^2}(\vec{\mathcal{H}}^2)^2$$

$$\mathcal{L}_{eff} \approx -\frac{\mathcal{H}^2}{2} + (\frac{e^2}{4\pi\hbar c})\frac{\mathcal{H}^2}{12\pi}\ln(\frac{e\hbar\mathcal{H}}{m^2c^3})^2$$

Moscow zero

Heisenberg-Euler Effective Lagrangian

Limit of massless Fermions GS Ann.Phys.2018

$$\mathcal{L}_e = -\mathcal{F} + \frac{e^2 \mathcal{F}}{24\pi^2}\Big[\ln(\frac{2e^2\mathcal{F}}{\mu^4}) - 1\Big], \qquad \mathcal{F} = \frac{\vec{\mathcal{H}}^2 - \vec{\mathcal{E}}^2}{2}, \quad \mathcal{G} = \vec{\mathcal{E}}\vec{\mathcal{H}} = 0,$$

the energy momentum tensor by using the formula derived by Schwinger in [5]:

$$T_{\mu\nu} = (F_{\mu\lambda}F_{\nu\lambda} - g_{\mu\nu}\frac{1}{4}F^2_{\lambda\rho})\frac{\partial\mathcal{L}}{\partial\mathcal{F}} - g_{\mu\nu}(\mathcal{L} - \mathcal{F}\frac{\partial\mathcal{L}}{\partial\mathcal{F}} - \mathcal{G}\frac{\partial\mathcal{L}}{\partial\mathcal{G}}).$$

In massless QED using the one-loop expression (1.2) for $T_{\mu\nu}$ one can get

$$T_{\mu\nu} = T^M_{\mu\nu}\Big[1 - \frac{e^2}{24\pi^2}\ln\frac{2e^2\mathcal{F}}{\mu^4}\Big] + g_{\mu\nu}\frac{e^2}{24\pi^2}\mathcal{F}, \qquad \mathcal{G} = 0.$$

Effective Lagrangian in Yang-Mill theory

The YM effective Lagrangian take the following form

$$\mathcal{L}^{(1)} = -\frac{1}{8\pi^2}\int \frac{ds}{s^3} e^{-i\mu^2 s} \frac{(gF_1 s)\ (gF_2 s)}{\sinh(gF_1 s)\ \sinh(gF_2 s)} - $$
$$ - \frac{1}{4\pi^2}\int \frac{ds}{s^3} e^{-i\mu^2 s} (gF_1 s)\ (gF_2 s) [\frac{\sinh(gF_1 s)}{\sinh(gF_2 s)} + \frac{\sinh(gF_2 s)}{\sinh(gF_1 s)}]$$

$$F_1^2 = -\mathcal{F} - (\mathcal{F}^2 + \mathcal{G}^2)^{1/2}, \qquad F_2^2 = -\mathcal{F} + (\mathcal{F}^2 + \mathcal{G}^2)^{1/2}$$

Bartalin, Matinyan and Savvidy 1976
Savvidy 1977
Matinyan and Savvidy 1978

Vanyashin and Terentev 1965
Duff and Ramon-Medrano 1975
Skalozub 1976

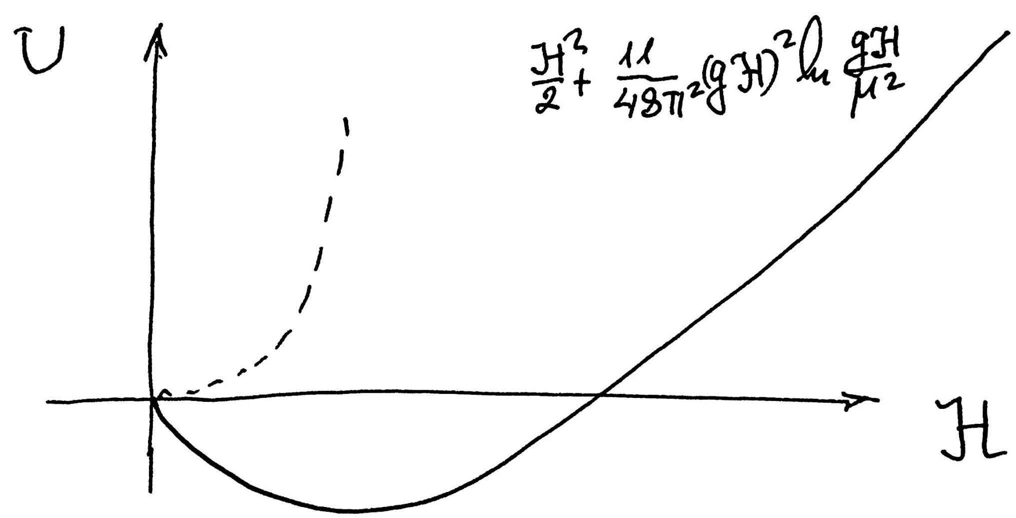
N.Nielsen and Olesen 1978
Ambjorn, N.Nielsen and Olesen 1979
H.Nielsen and Ninomia 1979
H.Nielsen and Olesen 1979
Ambjorn and Olesen1980

Chromomagnetic Condensate

G.Savvidy 1977

$$\mathcal{L}_g = -\mathcal{F} - \frac{11N}{96\pi^2} g^2 \mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big) , \qquad \mathcal{F} = \frac{\vec{\mathcal{H}}_a^2 - \vec{\mathcal{E}}_a^2}{2} > 0, \quad \mathcal{G} = \vec{\mathcal{E}}_a \vec{\mathcal{H}}_a = 0 \ .$$

$$\mathcal{L}_q = -\mathcal{F} + \frac{N_f}{48\pi^2} g^2 \mathcal{F}\Big[\ln(\frac{2g^2\mathcal{F}}{\mu^4}) - 1\Big]$$



$$2g^2\mathcal{F}_{vac} = \mu^4 \exp\,(-\frac{96\pi^2}{b\ g^2(\mu)}) = \Lambda_{YM}^4,$$

where $b = 11N - 2N_f$.

$$T_{\mu\nu} = T_{\mu\nu}^{YM}\Big[1 + \frac{b\ g^2}{96\pi^2}\ln\frac{2g^2\mathcal{F}}{\mu^4}\Big] - g_{\mu\nu}\frac{b\ g^2}{96\pi^2}\mathcal{F}, \qquad \mathcal{G} = 0.$$

Quantum Energy Momentum Tensor

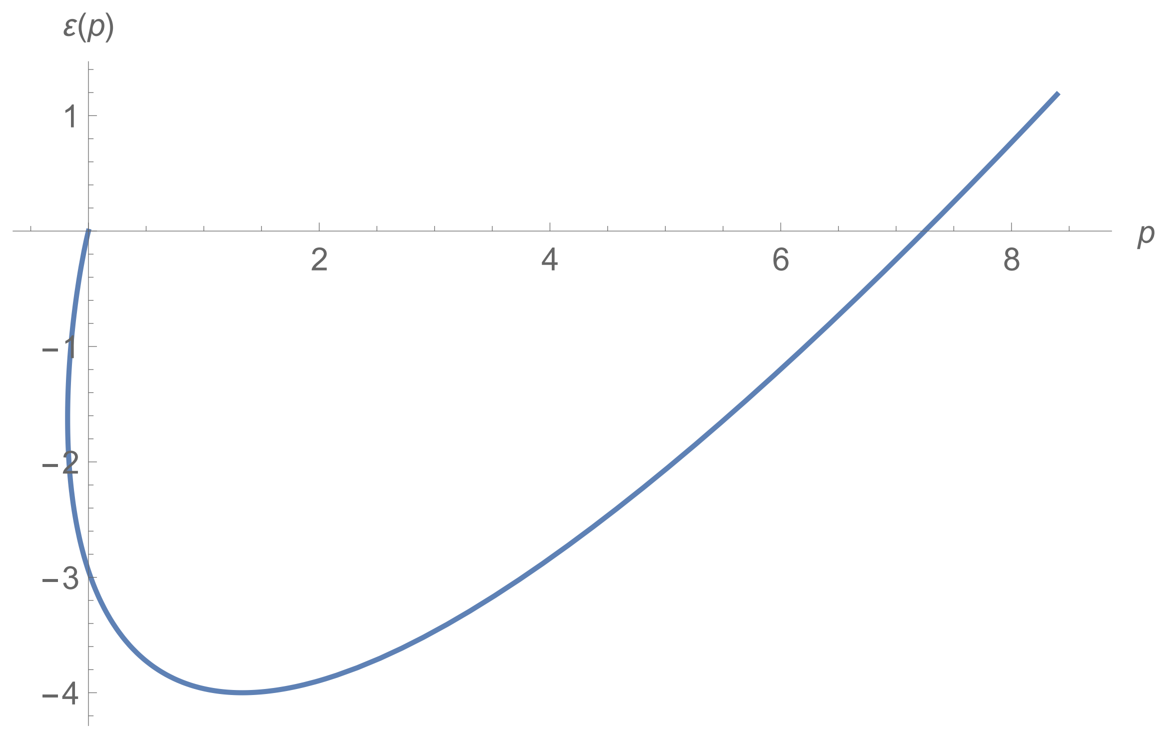
$$T_{\mu\nu} = T_{\mu\nu}^{YM}\Big[1 + \frac{b\ g^2}{96\pi^2}\ln\frac{2g^2\mathcal{F}}{\mu^4}\Big] - g_{\mu\nu}\frac{b\ g^2}{96\pi^2}\mathcal{F}, \qquad \mathcal{G} = 0,$$

$$\epsilon(\mathcal{F}) = \ \mathcal{F} + \frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big), \qquad p(\mathcal{F}) = \frac{1}{3}\mathcal{F} + \frac{1}{3}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} + 3\Big).$$

Quantum Equation of State

$$\epsilon(\mathcal{F}) = \ \frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4} - 1\Big), \qquad p(\mathcal{F}) = \ \frac{1}{3}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4} + 3\Big).$$

Quantum Equation of State



$$p = \frac{1}{3}\epsilon + \frac{4}{3}\frac{b\ g^2\mathcal{F}}{96\pi^2}\Lambda_{YM}^4 \qquad \text{and} \qquad w = \frac{p}{\epsilon} = \frac{\ln\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4} + 3}{3\left(\ln\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4} - 1\right)}$$

general parametrisation of the equation of state $p = w\epsilon$

Einstein Evolution Equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} \Big[T^{YM}_{\mu\nu} \Big(1 + \frac{b\ g^2}{96\pi^2} \ln \frac{2g^2 \mathcal{F}}{\mu^4} \Big) - g_{\mu\nu} \frac{b\ g^2}{96\pi^2} \mathcal{F} \Big].$$

$$\Lambda_{eff} = \frac{8\pi G}{3c^4}\ \epsilon_{vac} = -\frac{8\pi G}{3c^4} \frac{b}{192\pi^2} 2g^2 \mathcal{F}_{vac} = -\frac{8\pi G}{3c^4} \frac{b}{192\pi^2} \Lambda^4_{YM}\ .$$

Freidmann Evolution Equations

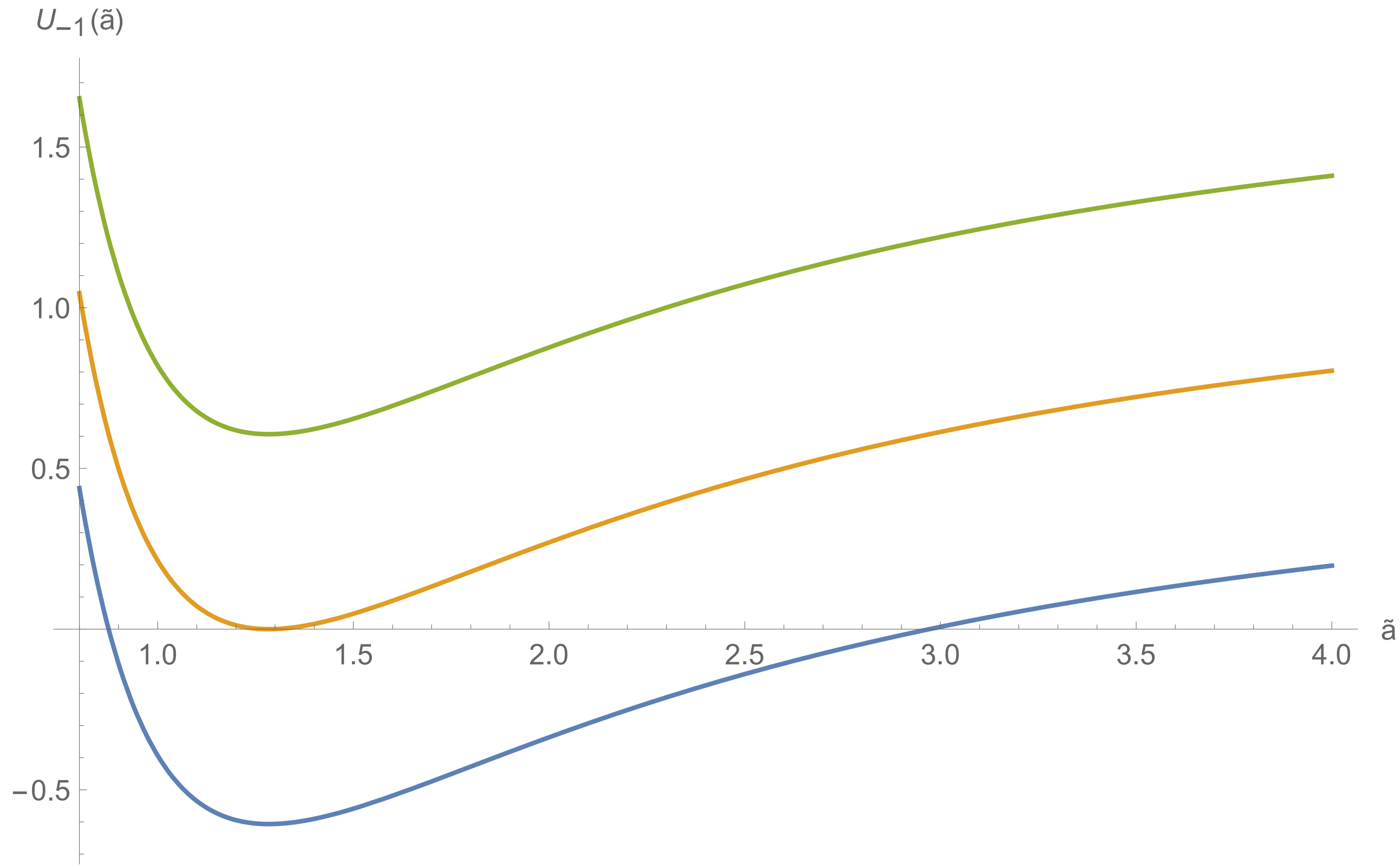
$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + p) = 0, \longrightarrow \quad \epsilon + p = \frac{4\mathcal{A}}{3}\ (2g^2\mathcal{F})\ \log\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4},$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^4}(\epsilon + 3p). \longrightarrow \quad \epsilon + 3p = 2\mathcal{A}\ (2g^2\mathcal{F})\ \Big(\log\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4} + 1\Big).$$

$$\frac{1}{L^2} = \frac{8\pi G}{3c^4}\ \mathcal{A}\ \Lambda_{YM}^4 \equiv \Lambda_{eff}\ , \qquad a(\tau) = a_0\ \tilde{a}(\tau), \qquad ct = L\ \tau,$$

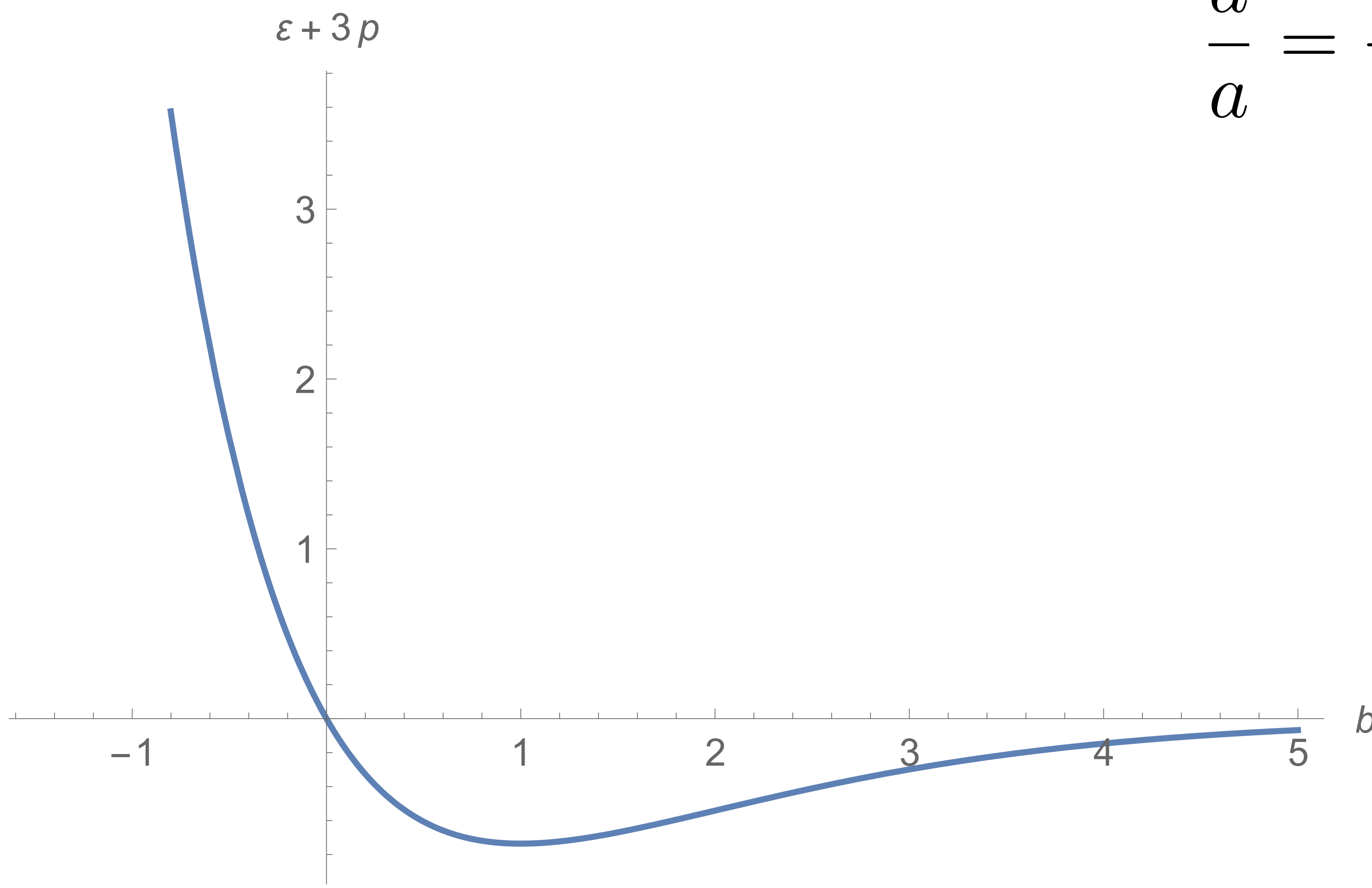
$$\frac{d\tilde{a}}{d\tau} = \pm\sqrt{\frac{1}{\tilde{a}^2}\Big(\log\frac{1}{\tilde{a}^4} - 1\Big) - k\gamma^2}, \qquad k = 0, \pm 1, \qquad \gamma^2 = \Big(\frac{L}{a_0}\Big)^2.$$

$$U_{-1}(\tilde{a}) \equiv \frac{1}{\tilde{a}^2}\left(\log\frac{1}{\tilde{a}^4} - 1\right) + \gamma^2.$$



Strong Energy Dominance Condition is Violated

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^4}(\epsilon + 3p).$$



$\epsilon + 3p$ of the Friedmann acceleration equation is positive when $b < 0$ and is negative when $b > 0$.

Evolution of Energy Density,Pressure, Hubble parameter, Deceleration

$$2g^2\mathcal{F} \;=\frac{\Lambda_{YM}^4}{\tilde{a}^4(\tau)}$$

$$\epsilon=\frac{\mathcal{A}}{\tilde{a}^4(\tau)}\Big(\log\frac{1}{\tilde{a}^4(\tau)}-1\Big)\Lambda_{YM}^4,\qquad p=\frac{\mathcal{A}}{3\tilde{a}^4(\tau)}\Big(\log\frac{1}{\tilde{a}^4(\tau)}+3\Big)\Lambda_{YM}^4.$$

$$p=\frac{1}{3}\epsilon+\frac{4}{3}\frac{\mathcal{A}}{\tilde{a}^4(\tau)}\Lambda_{YM}^4,\qquad w=\frac{p}{\epsilon}=\frac{\log\frac{1}{\tilde{a}^4(\tau)}+3}{3\Big(\log\frac{1}{\tilde{a}^4(\tau)}-1\Big)}.$$

$$L^2H^2=L^2\Big(\frac{\dot{a}}{a}\Big)^2=\frac{1}{\tilde{a}^2}\Big(\frac{d\tilde{a}}{d\tau}\Big)^2=\frac{1}{\tilde{a}^4(\tau)}\Big(\log\frac{1}{\tilde{a}^4(\tau)}-1\Big)-\frac{k\gamma^2}{\tilde{a}^2(\tau)}$$

$$q=\frac{\frac{1}{\tilde{a}^4}\Big(\log\frac{1}{\tilde{a}^4}+1\Big)}{\frac{1}{\tilde{a}^4}\Big(\log\frac{1}{\tilde{a}^4}-1\Big)-\frac{k\gamma^2}{\tilde{a}^2}}\qquad \Omega_{vac}\equiv\frac{8\pi G}{3c^4}\frac{\epsilon}{H^2}=\frac{1}{L^2H^2}\;\frac{1}{\tilde{a}^4}\Big(\log\frac{1}{\tilde{a}^4}-1\Big),$$

Type II Solution — Initial Acceleration of Finite Duration $0 \leq \gamma^2 < \frac{2}{\sqrt{e}}$ and $\tilde{a} \geq \mu_2$.

$$\tilde{a}^4 = \mu_2^4 e^{b^2}, \qquad b \in [0, \infty], \qquad\qquad \frac{db}{d\tau} = \frac{2}{\mu_2^2}\, e^{-\frac{b^2}{2}} \Big(\frac{\gamma^2 \mu_2^2}{b^2} (e^{\frac{b^2}{2}} - 1) - 1 \Big)^{1/2}.$$

$$\mu_2^2 = -\frac{2}{\gamma^2}\, W_-\Big(-\frac{\gamma^2}{2\sqrt{e}} \Big), \qquad\qquad \sqrt{e} < \mu_2^2 \leq \infty, \qquad 2 < \gamma^2 \mu_2^2.$$

The deceleration parameter of the Type II solution is always negative:

$$q_{II} = \frac{b^2 + \gamma^2 \mu_2^2 - 2}{b^2 + \gamma^2 \mu_2^2 (1 - e^{b^2/2})} < 0 \qquad q_{II} \propto -\frac{2}{b^2} \qquad q_{II} \propto -\frac{b^2}{\gamma^2 \mu_2^2} e^{-b^2/2} \to 0.$$

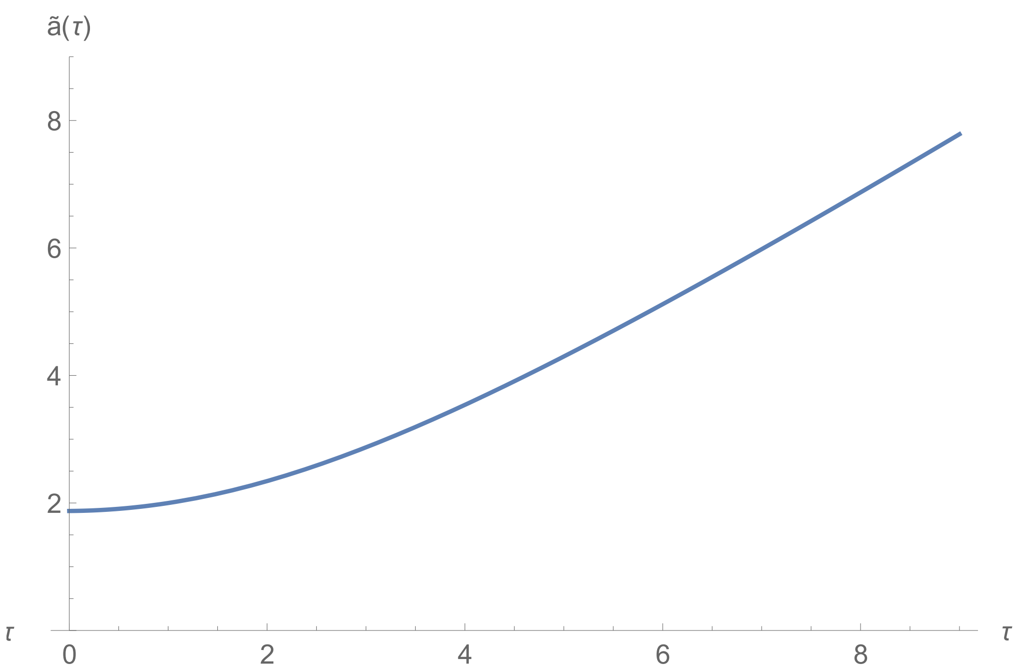
For the equation of state $p = w\epsilon$ one can find the behaviour of the effective parameter w

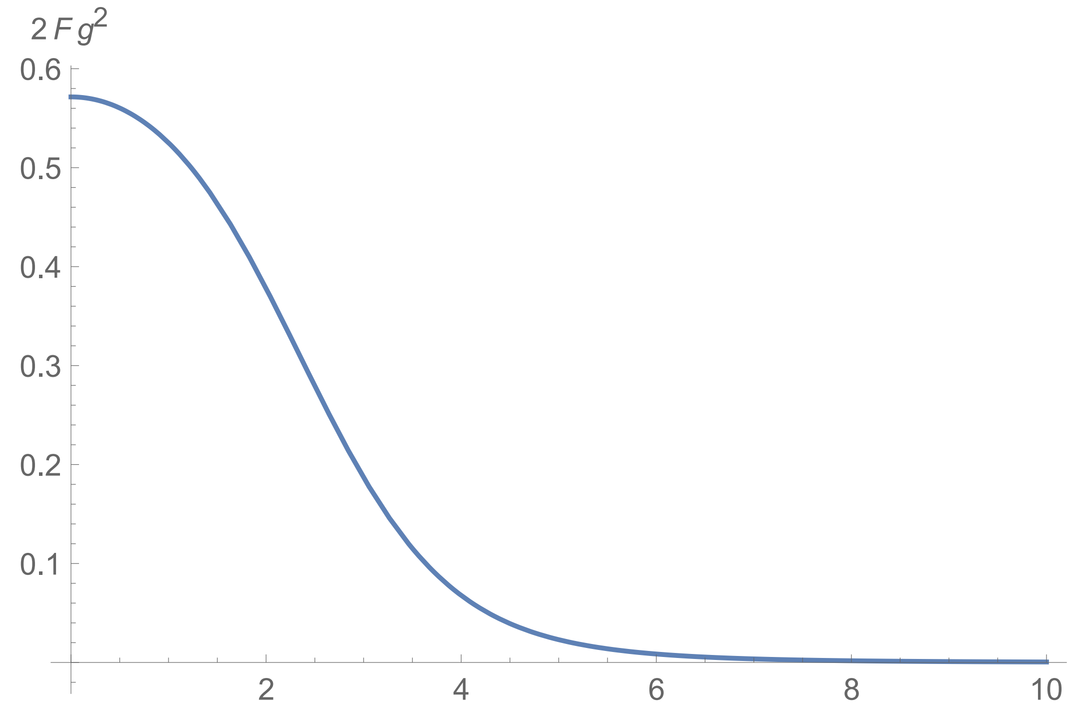
$$w_{II} = \frac{b^2(\tau) + \gamma^2 \mu_2^2 - 4}{3\Big(b^2(\tau) + \gamma^2 \mu_2^2\Big)}, \qquad -1 \leq w_{II},$$

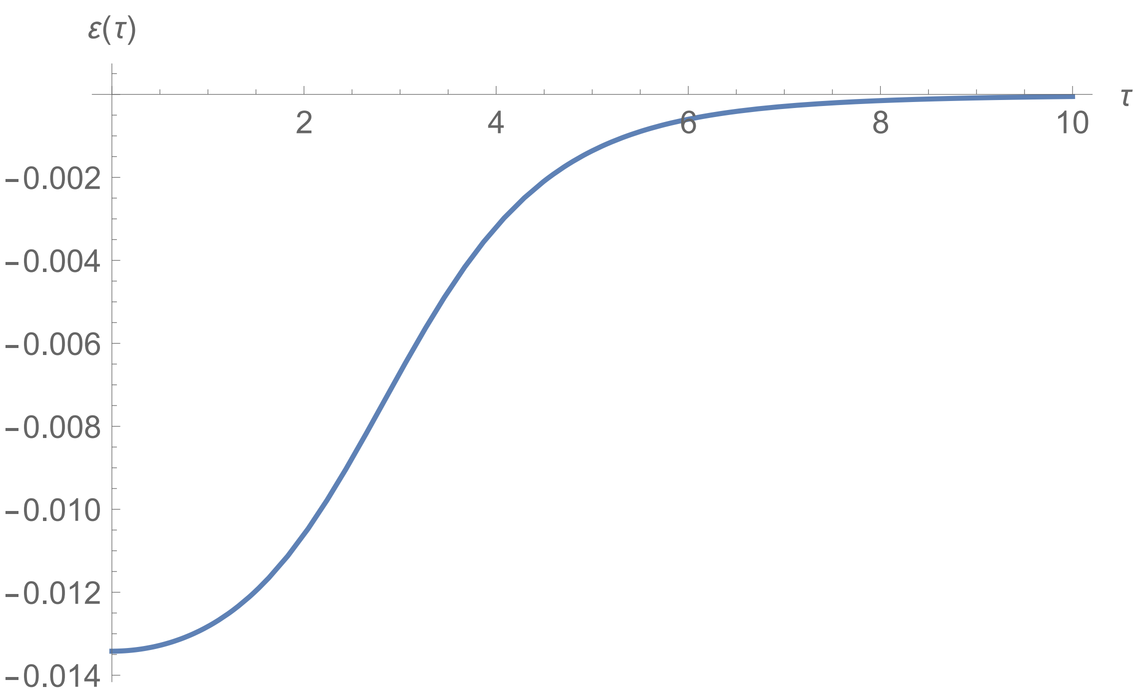
$$L^2 H^2 = \frac{e^{-b^2}}{\mu_2^4} \Big(\gamma^2 \mu_2^2 (e^{b^2/2} - 1) - b^2 \Big). \qquad\qquad \Omega_{vac} - 1 = -\frac{\gamma^2}{(\frac{d\tilde{a}}{d\tau})^2} = -\frac{\gamma^2 \mu_2^2 e^{b^2/2}}{\gamma^2 \mu_2^2 (e^{b^2/2} - 1) - b^2}$$

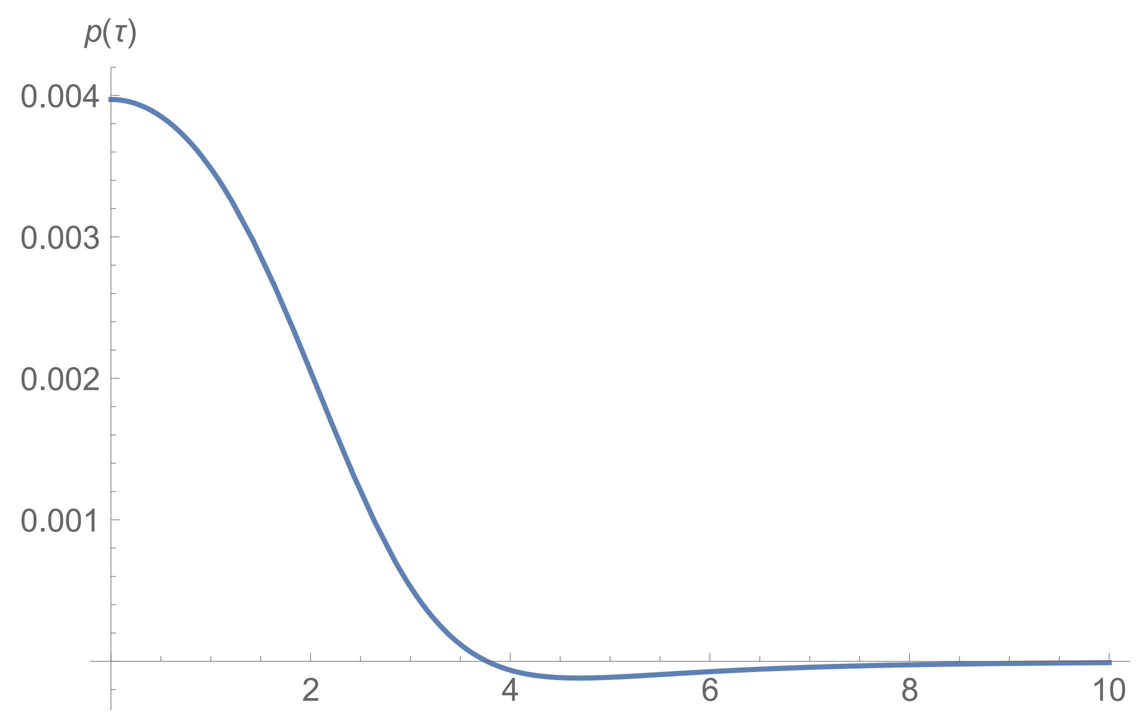
Type II Solution *Initial Acceleration of Finite Duration*

$$\frac{db}{d\tau} = \frac{2}{\mu_2^2}\; e^{-\frac{b^2}{2}} \left(\frac{\gamma^2 \mu_2^2}{b^2} (e^{\frac{b^2}{2}} - 1) - 1 \right)^{1/2}. \qquad \tilde{a}^4 = \mu_2^4 e^{b^2}, \qquad b \in [0, \infty],$$



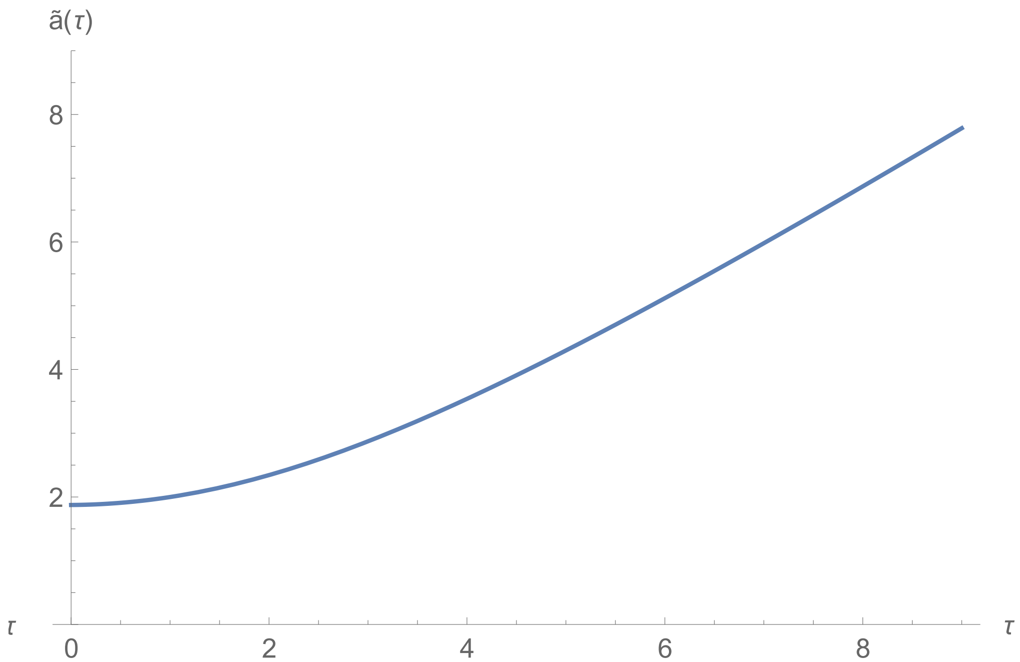


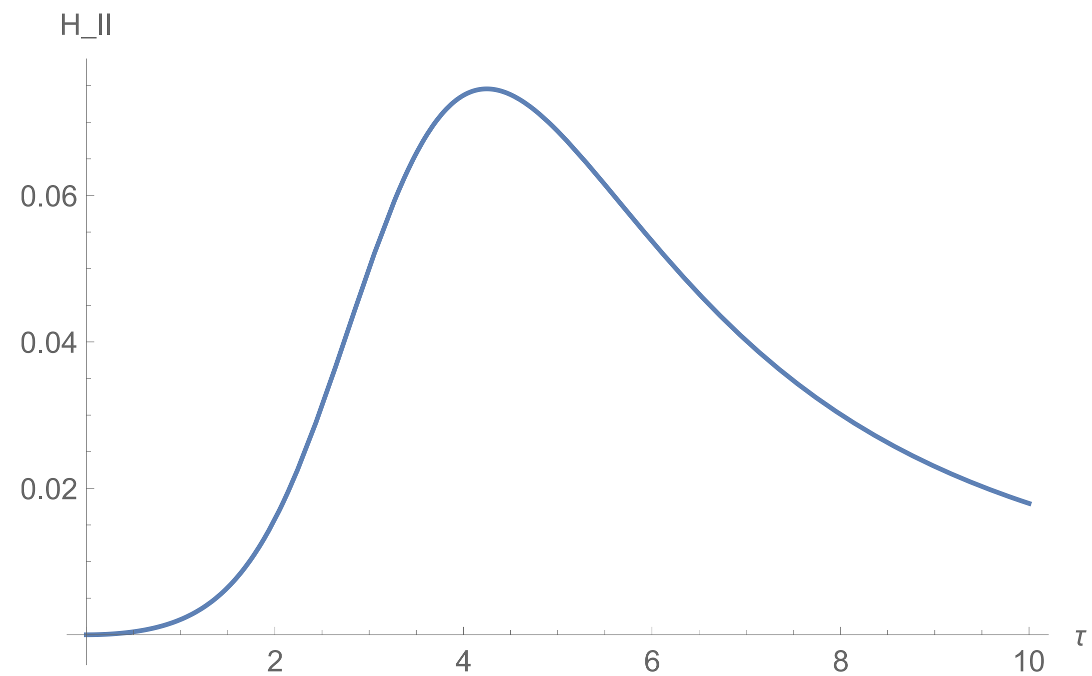


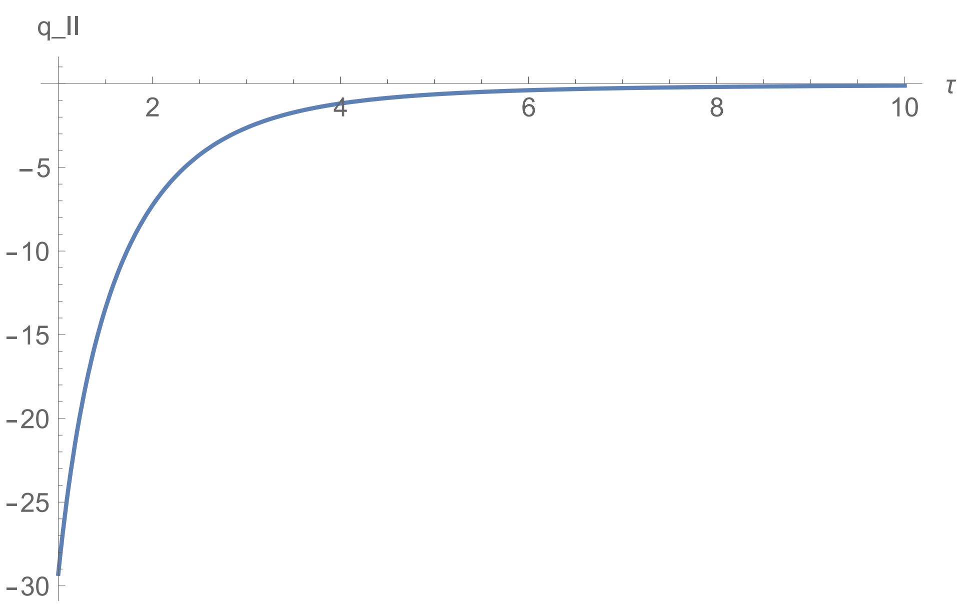


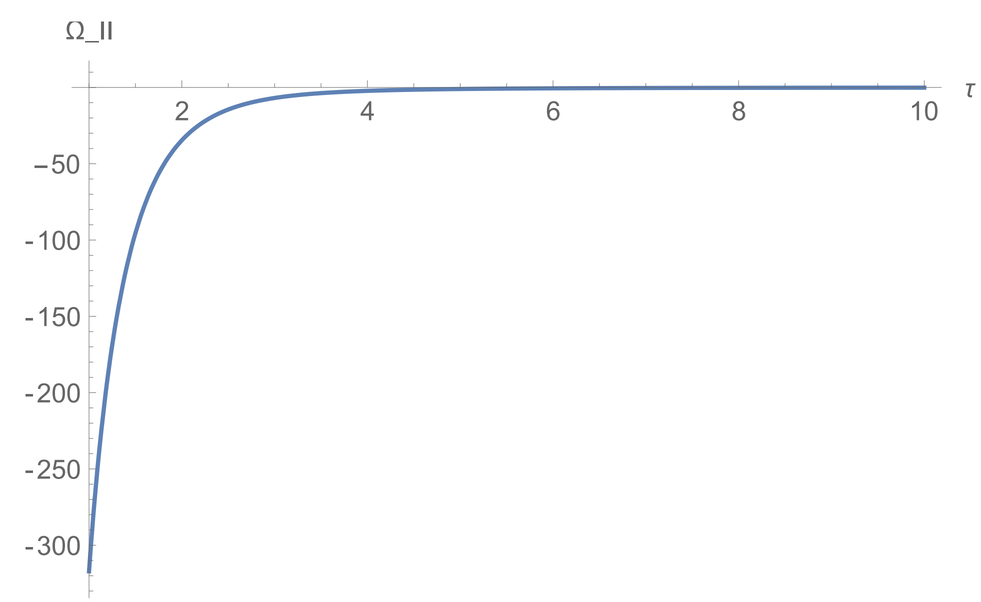
Type II Solution *Initial Acceleration of Finite Duration*

$$\frac{db}{d\tau} = \frac{2}{\mu_2^2}\ e^{-\frac{b^2}{2}}\left(\frac{\gamma^2\mu_2^2}{b^2}(e^{\frac{b^2}{2}}-1)-1\right)^{1/2}. \qquad \tilde{a}^4 = \mu_2^4 e^{b^2}, \qquad b \in [0,\infty],$$



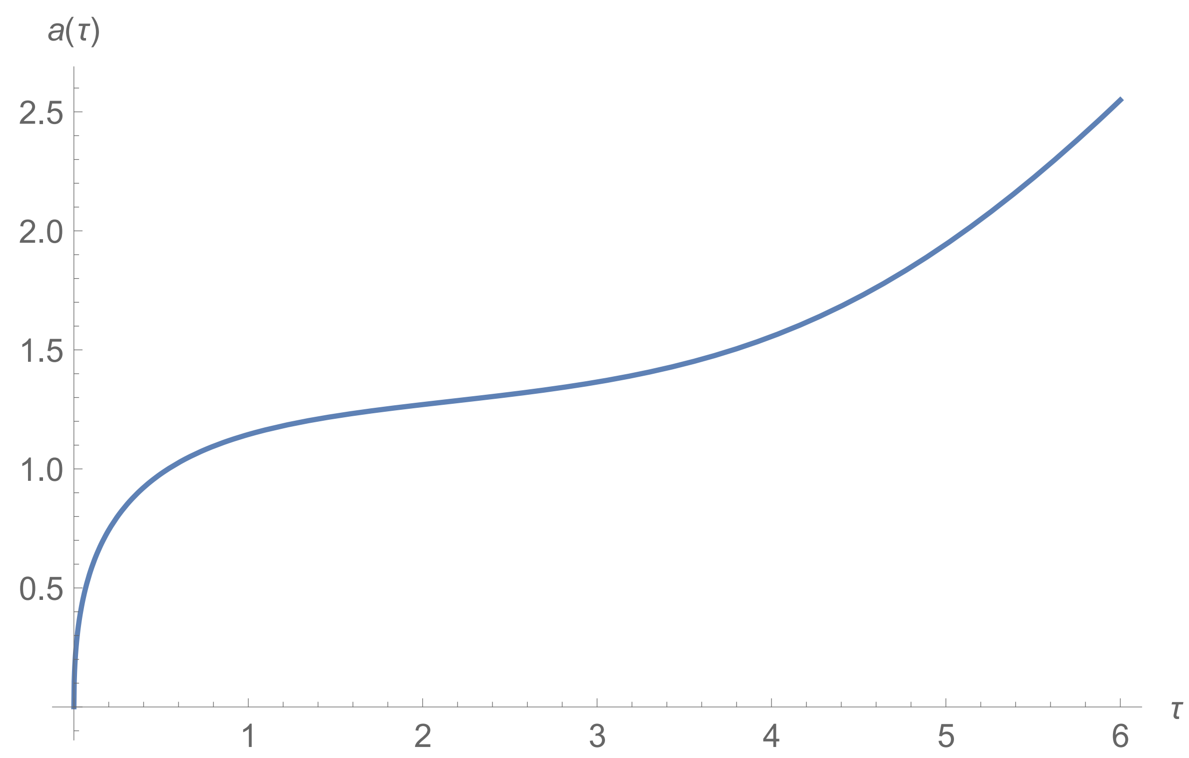






Type IV Solution - Late time Acceleration

The Type IV solution is defined in the region $\gamma^2 > \gamma_c^2$



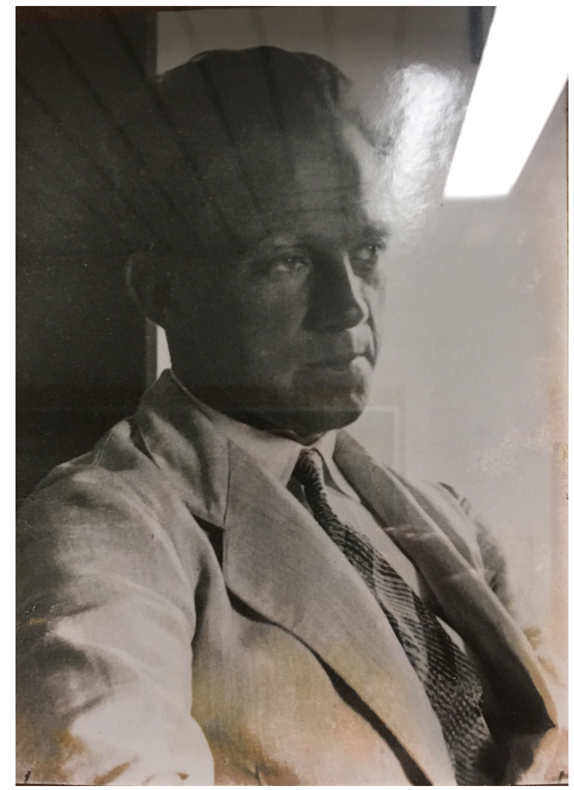
$$\gamma_c^2 = \frac{2}{\sqrt{e}},$$

$$q_{IV} = \frac{b}{b + \frac{1}{2}(1 - \frac{\gamma^2}{\gamma_c^2} e^{2b})},$$

$$H = \sqrt{\frac{2}{e}} \frac{e^{-2b}}{L} \Big(\frac{\gamma^2}{\gamma_c^2} e^{2b} - 1 - 2b \Big)^{1/2} \simeq \frac{1}{ct}.$$

$$\Omega_{vac} = 1 - \frac{\gamma^2}{(\frac{d\tilde{a}}{d\tau})^2} = 1 - \frac{\gamma^2 e^{2b}}{\gamma_c^2 \Big(\frac{\gamma^2}{\gamma_c^2} e^{2b} - 1 - 2b \Big)} \to 0.$$

Euler and Kockel 1935.
Heisenberg and Euler 1936



Hans Euler

Werner Heisenberg

Pair Creation in Electric Field

Arnold Sommerfeld   Werner Heisenberg

Werner Heisenberg in Demokritos National Research Center
Athens, 1956 -1957