Status of electroweak precision measurements

J. Alcaraz (CIEMAT-Madrid)
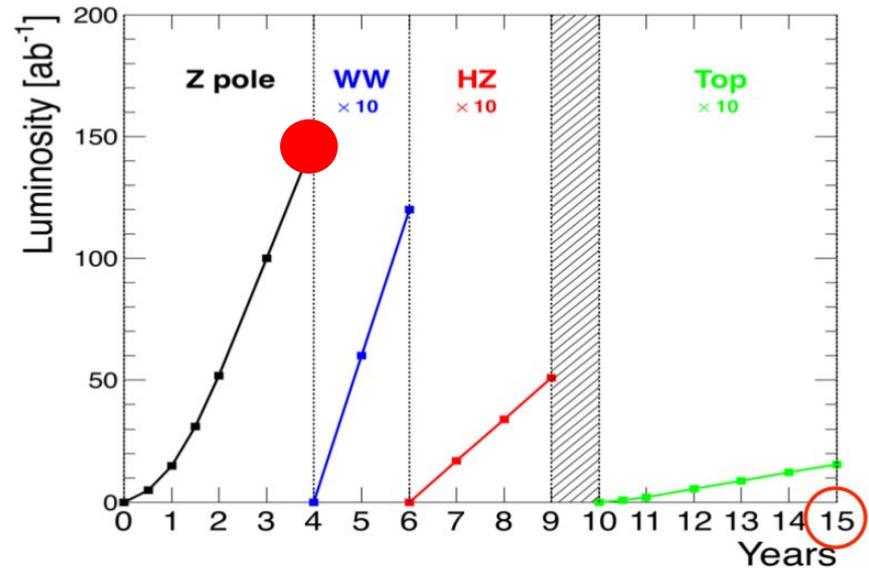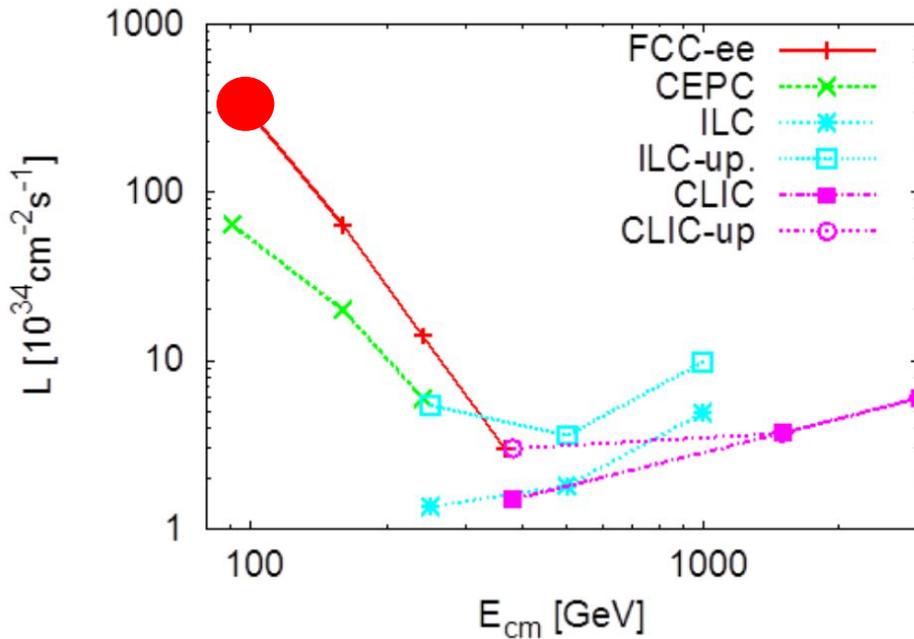
(Table updates: J.A. + A. Blondel + P. Janot + R. Tenchini)

FCC Physics Workshop
11 February 2022

FCC-ee context

- Key point 1: huge statistics → 150 ab^{-1}, 5 x 10^{12} Z decays in ≈ 4 years of running at / around the Z pole, ≈10^3 reduction in statistical uncertainties w.r.t. LEP
- Key point 2: extraordinary √s precision → 100 keV at the Z, 300 keV at WW threshold → exquisite control of beam uncertainties (average, width, systematics)
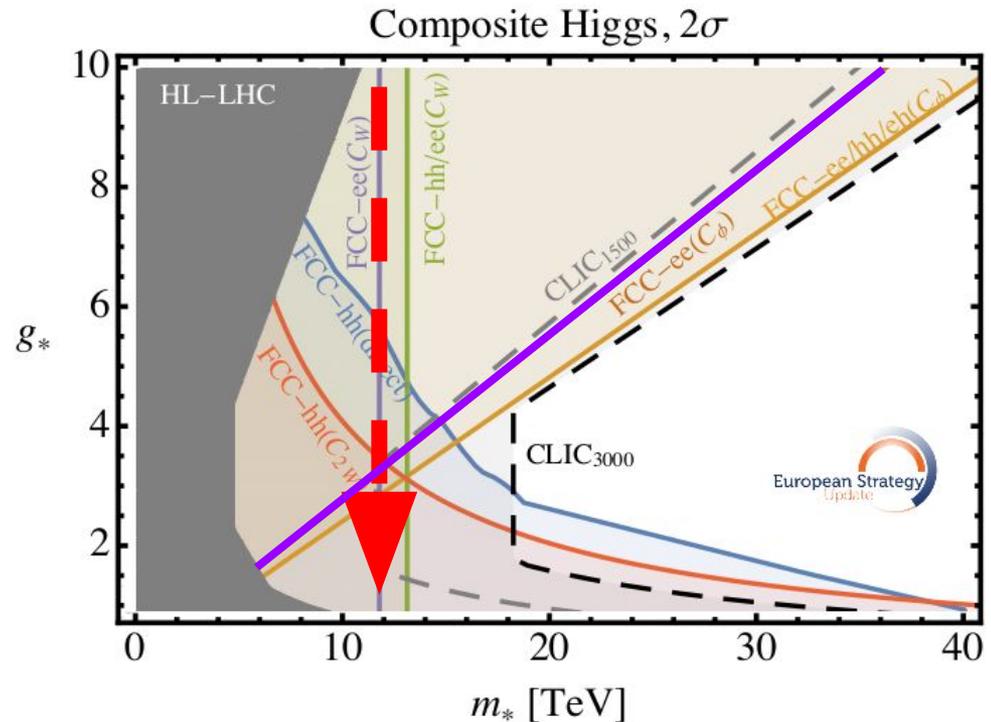- Aiming for up to ≈ 20 to100 times better precision than LEP/SLD on electroweak precision observables (EWPO)

Access to the deca-TeV scale: Higgs compositeness

Simple back-of-the-envelope calculation:

S parameter precision from LEP (PDG 18) $\Rightarrow$
   $m^* > 2.5$ TeV

Factor of 20-100 increase in precision $\Rightarrow$
   $m^* > 11$-25 TeV

European Strategy, J.A., A. Wulzer
in BSM discussion, **arXiv:1910.11775**

Composite Higgs, 2σ

- **Higgs compositeness $\Rightarrow$ S parameter modified effects (= $O_W + O_B$ operator in SILH scenario, related to 2f-2boson contact interactions):**
  - **Sensitive to compositeness scales > 10 TeV, independently of the strength of the g* coupling**
  - Complementary to measurements in the Higgs sector (cross section scaling $\propto g^{*2}$)

Uncertainties / challenges

LEP1:
hep-ex/0509008

- **Expected precisions in a nutshell:**
  - **≈ 10^{-6} statistical uncertainties (≈ 1/ √N) on relative measurements**
    - forward-backward charge asymmetries, cross section ratios, …
  - **≈ 10^{-4} on cross sections from luminosity unc. (Bhabhas at very low angle)**
    - Tight requirements on positioning (≈ μm level), but feasible
    - possibility to still improve by up to one order of magnitude using σ (ee→$\gamma\gamma$) at larger angles (negligible hadr. corrections, studies ongoing)
  - **Uncertainties would be dominated by systematics with today's knowledge, but large improvements expected in coming years:**
    - on theory side
    - on experimental side thanks to "Tera" Z samples:
      - alternative strategies, inclusive → exclusive, huge control samples in more specific kinematic regions, …

Uncertain systematics today ⇒ logic:

- **Two set of estimates:**

  - **1: only statistical uncertainties**
    - This sets our maximal sensitivity.
    - In many cases it will be close to the final total uncertainty, as experience has shown in the past (systematics follows statistics)
    - It is also sets the size of the challenge: how much we need to improve theory predictions, detectors and techniques ⇒ extremely useful to catalyze activities

  - **2: systematic uncertainties**
    - According to current knowledge, largely based on LEP experience
    - Assumptions driven by expected improvements on the theory side
    - Educated guess on reduction of experimental systematics due to improved detector, techniques and the availability of control samples ≈ 10^6 larger than those used at LEP

Our tables (being updated)

Column with previous numbers (stat. + syst.)

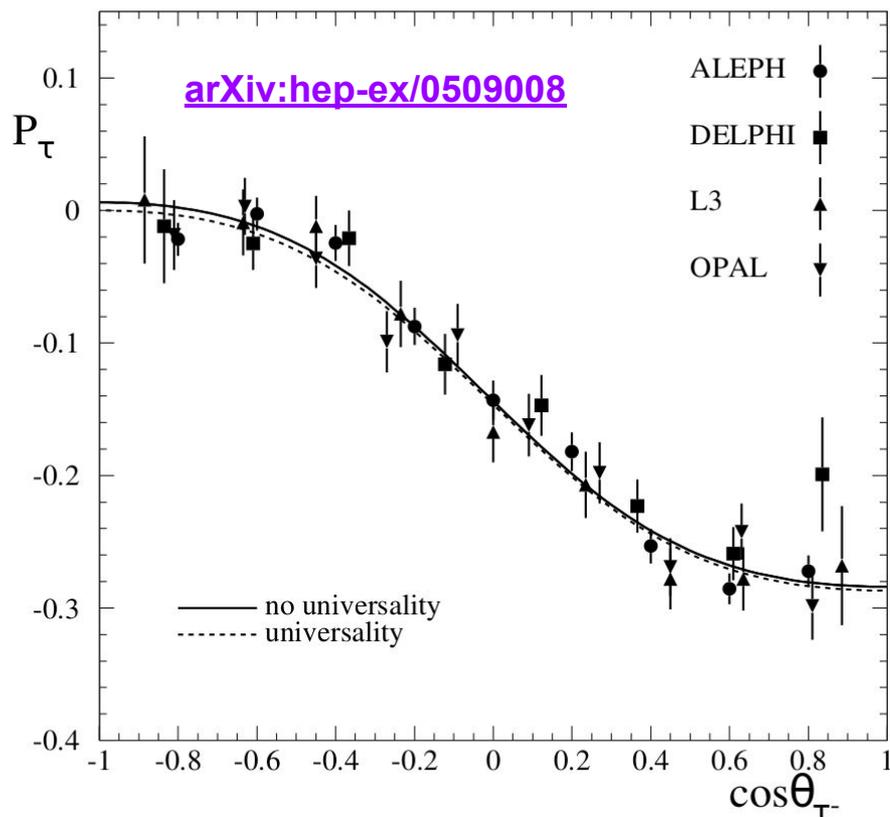|		FCC errors (2 exp)		
		as of table	stat	current exp syst
$\Delta\alpha^{-1}$		0.00387	0.0038	0.0012
Δm_W (MeV)		0.4	0.25	0.3
Δm_Z (MeV)		0.1	0.004	0.1
Δm_H (MeV)		11	2.5	2
$\Delta\Gamma_W$ (MeV)		1.2	1.2	0.3
$\Delta\Gamma_Z$ (MeV)		0.025	0.004	0.025
ΔA_e		0.000017	7.00E-06	2.00E-05
ΔA_μ		0.000023	2.31E-05	2.20E-05
ΔA_τ		0.000045	5.00E-06	2.00E-04
ΔA_τ			1.00E-05	1.30E-04
$\Delta\sin^2\Theta_{lept}$		--	1.40E-06	1.40E-06

• • •

I will focus on:

- **Relevant changes in tables**

- **A few "selected" points in the "to do next" list**

- **Brief discussion on other inputs can be found in backup**

Lepton asymmetries: A_e, A_μ, A_τ

Measured P_τ vs $\cos\theta_{\tau^-}$

arXiv:hep-ex/0509008

ALEPH
DELPHI
L3
OPAL

— no universality
---- universality

$$P(\cos\theta) = \frac{\mathcal{A}_\tau(1+\cos^2\theta) + 2\mathcal{A}_e\cos\theta}{(1+\cos^2\theta) + 2\mathcal{A}_e\mathcal{A}_\tau\cos\theta}$$
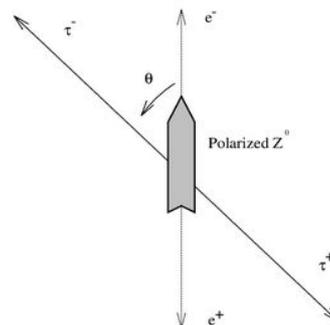
- **IMPORTANT: the FCC-ee baseline does not use longitudinal beam polarization:**
  - Although feasible, It would reduce too much the available luminosity
  - Not needed: tau polarization input is enough to measure A_e, thus facilitating precise measurements of the L-R asymmetry parameters for all fermions: $A_e, A_\mu, A_\tau, A_b, A_c$

$$A_{FB} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f$$

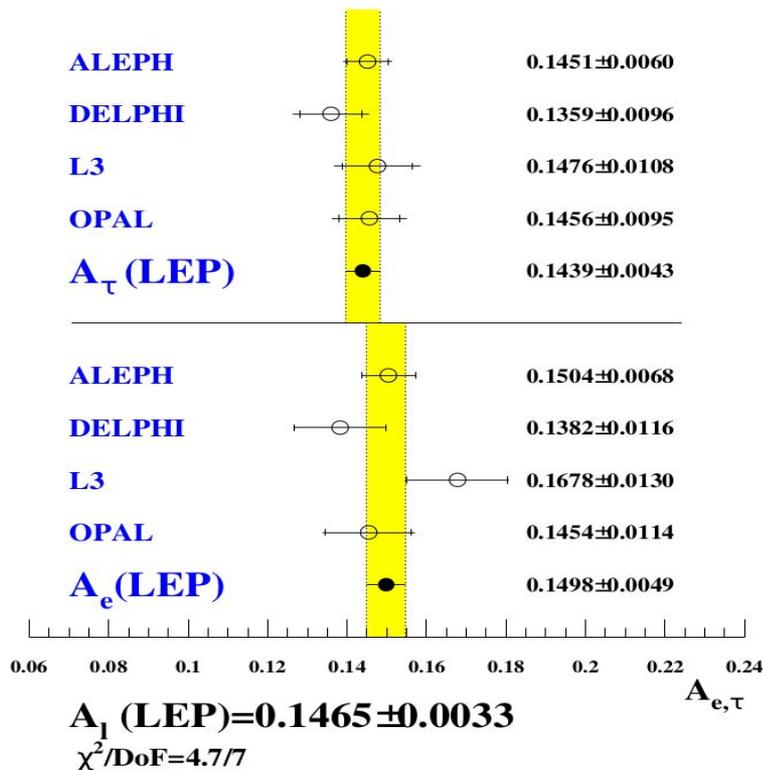- **A_e = Z polarization**
- **FB tau polarization asymmetry:**

$$\mathcal{P}_\tau^{FB} = -\tfrac{3}{4}\,\mathcal{A}_e$$

Polarized Z^0

A$_e$ is a safe measurement...

Experiment	$\mathcal{A}_\tau$	$\mathcal{A}_e$
ALEPH	$0.1451 \pm 0.0052 \pm 0.0029$	$0.1504 \pm 0.0068 \pm 0.0008$
DELPHI	$0.1359 \pm 0.0079 \pm 0.0055$	$0.1382 \pm 0.0116 \pm 0.0005$
L3	$0.1476 \pm 0.0088 \pm 0.0062$	$0.1678 \pm 0.0127 \pm 0.0030$
OPAL	$0.1456 \pm 0.0076 \pm 0.0057$	$0.1454 \pm 0.0108 \pm 0.0036$
LEP	$0.1439 \pm 0.0035 \pm 0.0026$	$0.1498 \pm 0.0048 \pm 0.0009$

ALEPH   0.1451±0.0060
DELPHI   0.1359±0.0096
L3   0.1476±0.0108
OPAL   0.1456±0.0095
A$_\tau$ (LEP)   0.1439±0.0043

ALEPH   0.1504±0.0068
DELPHI   0.1382±0.0116
L3   0.1678±0.0130
OPAL   0.1454±0.0114
A$_e$(LEP)   0.1498±0.0049

0.06 0.08 0.1 0.12 0.14 0.16 0.18 0.2 0.22 0.24   A$_{e,\tau}$

A_l (LEP)=0.1465±0.0033
χ^2/DoF=4.7/7

- **The FB tau polarization asymmetry (=$\mathcal{A}_e$) is NOT affected by uncertainties on the knowledge of polarization distributions / migrations (unless they are both F-B asymmetric and charge dependent)**

- **Dominant systematic uncertainty should be non-tau backgrounds: assume an order of magnitude reduction w.r.t. LEP: huge control samples, reduction via cuts, ...**

A_e measurement

ΔAe		0.000017	7.00E-06	2.00E-05

From AFBpol(tau), roughly assuming one order of magnitude reduction w.r.t. LEP in the uncertainty from non-tau backgrounds (dominant systematic contribution, it will be estimated from huge data control samples at FCC-ee)

- **Statistical uncertainty = LEP uncertainty * 10^{-3}**
- **ΔA_e (stat.+syst.) = 0.000021**

A$_\mu$ (without lepton universality)

ΔAμ		0.000023	2.31E-05	2.20E-05	

- **Revised estimate from A$_{FB}$($\mu\mu$) and A$_e$**

$$A_{FB} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f$$

- **Statistical uncertainty assumes N=1.5 x 10^{11} selected events: ΔA$_{FB}$($\mu\mu$)≈ 1/ √N**
- **Systematics assumed to be dominated by point-to-point uncertainties: ΔA$_{FB}$($\mu\mu$)≈2.5 x 10^{-6}**
- **ΔA$_\mu$ (stat.+syst.) = 0.000032**

Derived measurement: $\sin^2\theta_l$

Δsin2Theta_lept		--	1.40E-06	1.40E-06

from AFB(mumu) alone. syst being improved. It assumes lepton universality. AFBpol(tau) provides a result with similar/competitive uncertainty

- **Here we obviously assume lepton universality:**
  - $\Delta\sin^2\theta_l \approx A_l/16 \; \Delta A_{FB}(\mu\mu) / A_{FB}(\mu\mu)$

    $\Delta\sin^2\theta_l(\text{stat.+syst.}) \approx 2.0 \times 10^{-6}$

- **From $A_{FB,pol}(\tau\tau)$ ($\Leftrightarrow A_e$ independent measurement):**

    $\Delta\sin^2\theta_l \approx 2.5 \times 10^{-6}$

A$_\tau$

		0.000045	5.00E-06	2.00E-04
ΔАτ			5.00E-06	2.00E-04
ΔАτ			1.00E-05	1.30E-04

- **From polarization analysis:**
  - **Stat. uncertainty: 10^3 reduction w.r.t. ALEPH analysis (160 pb^{-1})**
  - **Systematics: reduction of systematics by an order of magnitude w.r.t. LEP**

- **From A$_{FB}(\tau\tau)$ and A$_e$ (≈ independent measurement):**
  - **Stat. uncertainty: 10^3 reduction w.r.t. LEP**
  - **Systematics: reduction of systematics by an order of magnitude w.r.t. LEP (dominated by knowledge of Bhabha background)**

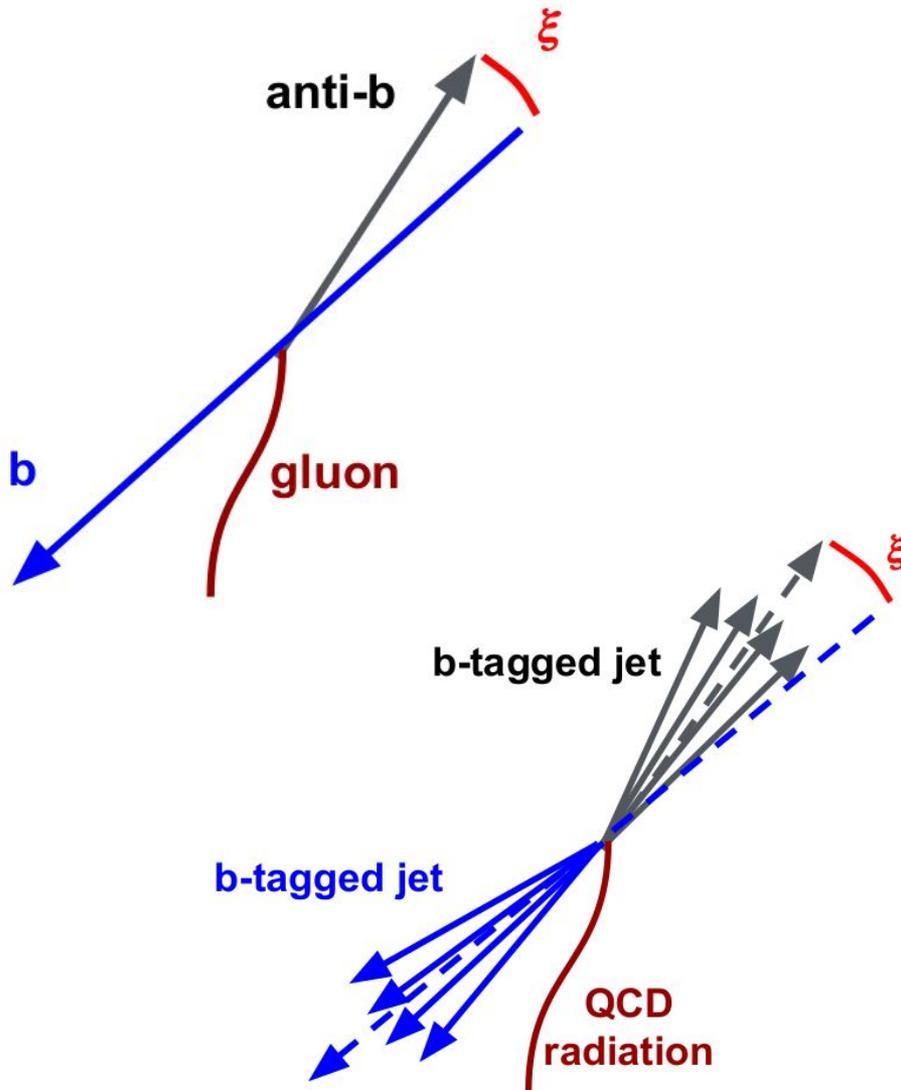$$A_{FB} = \frac{3}{4} \mathcal{A}_e \mathcal{A}_f$$

Present status of A_{FB}(Q)

- QCD corrections are the dominant source of correlated systematics between measurements

- Measurement (LEPEWWG reference):
  0.0992
  ± 0.0015 (stat.)
  ± 0.0007 (syst.)

- 1/2 syst. uncertainty using today's knowledge on modelling (MC tunes) (arXiv:2011.00530)

Source	R_b^0 $[10^{-3}]$	R_c^0 $[10^{-3}]$	$A_{FB}^{0,b}$ $[10^{-3}]$	$A_{FB}^{0,c}$ $[10^{-3}]$	$\mathcal{A}_b$ $[10^{-2}]$	$\mathcal{A}_c$ $[10^{-2}]$
statistics	0.44	2.4	1.5	3.0	1.5	2.2
internal systematics	0.28	1.2	0.6	1.4	1.2	1.5
QCD effects	0.18	0	0.4	0.1	0.3	0.2
$B(D \to \text{neut.})$	0.14	0.3	0	0	0	0
D decay multiplicity	0.13	0.6	0	0.2	0	0
B decay multiplicity	0.11	0.1	0	0.2	0	0
$B(D^+ \to K^- \pi^+ \pi^+)$	0.09	0.2	0	0.1	0	0
$B(D_s \to \phi \pi^+)$	0.02	0.5	0	0.1	0	0
$B(\Lambda_c \to p K^- \pi^+)$	0.05	0.5	0	0.1	0	0
D lifetimes	0.07	0.6	0	0.2	0	0
B decays	0	0	0.1	0.4	0	0.1
decay models	0	0.1	0.1	0.5	0.1	0.1
non incl. mixing	0	0.1	0.1	0.4	0	0
gluon splitting	0.23	0.9	0.1	0.2	0.1	0.1
c fragmentation	0.11	0.3	0.1	0.1	0.1	0.1
light quarks	0.07	0.1	0	0	0	0
beam polarisation	0	0	0	0	0.5	0.3
total correlated	0.42	1.5	0.4	0.9	0.6	0.4
total error	0.66	3.0	1.6	3.5	2.0	2.7

A$_{FB}$(b/c)

- New developments for A$_{FB}$(b/c): QCD corrections and uncertainties can be reduced significantly using acollinearity (ξ) cuts ⇒ important reduction in systematics, but how much ?
- Further improvements expected from better heavy flavor tagging capabilities and a more accurate measurement of the heavy quark flight direction
- **More sophisticated b/c tagging techniques => minimal charm/light background effects**
- **g->QQ splitting: huge control samples, smaller effect with back-to-back configuration and double tagging**
- Note that all these measurements can be done with exclusive decays. A Tera-Z facility will provide ≈10^8 B$^+$ exclusive decays

Semi-leptonic decays

- Evaluating the QCD corrections as a function of the momentum in semi-leptonic b decays, now with acollinearity cuts (generator level):

No acollinearity cuts $\xi < 0.3$

$$C_b \equiv 1 - \frac{A_{FB}}{A_{FB}^0}$$

- Typical ≈5-10 reduction factor (leptonic/inclusive tagging)
- More realistic analyses still to be done

A_b : final assumptions

ΔAb		0.0028	2.38E-05	1.29E-03

Derived from AFB(b) and Ae=AFBpol(tau) measurements: AFB(b) =3/4*Ae*Ab; systematics assumed to be dominated by modelling/tune uncertainties (Delta(AFB(b))=1e-4); QCD correction uncertainties estimated to be reduced by at least a factor of 5 w.r.t. LEP to Delta(AFB(b))=8e-5; gluon-splitting, charm/light backgrounds assumed to lead to negligible contributions due to very high b-tagging purity and huge control samples at FCC-ee

- ΔA_b **(stat.+syst.) = 0.0013**

- **We want to stay conservative for the time being: not clear how much QCD correction uncertainties and QCD modelling can be reduced at the end of the day: O(α_s^2) calculations/estimates including acollinearity cuts welcome (not disturbed by large gluon-splitting effects)**

A_c: no changes for the time being

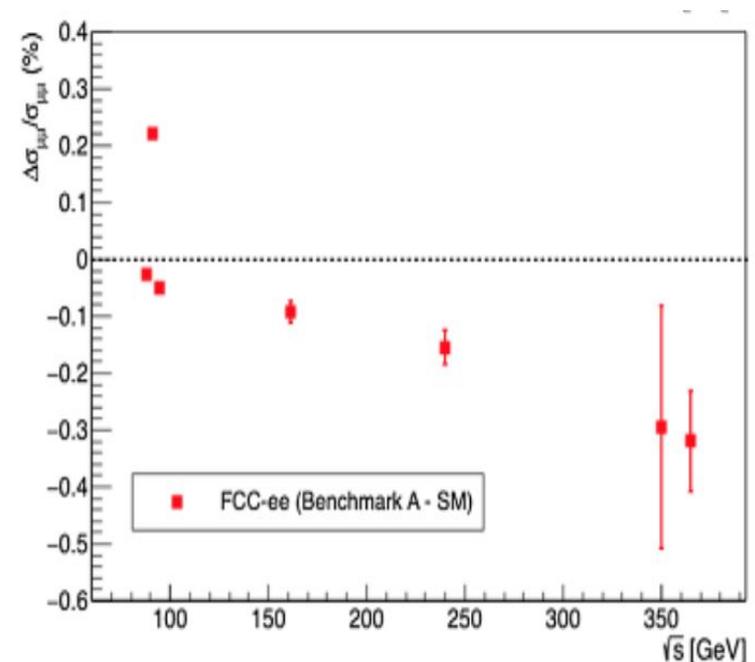| ΔAc	0.0053	2.00E-04	0.0053

Conservative extrapolation from LEP analyses assuming a factor of 3 reduction in charm modelling systematics and a factor of 5 reduction in QCD correction uncertainties

- ΔA_b (stat.+syst.) = 0.0053

- **Large improvements expected (better use of exclusive channels with such a large statistics, new strategies), but too early to assess uncertainties: dedicated studies necessary**

σ and A_{FB} away from Z peak

√s (GeV)	Relative error on XS	Absolute error on AFB
87.9	0.00010	1.8 E-5
91.2	0.00010	2.5 E-6
94.3	0.00010	1.6 E-5
161	0.00019	1.0 E-4
240	0.00030	2.0 E-4
350	0.00153	1.0 E-3
365	0.00088	6.2 E-4

- $\mu\mu$ **analysis (4D Higgs compositeness study); sensitivity to ‰ deviations in cross sections**

- **Uncertainties on account for:**
  - Statistical uncertainties + 10^{-4} (rel.) uncertainty from luminosity (only for σ)
  - No theory error assigned → needs progress on calculations
- **Why it is important:**
  - **Small signals with not so small widths at the LHC below 6-7 TeV (and not just "quarkfobic")**

m_W, Γ_W

ΔmW (MeV)		0.4	0.25	0.3

From cross section scan at WW threshold. Precise control of beam energy uncertainties via resonant depolarization. To be revised with 4 experiments instead of 2
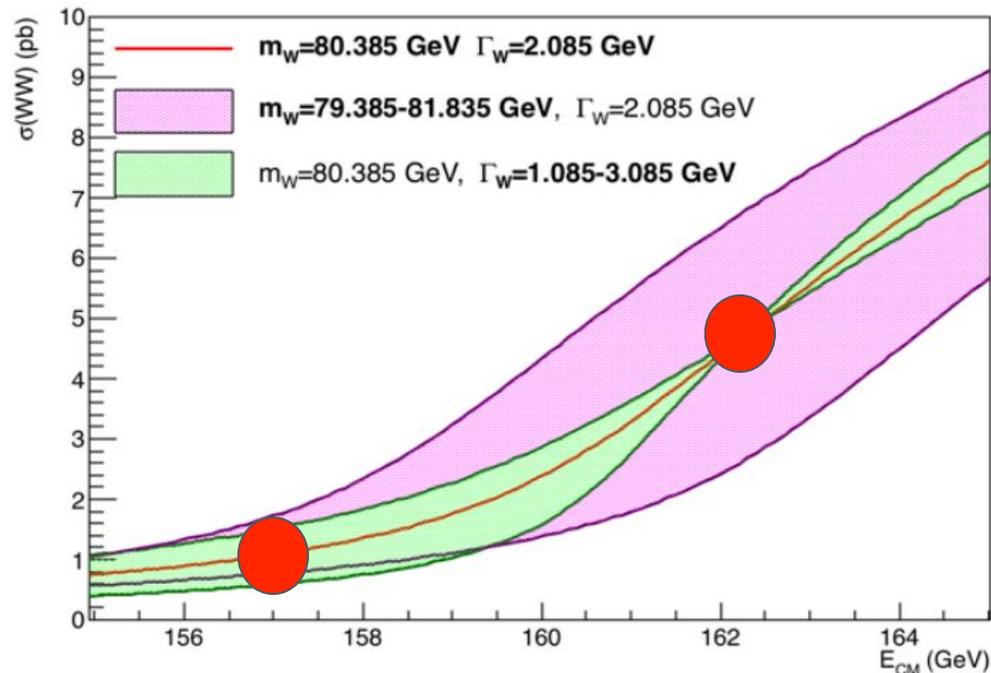
$\Delta\Gamma W$ (MeV)		1.2	1.2	0.3

From cross section scan at WW threshold (2 optimized points); potential improvement with direct reconstruction (under study)

Outlook

- **A few years of Tera-Z running should provide EWPO measurements with ≈20-100 times better than the current precision, thus giving early access to important/relevant/universal new physics effects at the deca-TeV scale**

- **Systematics will be the limiting factor in some these measurements, and they are difficult to estimate ⇒ more detailed studies/work needed. Reducing associated uncertainties via:**
    - **theory developments**
    - **new analysis strategies**
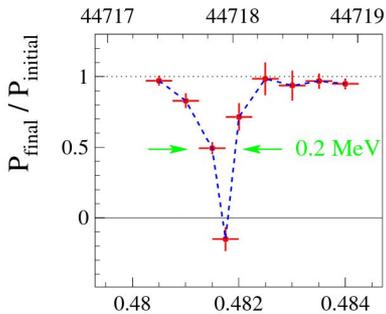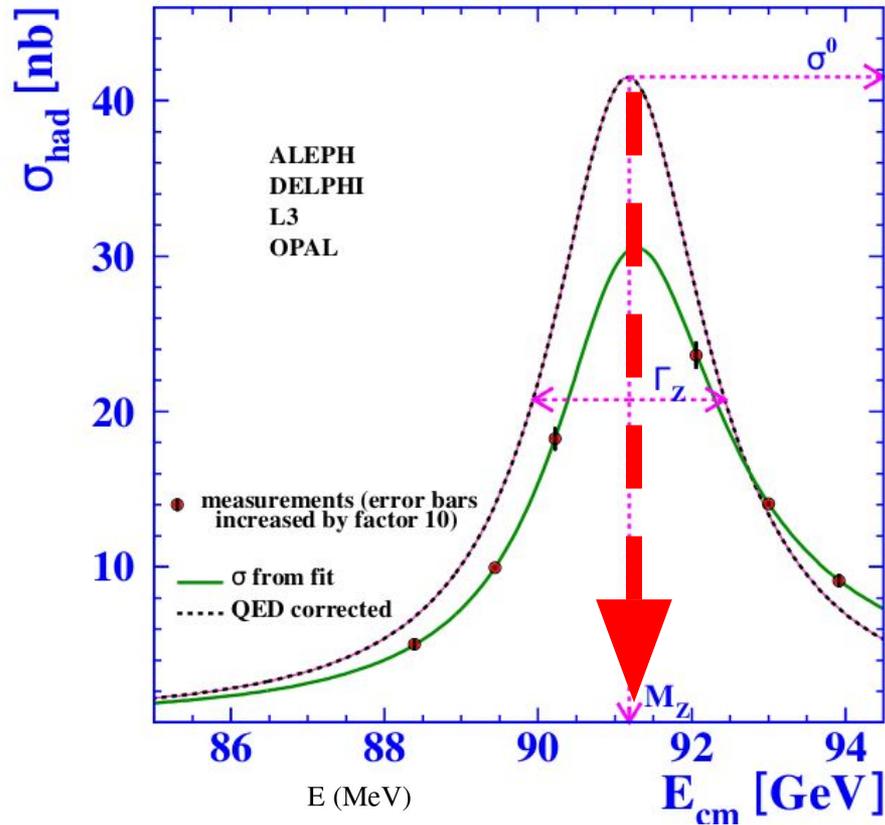   - **optimized detector design**

Backup

Our table (being updated)

Column with previous numbers
(stat. + syst.)

ΔAb		0.0028	2.38E-05	1.29E-03
ΔAc		0.0053	2.00E-04	0.0053
Δσhad (nb)	35	3.5	0.000035	0.0049
δRe		0.0003	3.61E-06	0.00001
δRμ		0.00005	2.58E-06	0.00001
δRτ		0.0001	3.10E-06	0.00001
δRb		<0.0003	1.39E-06	<0.0003
δRc		0.0015	1.50E-04	<0.0015

● ● ●

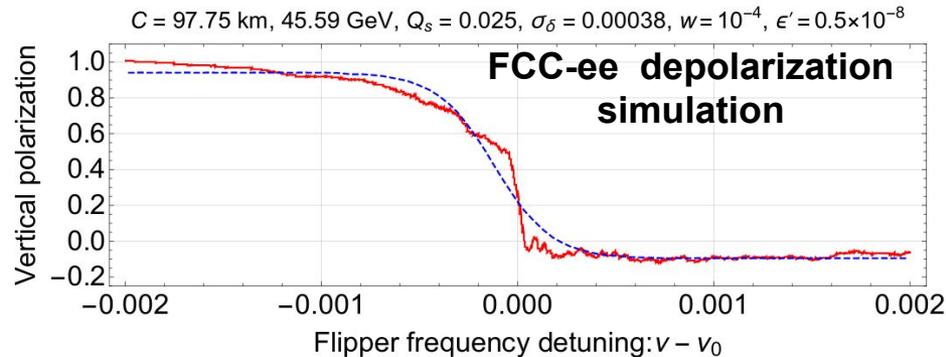# Z lineshape: mass

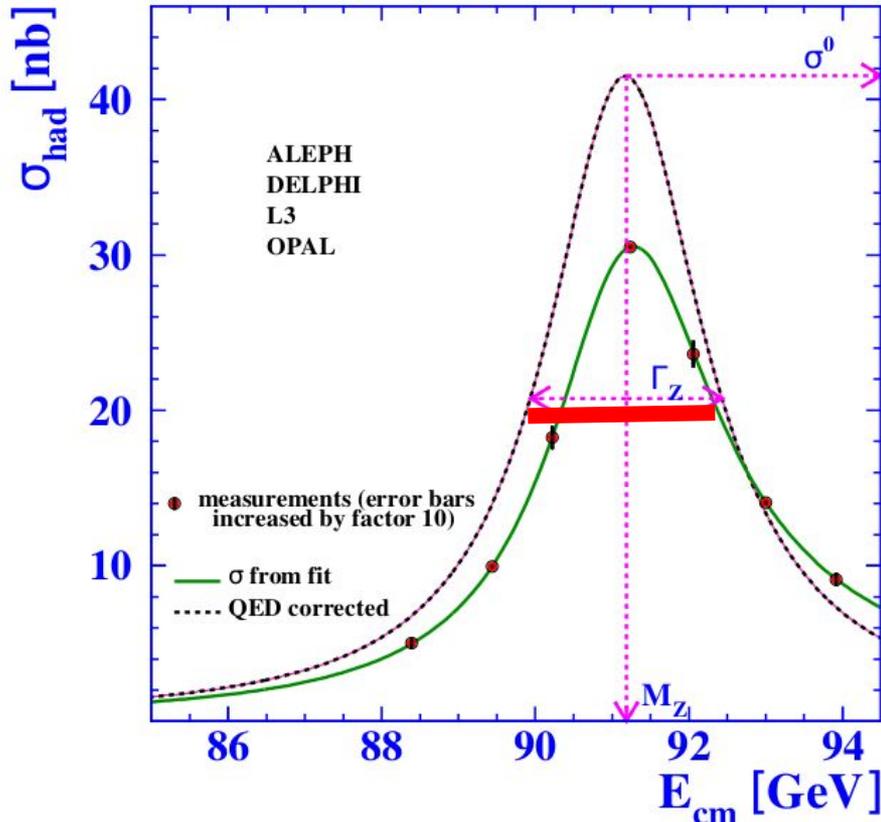

- m_Z: position of Z peak
- Beam energy measured with extraordinary precision ($\Delta\sqrt{s}\approx100$ keV) using resonant depolarization of transversely polarized beams (method already used at LEP, much better prepared now, calibrations in situ with pilot bunches, no energy extrapolations, …)
- Beam width/asymmetries studied analyzing the longitudinal boost distribution of the $\mu\mu$ system

Resonant depolarization at LEP
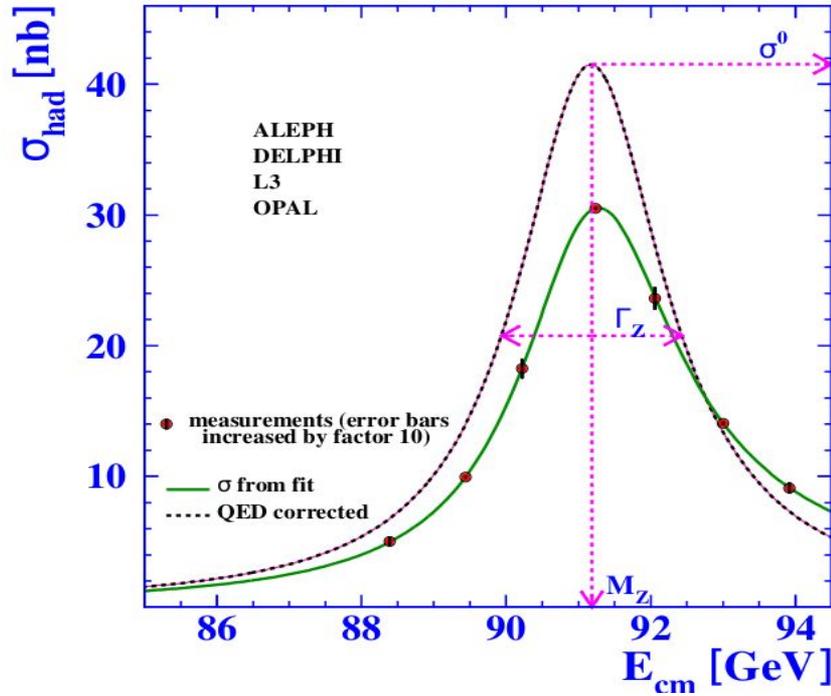
$C = 97.75$ km, 45.59 GeV, $Q_s = 0.025$, $\sigma_\delta = 0.00038$, $w = 10^{-4}$, $\epsilon' = 0.5\times10^{-8}$

FCC-ee depolarization simulation

arXiv:1909.12245

Γ_Z, N_ν

- **Total Z width → basically coming from the visible width of the lineshape**
- **Statistical precision of $\Delta\Gamma_Z \approx$ 4 keV** using hadronic lineshape
- Dominant systematics is the "point-to-point" beam uncertainty
- Study the point-to-point changes (3-5 points) using the invariant mass of dimuon events at each energy and realistic conditions at the beam interaction region: **current estimate is $\Delta\Gamma_Z \approx$ 25 keV**

- **A precise measurement of N_ν / invisible width requires a measurement of cross sections at the peak, not just Γ_Z → luminosity dependency → ≈ 10 times improvement over LEP (it will be measured with better precision using radiative recoil ratios: $\sigma(\nu\nu\gamma)/\sigma(ll\gamma)$)**
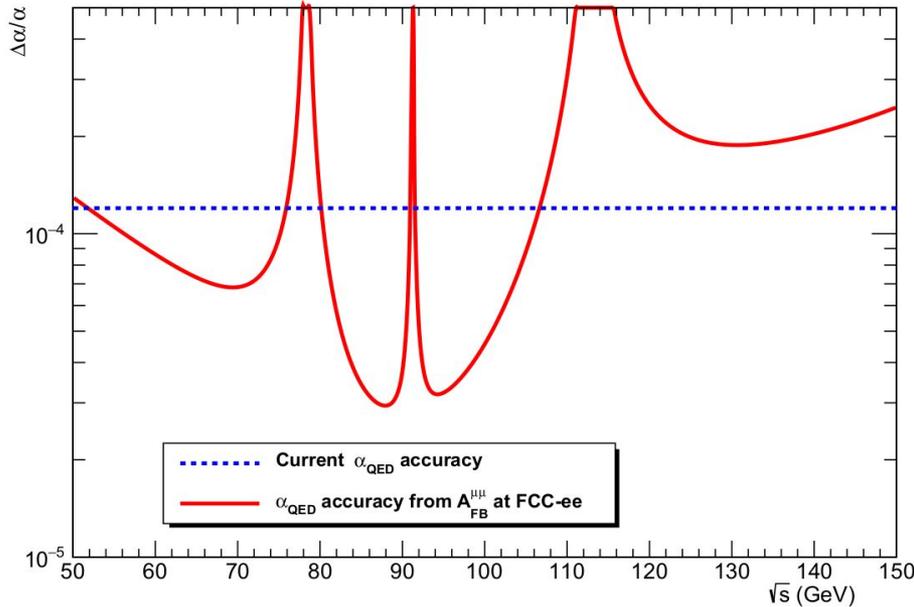
$R_l = \Gamma_{had}/\Gamma_l$

- Relative measurement, independent of luminosity: aiming for a 10^{-5} precision
- Extremely sensitive to new physics deviations (ϱ,T parameters: deviations of custodial symmetry)
- $\alpha_s(m^2_Z)$ modifies the hadronic partial width $\to R_l$ provides an ultra-precise measurement

- **Studies to define detector requirements to ensure negligible systematic uncertainties on acceptance (a priori more critical on leptons)**

δRe		0.0003	3.61E-06	0.00001
δRμ		0.00005	2.58E-06	0.00001
δRτ		0.0001	3.10E-06	0.00001

$\alpha_{QED}(m^2_Z)$

- $\alpha_{QED}(m^2_Z)$: off-peak/peak evolution of the asymmetry (due to interference with γ^* exchange)
- Measurement approaching the ultimate statistical sensitivity: 3×10^{-6} (relative)
- 3 energy points ($\approx$88, 91.2, 94 GeV)

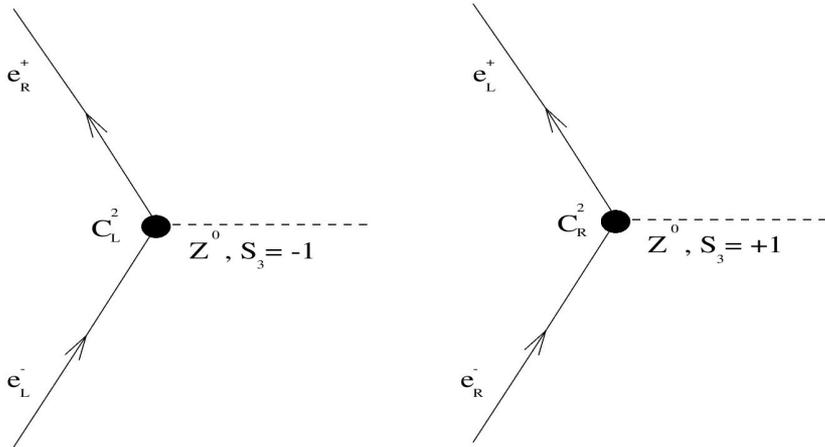| $\Delta\alpha$-1		0.00387	0.0038	0.0012

for this, mZ and Gamma Z, a line shape scan is assumed with 40 ab-1 for each of 2 off-peak energy points. to do: check stat improvements with other lepton channels and with 4 exp. Largest experimental uncertainty comes from point to point energy calibration in the mumu channel

Tau polarization: A_τ, A_e

Z: naturally polarized

$$\mathcal{P}_{Z^0} = \frac{C_R^2 - C_L^2}{C_R^2 + C_L^2} = -\frac{2v_e a_e}{v_e^2 + a_e^2} \equiv -\mathcal{A}_e$$

τ decay: excellent polarimeter

$$\mathcal{P}_\tau(\cos\theta) = -\frac{\mathcal{A}_\tau(1 + \cos^2\theta) + 2\mathcal{A}_e \cos\theta}{(1 + \cos^2\theta) + 2\mathcal{A}_e \mathcal{A}_\tau \cos\theta}$$

$$\langle \mathcal{P}_\tau \rangle = -\mathcal{A}_\tau$$

$$\mathcal{P}_\tau^{FB} = -\tfrac{3}{4}\mathcal{A}_e$$

Most sensitive channels

RUN NR 359206
EVENT NR 3586
29/10/91
22 00.51

$$\tau^- \longrightarrow \rho^- \nu_\tau \longrightarrow \pi^- \pi^0 \, \nu_\tau$$

$$\tau^- \longrightarrow \pi^- \, \nu_\tau$$

Analysis at LEP

(x=E_τ/E_{beam})

(Optimal variable)

- **Cross-talk between τ decay channels and the precise understanding of the helicity shape are main items to study to reduce systematics:**
  - ≈ 11% τ background from other decay channels in these plots
  - the tiny yellow shaded area is the non-τ background

A_τ to do: optimize channel separation

Table 2: Summary of the systematic uncertainties (%) on A_τ and A_e in the single-τ analysis.

ALEPH

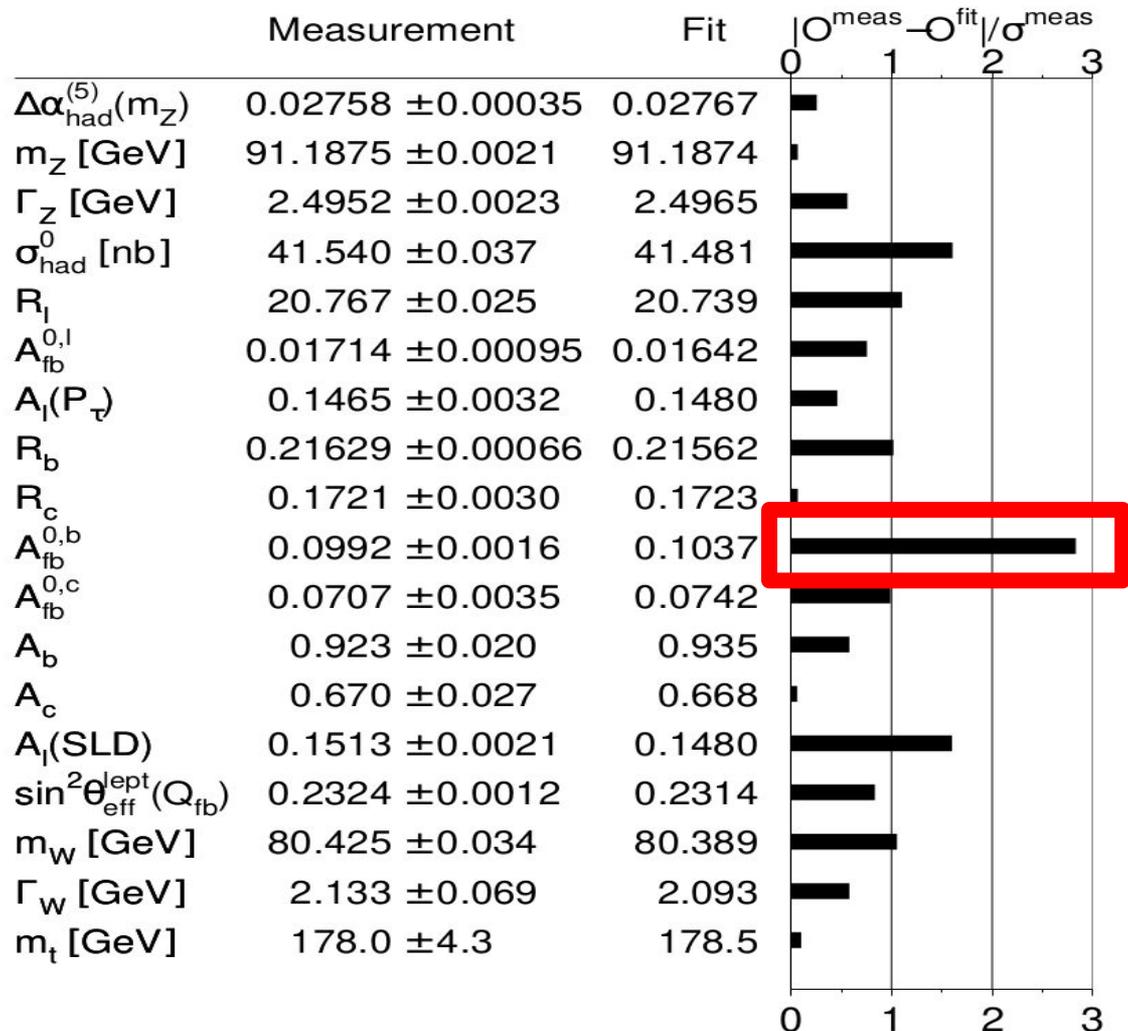| | | | | A_τ | | | |
Source	h	ρ	$3h$	$h\,2\pi^0$	e	μ	Incl. h
selection	-	0.01	-	-	0.14	0.02	0.08
tracking	0.06	-	0.22	-	-	0.10	-
ECAL scale	0.15	0.11	0.21	1.10	0.47	-	-
PID	0.15	0.06	0.04	0.01	0.07	0.07	0.18
misid.	0.05	-	-	-	0.08	0.03	0.05
photon	0.22	0.24	0.37	0.22	-	-	-
non-τ back.	0.19	0.08	0.05	0.18	0.54	0.67	0.15
τ BR	0.09	0.04	0.10	0.26	0.03	0.03	0.78
modelling	-	-	0.70	0.70	-	-	0.09
MC stat	0.30	0.26	0.49	0.63	0.61	0.63	0.26
TOTAL	0.49	0.38	1.00	1.52	0.96	0.93	0.87

- **ALEPH was the best detector for this: large tracking volume for separation, large magnetic field for bending, high granularity for π0 → $\gamma\gamma$ identification**
- **Photon separation / π^0 identification was still the dominant systematics**

Present status of A$_{FB}$(Q)

- Electroweak measurement presenting the largest deviations in the global SM fit (final LEPEWWG paper (2005))

$$A_{FB}(Q) = \frac{\sigma_F^Q - \sigma_B^Q}{\sigma_F^Q + \sigma_B^Q}$$

- New physics explanations require a substantial modification of Zbb right-hand couplings (arxiv:0610173)

	Measurement	Fit	$|O^{meas} - O^{fit}|/\sigma^{meas}$ 0 1 2 3
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ±0.00035	0.02767	
m_Z [GeV]	91.1875 ±0.0021	91.1874	
Γ_Z [GeV]	2.4952 ±0.0023	2.4965	
σ_{had}^0 [nb]	41.540 ±0.037	41.481	
R_l	20.767 ±0.025	20.739	
$A_{fb}^{0,l}$	0.01714 ±0.00095	0.01642	
$A_l(P_\tau)$	0.1465 ±0.0032	0.1480	
R_b	0.21629 ±0.00066	0.21562	
R_c	0.1721 ±0.0030	0.1723	
$A_{fb}^{0,b}$	0.0992 ±0.0016	0.1037	
$A_{fb}^{0,c}$	0.0707 ±0.0035	0.0742	
A_b	0.923 ±0.020	0.935	
A_c	0.670 ±0.027	0.668	
A_l(SLD)	0.1513 ±0.0021	0.1480	
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ±0.0012	0.2314	
m_W [GeV]	80.425 ±0.034	80.389	
Γ_W [GeV]	2.133 ±0.069	2.093	
m_t [GeV]	178.0 ±4.3	178.5	

$A_{FB}(b/c)$, $R_{b/c} = \Gamma_{b/c}/\Gamma_{had}$

J.A., arXiv:2010.08604

FCC-ee simulation, 7×10^7 $e^+e^- \to b\bar{b}(g)$ events

Double b-tag (tracking)

$A_{FB,0}$ (gen. level, QED corr.)
stat. unc.
$A_{FB} = 0.10379 \pm 0.00011$ (stat.)

Pseudo-data
tune $\oplus$ QCD unc.
stat. $\oplus$ tune $\oplus$ QCD unc.

A_{FB}, (gen. level, no ξ cut)
stat. $\oplus$ QCD unc.
$A_{FB} = 0.09957 \pm 0.00032$

Jet acollinearity cut ξ_0 ($\xi < \xi_0$)

Lepton tag, no cut

$\xi < 0.3$

- New developments for $A_{FB}(b/c)$: QCD corrections and uncertainties can be reduced significantly using acollinearity (ξ) cuts $\Rightarrow$ not a limiting factor anymore to reach the $\lesssim 0.1\%$ precision level
- Further improvements expected from better heavy flavor tagging capabilities and a more accurate measurement of the b flight direction
- **Performing a realistic measurement with more sophisticated b-tagging techniques $\to$ detector requirements**
- Studies to be extended to R_b, R_c double-tag measurements: increasing tag purity, better understanding of gluon-splitting and hemisphere correlations, …

Reduction of QCD uncertainties

- Detailed table of central values and uncertainties:

stat. unc. for 7×10^7 $Z \to b\bar{b}$ events

ξ_0 cut	Measured A_{FB}	ΔA_{FB}(stat)	ΔA_{FB}(tune)	ΔA_{FB}(theo. QCD corr)
No cut	0.0998 ± 0.0004	0.00008	0.00014	0.00033
1.50	0.1003 ± 0.0003	0.00011	0.00014	0.00023
1.00	0.1011 ± 0.0002	0.00011	0.00010	0.00016
0.50	0.1023 ± 0.0002	0.00011	0.00010	0.00007
0.30	0.1030 ± 0.0002	0.00011	0.00010	0.00003
0.20	0.1033 ± 0.0001	0.00011	0.00005	0.00002
0.10	0.1035 ± 0.0002	0.00016	0.00005	0.00001

Table 9: Central values and components of the uncertainty in the measurement of the A_{FB} asymmetry with 7×10^7 $e^+e^- \to b\bar{b}(g)$ events at the Z pole, for different $\xi < \xi_0$ cuts at the reconstructed level.

≲ 0.1% relative systematic uncertainties for $\xi \lesssim 0.3$

R_b, R_c

$$R_b = \frac{\Gamma_{b\bar{b}}}{\Gamma_{had}}, \quad R_c = \frac{\Gamma_{c\bar{c}}}{\Gamma_{had}}$$

- Measured at LEP/SLC very precisely using single and double-tag event fractions for the b case:

Single

b-tagged hemisphere

Non-b-tagged hemisphere

Double

b-tagged hemisphere

b-tagged hemisphere

No Bckgd, no hemisphere correlations $\Rightarrow \mathbf{R_b} = \dfrac{\mathbf{f_{single}^2}}{\mathbf{f_{double}}}$

Real life:

$$f_{single} = R_b \epsilon_b + R_c \epsilon_c + (1 - R_b - R_c)\epsilon_{uds}$$

$$f_{double} = c_b\, R_b \epsilon_b^2 + c_c\, R_c \epsilon_c^2 + c_{uds}\,(1 - R_b - R_c)\epsilon_{uds}^2$$

$$c_b = c_c = c_{uds} = 1 \text{ if no hemisphere correlations}$$
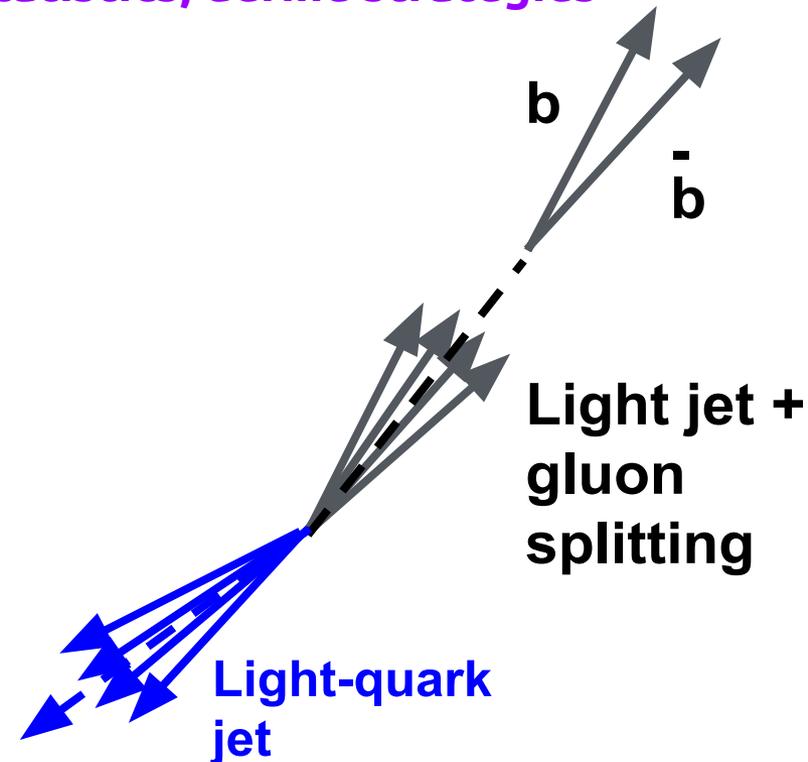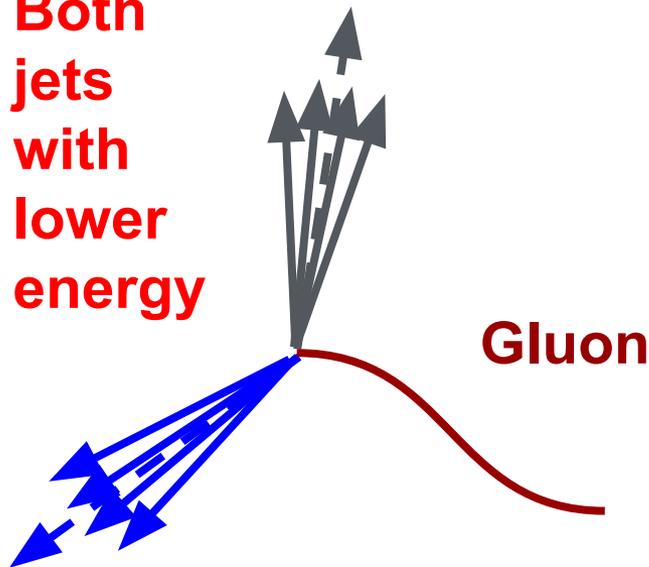
Present status of Rb, Rc

- **Hemisphere correlation effects (QCD) and gluon splitting are large sources of correlated uncertainty among experiments**

  - LEPEWWG result: $R_b = 0.21629 \pm 0.00066$

- **Aiming for a $\lesssim 3\times10^{-4}$ precision measurement on R_b at FCC-ee: one order of magnitude improvement**

- **R_c to be re-studied for a Tera-Z factory via exclusive / inclusive single+double-tag methods (SLD way, not LEP main way)**

Source	R_b^0 $[10^{-3}]$	R_c^0 $[10^{-3}]$	$A_{FB}^{0,b}$ $[10^{-3}]$	$A_{FB}^{0,c}$ $[10^{-3}]$	$\mathcal{A}_b$ $[10^{-2}]$	$\mathcal{A}_c$ $[10^{-2}]$
statistics	0.44	2.4	1.5	3.0	1.5	2.2
internal systematics	0.28	1.2	0.6	1.4	1.2	1.5
QCD effects	0.18	0	0.4	0.1	0.3	0.2
$B(D \to neut.)$	0.14	0.3	0	0	0	0
D decay multiplicity	0.13	0.6	0	0.2	0	0
B decay multiplicity	0.11	0.1	0	0.2	0	0
$B(D^+ \to K^- \pi^+ \pi^+)$	0.09	0.2	0	0.1	0	0
$B(D_s \to \phi\pi^+)$	0.02	0.5	0	0.1	0	0
$B(\Lambda_c \to p K^- \pi^+)$	0.05	0.5	0	0.1	0	0
D lifetimes	0.07	0.6	0	0.2	0	0
B decays	0	0	0.1	0.4	0	0.1
decay models	0	0.1	0.1	0.5	0.1	0.1
non incl. mixing	0	0.1	0.1	0.4	0	0
gluon splitting	0.23	0.9	0.1	0.2	0.1	0.1
c fragmentation	0.11	0.3	0.1	0.1	0.1	0.1
light quarks	0.07	0.1	0	0	0	0
beam polarisation	0	0	0	0	0.5	0.3
total correlated	0.42	1.5	0.4	0.9	0.6	0.4
total error	0.66	3.0	1.6	3.5	2.0	2.7

R_b, R_c

- **Important elements of the study:**
  - **Improvement of the b (and c) purity → better detectors**
  - **Reduction of hemisphere correlations and syst. uncertainties:**
    - **Common vertex correlations (smaller in future detectors)**
    - **QCD effects (reduction with acollinearity cuts like in $A_{FB}(Q)$?)**
    - **Gluon splitting → huge available statistics, define strategies**

Both jets with lower energy

Gluon

b

$\bar{b}$

Light jet + gluon splitting

Light-quark jet

R$_b$, R$_c$

δRb		<0.0003	1.39E-06	<0.0003
δRc		0.0015	1.50E-04	<0.0015

extrapolation from SLD <=> reduction by a factor of 3 in correlation
systematics w.r.t. LEP. Expect big reductions of systematics

m_H
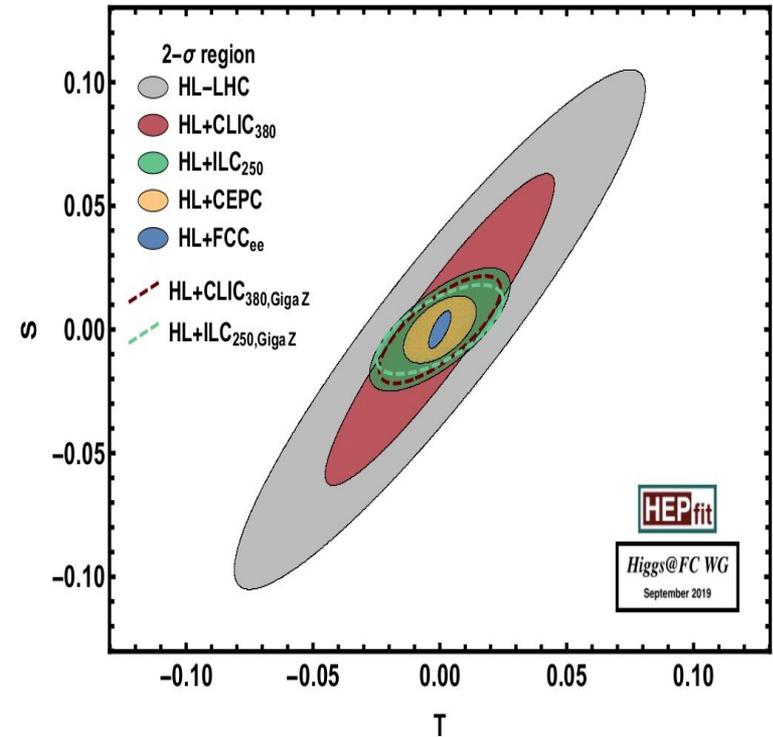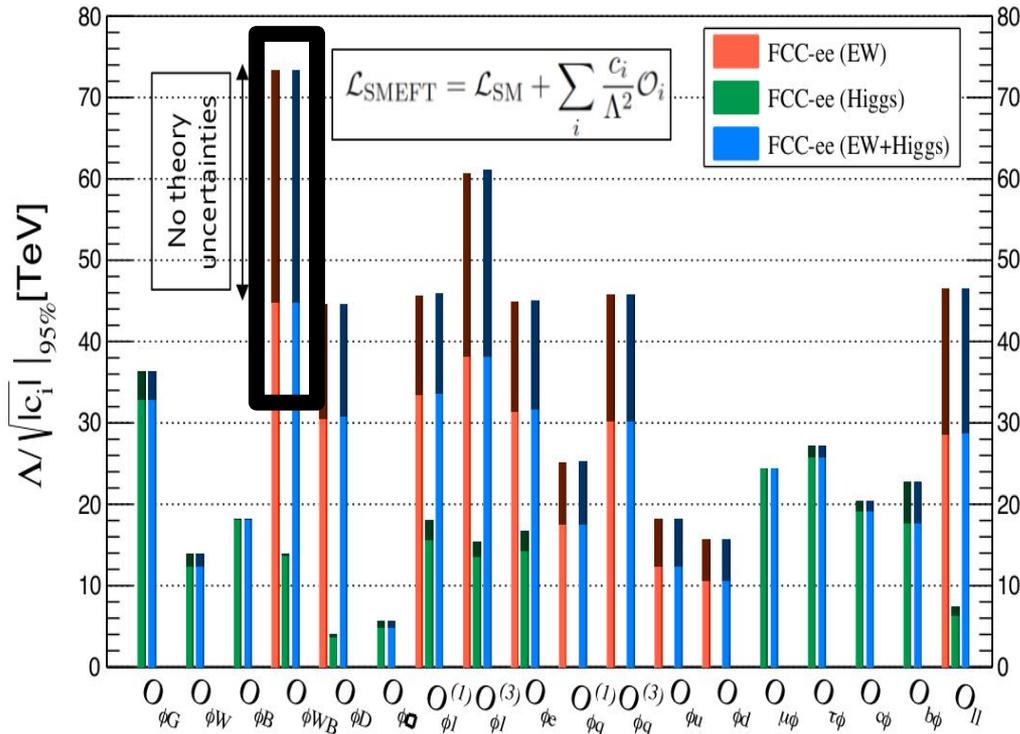
ΔmH (MeV)		11	2.5		2

might improve with four experiments and all channels. Systematic uncertainty included here is the in situ energy calibration with Zgamma events.

At $\sqrt{s} = 240\,\text{GeV}$, the same method still enjoys 70 million $Z\gamma$ events (with a twice smaller luminosity, a twice smaller cross section, and a twice smaller acceptance due to the larger longitudinal boost than at the WW threshold), which suffice for a determination of the average centre-of-mass energy with a precision of 1.7 MeV.

- **Uncertainty < 4 MeV necessary to define a potential running at √s = 125 GeV to test Yukawa coupling to electrons**
- **Also external input in many high precision calculations**

Physics potential: deca-TeV scale

- **Probing (really) the deca-TeV scale for universal new-physics effects with just a few years of FCC-ee EW running:**
  - Strong constraints on the S parameter ($O_{\phi WB}$ operator)
  - and on the T parameter (violations of custodial symmetry)

Eur. Strat.: arXiv:1910.11775