Hadronization corrections in event shapes

Silvia Ferrario Ravasio
University of Oxford

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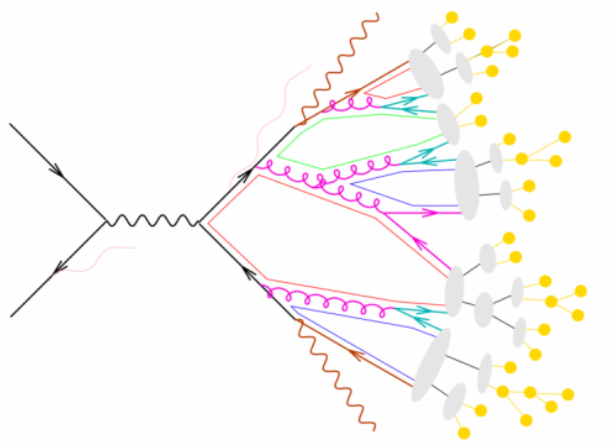
Based on
"*On linear power corrections in certain collider observables*", JHEP **01** (2022), 093,
F. Caola, S.F.R., G. Limatola, K. Melnikov, P. Nason
"*Linear power corrections to e^+e^- shape variables in the 3-jet region*", to appear soon
F. Caola, S.F.R., G. Limatola, K. Melnikov, P. Nason, **M. A. Ozcelik**

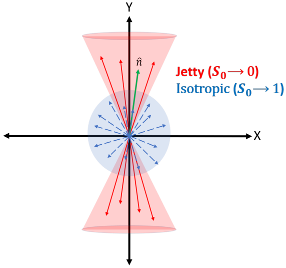
A typical FCC-ee event



Ingredients to describe a lepton collision

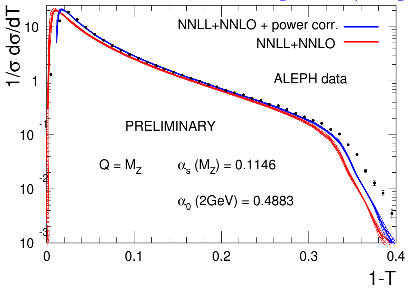
- Hard process ($Q \sim 100$ GeV): fixed order expansion in the strong coupling $\alpha_s(Q)$
- multiple soft and/or collinear emissions, whose contribution is logarithmically enhanced $L = \ln Q/k_\perp$, $Q > k_\perp > \Lambda$, with $\Lambda \sim 1$ GeV. Tools: **analytic resummation** (more accurate) or **parton shower algorithms** (more flexible)
- Hadronization corrections: **phenomenological models** (Lund or cluster) from **MC** event generators, or **analytic models**

Hadronization models for shape observables (I)



- State-of the art **most precise calculations** (NNLO, NNLL, N^{3}LL, ...) are not interfaced to parton showers.
- This is the case for for **Event shapes**, which characterize the geometry of a collision.
- Event shapes are among the most precise fits to e^+e^- hadronic final-state data and are use to perform **precise measurements of α_s**.
  $\Rightarrow$ per-mil targetted precision at FCC-ee

[Monni, '12]



- **Non-perturbative linear power corrections $\propto 1/Q$** must be provided in order to fit the data!
- **Analytic models:** shift the peturbative prediction by a **constant amount** $\propto 1/Q$

$$\Sigma(O) \to \Sigma(\,O - \underbrace{\mathcal{N}}_{\text{universal}}\ \underbrace{\Delta O}_{\text{Independent of } O(\Phi)}\,)$$

We need to control linear NP corrections if we want percent or permille precision at $Q \approx$100 GeV!

Hadronization models for shape observables (II)

- Using an **analytic model** for the hadronization $\Sigma^{\rm full}(O) = \Sigma^{\rm pert}(O - \mathcal{N}\Delta_O; \alpha_s)$ possibility to fit a **non-perturbative parameter** and the **perturbative coupling** in a **single, consistent framework**
- Unfortunately the determinations are several std away from the world average $\mathbf{0.1179 \pm 0.0010}$

$$\alpha_s = \mathbf{0.1135 \pm 0.0010} \qquad \text{thrust, [Abbate et al., 2010]}$$
$$\alpha_s = \mathbf{0.1123 \pm 0.0015} \qquad C-\text{parameter, [Hoang et. al, 2015]}$$

- Hadronizaton from **MC** is tuned on less accurate parton showers, depends on the shower cutoff but leads to better results!!
- Analytic hadronization models are derived only for events with **two collimated jets**.

Analytic models suited for generic final states are required. Besides the impact on α_s from LEP or FCC-ee data, this is the simplest context to investigate the effectiveness of pQCD and the need for power corrections in a general way, leading possibly to important implications for LHC and FCC-hh.

- FCC-ee will allow for clean **non-perturbative QCD studies** and give us a handle on quark and **gluon fragmentation** (see Grojean's talk!); many hadronic observables ($H \to b\bar{b}$, $H \to gg$), which requires hadronization corrections, are aimed to be measured at (sub) percent precision!

C-parameter in the two- and three-jet limit

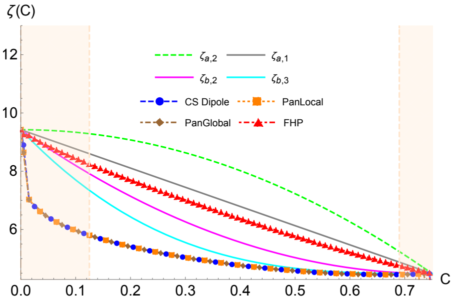
- N.P. corrections boil down to the calculation of the **shift** Δ_O, which corresponds to the average change in the observable induced by a **soft emission**:

$$\Delta_O = \int \frac{d\varphi}{2\pi} dy \left[O(p_1, \ldots, p_n) - O(p'_1, \ldots, p'_n, k) \right] \propto \frac{1}{Q}$$

- The calculation of the shift Δ_O depends on how we build the p'_i momenta if we are away from the 2 jet limit, and hence is **ambiguous**!, but the experimental data can also come from the **three-jet region**!
- Usually calculate Δ_O for $\mathbf{O = 0}$ and then assumes is **constant** across the whole spectrum;
- The ***C*-parameter** distribution has a Sudakov shoulder at $\mathbf{C = 3/4}$: also here ΔC can be computed unambigously!
- Luisoni, Monni, Salam ('20) find

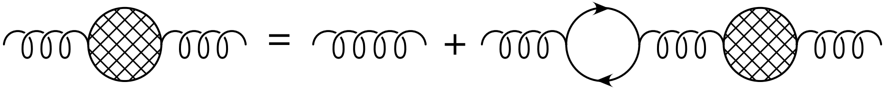
$$\frac{\delta C(0.75)}{\delta C(0)} = \frac{C_A/2 + C_F}{C_F} \times 0.224 \approx 0.48$$

- Better analytic models can provide a better value for α_s?



Large n_f limit

- We want to include the **exact kinematic dependence** in Δ_O.
- We rely on the **large-n_f** limit to study the all-order behaviour of the theory and hence also its non-perturbative ambiguity (see backup for more details)
- The dominant contributions come from the insertions of fermionic bubbles into gluon lines



- Recent developments: linear power corrections are not present for observables inclusive with respect to QCD radiation [Caola, S.F.R., Limatola, Melnikov, Nason, '21]
- This helped us to find a prescription to solve the "mapping ambiguity" in the $\Delta_O(\boldsymbol{\Phi})$ definition entering the NP shift:

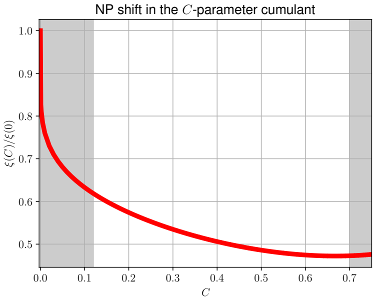
$$\Sigma(o) \to \Sigma(o - \mathcal{N}\langle \boldsymbol{\Delta_O(\Phi)}\rangle_{O(\Phi)=o}), \quad \Delta_O(\Phi) = \int \frac{d\varphi}{2\pi} dy \left[O(p_1, \ldots, p_n) - O(p_1', \ldots, p_n', k)\right]$$

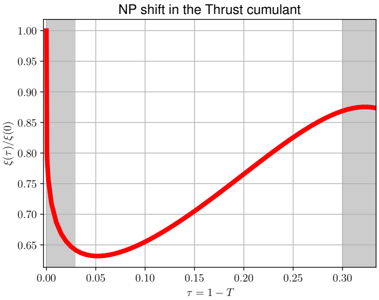
$\Delta_O(\boldsymbol{\Phi})$ must be computed using a **smooth mapping**

In the soft limit: $[p_i' = p_i + M_i(\{p\}) \cdot k]_{\text{Longitudinal components}}$

Results [Caola, S.F.R., Limatola, Melnikov, Nason, Ozcelik, preliminary]

Ratio between the exact non-perturbative shift and its value in the two jet limit $O = 0$ (*i.e.* its currend used value)





- for the ***C*-parameter**, NP corrections lie between 45% and 70% of what is currently used;

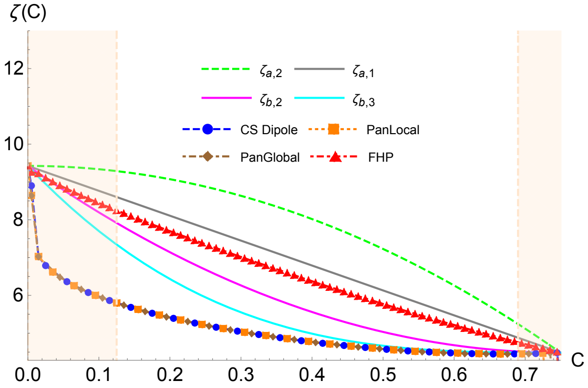
$$\alpha_s = 0.1123(15)$$

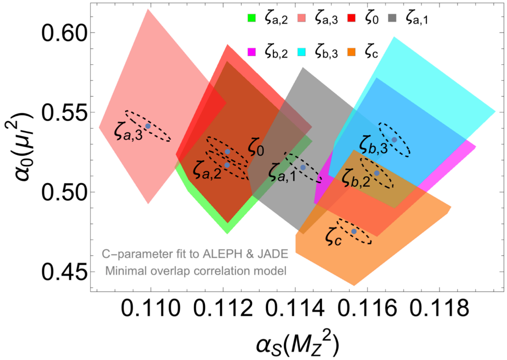
- for the **thrust**, NP corrections lie between 60% and 90% of what is currently used;

$$\alpha_s = 0.1135(10)$$

Extraction of the strong coupling constant

Our results for the C-parameter coincide with the **Catani-Seymour**/**PanLocal**/**PanGlobal** curve of LMS (Lusioni-Monni-Salam '20), *i.e.* the ones which employ a smooth phase space mapping!





This curve is "quite similar" to the $\zeta_{b,3}$ **curve** considered in LMS, which leads $\alpha_s \sim 0.117$. Much closer to 0.118 than the flat (ζ_0) assumption ($\alpha_s \sim 0.112$).

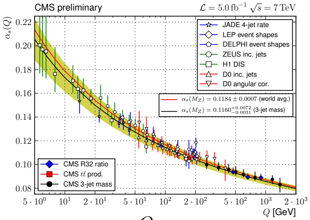
Summary and outlooks

- We have provided a recipe to esily evaluate **NP corrections** for a broad class event shapes in lepton collisions for **any final state**.
- For the **thrust** and C**-parameter** the "true" NP shift in the cumulative and differential distributions are heavily overestimated.
- This will potentially enable us to provide more accurate **estimate of** α_s from LEP and, especially, from the **FCC-ee** data.
- It will be interesting to see if **MC hadronization models** lead to the same NP shift once they act on a showered event. If not, one can use this **analytic insights** to improve them, possibly reducing the hadronization uncertainties affecting several collider measurements.

THANKS FOR THE ATTENTION!

How do we obtain an analytic model for hadronization?

- There are several sources of non-perturbative corrections, one of them lies in pQCD itself:



$$\alpha_s(Q) = \frac{1}{2b_0 \log\left(\frac{Q}{\Lambda}\right)}; \qquad b_0 = \frac{11C_A}{12\pi} - \frac{n_l T_R}{3\pi} > 0$$

the **Landau pole** Λ in α_s leads to an **instrinsic ambiguity** when integrating over the soft momenta.

$$\underbrace{\int_0^Q \mathrm{d}k\, k^{p-1}\alpha_s(Q)}_{\text{NLO}} \Longrightarrow \underbrace{\int_0^Q \mathrm{d}k\, k^{p-1}\alpha_s(k)}_{\text{all orders}} = \boxed{Q^p \times \frac{p}{2\,b_0}\sum_{n=0}^{\infty}\left(\frac{2\,b_0}{p}\alpha_s(Q)\right)^{n+1} n!}$$

- **Asymptotic series**, which we truncate at the minimal term, which is the estimate of the ambiguity

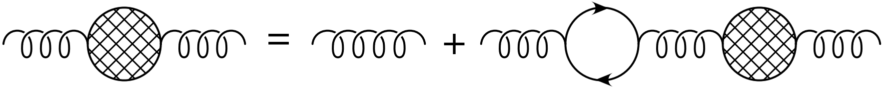
$$\boxed{\sqrt{\frac{\alpha_s(Q)p\pi}{b_0}}\Lambda^p}$$

for $Q \sim 100\mathrm{GeV}$, only **linear** power corrections are worrysome.

- Since this ambiguity has to cancel with contributions arising from physics beyond perturbation theory, it can be used to estimate some **non-perturbative effects**.

Large-n_f limit

- Ambiguity related to the appearance of the Landau pole can be studied in the large number of flavour n_f limit, which allows to perform all-orders computations exactly.



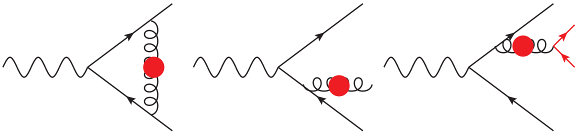
$$\frac{-ig^{\mu\nu}}{k^2+i\eta} \to \frac{-ig^{\mu\nu}}{k^2+i\eta} \times \frac{1}{1+\Pi(k^2+i\eta,\mu^2,\epsilon)-\Pi_{\rm ct}}$$

$$\Pi(k^2+i\eta,\mu^2)-\Pi_{\rm ct} = \alpha_s(\mu)\left(-\frac{n_f T_R}{3\pi}\right)\left[\log\left(\frac{|k^2|}{\mu^2}\right) - i\pi\theta(k^2) - \frac{5}{3}\right] + \mathcal{O}(\epsilon)$$

- **naive non-abelianization** at the end of the computation (large b_0)

$$\Pi(k^2+i\eta,\mu^2)-\Pi_{\rm ct} \to \alpha_s(\mu)\underbrace{\left(\frac{11C_A}{12\pi} - \frac{n_l T_R}{3\pi}\right)}_{b_0}\left[\log\left(\frac{|k^2|}{\mu^2}\right) - i\pi\theta(k^2) - C\right]$$

Large-n_f approximation for "complex" collider processes



We obtained an expression that can be used for generic processes without gluons at LO to evaluate **any arbitrarily complex** infrared safe observable [S.F.R, Nason, Oleari '18]
The large-n_f limit is a **rigorous** approach!

$$O = \int d\Phi \frac{d\sigma(\Phi)}{d\Phi} O(\Phi) = O_{\rm LO} - \frac{1}{\pi b_0} \int_0^\infty d\lambda \frac{d}{d\lambda}\left[\frac{T(\lambda)}{\alpha_s(\mu)}\right] \overbrace{\arctan\left[\pi\, b_0\, \alpha_s\left(\lambda e^{-C/2}\right)\right]}^{\alpha_{s,\rm eff}(\lambda)\ \text{[Beneke, '98]}}$$

$$T(\lambda) = \underbrace{\int d\Phi_b \frac{d\sigma_v^{(1)}(\Phi_b;\lambda)}{d\Phi_b} O(\Phi_b) + \int d\Phi_{g^*} \frac{d\sigma_r^{(1)}(\Phi_{g^*};\lambda)}{d\Phi_{g^*}} O(\Phi_{g^*})}_{O_{\rm NLO}(\lambda)}$$

$$+ \underbrace{\frac{3\lambda^2}{2 T_R \alpha_s} \int d\Phi_{g^*} d\Phi_{\rm dec} \frac{d\sigma_{q\bar q}^{(2)}(\lambda,\Phi)}{d\Phi}\Big[O(\Phi) - \underbrace{O(\Phi_{g^*})}_{q\bar q \to g^*}\Big]}_{\Delta_{q\bar q}\ \text{[Nason, Seymour '95]}}$$

Linear λ terms in $T(\lambda)$ $\Leftrightarrow$ Linear power corrections

Non-perturbative approach for shape observables

- A broad class of shape observable is **suppressed in the collinear limit** (thust, C-parameter, heavy jet mass), *i.e.* an emission modifies the observable by an amount

$$\Delta O = \frac{p_\perp}{Q} \times f(y,\varphi) + \mathcal{O}\left(\frac{p_\perp^2}{Q^2}\right) \qquad \text{with} \lim_{y\to\pm\infty} \int \frac{d\varphi}{2\pi} f(y,\varphi) = 0$$

- Dokshitzer, Lucenti, Marchesini, Salam ('98) showed that the exact large-n_f result of Nason and Seymour '95 can be obtained multiplying the result for the emission of a **soft gluon** of fixed $p_\perp =$ by the **Milan factor**

$$\frac{\partial\Sigma(O<o)}{\partial\lambda} = \frac{d\sigma}{do}\left[\mathcal{M}\frac{2C_F\alpha_s}{\pi}\int\frac{dp_\perp}{p_\perp}dy\frac{d\varphi}{2\pi}\Delta O\,\delta(p_\perp-\lambda)\right]=\frac{d\sigma}{do}\langle\Delta O\rangle \qquad \mathcal{M}=\frac{15\pi^2}{128}$$

[Dasgupta, Magnea, Smye '99]

- Although not rigorous, it is phenomenologically reasonable to include separately also the $g\to gg$ splitting, assuming the naive non-abelianization does not capture the fact that these splittings have a kinematic different from $g\to q\bar{q}$

$$\mathcal{M} = \frac{3C_A\left[128\pi(1+\log(2))-35\pi^2\right]-15\pi^2 n_f}{64(11C_A-2n_f)}$$

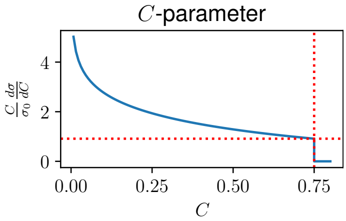
[Smye, '01]

- Non-perturbative corrections amount to a **shift** in the perturbative distribution [Dokshitzer, Webber '97]

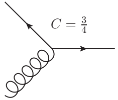
$$O \to O - \langle\Delta O\rangle\left[\int_0^{\mu_I} dp_\perp \tilde{\alpha}_{\rm eff}(p_\perp) - \alpha_s^{\rm CMW}(Q) - \frac{\alpha_s(Q)^2 b_0}{2}\ln\frac{Q}{\mu_I} + \ldots\right]$$

C-parameter in the two- and three-jet limit

- The calculation of ΔO is **ambiguous** away from the two jet limit as it depends on the **phase space parametrization**: $\Phi_n \to \Phi_{n+1}(\Phi_n, y, p_\perp, \varphi)$.
- The experimental data however can also come from the three-jet region!
- Usually calculates ΔO for $\mathbf{O = 0}$ and then assumes is constant across the whole spectrum;



- The C-parameter distribution has a Sudakov shoulder at $\mathbf{C = 3/4}$: also here ΔC can be computed unambigously!



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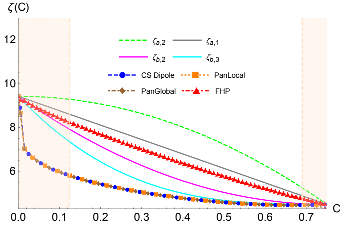
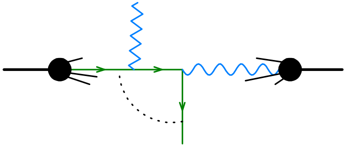


- Better analytic models can provide a better value for α_s?

Back to the large-n_f limit

- Since we **cannot deal with gluons**, to study NP corrections away from the two-jet limit we consider the process $\mathbf{Z \to q\bar{q}\gamma}$.



- This process shares many similarities with $\mathbf{q\gamma \to Zq}$, where we saw no power corrections affect the **transverse momentum distribution of the Z**.
[S.F.R., Limatola, Nason, '20]

Caola, S.F.R., Limatola, Melnikov, Nason, '21

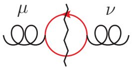
(I) The **linear mass dependence cancels** in an (abelian) theory with massive gluons, in the context of a single gluon emission or exchange, if the observable is **inclusive** with respect to QCD radiation.
(II) can we use this to simplify the calculation of NP corrections for **shape observables**?

Simplifying the calculation with a suitable phase space mapping

- To expose linear corrections for a **generic infrared-safe observable** in our simplified abelian theory, the real emission corrections can be computed using the **next-to-eikonal** approximation;
- For observables sensitive only to **soft radiation** (thrust, C-parameter, heavy jet mass ...)

$$R^{(\lambda)}(\Phi_{n+1}) \approx 4g_s^2 C_F B(\Phi_n) \times J_\mu J_\nu (-g^{\mu\nu}) \quad \text{with} \quad \underbrace{J^\mu = \frac{p_1^\mu}{(p_1+k)^2} - \frac{p_2^\mu}{(p_2+k)^2}}_{\text{Eikonal current}}$$

$$V^{(\lambda)}(\Phi_{n+1}) \approx \int [dk] R^{(\lambda)}(\Phi_{n+1}) \qquad R_{q\bar{q}}^{(\lambda)}(\Phi_{n+2}) \approx B(\Phi_n) J_\mu J_\nu P_{\text{split}}^{\mu\nu}(\Phi_{\text{dec}}) \quad \text{with} \quad P_{\text{split}}^{\mu\nu} =$$

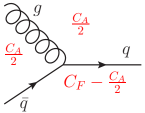


- These approximations requires the use of a **smooth mapping**,

$$k = z_1 p_1 + z_2 p_2 + k_\perp$$

$$P_{1,2} \approx (1 \mp z_{1,2}) p_{1,2}$$

and the way $k_\perp$ is redistributed does not matter!



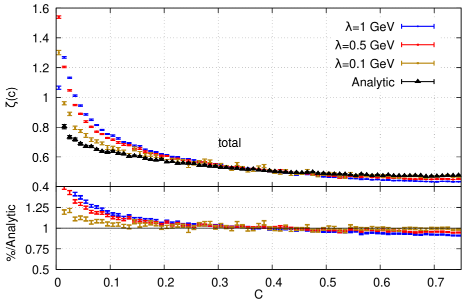
- We assume the same formulae for more complex final states, with C_F replaced by the proper color factor for each dipole

We can try to investigate the shift in the thrust and the C-parameter also away from Sudakov shoulders!

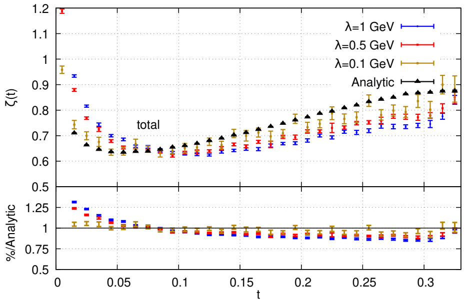
Caola, S.F.R., Limatola, Melnikov, Nason, Ozcelik, TO APPEAR SOON!

- We managed to obtain **analytic results** for the power corrections in the **thrust** and ***C*-parameter distribution**, which confirms our previous numeric findings
- We also re-obtained the **factorized form** for the shift in the cumulant in terms of the Milan factor, provided a **smooth phase space mapping** is employed

C-parameter



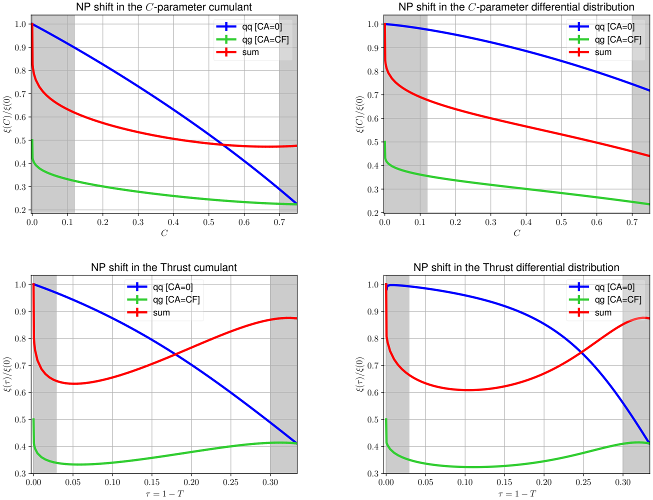
Thrust



Notice that these results coincide with the Dipole/Antenna/PanGlobal results from [Luisoni, Monni, Salam '20]

Cumulant vs differential

$$\Sigma^{NP}(\lambda, O < o) = \left[\langle \Delta O \rangle \frac{d\sigma}{dO}\right]_{O=o} \qquad \sigma^{NP}(\lambda, O = o) = -\left[\frac{d}{dO}\left(\langle \Delta O \rangle \frac{d\sigma}{dO}\right)\right]_{O=o} = \left[\langle \Delta' O \rangle \frac{d^2\sigma}{dO^2}\right]_{O=o}$$



In the fiducial range used for α_s extractions:

- for the C-**parameter**, NP corrections roughly lie between 45% and 70% of what is currently used;

$$\alpha_s = 0.1123(15)$$

- for the **thrust**, they lie between 60% and 90% of what is currently used.

$$\alpha_s = 0.1135(10)$$

Summary and outlooks [extended]

- We have provided a recipe to esily evaluate **NP corrections** for a broad class event shapes in lepton collisions for **any final state**.
- The derivation of the abelian contribution is rigourous and directly follows from the **large-n_f limit**.
- The full NP shift comprises also a non-abelian contribution, whose derivation is phenomenologically well-motivated (and it is analogous to the one related to the "full" Milan factor).
- We provide explicit formulae for the **thrust** and **C-parameter**, seeing that the "true" NP shift in the cumulative and differential distributions are heavily overestimated.
- This will potentially enable us to provide more accurate **estimate of α_s** from LEP and, especially, from the **FCC-ee** data.
- It will be interesting to see if **MC hadronization models** lead to the same NP shift once they act on a showered event. If not, one can use this **analytic insights** to improve them, possibly reducing the hadronization uncertainties affecting several collider measurements.
- TODO: include a dedicated treatment of **mass effects** (pole mass renormalons were specifically addressed in [S.F.R., Nason, Oleari '19].