The path to precision

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Need for theory input

- Comparison of EWPOs with SM to **probe new physics**
  $\longrightarrow$ multi-loop corrections in full SM

- Extraction of EWPOs (**pseudo-observables**) from **real observables**
  $\longrightarrow$ backgrounds (in full SM), QED/QCD, MC tools

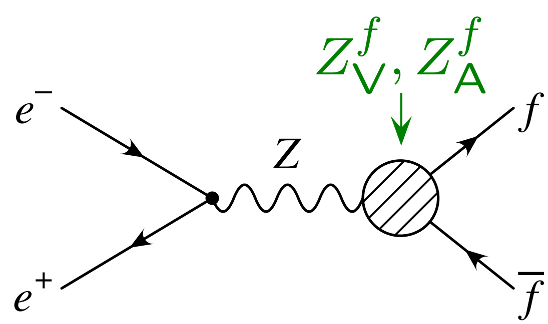
- “Other” eletroweak parameters (**“input” parameters**)
  $\longrightarrow$ $m_{\rm t}$, $\alpha_{\rm s}$, etc. extracted from other processes

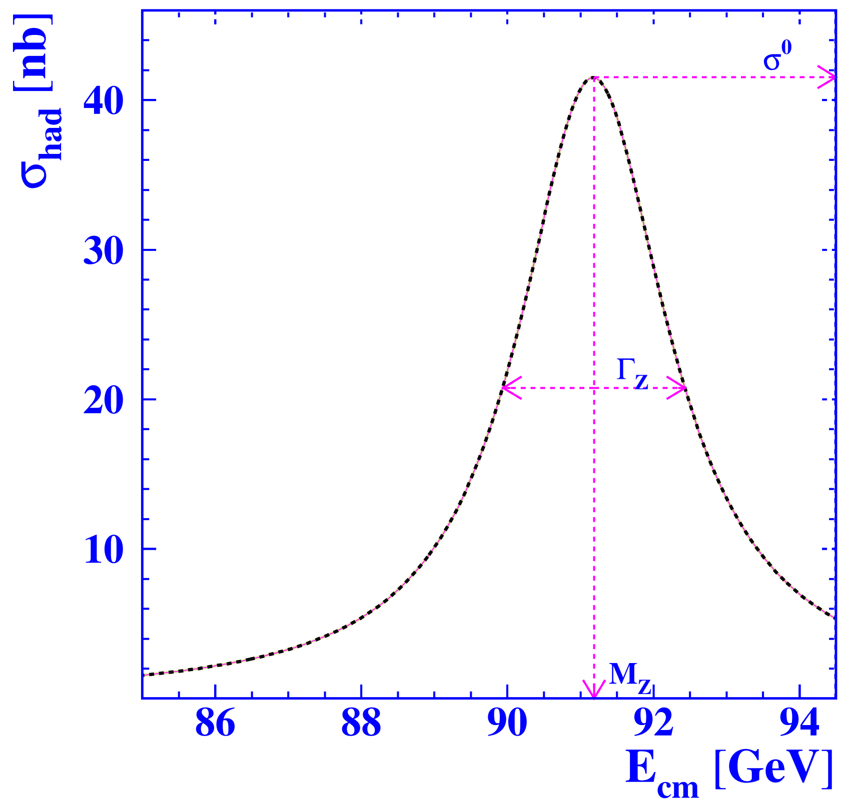
Z cross section and branching fractions

$e^+e^- \to f\bar{f}$ for $E_{\text{CM}} \sim M_{\text{Z}}$:

- Mass M_{Z}
- Width $\Gamma_{\text{Z}} = \sum_f \Gamma_{ff}$
- Braching ratio $R_f = \Gamma_{ff}/\Gamma_{\text{Z}}$
- $\sigma^0 \approx \frac{12\pi\,\Gamma_{ee}\Gamma_{ff}}{(s-M_{\text{Z}}^2)^2+M_{\text{Z}}^2\Gamma_{\text{Z}}^2} = \frac{12\pi}{M_{\text{Z}}^2}R_e R_f$

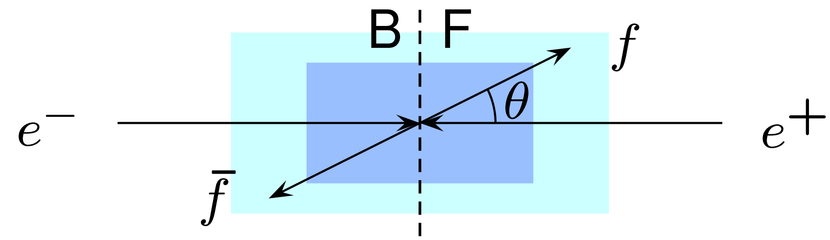
$$\Gamma_{ff} = C\left[(Z_{\text{V}}^f)^2 + (Z_{\text{A}}^f)^2\right]$$





Z-pole asymmetries

Forward-backward asymmetry:



$$A_{\mathrm{FB}} \equiv \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}} = \frac{3}{4}\mathcal{A}_e \mathcal{A}_f$$

$$\mathcal{A}_f = \frac{2(1 - 4\sin^2\theta^f_{\mathrm{eff}})}{1 + (1 - 4\sin^2\theta^f_{\mathrm{eff}})^2} \qquad \sin^2\theta^f_{\mathrm{eff}} = \frac{1}{4|Q_f|}\left[1 - \frac{Z^f_{\mathrm{V}}}{Z^f_{\mathrm{A}}}\right]$$

Left-right asymmetry:

With polarized e^- beam:

$$A_{\mathrm{LR}} \equiv \frac{\sigma_{\mathrm{L}} - \sigma_{\mathrm{R}}}{\sigma_{\mathrm{L}} + \sigma_{\mathrm{R}}} = \mathcal{A}_e$$

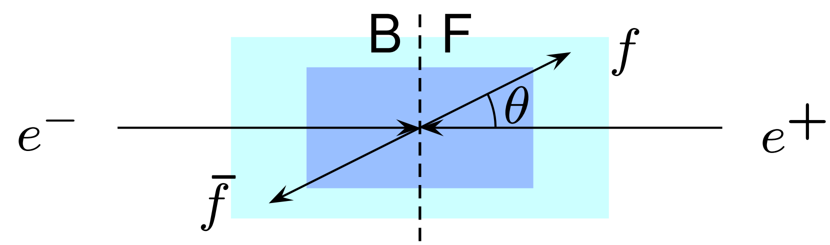
Polarization asymmetry:

Average τ pol. in $e^+e^- \to \tau^+\tau^-$:

$$\langle \mathcal{P}_\tau \rangle = -\mathcal{A}_\tau$$

Z-pole asymmetries

Forward-backward asymmetry:



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Average τ pol. in $e^+e^- \to \tau^+\tau^-$: $$\langle \mathcal{P}_\tau \rangle = -\mathcal{A}_\tau$$

Decay widths in terms of $\sin^2\theta^f_{\text{eff}}$:

$$\Gamma_{ff} = C\left[F^f_V + F^f_A\right] = C\left[F^f_A(1 - 4|Q_f|\sin^2\theta^f_{\text{eff}})^2 + F^f_A\right]$$

Z lineshape

- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\rm ini}(s,s') \otimes \sigma_{\rm hard}(s')$$

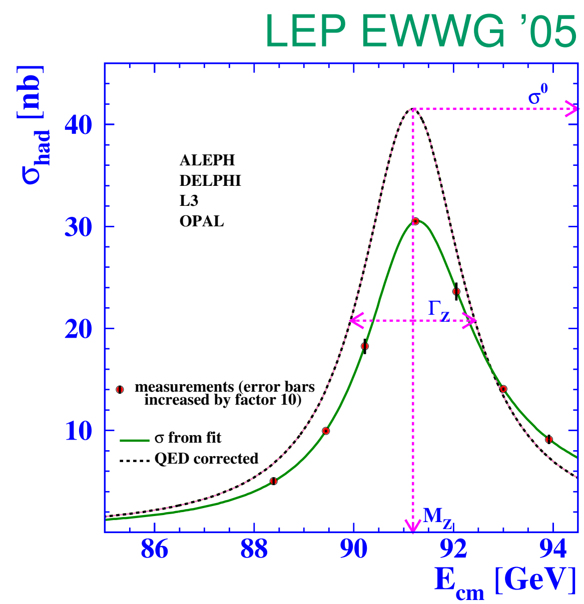
- Subtraction of γ-exchange, γ–Z interference, box contributions:

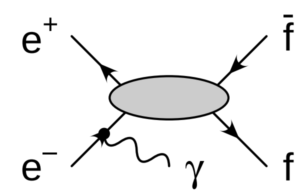
$$\sigma_{\rm hard} = \sigma_{\rm Z} + \sigma_\gamma + \sigma_{\gamma \rm Z} + \sigma_{\rm box}$$

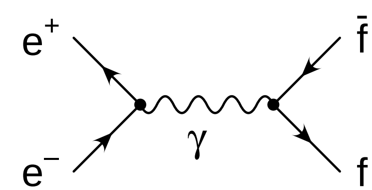
- Z-pole contribution:

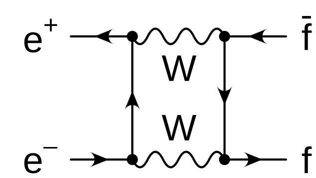
$$\sigma_{\rm Z} = \frac{R}{(s-\overline{M}_{\rm Z}^2)^2 + \overline{M}_{\rm Z}^2\Gamma_{\rm Z}^2} + \sigma_{\rm non-res}$$

σ_γ, $\sigma_{\gamma \rm Z}$, $\sigma_{\rm box}$, $\sigma_{\rm non-res}$ known at NLO
$\longrightarrow$ need consistent pole expansion framework
$\longrightarrow$ higher orders needed for FCC-ee









Pole expansion

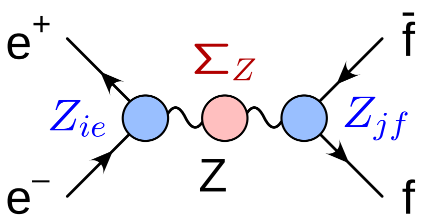
Expand amplitude for $e^+e^- \to f\bar{f}$ about **complex pole** $s_0 \equiv \overline{M}_Z^2 + i\overline{M}_Z\overline{\Gamma}_Z$:

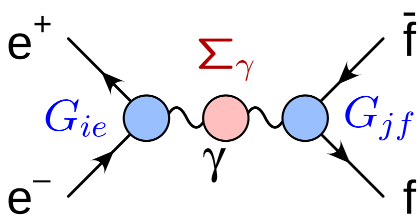
$$\mathcal{M}_{ij} = \frac{R_{ij}}{s - s_0} + S_{ij} + (s - s_0)S'_{ij} + \ldots \qquad (i,j = V, A)$$

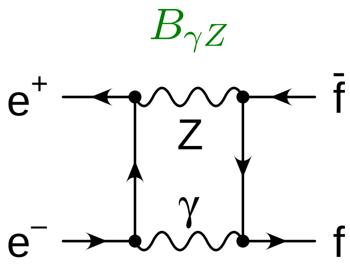
$$R_{ij} = \left.\frac{Z_{ie}Z_{jf}}{1 + \Sigma'_Z}\right|_{s=s_0} + B^R_{\gamma Z,ij} + B^{RL}_{\gamma Z,ij}\ln(1 - \tfrac{s}{s_0})$$

$$S_{ij} = \left[\frac{Z_{ie}Z'_{jf} + Z'_{ie}Z_{jf}}{1 + \Sigma'_Z} - \frac{Z_{ie}Z_{jf}\Sigma''_Z}{2(1 + \Sigma'_Z)^2} + \frac{G_{ie}G_{jf}}{s + \Sigma_\gamma} + B_{ij}\right]_{s=s_0} + B^S_{\gamma Z,ij} + B^{SL}_{\gamma Z,ij}\ln(1 - \tfrac{s}{s_0})$$

$$S'_{ij} = \ldots$$







Pole expansion

Express R_{ij} in terms of $\sin^2\theta^f_{\text{eff}}$ and F^f_A:

$$R_{ij} = 4 I^3_e I^3_f \sqrt{F^e_A F^f_A} \Big[Q^e_i Q^f_j \Big(1 + i\, r^I_{AA} - \tfrac{1}{2}(r^I_{AA})^2 + \tfrac{1}{2}\delta\overline{X}_{(2)} \Big) + (Q^e_i I_{j,f} + Q^f_j I_{i,e})(i - r^I_{AA}) - I_{i,e} I_{j,f} \Big] + M_Z \Gamma_Z\, Z_{ie(0)} Z_{jf(0)}\, x^I_{ij}\,,$$

$$Q^f_V = 1 - 4|Q_f| \sin^2\theta^f_{\text{eff}}, \qquad Q^f_A = 1$$

$$I_{V,f} = \frac{1}{(a^Z_{f(0)})^2} \Big[a^Z_{f(0)}\, \text{Im}\, Z_{Vf(1)} - v^Z_{f(0)}\, \text{Im}\, Z_{Af(1)} \Big], \qquad I_{A,f} = 0$$

$$\delta\overline{X}_{(2)} = -(\text{Im}\,\Sigma'_{Z(1)})^2 + 2\,\overline{b}^R_{\gamma Z(1)},$$

$$r^I_{ij} = \frac{\text{Im}\, Z_{ie(1)}}{Z_{ie(0)}} + \frac{\text{Im}\, Z_{jf(1)}}{Z_{jf(0)}} - \text{Im}\,\Sigma'_{Z(1)},$$

$$x^I_{ij} = \frac{\text{Im}\, Z'_{ie(1)}}{Z_{ie(0)}} + \frac{\text{Im}\, Z'_{jf(1)}}{Z_{jf(0)}} - \frac{1}{2}\,\text{Im}\,\Sigma''_{Z(1)},$$

C++ library GRIFFIN (in preparation)

Chen, Freitas '22

Pole expansion

Expand amplitude for $e^+e^- \to f\bar{f}$ about **complex pole** $s_0 \equiv \overline{M}_Z^2 + i\overline{M}_Z\overline{\Gamma}_Z$:

$$\mathcal{M}_{ij} = \frac{R_{ij}}{s - s_0} + S_{ij} + (s - s_0)S'_{ij} + \dots \qquad (i,j = V,A)$$

Current state of art: R @ NNLO + leading higher orders
S @ NLO
S' @ (N)LO

For FCC-ee: (at least) one order more!

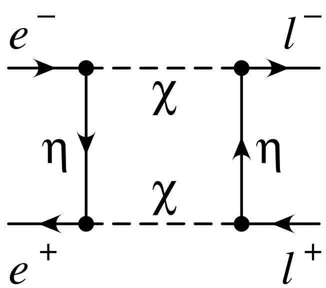
$\rightarrow$ also matching to Monte-Carlo for QED/QCD ISR/FSR/IFI

$ee \to ff$ above Z pole

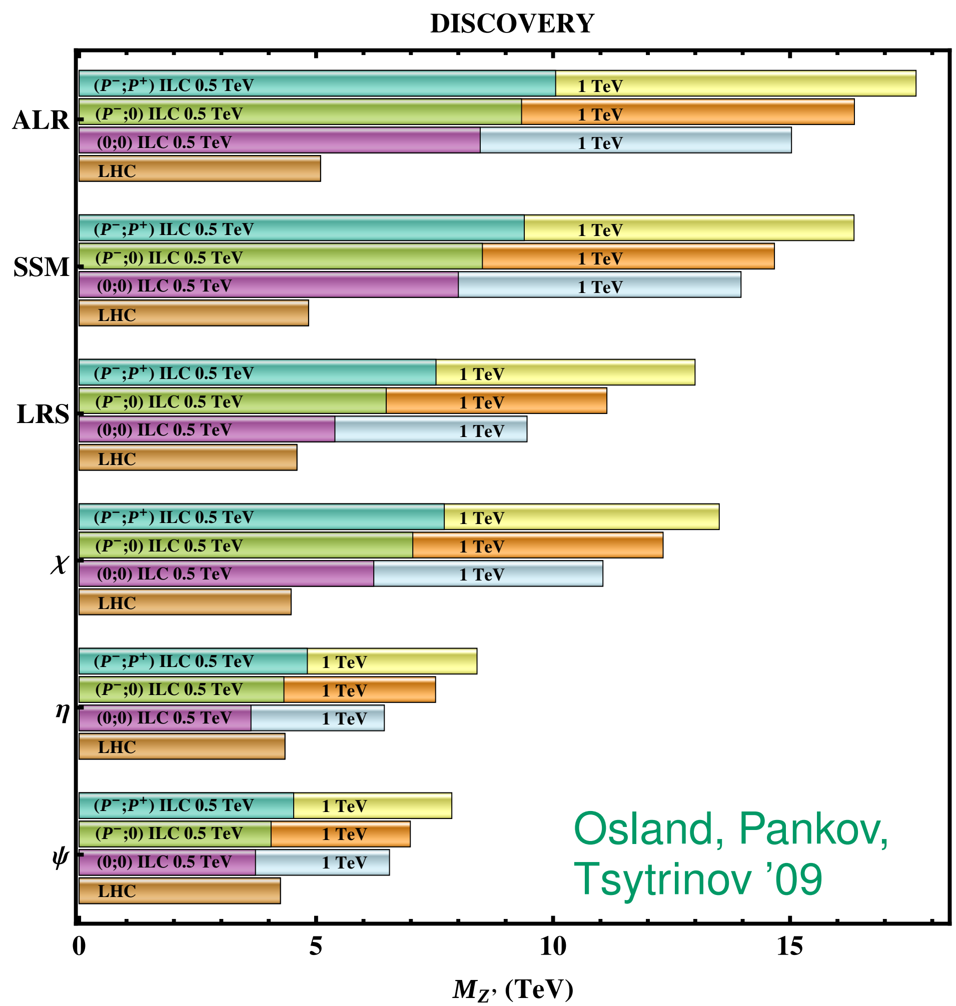
With $\mathcal{O}(\mathrm{ab}^{-1})$ at $\sqrt{s} \sim 161$ GeV and $\sqrt{s} \sim 240$ GeV:

→ $< 10^{-3}$ precision for $\sigma[e^+e^- \to f\bar{f}]$ (similar for A_{FB})

→ strong sensitivity to new physics, e.g. Z' bosons or lepto-philic DM



Freitas, Westhoff '14



Osland, Pankov, Tsytrinov '09

→ NNLO corrections for full process $e^+e^- \to f\bar{f}$ needed (+ partial higher orders)

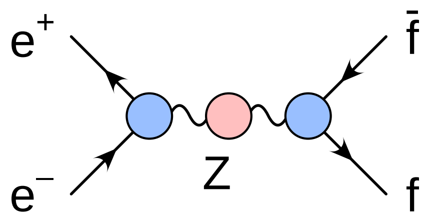
SMEFT operators

SMEFT: Gauge-invariant operators with SU(2) Higgs doublet

$$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i + \mathcal{O}(\Lambda^{-3}) \qquad (\Lambda \gg M_Z)$$

Modified propagators:

e.g. $\mathcal{O}_{\phi 1} = (D_\mu \Phi)^\dagger \Phi \, \Phi^\dagger (D^\mu \Phi)$
$\mathcal{O}_{BW} = \Phi^\dagger B_{\mu\nu} W^{\mu\nu} \Phi$

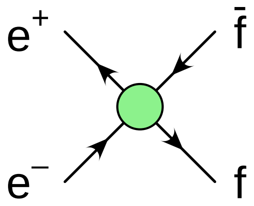


Modified Z-fermion couplings:

e.g. $O^f = i(\Phi^\dagger \overset{\leftrightarrow}{D}_\mu \Phi)(\bar{f}\gamma^\mu f) \qquad f = e, \mu\, \tau, b, \ldots$

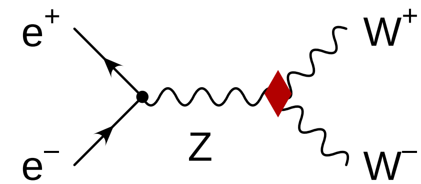
4-fermion operators:

e.g. $O^{(1)}_{ff'} = (\bar{f}\gamma_\mu f)(\bar{f}'\gamma^\mu f') \qquad f, f' = e, \mu\, \tau, b, \ldots$



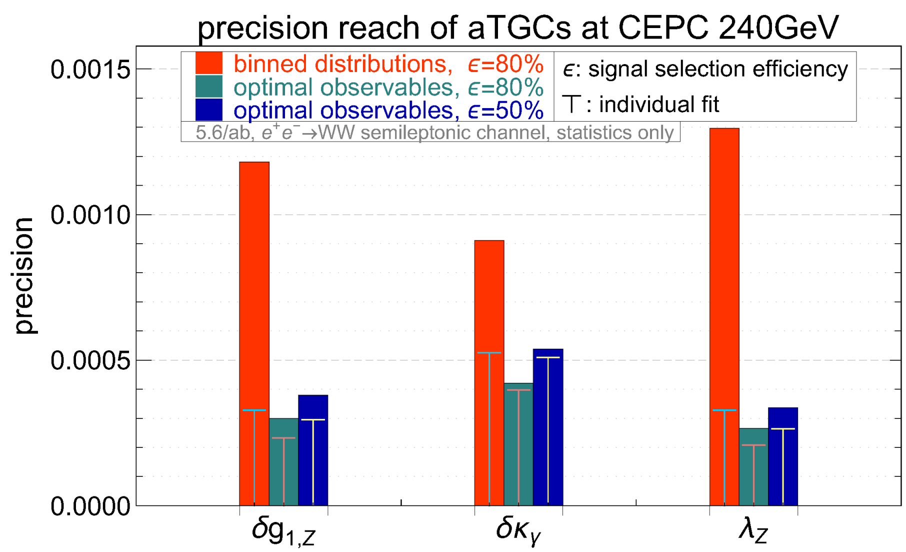
WW production

$e^+e^- \to W^+W^-$ at $\sqrt{s} = 240$ GeV: test of aGC



$\sigma[ee \to WW] \sim 15$ pb $\quad \Rightarrow \quad \mathcal{O}(10^8)$ events

$< 10^{-3}$ precision for aGC



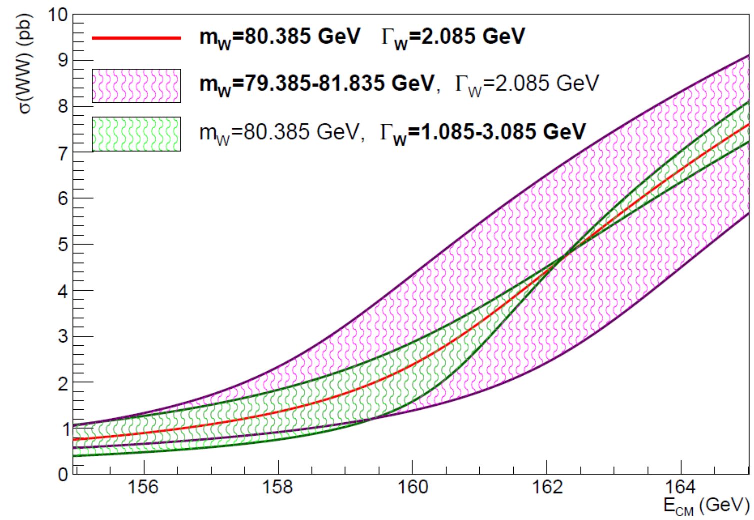
de Blas, Durieux, Grojean, Gu, Paul '19

→ NNLO corrections for full process $e^+e^- \to W^+W^-$ needed (+ partial higher orders)

WW threshold

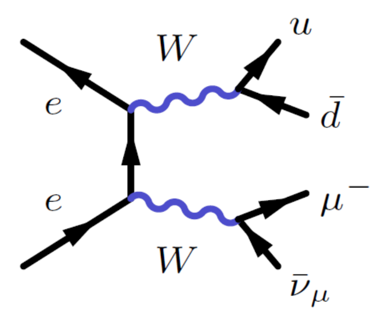
- High-precision measurement of M_W from $e^+e^- \to W^+W^-$ at threshold

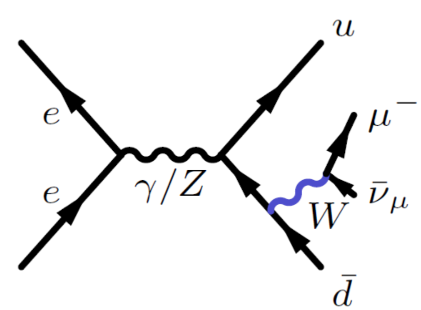
- a) Corrections near threshold enhanced by
  $1/\beta$ and $\ln\beta$
  $$\beta \sim \sqrt{1 - 4\frac{M_W^2 - iM_W\Gamma_W}{s}} \sim \sqrt{\Gamma_W/M_W}$$
  b) Non-resonant contributions are important



- Full $\mathcal{O}(\alpha)$ calculation of $e^+e^- \to 4f$
  Denner, Dittmaier, Roth, Wieders '05

- EFT expansion in $\alpha \sim \Gamma_W/M_W \sim \beta^2$
  Beneke, Falgari, Schwinn, Signer, Zanderighi '07
  - NLO corrections with NNLO Coulomb correction ($\propto 1/\beta^n$): $\delta_{th}M_W \sim 3$ MeV
    Actis, Beneke, Falgari, Schwinn '08
  - Adding NNLO corrections to $ee \to WW$ and $W \to f\bar{f}$ and NNLO ISR: $\delta_{th}M_W \lesssim 0.6$ MeV





Comparison of EWPOs with theory

- To probe new physics, compare EWPOs with SM theory predictions
- Need to take theory error into account:

	Current exp.	Current th.	CEPC	FCC-ee
M_{W}* [MeV]	15	4	1	0.5
Γ_Z [MeV]	2.3	0.4	0.025	0.025
$R_\ell = \Gamma_Z^{\mathrm{had}}/\Gamma_Z^\ell$ $[10^{-3}]$	25	5	2	1
$R_b = \Gamma_Z^b/\Gamma_Z^{\mathrm{had}}$ $[10^{-5}]$	66	10	4.3	6
$\sin^2\theta^\ell_{\mathrm{eff}}$ $[10^{-5}]$	16	4.5	<1	0.5

* computed from G_μ

Theory calculations: Status

- Many seminal works on 1-loop and leading 2-loop corrections

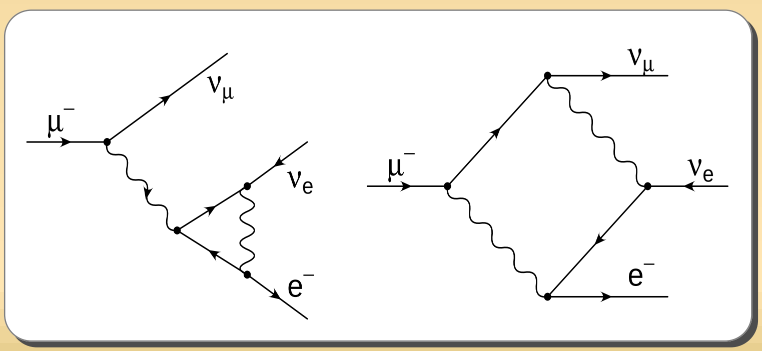
  Veltman, Passarino, Sirlin, Marciano, Bardin, Hollik, Riemann, Degrassi, Kniehl, ...

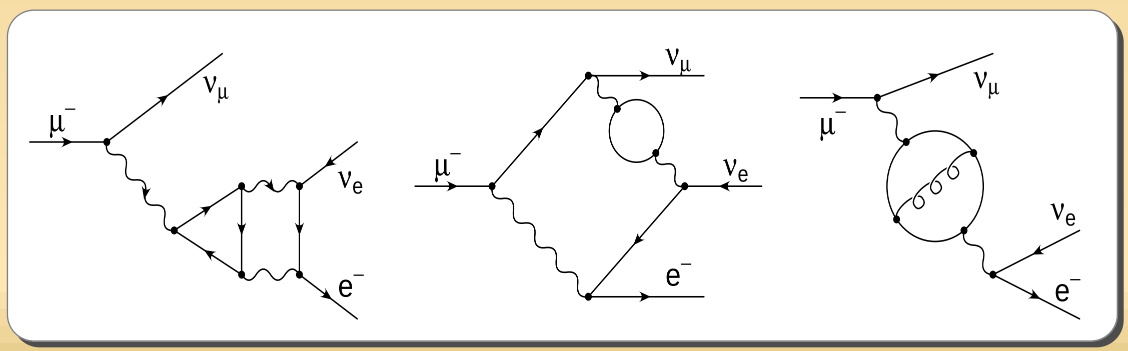
- Full 2-loop results for M_W, Z-pole observables

  Freitas, Hollik, Walter, Weiglein ’00
  Awramik, Czakon ’02
  Onishchenko, Veretin ’02
  Awramik, Czakon, Freitas, Weiglein ’04
  Awramik, Czakon, Freitas ’06
  Hollik, Meier, Uccirati ’05,07
  Awramik, Czakon, Freitas, Kniehl ’08
  Freitas ’14
  Dubovyk, Freitas, Gluza, Riemann, Usovitsch ’16,18

- Approximate 3- and 4-loop results (enhanced by Y_t and/or N_f)

  Chetyrkin, Kühn, Steinhauser ’95
  Faisst, Kühn, Seidensticker, Veretin ’03
  Boughezal, Tausk, v. d. Bij ’05
  Schröder, Steinhauser ’05
  Chetyrkin et al. ’06
  Boughezal, Czakon ’06
  Chen, Freitas ’20





Theory and parametric uncertainties

	FCC-ee	perturb. error with 3-loop†	Param. error*	main source
M_W [MeV]	0.5	1	0.6	$\Delta\alpha$
Γ_Z [MeV]	0.025	0.15	0.1	α_s
R_b [10^{-5}]	6	5	< 1	
$\sin^2\theta^\ell_{eff}$ [10^{-5}]	0.5	1.5	1	$\Delta\alpha$

† **Theory scenario:** $\mathcal{O}(\alpha\alpha_s^2)$, $\mathcal{O}(N_f\alpha^2\alpha_s)$, $\mathcal{O}(N_f^2\alpha^2\alpha_s)$, leading 4-loop

(N_f^n = at least n closed fermion loops)

Parametric inputs:

***FCC-ee:** $\delta m_t = 50$ MeV, $\delta\alpha_s = 0.0002$, $\delta M_Z = 0.5$ MeV,

$\delta(\Delta\alpha) = 3 \times 10^{-5}$

Reviews: 1906.05379, 2012.11642

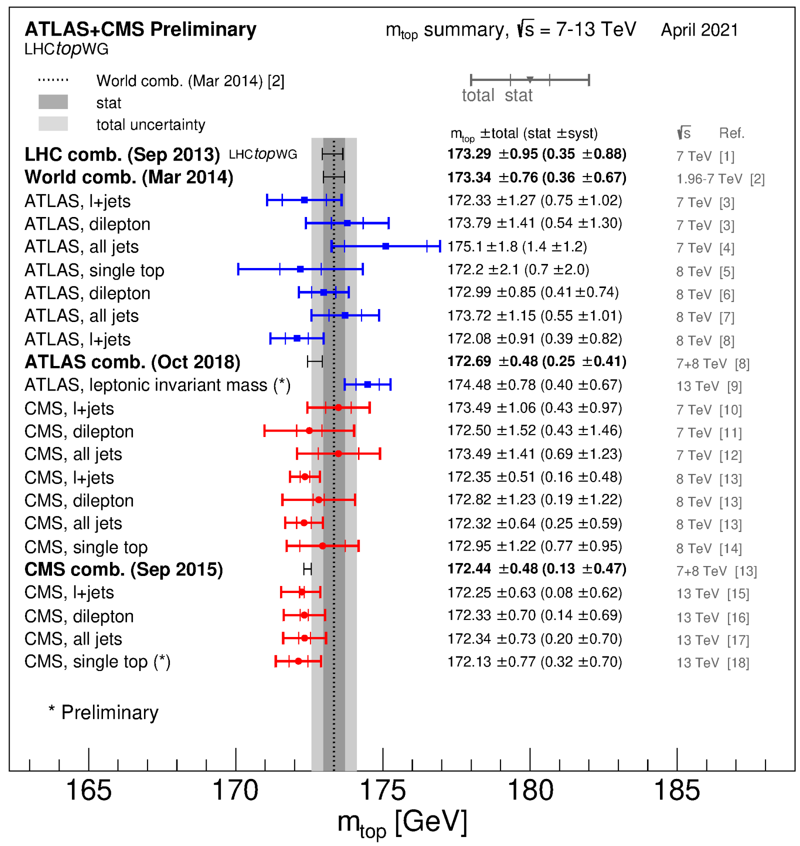
- M_Z, Γ_Z: From $\sigma(\sqrt{s})$ lineshape; δM_Z, $\delta\Gamma_Z \sim 0.1$ MeV at FCC-ee
  $\rightarrow$ Main theory uncertainties: QED ISR

- m_t: Most precise measurement at LHC: $\delta m_t \sim 0.3$ GeV PDG '20

  Theoretical ambiguity in mass def.:

  Hoang, Plätzer, Samitz '18

$$m_t^{\rm CB}(Q_0) - m_t^{\rm pole}$$
$$= -\tfrac{2}{3}\alpha_s(Q_0)\,Q_0 + \mathcal{O}(\alpha_s^2 Q_0)$$
$$\approx 0.5 \pm 0.2_{\rm pert.} \pm 0.2_{\rm np.}\,\text{GeV}$$



SM input parameters

Reviews: 1906.05379, 2012.11642

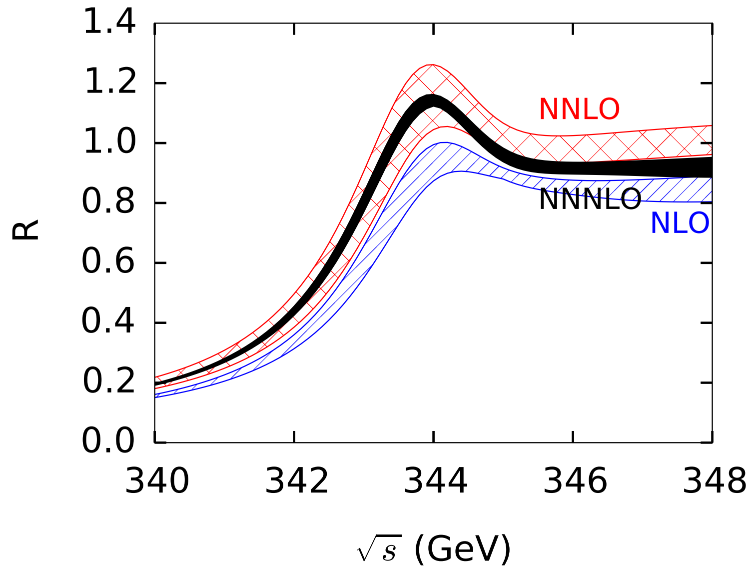
- $M_Z,\ \Gamma_Z$: From $\sigma(\sqrt{s})$ lineshape; $\delta M_Z,\ \delta\Gamma_Z \sim 0.1$ MeV at FCC-ee
  $\to$ Main theory uncertainties: QED ISR

- m_t: Most precise measurement
  at LHC: $\delta m_t \sim 0.3$ GeV

  From $e^+e^- \to t\bar{t}$ at $\sqrt{s} \sim 350$ GeV:

  Impact of theory modelling:

$$\delta m_t^{\overline{\text{MS}}} = [\quad]_{\text{exp}} \oplus [50\ \text{MeV}]_{\text{QCD}} \oplus [10\ \text{MeV}]_{\text{mass def.}} \oplus [70\ \text{MeV}]_{\alpha_s} > 100\ \text{MeV}$$



Beneke et al. '15

SM input parameters

Reviews: 1906.05379, 2012.11642

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- m_t: Most precise measurement
  at LHC: $\delta m_t \sim 0.3$ GeV

  From $e^+e^- \to t\bar{t}$ at $\sqrt{s} \sim 350$ GeV:

Impact of theory modelling:	future improvements:	
$\delta m_t^{\overline{\text{MS}}} = [\quad]_{\text{exp}}$	$[20\text{ MeV}]_{\text{exp}}$	
$\oplus\, [50\text{ MeV}]_{\text{QCD}}$	$\oplus\, [30\text{ MeV}]_{\text{QCD}}$	(h.o. resummation)
$\oplus\, [10\text{ MeV}]_{\text{mass def.}}$	$\oplus\, [10\text{ MeV}]_{\text{mass def.}}$	
$\oplus\, [70\text{ MeV}]_{\alpha_s}$	$\oplus\, [15\text{ MeV}]_{\alpha_s}$	$(\delta\alpha_s \lesssim 0.0002)$
> 100 MeV	$\lesssim 50$ MeV	

- α_s: d'Enterria, Skands, et al. '15
  - Most precise determination using Lattice QCD:

    $\alpha_s = 0.1184 \pm 0.0006$ HPQCD '10

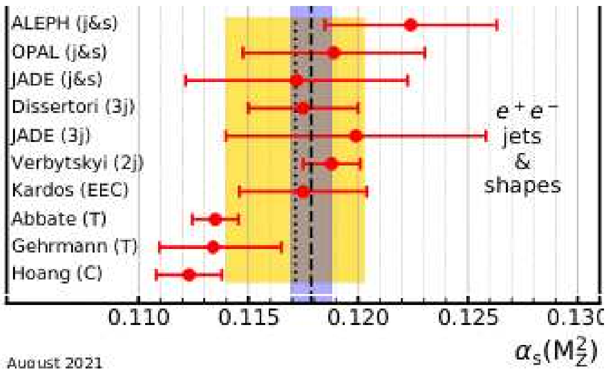
    $\alpha_s = 0.1185 \pm 0.0008$ ALPHA '17

    $\alpha_s = 0.1179 \pm 0.0015$ Takaura et al. '18

    $\alpha_s = 0.1172 \pm 0.0011$ Zafeiropoulos et al. '19

    $\rightarrow$ Difficulty in evaluating systematics

PDG '21



  - e^+e^- event shapes: $\alpha_s \sim 0.113...0.119$

    $\rightarrow$ Large non-pertubative power corrections

    $\rightarrow$ Systematic uncertainties?

  - Hadronic τ decays: $\alpha_s = 0.119 \pm 0.002$ PDG '18

    $\rightarrow$ Non-perturbative uncertainties in OPE and from duality violation

    Pich '14; Boito et al. '15,18

Strong coupling

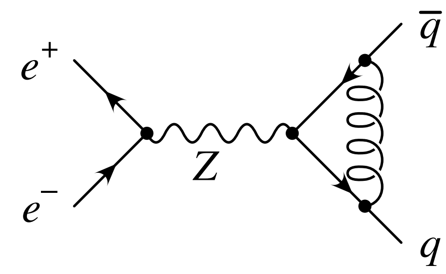
- α_s:
  - Electroweak precision ($R_\ell = \Gamma_Z^{had}/\Gamma_Z^{\ell}$):
    $\alpha_s = 0.120 \pm 0.003$ PDG '18

    $\rightarrow$ No (negligible) non-perturbative QCD effects

    FCC-ee: $\delta R_\ell \sim 0.001$
    $\Rightarrow \quad \delta\alpha_s < 0.0001$

    Theory input: N^{3}LO EW corr. + leading N^{4}LO
    to keep $\delta_{th} R_\ell \lesssim \delta_{exp} R_\ell$

    Caviat: R_ℓ could be affected by new physics



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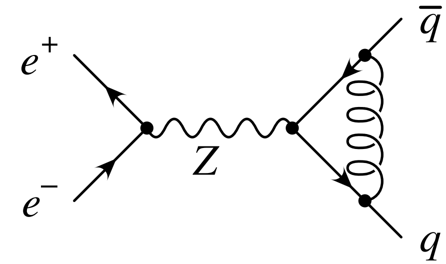
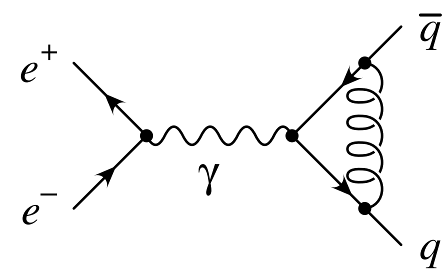
    

    $\rightarrow$ No (negligible) non-perturbative QCD effects

    FCC-ee: $\delta R_\ell \sim 0.001$
    $\Rightarrow \quad \delta\alpha_s < 0.0001$

    Caviat: R_ℓ could be affected by new physics

  - $R = \frac{\sigma[ee\to\text{had.}]}{\sigma[ee\to\mu\mu]}$ at lower $\sqrt{s}$

    e.g. CLEO ($\sqrt{s} \sim 9$ GeV): $\alpha_s = 0.110 \pm 0.015$
    Kühn, Steinhauser, Teubner '07

    $\rightarrow$ dominated by s-channel photon, less room for new physics
    $\rightarrow$ QCD still perturbative

    naive scaling to 50 ab^{-1} (BELLE-II): $\delta\alpha_s \sim 0.0001$

Calculational techniques

Analytical techniques:

- Computational intensive reduction to master integrals (MIs)
- Not fully understood function space of MIs
- Works best for problems with few (no) masses

Numerical techniques:

- Large computing time
- Numerical instabilites, in particule for diagrams with physical cuts
- Works best for problems with many masses

New techniques:

- Numerical reduction to MIs, numerical MIs via differential equations (DEs)
  Mandal, Zhao '18, Czakon, Niggetiedt '20
- DEs with respect to auxialiary parameter, $\frac{1}{k_i^2 - m_i^2 + i\epsilon}$ Liu, Ma, Wang '17 Liu, Ma '18,21,22
- Series solutions of DEs Moriello '19, Hidding '20

$\longrightarrow$ talks by J. Gluza, J. Usovitsch

Summary

- Electroweak precision tests require theory input for **measurements of pseudo-observables** (BRs, widths, masses, cross-sections, ...) and their **SM/BSM interpretation**
- **Future e^+e^- colliders (FCC-ee)** improve precision by 1–2 orders of magnitude
- Uncertainties from **perturbative** and **non-perturbative** theory and **input parameters** require much work, but ongoing progress in calculational techniques
- Theory progress needed both for **fixed-order loop corrections** as well as **MC tools**

Backup slides

Z lineshape

- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s,s') \otimes \sigma_{\text{hard}}(s')$$

- Subtraction of γ-exchange, γ–Z interference, box contributions:

$$\sigma_{\text{hard}} = \sigma_{\text{Z}} + \sigma_{\gamma} + \sigma_{\gamma\text{Z}} + \sigma_{\text{box}}$$

- Z-pole contribution:

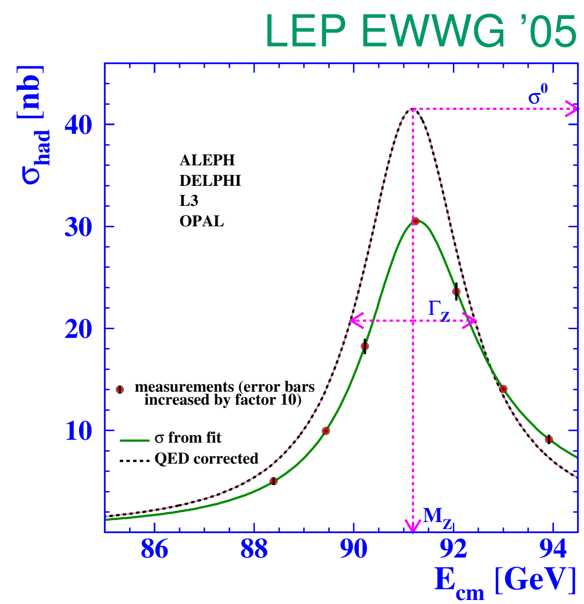
$$\sigma_{\text{Z}} = \frac{R}{(s-\overline{M}_{\text{Z}}^2)^2 + \overline{M}_{\text{Z}}^2\overline{\Gamma}_{\text{Z}}^2} + \sigma_{\text{non-res}}$$

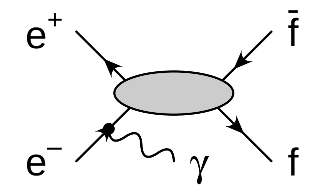
- In experimental analyses:

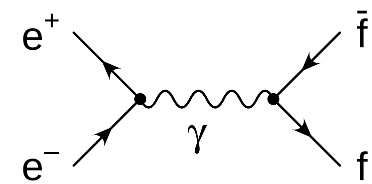
$$\sigma \sim \frac{1}{(s-M_{\text{Z}}^2)^2 + s^2\Gamma_{\text{Z}}^2/M_{\text{Z}}^2}$$

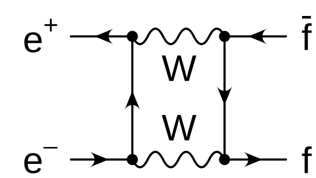
$$\overline{M}_{\text{Z}} = M_{\text{Z}} \Big/ \sqrt{1+\Gamma_{\text{Z}}^2/M_{\text{Z}}^2} \approx M_{\text{Z}} - 34 \text{ MeV}$$

$$\overline{\Gamma}_{\text{Z}} = \Gamma_{\text{Z}} \Big/ \sqrt{1+\Gamma_{\text{Z}}^2/M_{\text{Z}}^2} \approx \Gamma_{\text{Z}} - 0.9 \text{ MeV}$$





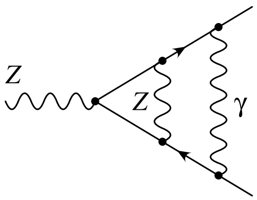




Z decay

Factorization of massive and QED/QCD FSR:

$$\overline{\Gamma}_f \approx \frac{N_c \overline{M}_\mathrm{Z}}{12\pi}\left[\left(\mathcal{R}_V^f |g_V^f|^2 + \mathcal{R}_A^f |g_A^f|^2\right)\frac{1}{1+\mathrm{Re}\,\Sigma'_\mathrm{Z}}\right]_{s=\overline{M}_\mathrm{Z}^2}$$



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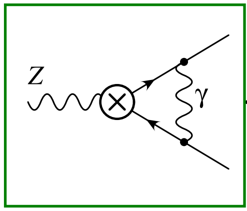
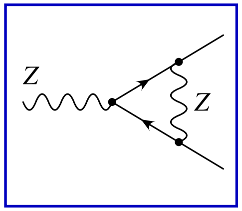


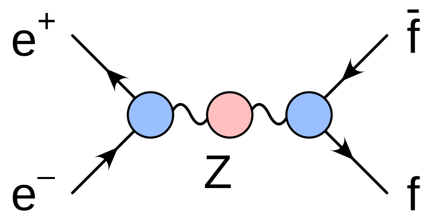
$+$ finite, with $\otimes =$



$\mathcal{R}_V^f$, $\mathcal{R}_A^f$: Final-state QED/QCD radiation;
known to $\mathcal{O}(\alpha_\mathrm{s}^4)$, $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_\mathrm{s})$

Kataev '92
Chetyrkin, Kühn, Kwiatkowski '96
Baikov, Chetyrkin, Kühn, Rittinger '12

g_V^f, g_A^f, Σ'_Z: Electroweak corrections



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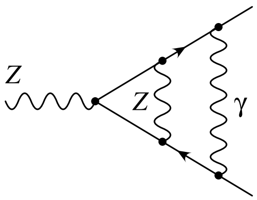

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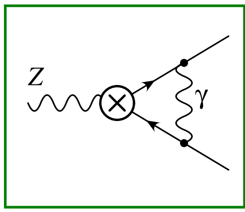
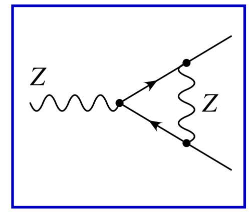
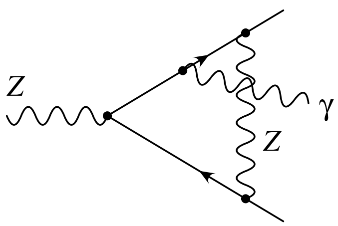


$+$ finite, with $\otimes =$



Additional non-factorizable contributions, e.g.



→ Known at $\mathcal{O}(\alpha\alpha_s)$ Czarnecki, Kühn '96
Harlander, Seidensticker, Steinhauser '98

→ Currently not known at $\mathcal{O}(\alpha^2)$ and beyond

→ $\mathcal{O}(0.01\%)$ uncertainty on Γ_Z, σ_Z, maybe larger for A_b

→ How to account for in MC simulations?

Theory calculations: Uncertainties

	Experiment	Theory error	Main source
M_W	80.379 ± 0.012 MeV	4 MeV	α^3, $\alpha^2\alpha_s$
Γ_Z	2495.2 ± 2.3 MeV	0.4 MeV	α^3, $\alpha^2\alpha_s$, $\alpha\alpha_s^2$
R_ℓ	20.767 ± 0.025	0.005	α^3, $\alpha^2\alpha_s$
R_b	0.21629 ± 0.00066	0.0001	α^3, $\alpha^2\alpha_s$
$\sin^2\theta^\ell_{eff}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_s$

- Theory error estimate is not well defined, ideally $\Delta_{th} \ll \Delta_{exp}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence

Example: Error estimation for Γ_Z

- Goemetric perturbative series $\alpha_t = \alpha m_t^2$

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim 0.26 \text{ MeV}$$

$$\mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.30 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.23 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\mathcal{O}(\alpha_{bos}^2) \sim \mathcal{O}(\alpha_{bos})^2 \sim 0.1 \text{ MeV}$$

- Parametric prefactors: $\mathcal{O}(\alpha_{bos}^2) \sim \Gamma_Z\alpha^2 \sim 0.1 \text{ MeV}$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\alpha n_{lq}}{\pi}\alpha_s^2 \sim 0.29 \text{ MeV}$$

Total: $\delta\Gamma_Z \approx 0.5$ MeV