Electroweak pseudo-observables and Z-boson form factors at two-loop accuracy

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in collaboration with:

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09. February 2022

Outline

Asymmetries measured at the Z pole

We study the process $e^+e^- \to (Z) \to b\bar{b}$

Pseudo-observables, unfolded at the Z peak

forward-backward asymmetry $A_{\mathrm{FB}}^{\mathrm{b\bar{b}},0} = \frac{3}{4} A_{\mathrm{e}} A_{\mathrm{b}}$

f-b left-right asymmetry $A_{\mathrm{FB,LR}}^{\mathrm{b\bar{b}},0} = \frac{3}{4} P_{\mathrm{e}} A_{\mathrm{b}}$, P_{e} is the electron polarization

$$A_{\mathrm{b}} = \frac{2\Re e \frac{v_b}{a_b}}{1 + \left(\Re e \frac{v_b}{a_b}\right)^2} = \frac{1 - 4|Q_b|\sin^2\theta_{\mathrm{eff}}^{\mathrm{b}}}{1 - 4|Q_b|\sin^2\theta_{\mathrm{eff}}^{\mathrm{b}} + 8Q_b^2(\sin^2\theta_{\mathrm{eff}}^{\mathrm{b}})^2} \quad (1)$$

Definition of the effective weak mixing angle

$$\sin^2\theta_{\mathrm{eff}}^{\mathrm{b}} = \frac{1}{4|Q_b|}\left(1 - \Re e \frac{v_b}{a_b}\right) \quad (2)$$

v_b and a_b are effective vector coupling and axial-vector coupling of the $Zb\bar{b}$ vertex

Vertex form factor

- In the pole scheme, near the Z pole, the amplitude is written as

$$\mathcal{A}^{e^+e^-\to b\bar{b}} = \frac{R}{s-s_0} + S + (s-s_0)S' + \ldots, \quad s_0 = \overline{M}_Z^2 - i\overline{M}_Z\overline{\Gamma}_Z \tag{3}$$

- The Residue R of $\mathcal{A}^{[e^+e^-\to b\bar{b}]}$ factorizes into initial- and final state vertex form factors and Z-propagator corrections

$$V_\mu^{Zb\bar{b}} = \gamma_\mu[\hat{v}_b(s) - \hat{a}_b(s)\gamma_5] \tag{4}$$

- The effective vector and axial-vector components can be projected via

$$\hat{v}_b(s) = \frac{1}{2(2-D)s}\mathrm{Tr}[\gamma^\mu \not{p}_1 V_\mu^{Zb\bar{b}} \not{p}_2], \quad \hat{a}_b(s) = \frac{1}{2(2-D)s}\mathrm{Tr}[\gamma_5\gamma^\mu \not{p}_1 V_\mu^{Zb\bar{b}} \not{p}_2] \tag{5}$$

- $D = 4 - 2\epsilon$ is the space-time dimension
- $p_{1,2}$ are the momenta of the external b-quarks and $s = (p_1 + p_2)^2$
- The hat in $\hat{v}_b(s)$ and $\hat{a}_b(s)$ denotes the $Z-\gamma$ mixing [$\to$ see Ayres talk for more details]

Historical time stamps for electroweak $\sin^2\theta_{\rm eff}^{\rm b}$

- One-loop corrections to the $\sin^2\theta_{\rm eff}^{\rm b}$ [A. Akhundov, D. Bardin, T. Riemann, Electroweak one loop corrections to the decay of the neutral vector boson, Nucl. Phys. B276 (1986) 1.] [W. Beenakker, W. Hollik, The width of the Z boson, Z. Phys. C40 (1988) 141.]
- Two-loop electroweak corrections to the $\sin^2\theta_{\rm eff}^{\rm b}$ [Awramik, M. Czakon, A. Freitas, B. Kniehl, Two-loop electroweak fermionic corrections to $\sin^2\theta_{\rm eff}^{\rm b}$, Nucl. Phys. B813 (2009) 174â187.] [I. Dubovyk, A. Freitas, J. Gluza, T. Riemann, J. Usovitsch, The two-loop electroweak bosonic corrections to $\sin^2\theta_{\rm eff}^{\rm b}$, Phys. Lett. B762 (2016) 184â189.]
- We should focus to deliver three-loop contributions faster than in a 30 years time frame.
- The emergement of advanced tools helps us to achieve this goal to compute Feynman integrals and amplitudes efficiently

Z-boson form factors at two-loop accuracy

[I. Dubovyk, A. Freitas, J. Gluza, T. Riemann, J. Usovitsch, Electroweak pseudo-observables and Z-boson form factors at two-loop accuracy, JHEP 08 (2019) 113.]

Form fact.	Born	$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha\alpha_s)$	$\mathcal{O}(\alpha\alpha_s)$ non-fact.	$\mathcal{O}(\alpha_t\alpha_s^2,\ \alpha_t\alpha_s^3,\ \alpha_t^2\alpha_s,\ \alpha_t^3)$	$\mathcal{O}(N_f^2\alpha^2)$	$\mathcal{O}(N_f\alpha^2)$	$\mathcal{O}(\alpha_{bos}^2)$
F_V^ℓ $[10^{-5}]$	39.07	−24.86	2.41	–	0.35	1.47	2.37	0.27
F_A^ℓ $[10^{-5}]$	3309.44	118.59	9.46	–	1.22	8.60	2.60	0.45
$F_{V,A}^\nu$ $[10^{-5}]$	3309.44	127.56	9.46	–	1.22	8.60	3.83	0.39
$F_V^{u,c}$ $[10^{-5}]$	544.88	−44.80	7.29	−0.39	1.02	−1.67	3.25	0.33
$F_A^{u,c}$ $[10^{-5}]$	3309.44	120.79	9.46	−0.98	1.22	8.60	3.27	0.44
$F_V^{d,s}$ $[10^{-5}]$	1635.01	5.84	9.64	−0.80	1.32	0.71	3.45	0.37
$F_A^{d,s}$ $[10^{-5}]$	3309.44	123.78	9.46	−1.14	1.22	8.60	3.11	0.42
F_V^b $[10^{-5}]$	1635.01	−26.16	9.64	3.13	1.32	0.71	1.77	1.05
F_A^b $[10^{-5}]$	3309.44	78.26	9.46	4.45	1.22	8.60	0.13	1.18

Table: *Contributions of different perturbative orders to the Z vertex form factors. A fixed value of M_W has been used as input, instead of G_μ. N_f^n refers to corrections with n closed fermions loops, whereas α_{bos}^2 denotes corrections without closed fermions loops. Furthermore, $\alpha_t = y_t/(4\pi)$ where y_t is the top Yukawa coupling.*

Electroweak Precision Physics

	Experiment	**Theory uncertainty**	**Main source**
$M_W\,[\mathrm{MeV}]$	80385 ± 15	4	$N_f^2\alpha^3,\ N_f\alpha^2\alpha_s$
$\sin^2\theta^{\mathrm{l}}_{\mathrm{eff}}\,[10^{-5}]$	23153 ± 16	4.5	$N_f^2\alpha^3,\ N_f\alpha^2\alpha_s$
$\Gamma_Z\,[\mathrm{MeV}]$	2495.2 ± 2.3	0.4	$N_f^2\alpha^3,\ N_f\alpha^2\alpha_s,\ \alpha\alpha_s^2$
$\sigma^0_{\mathrm{had}}\,[\mathrm{pb}]$	41540 ± 37	6	$N_f^2\alpha^3,\ N_f\alpha^2\alpha_s$
$R_b = \Gamma_Z^b/\Gamma_Z^{\mathrm{had}}\,[10^{-5}]$	21629 ± 66	15	$N_f^2\alpha^3,\ N_f\alpha^2\alpha_s$

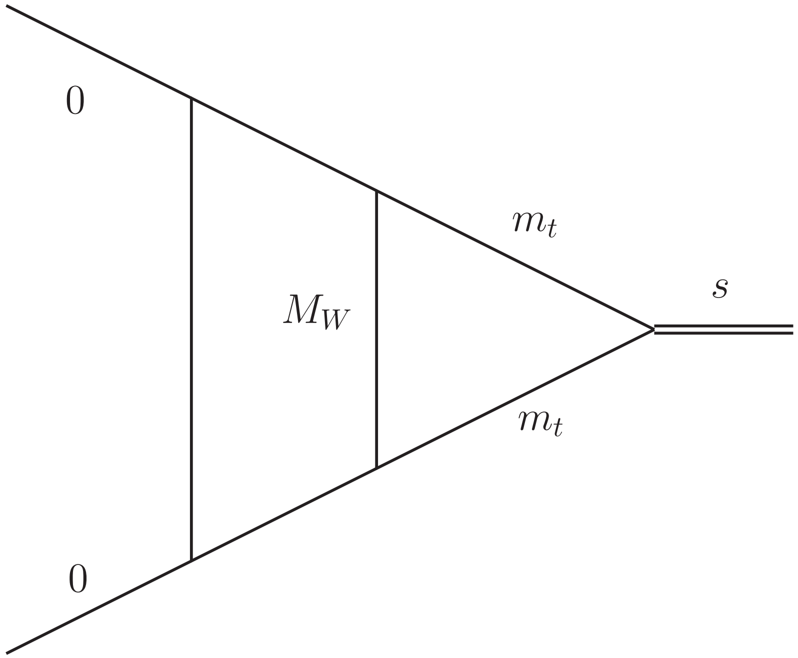
- The number of Z-bonos collected at LEP is 1.7×10^7
- Many pseudo observables are determined with high precision
- Present theoretical predictions (at least one order of magnitude better) are accurate enough to fullfill experimental demands

Overview Experiment Future

	Experiment uncertainty			Theory uncertainty	
	ILC	CEPC	FCC-ee	Current	Future
M_W[MeV]	3-4	3	1	4	1
$\sin^2\theta^{\mathrm{l}}_{\mathrm{eff}}[10^{-5}]$	1	2.3	0.6	4.5	1.5
Γ_Z[MeV]	0.8	0.5	0.1	0.4	0.2
$R_b[10^{-5}]$	14	17	6	15	7

- The concepts for the new experiments will have new demands to the theoreticle predictions
- The projection to the theory errors in the future assumes that **the tower of missing corrections** $\alpha\alpha_{\mathrm{s}}^2$, $N_f^2\alpha^3$, $N_f\alpha^2\alpha_{\mathrm{s}}$ will become available
- Theoretical computations are universal

State of the art 6 years ago



- In physical regions $(s, M_W^2, m_t^2) = (1, (\frac{401925}{4559382})^2, (\frac{433000}{227969})^2)$
- Arbitrary kinematic point, but with restricted accuracy
- A complementary mixture of Mellin-Barnes integral and sector decomposition methods [$\rightarrow$ see Janusz's talk]

$$\text{soft13}^{d=4-2\epsilon}[1,1,1,1,1,1,0] = 0.93453624 + 0.54089756\, i \quad (6)$$
$$+(0.1901137256 - 0.6583157563\, i)/\epsilon - 0.209548413480837 0/\epsilon^2$$

The state of the art - automatic computations

- With the program AMFlow [Liu, Xiao and Ma, Yan-Qing, AMFlow: a Mathematica Package for Feynman integrals computation via Auxiliary Mass Flow,arXiv:2201.11669] and the program Caesar

$$\begin{aligned}
&\text{soft13}^{d=4-2\epsilon}[1,1,1,1,1,1,0] \\
&= (0.934536247523241 + 0.540897568924577\, i) \\
&+(0.190113725674667 - 0.658315756362794\, i)1/\epsilon \\
&-0.209548413480837 0/\epsilon^2
\end{aligned} \tag{7}$$

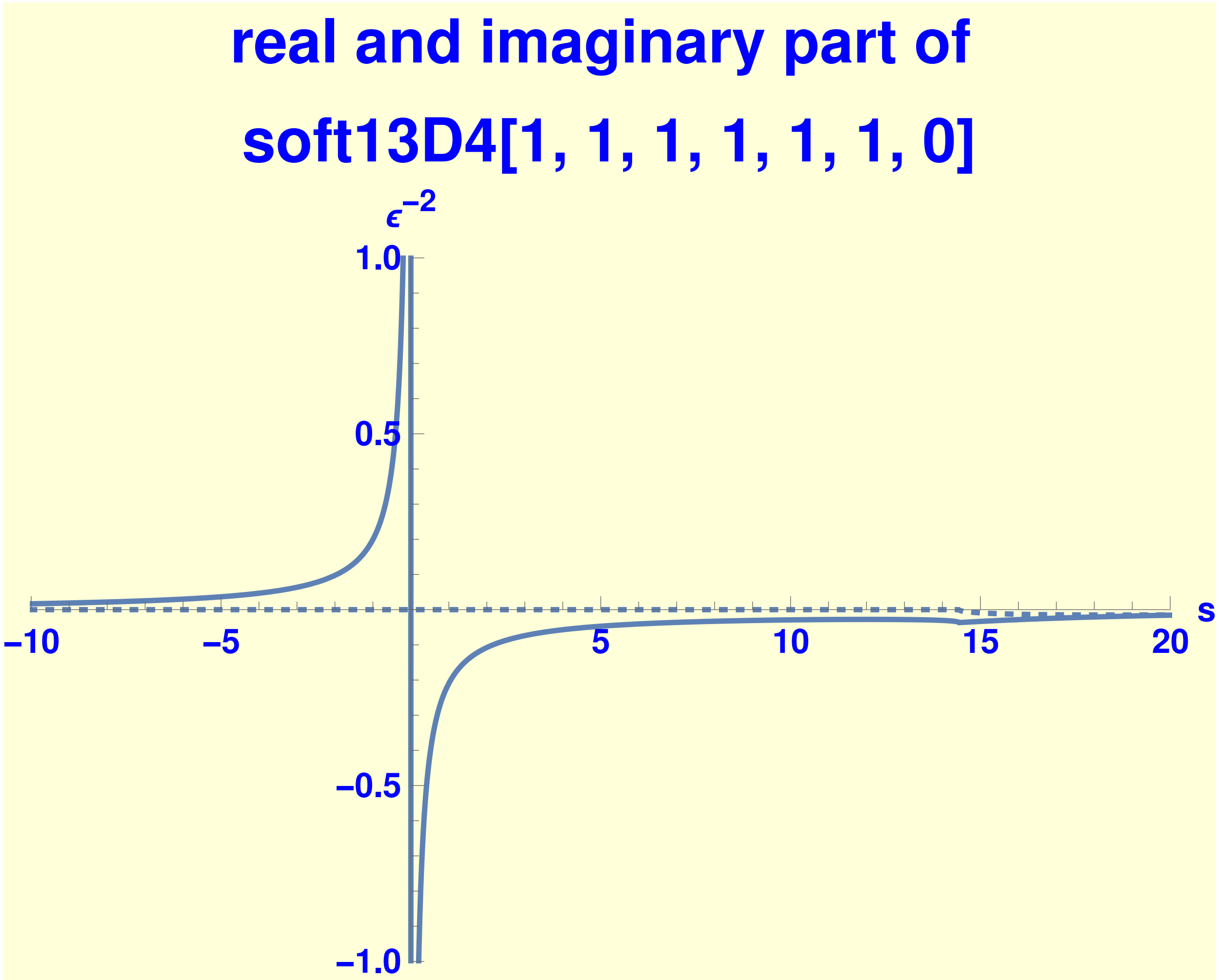
- Arbitrary kinematic point but with arbitrary accuracy
- AMFlow is based on auxiliary mass flow introduced in [arXiv:1711.09572, arXiv:1912.09294, arXiv:2107.01864]
- Caesar is a program, which is a combination of different set of tools based on [arXiv:2201.02576]

Caesar: blueprint for numerical evaluation of Feynman integrals

- Developers team: Martijn Hidding and me.
- Basic idea: Caesar has an interface to Kira [Jonas Klappert, Fabian Lange, Philipp Maierhöfer, J.U], Reduze 2 [Von Manteuffel, Studerus, 2012], (pySecDec [Borowka et al., 2018] or AMFlow [Xiao Liu, Yan-Qing Ma, 2022]) and DiffExp [Martijn Hidding, 2021].
- Kira - the backbone / major bottleneck of the Caesar project - solves linear system of equations
- Reduze 2 - finds candidates for a finite basis of master integrals
- pySecDec - computes these master integrals in Euclidean regions - boundary terms for the system of differential equations
- AMFlow - computes these master integrals in physical regions - boundary terms for the system of differential equations
- DiffExp - transports the boundary terms to an arbitrary physical point
- Error estimate: repeat the chain of tools for different initial boundary terms

The state of the art - automatic computations

- With Caesar[AMFlow] [Martijn Hidding, J.U., in preparation]



- Arbitrary line in the space of all mass scales with arbitrary accuracy

The state of the art - automatic computations

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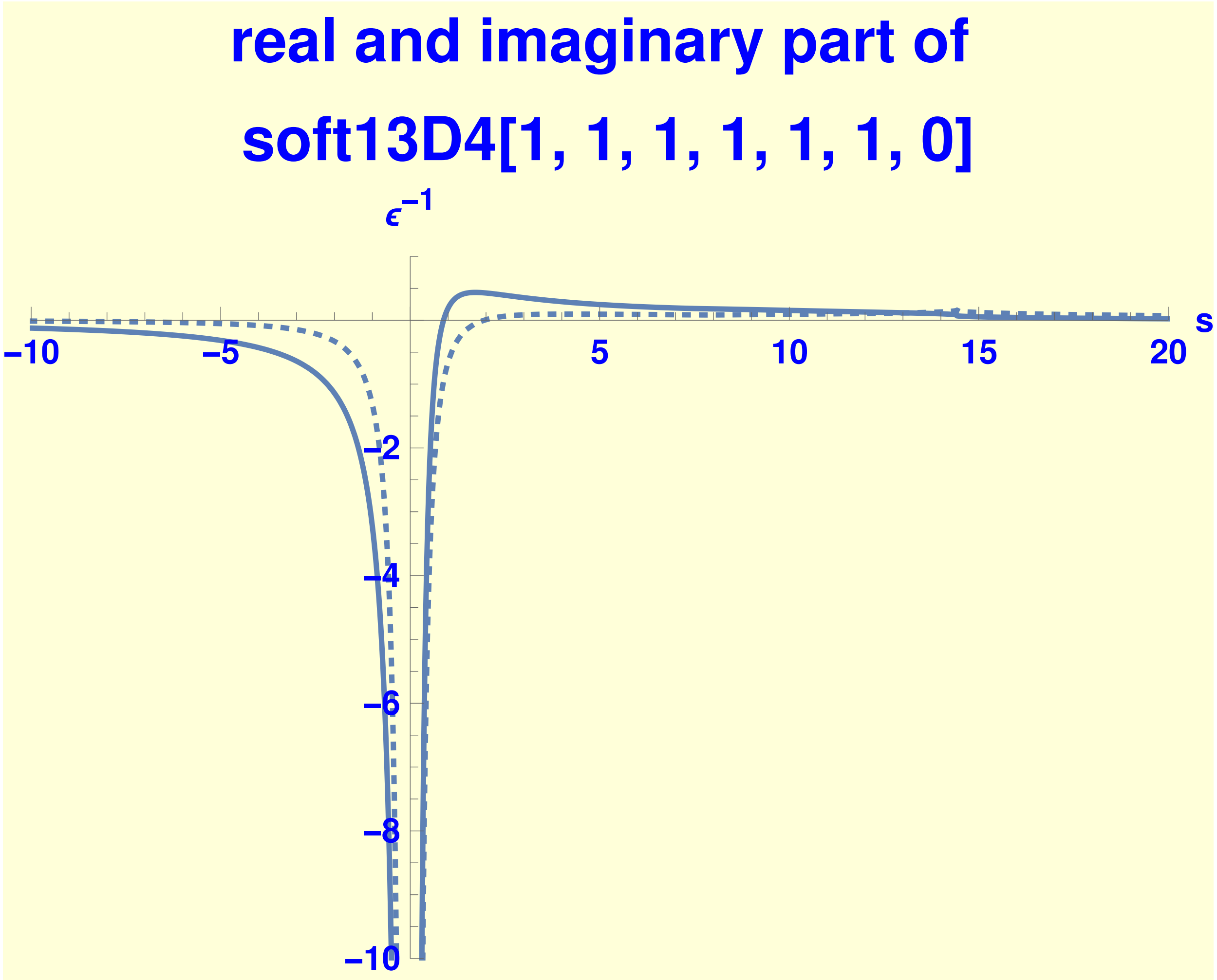

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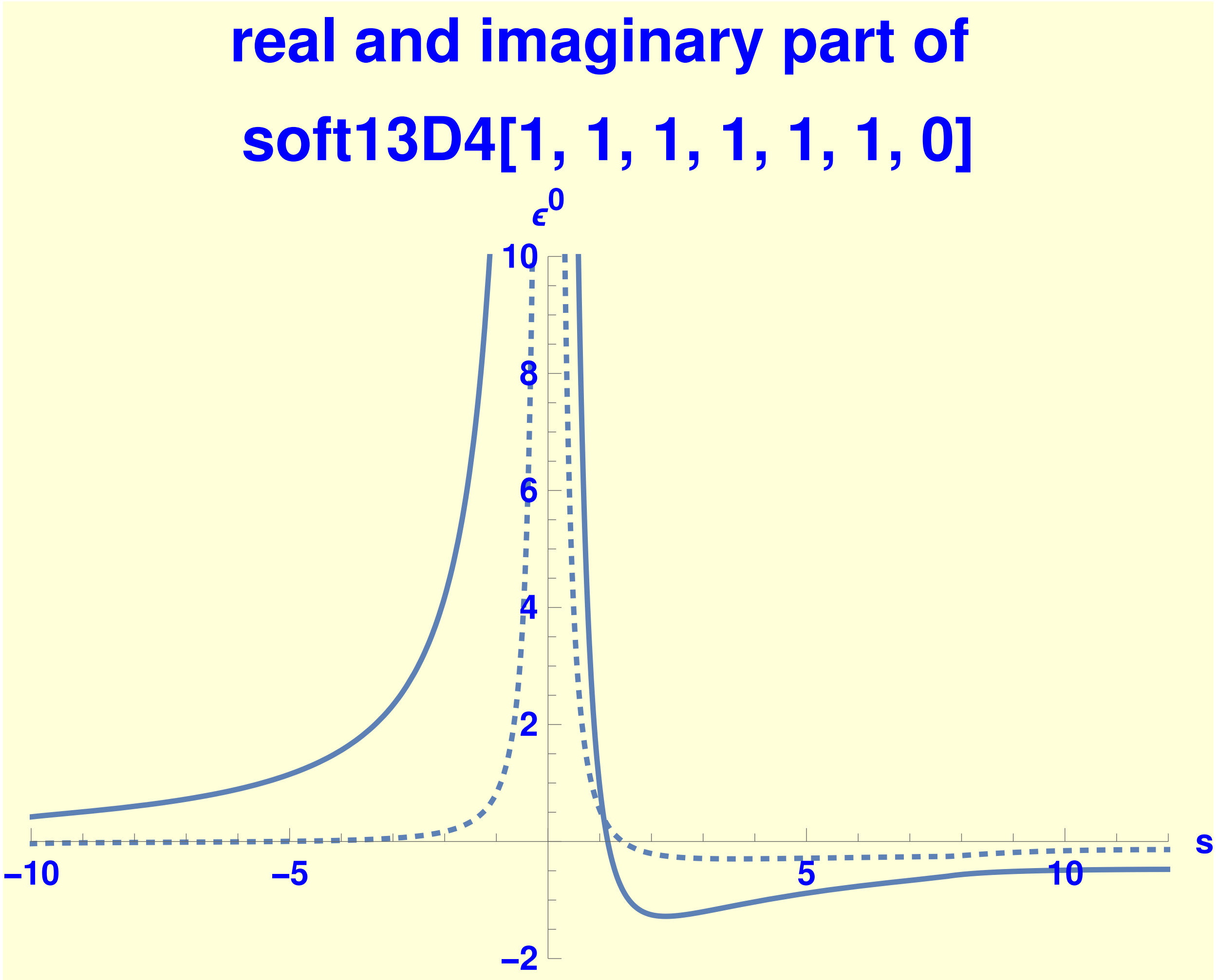

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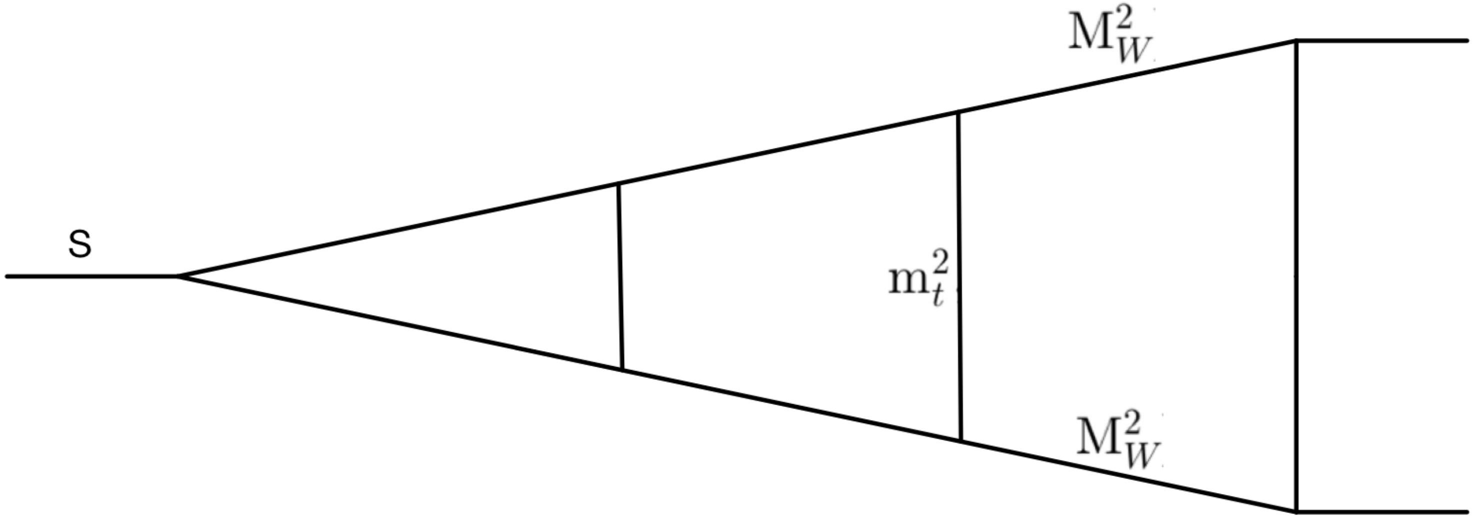


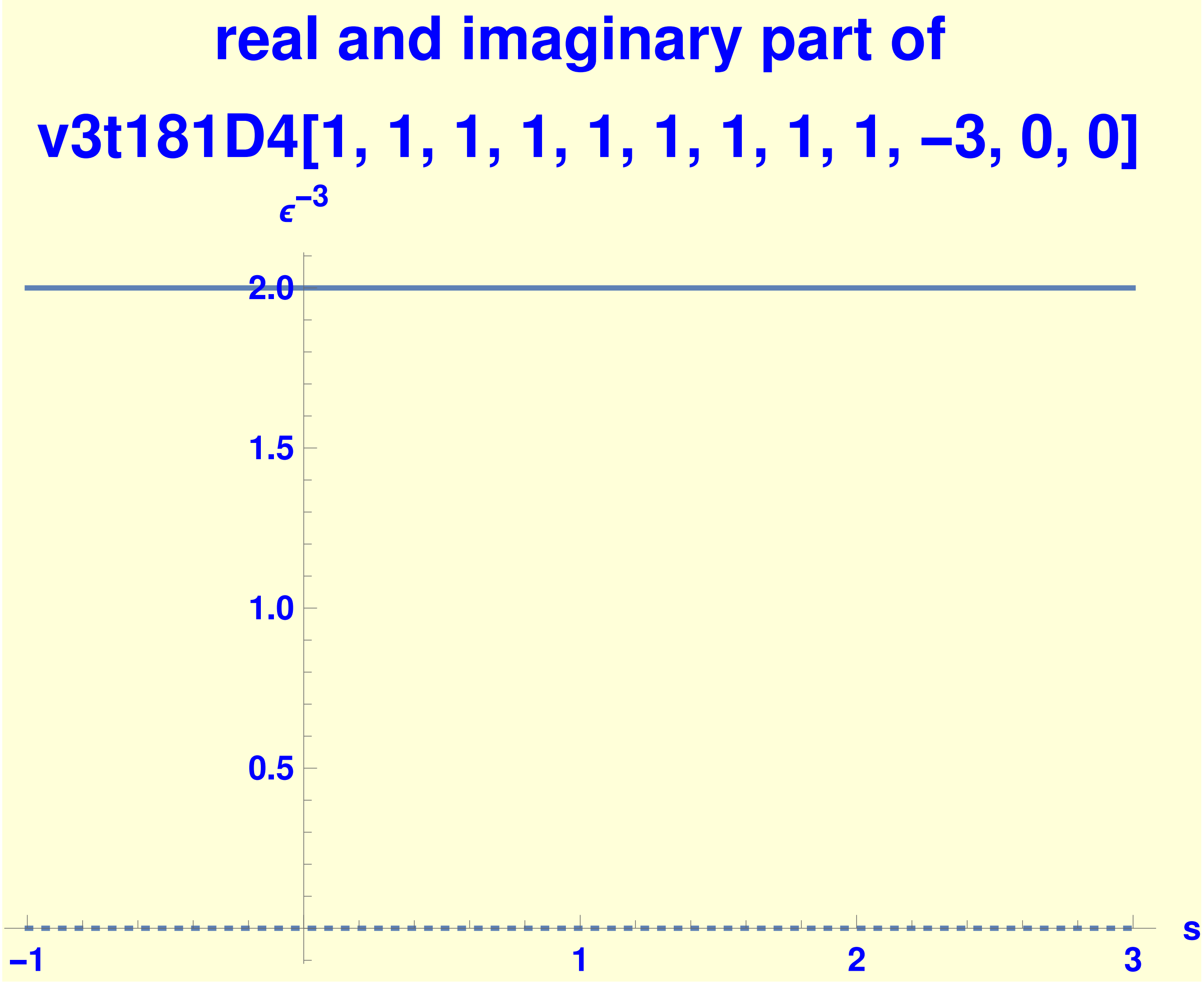
- In physical regions $(s, M_W^2, m_t^2) = (1, (\frac{401925}{4559382})^2, (\frac{433000}{227969})^2)$

-> v3t181$^{d=4-2\epsilon}$[1, 1, 1, 1, 1, 1, 1, 1, 1, -3, 0, 0] $=$

$$\frac{2.00000000000}{\epsilon^3} + \frac{9.8700393436 + 18.8495559213\, i}{\epsilon^2} - \frac{26.507336797 - 41.196707081\, i}{\epsilon} + (2.29574523 + 201.06880207\, i) + O(\epsilon)$$

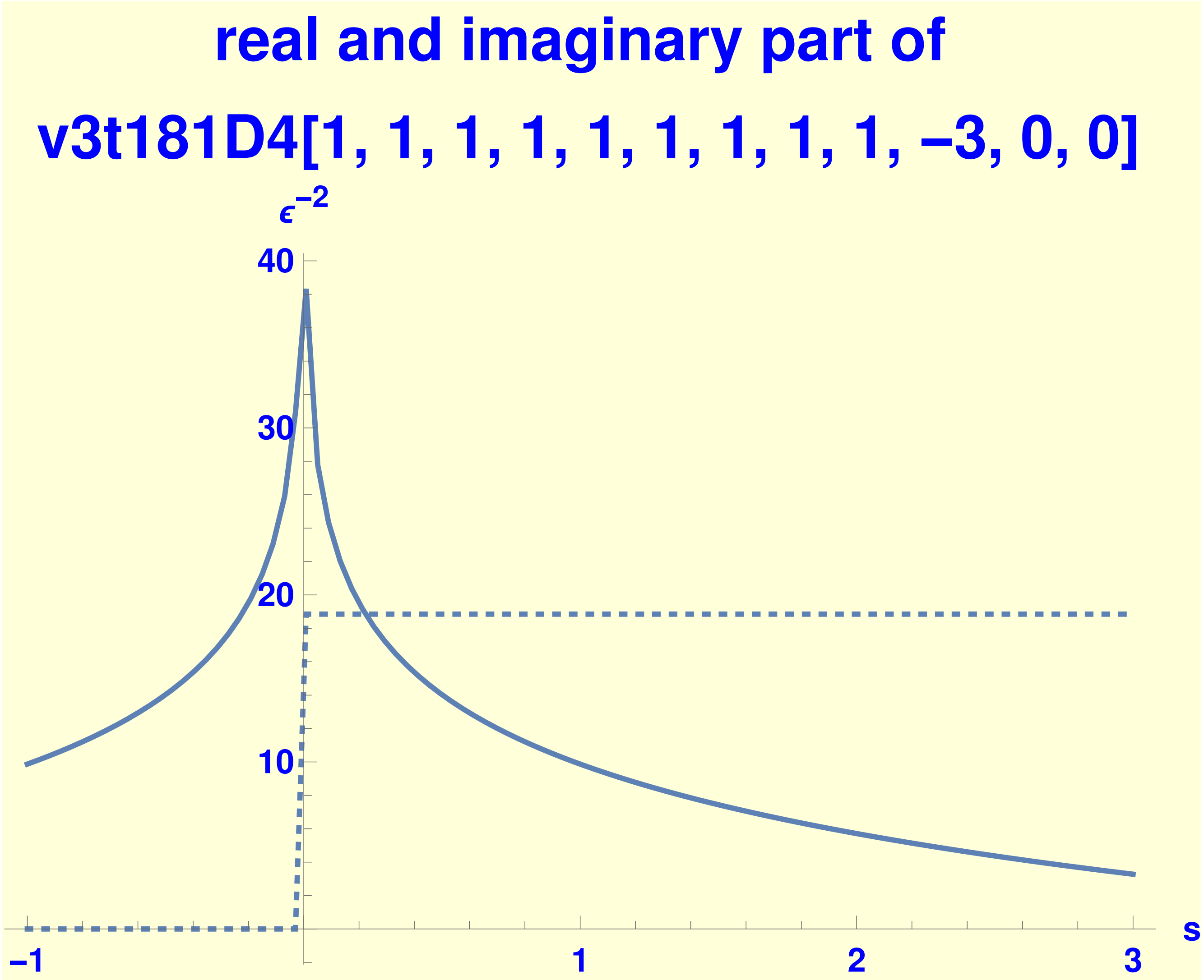
- Fully automated with Caesar[pySecDec]

The state of the art - automatic computations



- Fully automated with Caesar[AMFlow]

The state of the art - automatic computations



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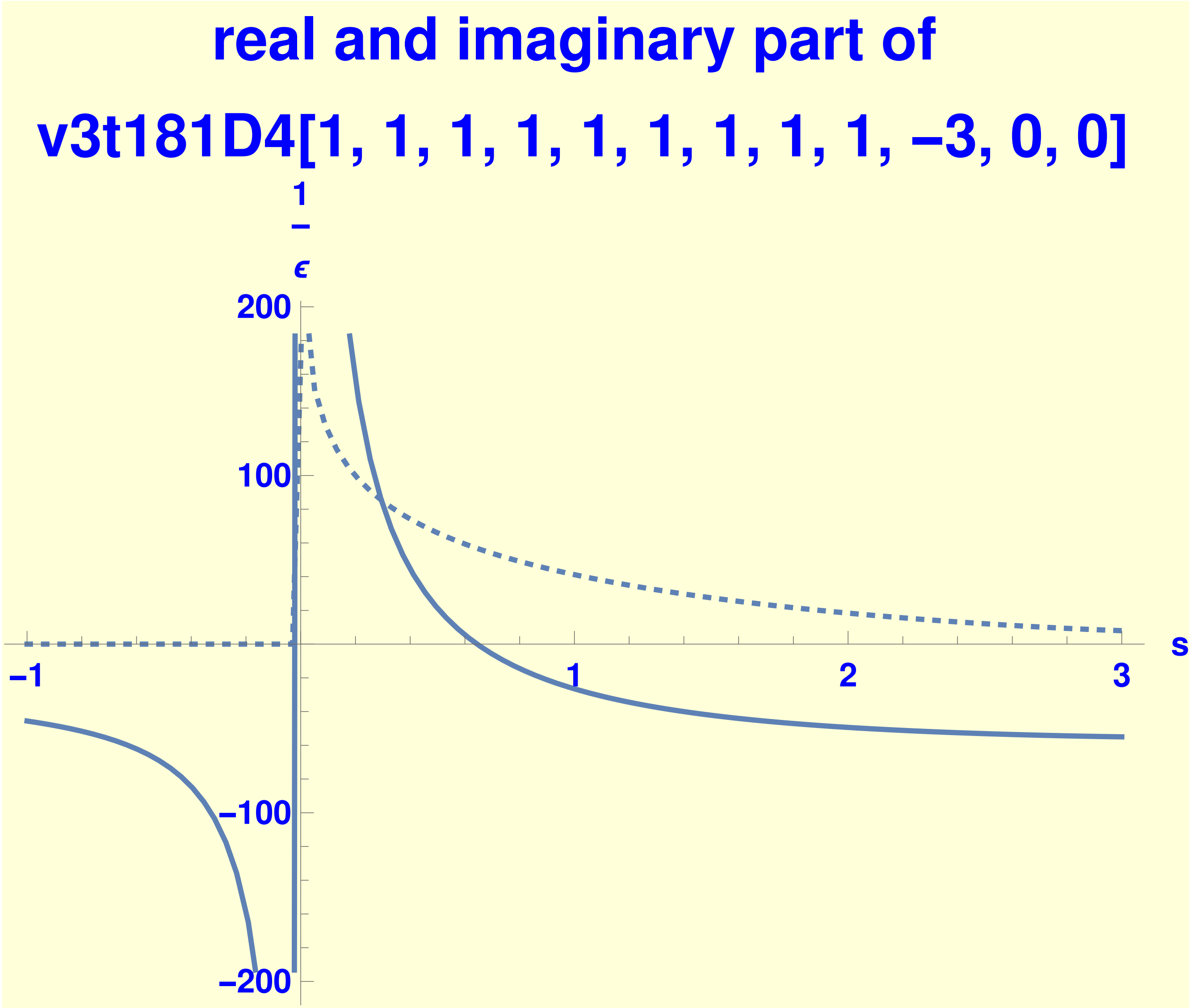
The state of the art - automatic computations



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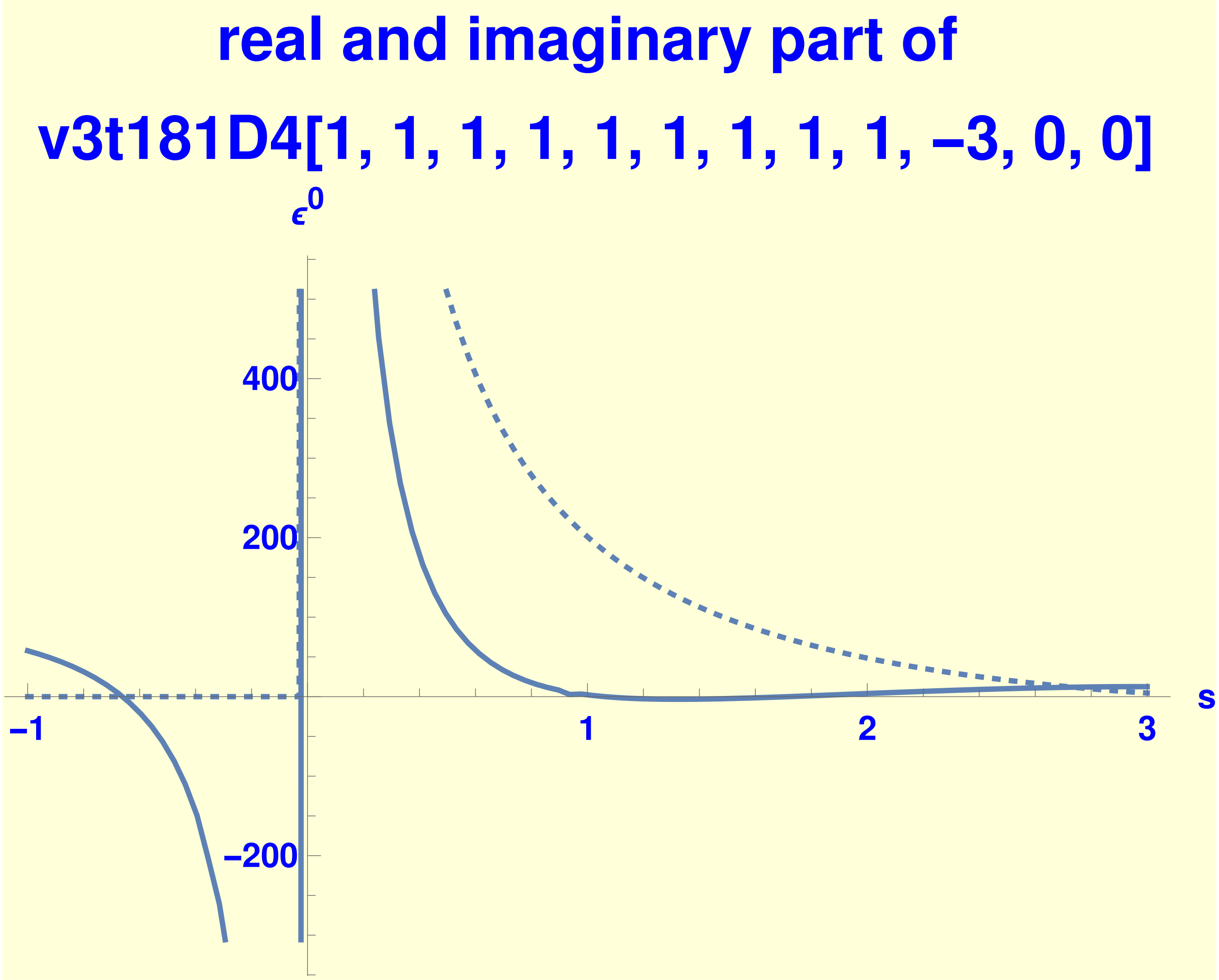

- Fully automated with Caesar[AMFlow]

Outlook

- The first electroweak three-loop physics goals are in reach
- Important is the knowledge transfer and to get people motivated to engineer other methods for practical applications
- Without spending significant effort on simplification of the basis, we can numerically solve the differential equations of non-trivial 3-loop Feynman integrals.
- We find that the precision of the boundary conditions in the Euclidean region carries over to the physical region.
- The process is fully automated.
- In this talk presented fully automated methods AMFlow and Caesar have one common **bottleneck – integration-by-parts reductions** -> There are new methods already in the making to boost the IBP reductions