FUTURE CIRCULAR COLLIDER

5th FCC PHYSICS WORKSHOP

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www.cern.ch/FCCPhysics2022

Tau Physics at FCC

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FCC-ee(Z) promises extraordinary opportunities for Tau Physics measurements

many tau pairs

	Belle	Belle II	SCT	STCF	CEPC(Z)	FCC-ee(Z)
E_{CM} [GeV]	$\sim$10.58		$2-6$	$2-7$	92	
luminosity for tau pairs [ab^{-1}]	1	50	10			
tau pairs	$9.2\cdot 10^9$	$46\cdot 10^9$	$30\cdot 10^9$		$30\cdot 10^9$	$165\cdot 10^9$

note: SCT & SCFT tau pairs estimate assuming 10 years of tau-pairs-optimized CM energies running

e^+e^- collisions at Z peak offer by far best experimental conditions per produced tau-pair

- LEP experiments often outperform lower energy experiments with much smaller samples, e.g.
  - leptonic and in general large branching fractions
  - spectral functions
- at Z peak
  - pure and efficient tau pair selection selecting on just one of the two taus
  - track multiplicity separates very well $\tau^+\tau^-$ from $q\bar{q}$
  - high momenta reduce multiple scattering uncertainty in impact parameter measurements
- general purpose hermetic detector was OK at LEP, much higher FCC luminosity demands more

Two areas of Tau Physics

Core Tau Physics topics

- tau properties (mass, lifetime, $g-2$, EDM, CPV ...)
- tau decays measurements and searches for
  - EW precision measurements: SM lepton universality tests, B anomalies models tests
  - QCD measurements: $|V_{us}|$ and CKM unitarity test, $\alpha_s(m_\tau)$, low energy QCD tests
  - LFV

Additional Physics topics where tau leptons are relevant

- processes where the tau is a decay product
  - $e^+e^- \to Z \to \tau^+\tau^-$ process (EW)
  - $H, Z \to \tau^+\tau^-$ decays (EW)
  - $H, Z \to \mu^+\tau^-$ (LFV in EW heavy boson decays)
  - $W^- \to \tau^-\bar{\nu}_\tau$ (EW)
  - $B \to D^{(*)}\tau\nu$ (B Physics anomalies)
  - heavy flavour hadrons decays with LFV and that involve also tau leptons

- core Tau Physics measurement best done at FCC-ee at Z peak
- in the following, FCC mostly means FCC-ee(Z)

Past reports

Past reports (non exhaustive)

- FCC CDR
- Mogens Dam: Tau 2018, FCC Jan 2019, Tau 2021
  - M. Dam, SciPost Phys. Proc. 1, 041 (2019)
  - M. Dam, Eur. Phys. J. Plus 136, 963 (2021) DOI:10.1140/epjp/s13360-021-01894-y
- A.L. ESG update 2019, FCC Jan 2020, Charm 2021, Tau 2021, FCC-Ita Dec 2021
  - European Strategy Update 2019, arXiv:1910.11775 [hep-ex].

Here

- summarize and update past presentations
- some updates on sensitivity estimates
- some thoughts on useful detector features and desirable improvements on Monte Carlo simulation

Note on Monte Carlo simulation $e^+e^- \to \tau^+\tau^-$

- KKMC "monopolistic" generator for $e^+e^- \to \tau^+\tau^-$
  - up to two hard radiated photons plus coherent exclusive exponentiation of soft photons (CEEX)
- Tauola simulates tau decays (tau pairs spin effects handled cooperatively with KKMC)
- PHOTOS adds 1st order radiative correction in tau decays
- estimated precision 0.2-1.0%
- KMCCee branch (C++) being developed for new facilities, S. Jadach, Tau2021
- lepton pair radiation added to PHOTOS for BelleII (arXiv:2111.05914 [hep-ph])
- for $\leq$ 0.2% precision, one more radiative order implementation on both PHOTOS and KKMC advised
  - large amount of work involved
  - PHOTOS improvements relevant for precision measurements of tau branching fractions
- a 2nd generator would help for checks and for estimating simulation uncertainties

bottom line

- for precision measuremens at FCC, non trivial Monte Carlo improvements are required

Hot / interesting Physics topics involving tau measurements

Lepton Flavour Violation

- natural extension of Standard Model, often present in New Physics models
- clean and unambiguous signal of New Physics
- large improvements coming on muon LFV (DeeMe, Mu2e, COMET, Project X/Mu2e, PRISM/PRIME)
- muon LFV present and future limits typically more constraining that tau LFV ones
- tau LFV limits impose more powerful constraints for some specific models
- tau LFV measurements very important to investigate LFV possibly discovered on muons

Heavy Flavour anomalies $R_{D^{(*)}}$ and $R_{K^{(*)}}$

- lepton universality tests based on tau measurements impose significant constraints

Unitarity violation on first row of CKM matrix

- tau measurements determine $|V_{us}|$, though with less precision than kaons
- $\tau \to X_s \nu$ inclusive $|V_{us}|$ determination independent of lattice QCD predictions
  - depends on Cabibbo-suppressed tau branching fractions and spectral functions

Other important physics topics with relations with Tau Physics

>4 σ discrepancy on muon gyromagnetic anomaly a_μ

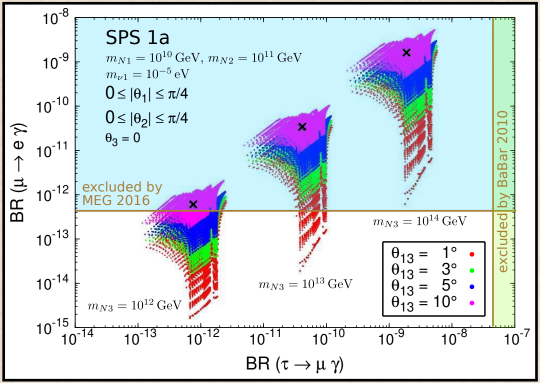
- tau spectral functions might complement e^+e^- data to compute HVP contribution

α_s from tau measurements

- spread of different determinations exceeds individual uncertainties
- different opinions on importance and treatment of duality violations
- tau spectral functions required to improve understanding and precision

Tau LFV searches probe & constrain New Physics models

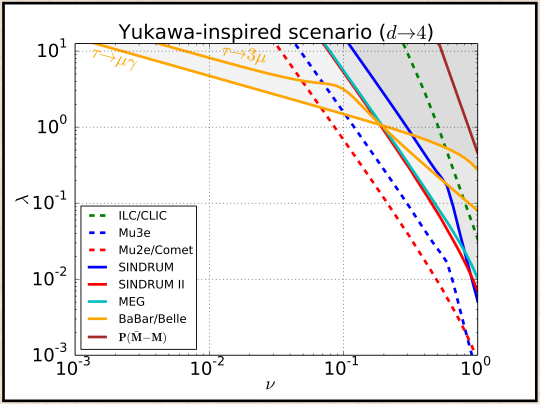
MSSM Seesaw
Antusch, Arganda, Herrero, Teixeira 2006



typical NP models

- $\mathcal{B}(\tau \to \mu\gamma) \sim 10{-}1000 \times \mathcal{B}(\mu \to e\gamma)$
- muon LFV searches more effective

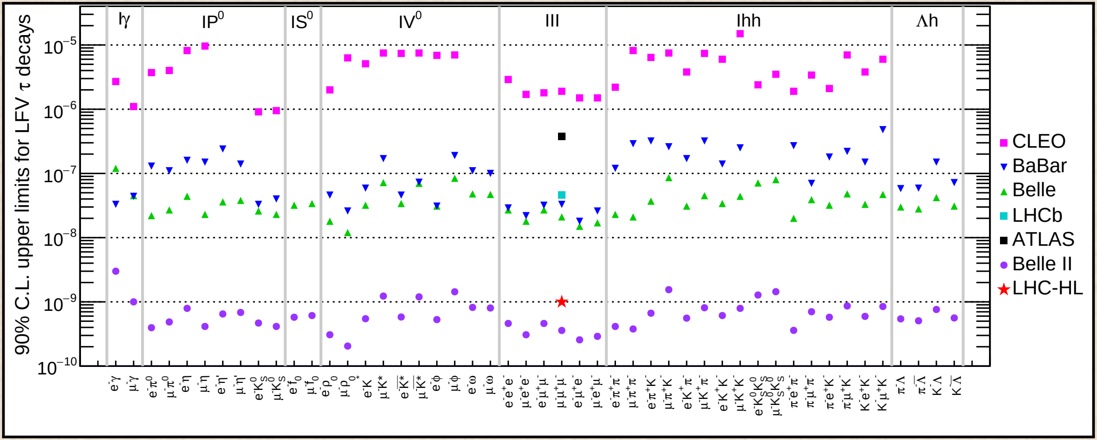
doubly charged scalar
Crivellin, Ghezzi, Panizzi, Pruna, Signer 2019



specific models / parameter space regions

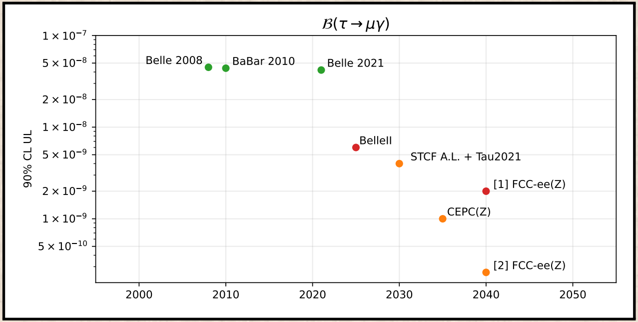
- part of plot only constrained by tau LFV limits

Tau LFV limits: present and future with Belle II and LHCb-HL



HL-LHC and HE-LHC opportunities, arXiv:1812.07638 [hep-ph]

FCC Tau LFV sensitivity for $\tau \to \mu\gamma$



FCC estimate for $\tau \to \mu\gamma$

[1] M. Dam simulation with 2% of full FCC statistics

[2] M. Dam 2021, guestimate with improved longitudinally segmented crystal EM calorimeter

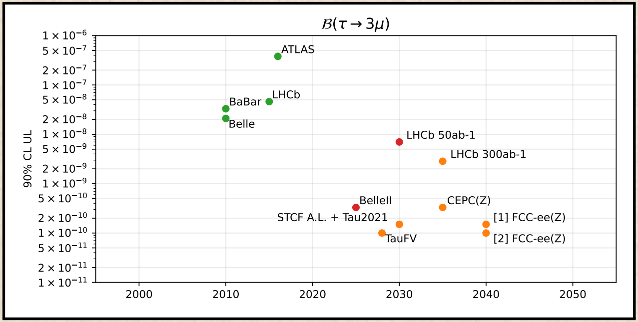
Other estimates

- ESG 2019 docs
- my extrapolation to 10y of SCTF limits presented at Tau2021

Plot notes

- Red more solid estimates
- Orange less solid estimates
- dates of future results are arbitrary, for plotting convenience

FCC Tau LFV sensitivity for $\tau \to 3\mu$



FCC estimate for $\tau \to \mu\mu\mu$

[1] my guestimate

[2] M. Dam, Tau2021

Other estimates

- ESG 2019 doc
- my extrapolation to 10y of SCTF limits presented at Tau2021

Guestimate of FCC 90% upper limit on $\tau \to \mu\mu\mu$

- $2.1{\cdot}10^{-8}$ published Belle limit at 0.782 ab^{-1}
- $/(50\ \text{ab}^{-1}/0.782\,\text{ab}^{-1}) = 3.3{\cdot}10^{-10}$, BelleII expected upper limit assuming background-free search
- FCC: $5{\cdot}10^{12}$ Z^0, 3.3% tau pair decays, $165{\cdot}10^{9}$ tau pairs, $\sim 3.6 \times 46{\cdot}10^{9}$ BelleII tau pairs
- estimate 4× better efficiency at FCC vs. BelleII
  - from DELPHI Phys.Lett. B359 (1995) 411-421 vs. BABAR Phys.Rev.Lett. 104 (2010) 021802
- muon PID efficiency and purity expected to be better for FCC
- in the improbable assumption that search remains backgroound free
  - $3.3{\cdot}10^{-10}/3.6/4.0 = 0.23{\cdot}10^{-10}$ estimated FCC 90% upper limit
- estimate / assume that
  - m_τ resolution comparable with B-factories
  - E resolution worse (850 MeV in M. Dam $\tau \to \mu\gamma$ study vs. 50-100 MeV $\approx$ 75 MeV in BABAR)
  - therefore search remains background free until $N^{\text{BelleII}}_{\tau^+\tau^-}/(850\,\text{MeV}/75\,\text{MeV})$
  - additional tau pairs improve upper limit proportionally to the square root (estimated bkg uncertainty)
- $3.3{\cdot}10^{-10} \cdot (850\,\text{MeV}/75\,\text{MeV})/\sqrt{[3.6 \cdot (850\,\text{MeV}/75\,\text{MeV})]}/4.0 \simeq 1.5{\cdot}10^{-10}$ FCC upper limit

Preparations and optimizations for tau LFV searches

- $\tau \to \mu\gamma$ reach improves with
  - energy resolution of EM calorimeter
  - angular precision (granularity) of EM calorimeter
  - efficiency & purity of muon PID
- $\tau \to 3\mu$ reach improves with
  - momentum resolution and tracking reconstruction accuracy
  - efficiency & purity of muon PID
  - other LFV searches profit from electron, pion, kaon PID
- existing Monte Carlo simulation technology seems sufficient

Lepton universality tests

HFLAV Tau 2018 report

$$\left(\frac{g_\tau}{g_\mu}\right) = \sqrt{\frac{\mathcal{B}_{\tau e}}{\mathcal{B}_{\mu e}}\frac{\tau_\mu m_\mu^5 f_{\mu e} R_\gamma^\mu R_W^\mu}{\tau_\tau m_\tau^5 f_{\tau e} R_\gamma^\tau R_W^\tau}} = 1.0009 \pm 0.0014 = \sqrt{\frac{\mathcal{B}_{\tau e}}{\mathcal{B}_{\tau e}^{\text{SM}}}}$$

$$\left(\frac{g_\tau}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\mu e}}\frac{\tau_\mu m_\mu^5 f_{\mu e} R_\gamma^\mu R_W^\mu}{\tau_\tau m_\tau^5 f_{\tau \mu} R_\gamma^\tau R_W^\tau}} = 1.0027 \pm 0.0014 = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\tau \mu}^{\text{SM}}}}$$

$$\left(\frac{g_\mu}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\tau e}}\frac{f_{\tau e}}{f_{\tau \mu}}} = 1.0019 \pm 0.0014$$

using Standard Model predictions for leptons $\lambda,\ \rho = e, \mu, \tau$ (Marciano 1988)

$$\Gamma[\lambda \to \nu_\lambda \rho \bar{\nu}_\rho(\gamma)] = \Gamma_{\lambda\rho} = \Gamma_\lambda \mathcal{B}_{\lambda\rho} = \frac{\mathcal{B}_{\lambda\rho}}{\tau_\lambda} = \frac{G_\lambda G_\rho m_\lambda^5}{192\pi^3} f\left(m_\rho^2/m_\lambda^2\right) R_W^\lambda R_\gamma^\lambda$$

$$G_\lambda = \frac{g_\lambda^2}{4\sqrt{2}M_W^2}\ ; \qquad f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x\ ; \qquad f_{\lambda\rho} = f\left(m_\rho^2/m_\lambda^2\right)$$

$$R_W^\lambda = 1 + \frac{3}{5}\frac{m_\lambda^2}{M_W^2} + \frac{9}{5}\frac{m_\rho^2}{M_W^2}\ ; \qquad R_\gamma^\lambda = 1 + \frac{\alpha(m_\lambda)}{2\pi}\left(\frac{25}{4} - \pi^2\right)\ ;$$

all statistical correlations included

Lepton universality tests with hadronic decays

HFLAV Tau 2018 report

$$\left(\frac{g_\tau}{g_\mu}\right)_\pi = 0.9960 \pm 0.0027 \; , \qquad \left(\frac{g_\tau}{g_\mu}\right)_K = 0.9858 \pm 0.0071 \; .$$

Averaging the three g_τ/g_μ ratios:

$$\left(\frac{g_\tau}{g_\mu}\right)_{\tau+\pi+K} = 0.9999 \pm 0.0014 \; .$$

using Standard Model predictions

$$\left(\frac{g_\tau}{g_\mu}\right)^2 = \frac{\mathcal{B}(\tau \to h\nu_\tau)}{\mathcal{B}(h \to \mu\bar{\nu}_\mu)} \frac{2m_h m_\mu^2 \tau_h}{(1+\delta_h)m_\tau^3 \tau_\tau} \left(\frac{1 - m_\mu^2/m_h^2}{1 - m_h^2/m_\tau^2}\right)^2 \qquad (h = \pi \text{ or } K)$$

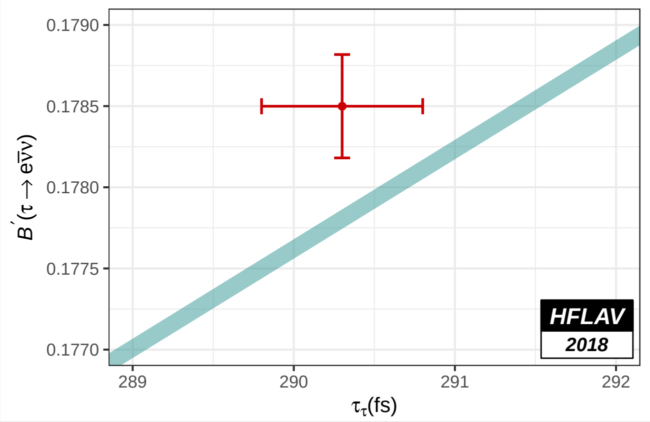
rad. corr. $\delta_\pi = (0.16 \pm 0.14)\%$, $\delta_K = (0.90 \pm 0.22)\%$ (Decker 1994)

note: electron tests less precise because $h \to e\nu$ decays are helicity-suppressed

tau hadronic decays radiative corrections

- at Tau2021 it has been remarked that improvements on radiative corrections are required to match possible future improvements in experimental precision

Canonical tau lepton universality test plot



$(g_\tau/g_{e\mu}) = 1.0018 \pm 0.0013$
$[g_{e\mu} = g_e = g_\mu$ assuming $g_e = g_\mu]$

$\Delta(g_\tau/g_{e\mu})$ contributions

input	Δinput	$\Delta(g_\tau/g_{e\mu})$
$\mathcal{B}'_{\tau\to e}$	0.180%	0.090%
τ_τ	0.172%	0.086%
m_τ	0.007%	0.017%
total		0.126%

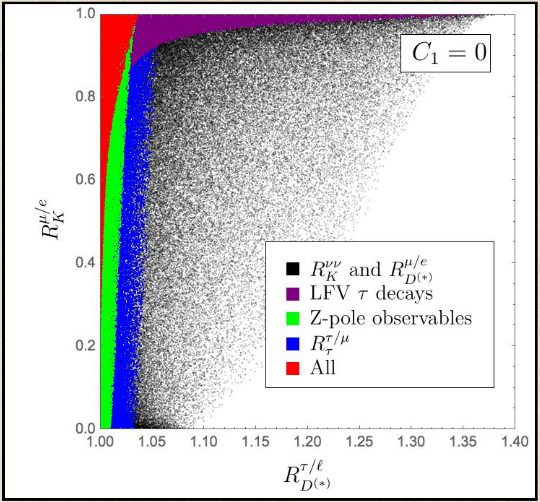
best measurements

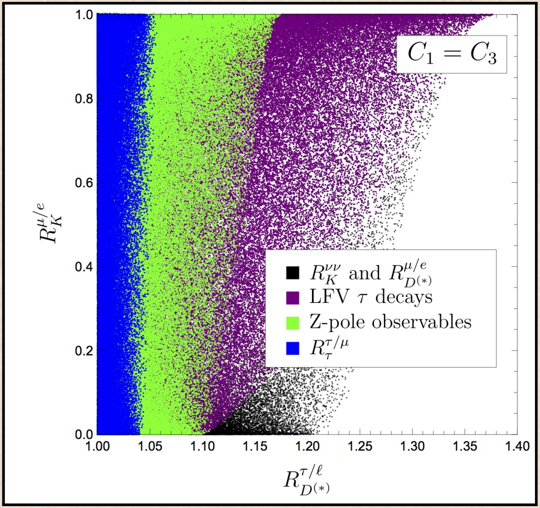
$\mathcal{B}'_{\tau\to e}$	ALEPH
τ_τ	Belle
m_τ	BES III

- $\mathcal{B}'(\tau \to e\bar{\nu}\nu) =$ average of $\begin{cases} \mathcal{B}(\tau \to e\bar{\nu}\nu) \\ \mathcal{B}(\tau \to \mu\bar{\nu}\nu) \cdot f_{\tau e}/f_{\tau\mu} \end{cases}$
- $\dfrac{\mathcal{B}'(\tau \to e\bar{\nu}\nu)\tau_\mu}{\mathcal{B}(\mu \to e\bar{\nu}\nu)\tau_\tau} = \dfrac{g_\tau^2}{g_{e\mu}^2} \dfrac{m_\tau^5 f_{\tau e} R_\gamma^\tau R_W^\tau}{m_\mu^5 f_{\mu e} R_\gamma^\mu R_W^\mu}$
- $\left(\dfrac{g_\tau}{g_{e\mu}}\right)^2 = \dfrac{\mathcal{B}'(\tau \to e\bar{\nu}\nu)}{\mathcal{B}(\mu \to e\bar{\nu}\nu)} \dfrac{\tau_\mu}{\tau_\tau} \dfrac{m_\mu^5}{m_\tau^5} \dfrac{f_{\mu e} R_\gamma^\mu R_W^\mu}{f_{\tau e} R_\gamma^\tau R_W^\tau}$

Tau Lepton universality constrains models for B $R^{\tau/\ell}_{D^{(*)}}$- $R^{\mu/e}_{K}$ anomalies

Feruglio, Paradisi, Pattori JHEP 09 (2017) 061
blue points correspond to parameter space region allowed by tau lepton universality





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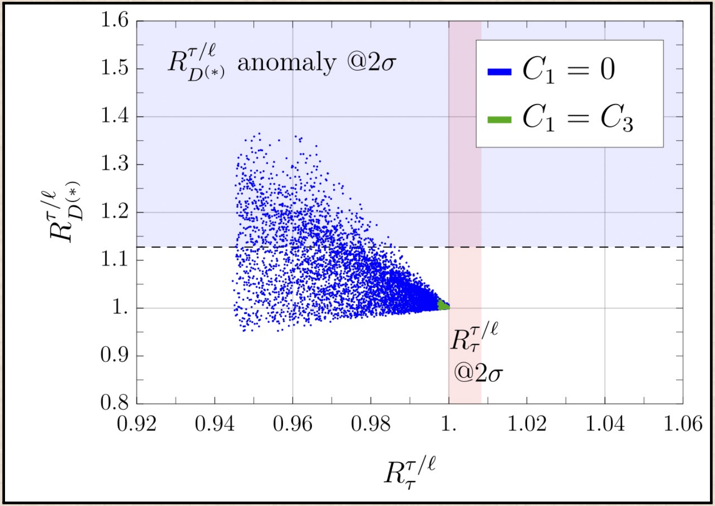
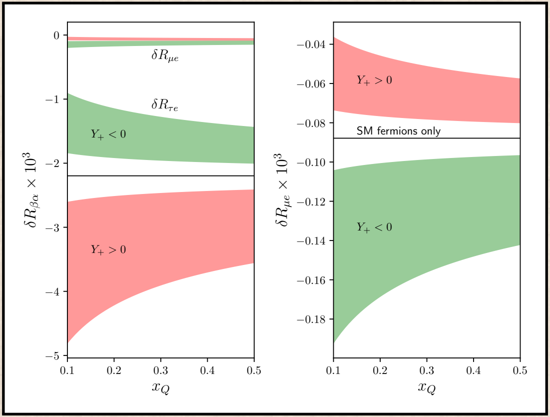
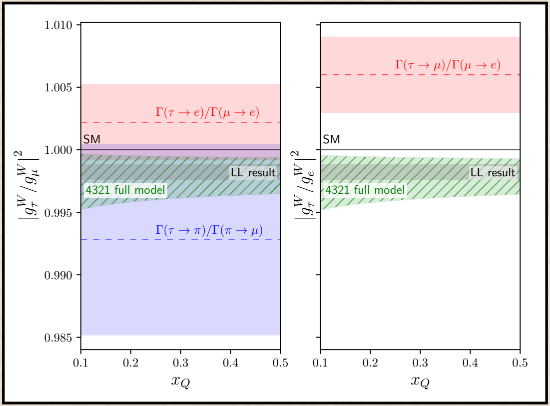


Tau Lepton universality constrains 4321 models for *B* $R^{\tau/\ell}_{D^{(*)}}$- $R^{\mu/e}_{K}$ anomalies

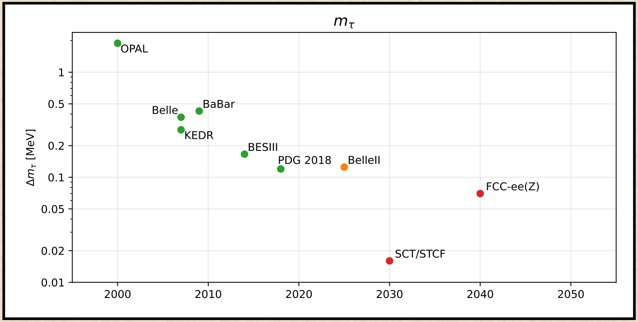
LFU violations in leptonic τ decays and *B*-physics anomalies

- Allwicher, Isidori, Selimovic, PLB 826 (2022) 136903
- finite 1-loop corrections for 4321 [$SU(4)\times SU(3)\times SU(2)\times U(1)$)] models from matching conditions at NP scale
- smaller impact on tau LU than "Effective Field Theory leading-log" calculations
  $\Rightarrow$ future precision measurements of leptonic τ decay widths important for testing 4321 models





FCC sensitivity for m_τ



FCC estimate

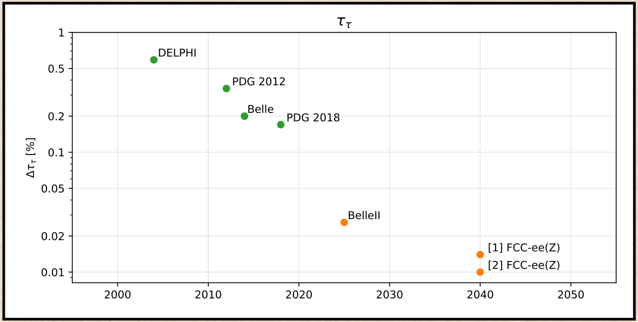
- M. Dam 11th FCC-ee Workshop 2019, limiting systematics of 0.1 MeV on pseudomass distribution modeling

Other estimates

- ESG 2019 docs

- best experimental facilities are e^+e^- at $\tau^+\tau^-$ threshold, then B-factories
- FCC
  - challenge is systematics from pseudomass distribution modeling
  - can use 5-prong decays (narrower pseudomass distribution drop)
  - attainable precision on momentum measurement scale appears not to be limiting

FCC sensitivity for τ_τ



FCC estimate

[1] M. Dam and CDR

[2] A.L. FCC Workshop Jan 2020

Other estimates

- ESG 2019 docs

- best measurement by Belle on 3-prong vs. 3-prong tau pairs
- expect limiting systematics from absolute length scale calibration on minivertex detector, 100 ppm
- 68 ppm systematics from Δm_τ at current precision
- potential systematics from modeling of measurement bias subtraction
- potential systematics from accuracy of simulation of average radiation energy loss
  - would profit from improvements of tau pairs generators
- profits from high-resolution vertex detector close to interation region

Tau Lifetime

τ MEAN LIFE

PDG 2019

VALUE (10^{-15} s)	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
290.3 ± 0.5	**OUR AVERAGE**				
290.17 ±0.53 ±0.33	1.1M	BELOUS	2014	BELL	711 fb^{-1} $E^{ee}_{\rm cm}$=10.6 GeV
290.9 ±1.4 ±1.0		ABDALLAH	2004T	DLPH	1991-1995 LEP runs
293.2 ±2.0 ±1.5		ACCIARRI	2000B	L3	1991--1995 LEP runs
290.1 ±1.5 ±1.1		BARATE	1997R	ALEP	1989--1994 LEP runs
289.2 ±1.7 ±1.2		ALEXANDER	1996E	OPAL	1990--1994 LEP runs
289.0 ±2.8 ±4.0	57.4k	BALEST	1996	CLEO	$E^{ee}_{\rm cm}$= 10.6 GeV

tau lifetime precision

precision (ppm)	
1700	PDG 2019
2100	Belle
5900	DELPHI
6400	ALEPH
7200	OPAL
260	Belle II guestimate, extrapolating from 0.711 ab^{-1} to 50 ab^{-1}
5	FCC, stat. only extrapolation from ALEPH (1e5) to FCC (1.65e11) tau pairs

$\Rightarrow$ what are the limiting systematics?

Tau Lifetime systematics at LEP

DELPHI main systematics, Eur.Phys.J.C36:283-296,200

- IP impact parameter difference on 1-1-prong tau pairs
  - trimming, backgrounds, impact parameter resolution, alignment
- MD miss-distance on 1-1-prong tau pairs
  - resolution on MD, bias, selection
- DL transverse decay length on 3-1 and 3-3 prong tau pairs
  - alignment

ALEPH main systematics, Phys.Lett.B414:362-372,1997

- MIPS, momentum-weighted impact parameter sum
  - resolution on impact parameter sum, bias (from MC)
- 3DIP 3D impact parameter, Z. Phys. C 74, 387–398 (1997)
  - bias (from MC), vertex chisq cut
- IPD, impact parameter difference
  - resolution and trimming of outliers
- DL, decay length
  - vertex chisq cut

expect that all these systematics scale with $1/\sqrt{N_{\text{events}}}$
including alignment systematics
although questionable if up to a factor $1/{\sim}1300$

Tau Lifetime systematics at FCC

Alignment systematic

- alignment calibration precision improves with statistics
- misalignment effects zero at first order for uniform azymuthal acceptance
  S.R.Wasserbaech, Nucl.Phys.Proc.Suppl. 76 (1999) 107-116
  - still, questionable how far this holds
- related systematic that does not scale
  absolute length scale of vertex detector average elements spacing, realiable to 10^{-4} or 100 ppm

Systematics from kinematics of tau decay

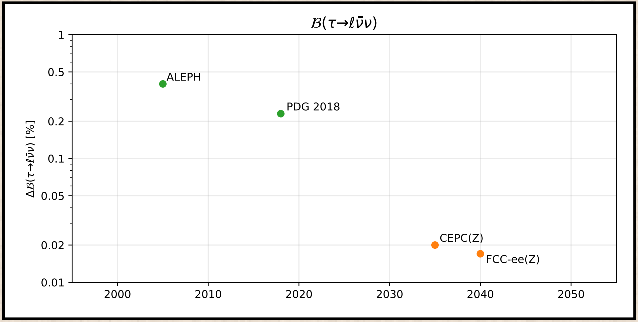
- $\tau_\tau = \lambda_\tau / \beta\gamma = \lambda_\tau / \frac{\sqrt{E_\tau^2 - m_\tau^2}}{m_\tau} = \lambda_\tau / \frac{\sqrt{(E_{\text{beam}} - E_{\text{rad}}^{\text{MC}})^2 - m_\tau^2}}{m_\tau}$

systematic [ppm]	
1	E_{beam}
68	m_τ PDG 2019
7	m_τ possible measurement at Super Charm-Tau Factories
?	MC accuracy on average radiation energy loss (*) (estimated 100 ppm for *BABAR*)

(*) depends on

- accuracy of generator, can be checked measuring momentum distribution of di-muon events
- accuracy of simulation of efficiency of selection procedure vs. E_τ (scales with luminosity)

FCC sensitivity for $\mathcal{B}(\tau \to \ell\bar{\nu}\nu)$



FCC estimate

- M. Dam Tau2018, Tau2021

Other estimates

- ESG 2019 docs

- sensitivity estimates very difficult, mostly guestimates
- best results from ALEPH global analysis of all tau decays
- important: PID efficiency, purity, accurate PID modeling with control samples
- efficiency, purity of π^0 reconstruction, accurate modeling with control samples
- improve current poor simulation of high multiplicity inv. mass distributions
- improvements on tau pairs Monte Carlo simulations highly desirable
- high statistics samples will help very much on first 3 points, but analyses will be very complex

Tau branching fractions

- world averages of large BRs still dominated by LEP
  - background separation from dileptons and hadrons much better
  - higher selection purity and efficiency
  - possible to tag single tau with good efficiency and purity and observe the other one
    $\Rightarrow$ wonderful base for reducing systematics using data, exploited in particular by ALEPH
- *B*-factories improved on small branching fractions using statistics
  $\Rightarrow$ FCC statistics 1300$\times$ ALEPH, 175$\times$ Belle, 3.5$\times$ BelleII (& better efficiency w.r.t. *B*-factories)
- FCC is best imaginable context for tau BR measurements
- what are the limiting systematics?

Systematics of main ALEPH tau BR paper, Phys. Rept. 421 (2005) 191

systematics

Total systematic errors for branching ratios measured from the 1994–1995 data sample

Topology	π^0	sel	bkg	pid	int	trk	dyn	mcs	Total
e	0.011	0.021	0.029	0.019	0.009	0.000	0.000	0.015	0.045
μ	0.004	0.020	0.020	0.021	0.008	0.000	0.000	0.015	0.039
h	0.071	0.016	0.010	0.022	0.022	0.014	0.000	0.019	0.083
$h\pi^0$	0.063	0.027	0.019	0.011	0.045	0.009	0.000	0.027	0.090
$h2\pi^0$	0.089	0.021	0.014	0.004	0.007	0.003	0.040	0.028	0.105
$h3\pi^0$	0.056	0.012	0.015	0.000	0.008	0.001	0.008	0.030	0.068
$h4\pi^0$	0.029	0.005	0.011	0.000	0.015	0.000	0.000	0.019	0.040
$3h$	0.047	0.021	0.018	0.004	0.012	0.014	0.006	0.015	0.059
$3h\pi^0$	0.033	0.017	0.029	0.002	0.041	0.009	0.007	0.018	0.066
$3h2\pi^0$	0.027	0.008	0.015	0.000	0.009	0.003	0.012	0.014	0.038
$3h3\pi^0$	0.010	0.012	0.002	0.000	0.002	0.001	0.010	0.006	0.019
$5h$	0.002	0.000	0.002	0.000	0.000	0.001	0.000	0.003	0.004
$5h\pi^0$	0.002	0.000	0.006	0.000	0.000	0.000	0.000	0.002	0.007
Class 14	0.013	0.003	0.022	0.002	0.024	0.000	0.000	0.011	0.037

All numbers are absolute in per cent. The labels are defined as follows: photon and π^0 reconstruction (π^0), event selection efficiency (sel), non-τ background (bkg), charged particle identification (pid), secondary interactions (int), tracking (trk), Monte Carlo dynamics (dyn), Monte Carlo statistics (mcs), total systematic uncertainty (total).

π^0 systematics

Total systematic errors for branching ratios measured from the 1994–1995 data sample

Topology	π^0	sel	bkg	pid	int	trk	dyn	mcs	Total
e	0.011	0.021	0.029	0.019	0.009	0.000	0.000	0.015	0.045
μ	0.004	0.020	0.020	0.021	0.008	0.000	0.000	0.015	0.039
h	0.071	0.016	0.010	0.022	0.022	0.014	0.000	0.019	0.083
$h\pi^0$	0.063	0.027	0.019	0.011	0.045	0.009	0.000	0.027	0.090
$h2\pi^0$	0.089	0.021	0.014	0.004	0.007	0.003	0.040	0.028	0.105
$h3\pi^0$	0.056	0.012	0.015	0.000	0.008	0.001	0.008	0.030	0.068
$h4\pi^0$	0.029	0.005	0.011	0.000	0.015	0.000	0.000	0.019	0.040
$3h$	0.047	0.021	0.018	0.004	0.012	0.014	0.006	0.015	0.059
$3h\pi^0$	0.033	0.017	0.029	0.002	0.041	0.009	0.007	0.018	0.066
$3h2\pi^0$	0.027	0.008	0.015	0.000	0.009	0.003	0.012	0.014	0.038
$3h3\pi^0$	0.010	0.012	0.002	0.000	0.002	0.001	0.010	0.006	0.019
$5h$	0.002	0.000	0.002	0.000	0.000	0.001	0.000	0.003	0.004
$5h\pi^0$	0.002	0.000	0.006	0.000	0.000	0.000	0.000	0.002	0.007
Class 14	0.013	0.003	0.022	0.002	0.024	0.000	0.000	0.011	0.037

All numbers are absolute in per cent. The labels are defined as follows: photon and π^0 reconstruction (π^0), event selection efficiency (sel), non-τ background (bkg), charged particle identification (pid), secondary interactions (int), tracking (trk), Monte Carlo dynamics (dyn), Monte Carlo statistics (mcs), total systematic uncertainty (total).

- many systematics but in general all limited only by data vs. MC comparisons
- non-trivial to extrapolate to 1300^2 more data

Main systematics of ALEPH tau BR paper, Phys. Rept. 421 (2005) 191

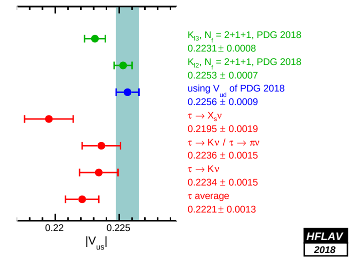
- non-tau backgrounds
  - estimated by varying MC estimate by 30%
  - does not trivially scale with luminosity, but can be improved
- tau pair selection
  - use break-mix method on data and MC, 0.1-0.2% uncertainties
    dominant systematics from data statistics of tau vs. hadron cut separation
  - scales with luminosity, but correlations between hemispheres limit how much
- PID
  - uncertainties from control samples studies
  - partially scales with luminosity, but limited by achievable purity of control samples
- photon efficiency
  - uncertainties from control samples studies data-MC comparisons
    - fit data using predicted MC fake and genuine photon distributions and compare number of genuine photons
    - compare photons $> 3\,\text{GeV}$ as function of separation from tracks
    - compare converted photons
    - compare hadron to electron misidentification
    - compare photon identification efficiency
    - photon energy scale calibrated with momentum measurement on high-energy e from tau decay
    - compare fake photons

Main systematics of ALEPH tau BR paper, Phys. Rept. 421 (2005) 191

- π^0 efficiency
  - compare data and MC D_{ij} distributions (probability γ_i, γ_j) of π^0 mass fit
- efficiency for π^0 with unresolved photons
  - compare data and MC 2nd moment of transverse evergy in calorimeter cells
- radiative and bremsstrahlung photons
  - compare data and MC distributions
  - compare PHOTOS vs. exact calculation for $\tau \to \pi\pi^0\nu$ with radiative $E_\gamma > 12\,\text{MeV}$
- tracking
  - compare data and MC on same sign events events (two tracks missing in one hemisphere)
- tau decay dynamic
  - reduced because acceptances are large and flat
  - will become important with higher statistics
  - can be partially addressed with iterative concurrent measurements where also invariant mass distributions are fitted on data (complicate)

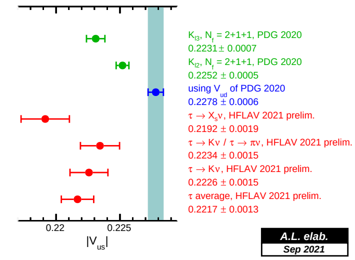
$|V_{us}|$-centric CKM matrix first row unitarity test

PDG 2018 review



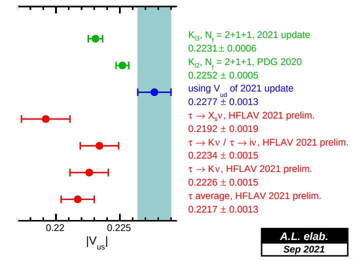
- CKM unitarity OK with kaons

PDG 2020 review



- new dispersive calculation of Δ_R^V inner or universal electroweak radiative corrections (RC) to super-allowed nuclear beta decays Seng, Gorchtein & Ramsey-Musolf, Phys. Rev. D 100, 013001 (2019)

2021 update



- J.C.Hardy & Ii.S.Towner, PRC 102, 045501 (2020)
  - inflated $|V_{us}|$ systematics
- Seng, Gorchtein & Ramsey-Musolf, 2021
- Seng, Galviz, Marciano, Meißner, 2021

- $\Delta\,|V_{us}|_\tau \approx \Delta\,|V_{us}|_{Vud}$ in 2021!

$|V_{us}|$ determinations can rely on τ branching fractions measurements

Using tau measurements and OPE, no lattice QCD

- $$\frac{R(\tau \to X_{\text{strange}}\,\nu)}{|V_{us}|^2} = \frac{R(\tau \to X_{\text{non-strange}}\,\nu)}{|V_{ud}|^2} - \delta R_{\tau,\text{SU3 breaking}},$$

$\tau \to X_s \nu$

Using tau measurements and lattice QCD

- $$\frac{\Gamma(\tau^- \to K^- \nu_\tau)}{\Gamma(\tau^- \to \pi^- \nu_\tau)} = \frac{|V_{us}|^2}{|V_{ud}|^2}\left(\frac{f_{K\pm}}{f_{\pi\pm}}\right)^2 \frac{\left(1 - m_K^2/m_\tau^2\right)^2}{\left(1 - m_\pi^2/m_\tau^2\right)^2}\frac{R_{\tau/K}}{R_{\tau/\pi}} R_{K/\pi}$$

$\tau \to K / \tau \to \pi$

- $$\Gamma(\tau^- \to K^- \nu_\tau) = \frac{G_F^2}{16\pi\hbar} f_{K\pm}^2 |V_{us}|^2 m_\tau^3 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 R_{\tau/K} R_{K\mu 2}$$

$\tau \to K$

Requirements

- Cabibbo-suppressed tau BRs
- tau spectral functions
- FCC high statistics is major advantage

Other core tau physics topics

- $\alpha_a(m_\tau)$, hadronic contribution from $\tau \to \pi\pi^0\nu$
  - requirements: tau spectral functions, tau branching fractions
  - FCC high statistics is big advantage
- Michel parameters (check SM charged weak current $V-A$ predictions)
  - requirements: reconstruction of tau decay modes and their kinematics
  - FCC high statistics is big advantage

Tau spectral functions

- reasonably complete sets only measured at LEP (ALEPH, OPAL)
- limited contributions from B-factories
- studies at the Z peak are by far the most favourable context
- significant improvements are possible at FCC especially for the poorly measurer rare modes
- analyses are complex and may be limited by manpower availability
- improvements on Monte Carlo simulation desirable

Conclusions

- FCC-ee(Z) (or comparable Z-peak e^+e^- facilities) are the best for tau Physics
- there are several interesting measurements to be improved
- FCC-ee(Z) permits impressive precision improvements
- fair share of systematics scale with luminosity if enough human effort is available
- identified possibly limiting systematics still allow large and interesting margins of improvement
- improvements on tau lifetime and leptonic BRs make desirable improvements on m_τ (SCTF?)

Thanks for your attention!

Backup Slides

α_s from tau decay measurements

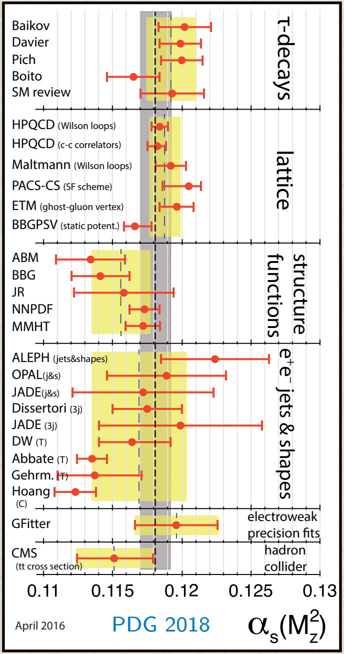
- $\alpha_s(m_\tau)$ from
  - $R_{VA} = \mathcal{B}(\tau \to X_d \nu)/\mathcal{B}(\tau \to e\bar{\nu}\nu)$
  - tau spectral functions
- extrapolation to M_Z competitive with other methods
- $\alpha_s(m_\tau)$ confirms running of α_s

Recent discussions on tau determinations

- FOPT and CIPT extractions get significantly different results
- different groups get significantly different results
- disagreement on treatment of duality violations
- Pich 2019
  Boito, Golterman, Maltman, Peris 2019
  Pich, Rojo, Sommer, Vairo 2018
  Boito, Golterman, Maltman, Peris 2017
  Pich, Rodríguez-Sánchez 2016

Requirements

- tau spectral functions
- tau branching fractions
- FCC high statistics is major advantage



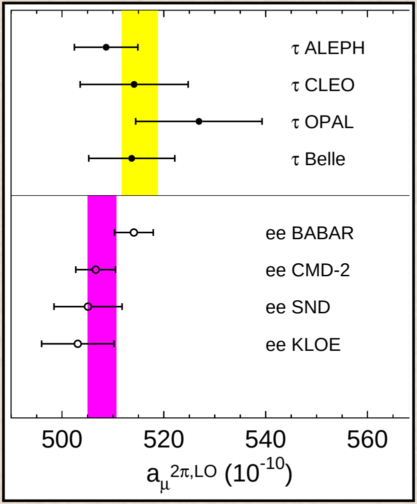
Muon $g-2$ hadronic contribution from tau

- $\alpha_\mu^{2\pi,\mathrm{LO}}$ from
  - $\tau \to \pi\pi^0\nu$ spectral function
  - normalization could come from $\mathcal{B}(\tau \to \pi\pi^0\nu)$, τ_τ
  - isospin rotation (associated theory systematics)
- tau data $\Rightarrow$ reduced discrepancy with exp.
- presently e^+e^- data more precise and complete

Requirements

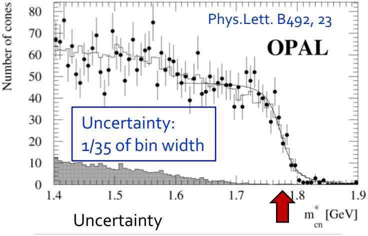
- improved isospin-violating and EM corrections for $\tau \to \pi^0\pi\nu_\tau$
- tau spectral functions
- tau branching fractions
- FCC high statistics is major advantage

Davier 2013



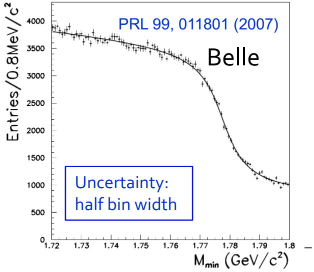
Tau Mass (from M.Dam, FCC-ee Workshop Jan 2019)

- **Current world average:** $m_\tau = 1776.86 \pm 0.12$ MeV
- Best in world: BES3 (threshold scan) $m_\tau = 1776.91 \pm 0.12$ (stat.) $^{+0.10}_{-0.13}$ (syst.) MeV
- Best at LEP: OPAL $m_\tau = 1775.1 \pm 1.6$ (stat.) ± 1.0 (syst.) MeV
  - About factor 10 from world's best
  - Main result from endpoint of distribution of pseudo-mass in $\tau \to 3\pi^{\pm}(n\pi^0)\nu_\tau$
  - Dominant systematics:
    - Momentum scale: 0.9 MeV
    - Energy scale: 0.25 MeV (including also π^0 modes)
    - Dynamics of τ decay: 0.10 MeV



- Same method from Belle
  - Main systematics
    - Beam energy & tracking system calib.: 0.26 MeV
    - Parameterisation of the spectrum edge: 0.18 MeV

$m_\tau = 1776.61 \pm 0.13$ (stat.) ± 0.35 (syst.) MeV



Pseudo-mass: $M_{min} = \sqrt{M_{3\pi}^2 + 2(E_{beam} - E_{3\pi})(E_{3\pi} - P_{3\pi})}$

Mogens Dam / NBI Copenhagen 11th FCC-ee Workshop, CERN 10/01/2019

Tau Mass (from M.Dam, FCC-ee Workshop Jan 2019)

- Prospects for FCC-ee:
  - 3 prong, 5 prongs, (perhaps even 7 prongs?)
  - Statistics 10^5 times OPAL: **δ_{stat} = 0.004 MeV**
  - Systematics:
    - At FCC-ee, E_{BEAM} known to better than 0.1 MeV (~ 1 ppm) from resonant depolarisation
      - Negligible effect on m_τ
    - Likely dominant experimental contribution comes from understanding of the mass scale
      - Use high stats $e^+e^- \rightarrow \mu^+\mu^-$ sample to fix momentum scale. Extrapolate down to momenta typical for $\tau \rightarrow 3\pi$.
      - Use $D^0 \rightarrow K^-\pi^+$ / $K^-\pi^+\pi^-\pi^-$ and $D^+ \rightarrow K^-\pi^+\pi^+$ to fix mass scale (m_D known to 50 keV)
    - Reduce uncertainty from parametrisation of spectrum edge by use of theoretical spectrum checked against high statistics data
    - Cross checks using 5-prongs
  - Suggested overall systematics: **δ_{syst} = 0.1 MeV**
    - Could potentially touch current precision but probably no substantial improvement ?

Non standard neutrino interactions

Babu, Bhupal Dev, Jana, Thapa, arXiv:1907.09498 [hep-ph]
blue region = model parameters' ($Y_{\tau e}$, $\sin\phi$, m_{h^+}) consistent with tau LU & lifetime, given $\epsilon_{\tau\tau}$

NSI parameters' definition

$$\mathcal{L}_{\rm NC} = -2\sqrt{2}G_F \sum_{f,P,\alpha,\beta} \varepsilon_{\alpha\beta}^{f,P} (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta)(\bar{f}\gamma_\mu P f)$$

$$\mathcal{L}_{\rm CC} = -2\sqrt{2}G_F \sum_{f,P,\alpha,\beta} \varepsilon_{\alpha\beta}^{f,P} (\bar{\nu}_\alpha \gamma^\mu P_L \ell_\beta)(\bar{f}\gamma_\mu P f')$$

where G_F is Fermi's constant and the ε terms quantify the size of the new interaction relative to the weak scale. The sum is over matter fermions, typically $f, f' \in \{e,u,d\}$ and $P \in \{P_L, P_R\}$ are the chirality projection operators. These projection operators can also be reparameterized

NSI model opens new tau decays