Facultad de Ciencias | Universidad Autónoma de Madrid | Instituto de Física Teórica UAM-CSIC

Z BOSON LFV DECAYS

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*with **Lorenzo Calibbi** and **Joydeep Roy***

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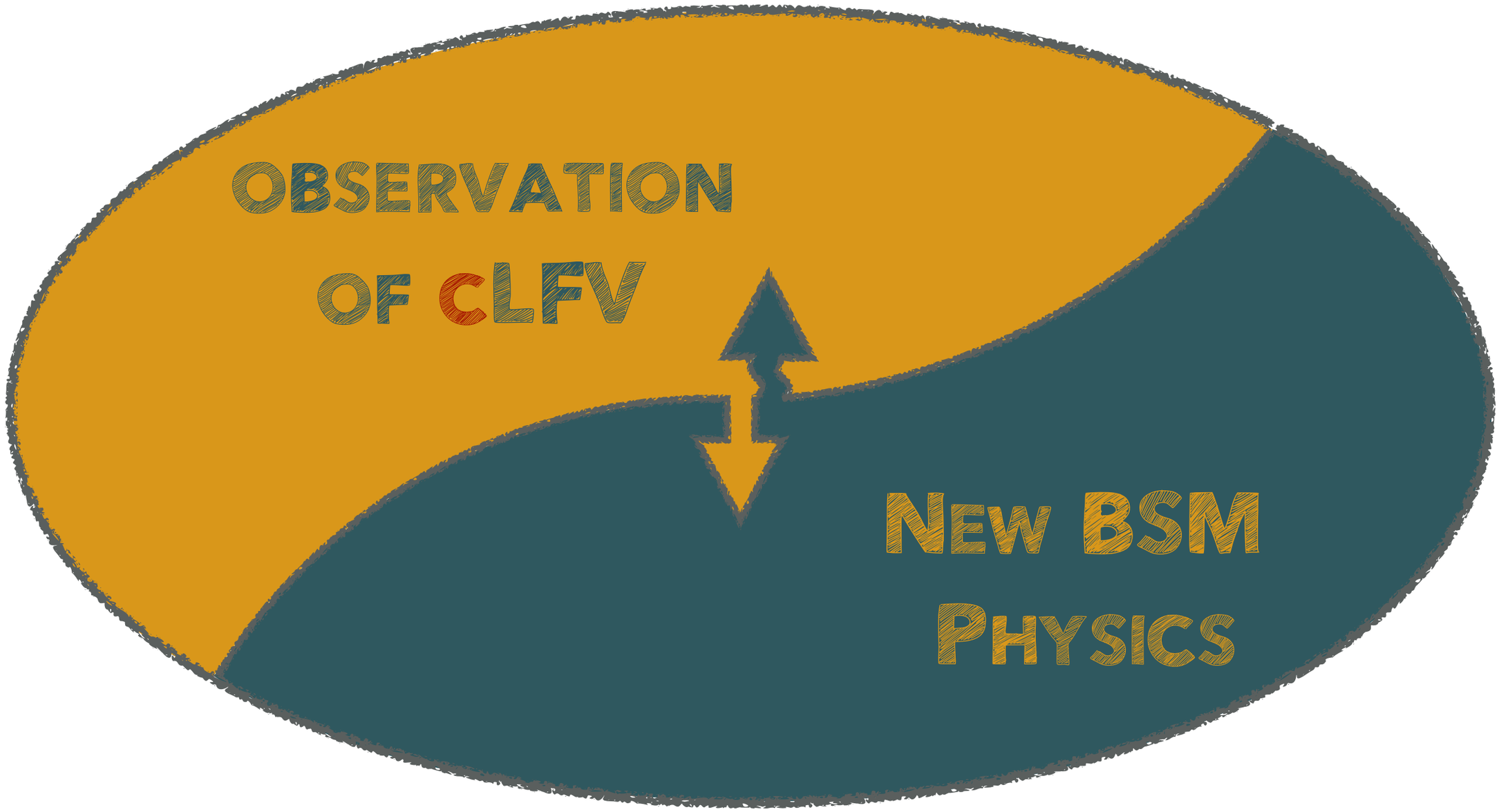
MINISTERIO DE CIENCIA E INNOVACIÓN | UNIÓN EUROPEA FONDO EUROPEO DE DESARROLLO REGIONAL "Una manera de hacer Europa" | AGENCIA ESTATAL DE INVESTIGACIÓN

HIDDeν Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

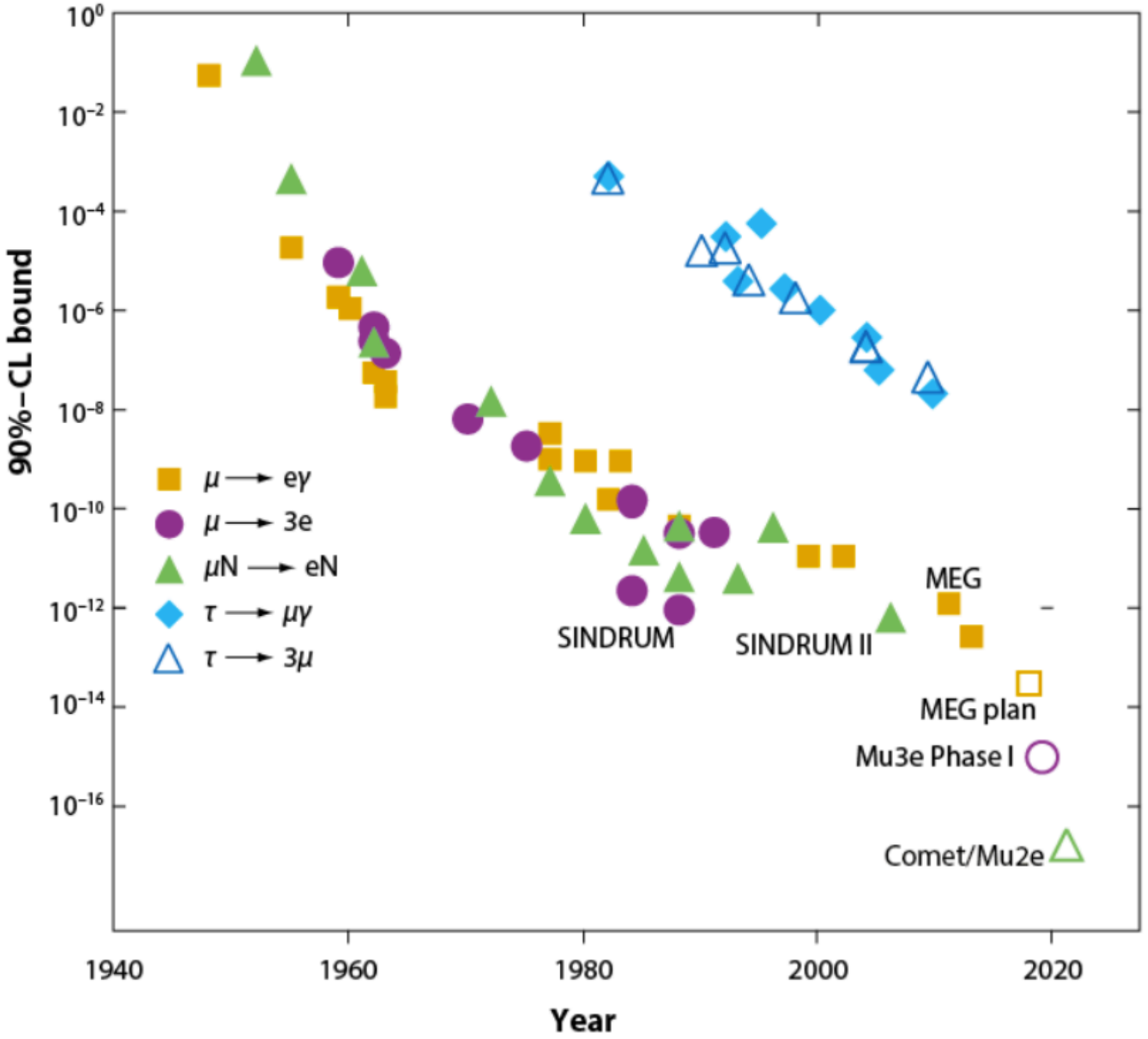
EXCELENCIA SEVERO OCHOA

WHY LEPTON FLAVOR VIOLATION?



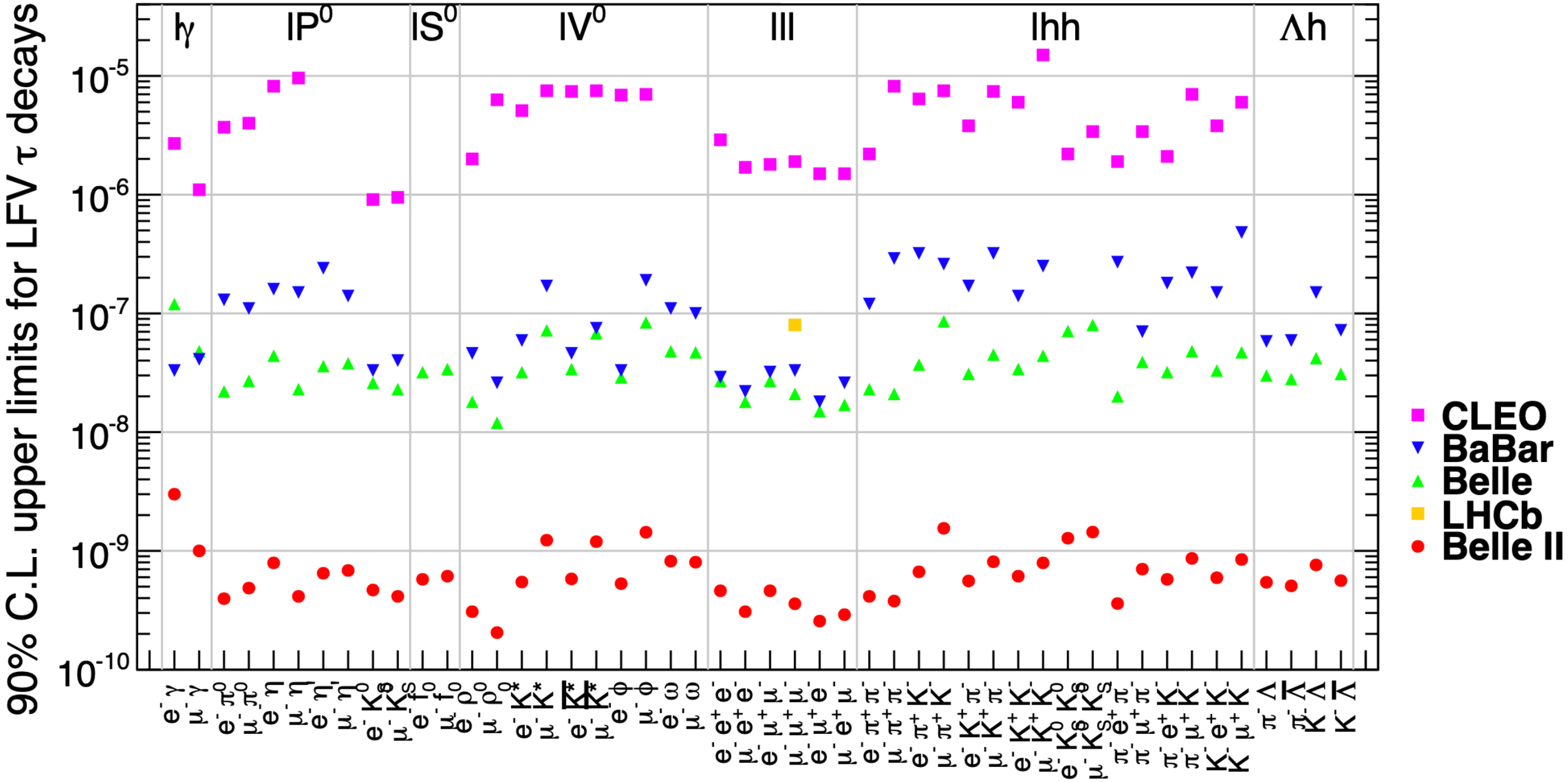
Long story

Bravar — ICHEP 2016



Long story

Belle II Physics Book — 1808.10567



Long story

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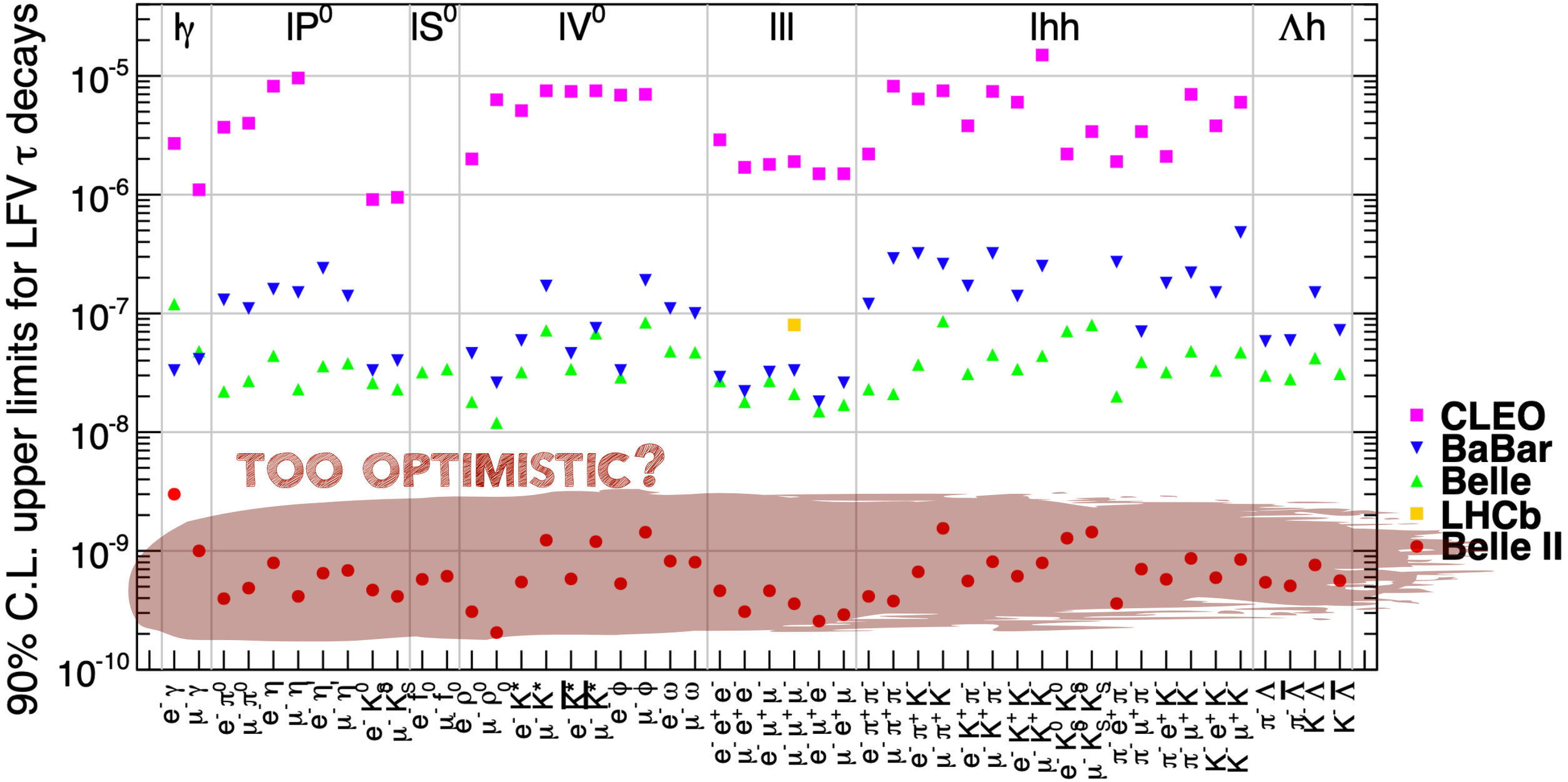
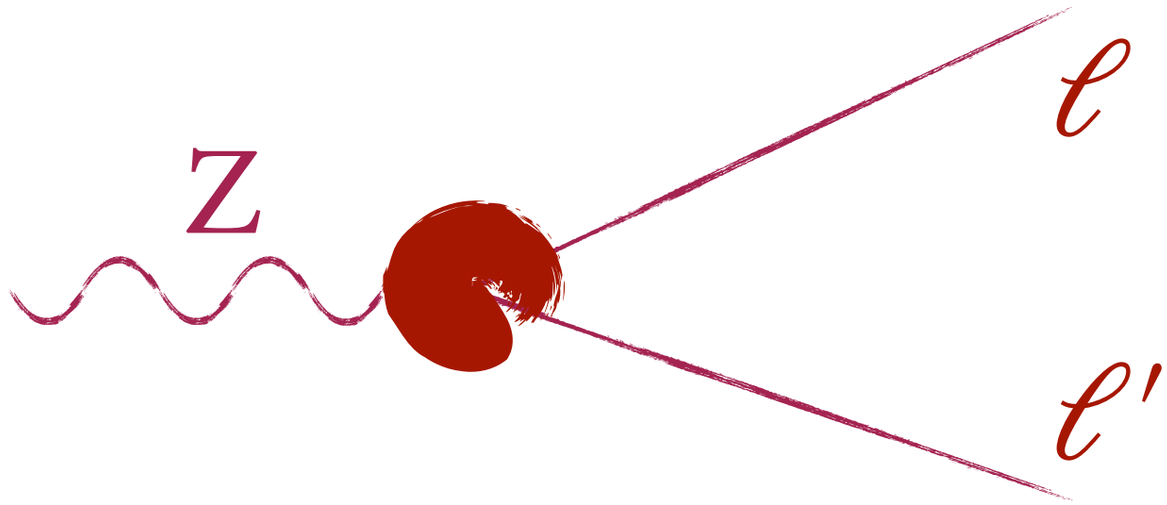
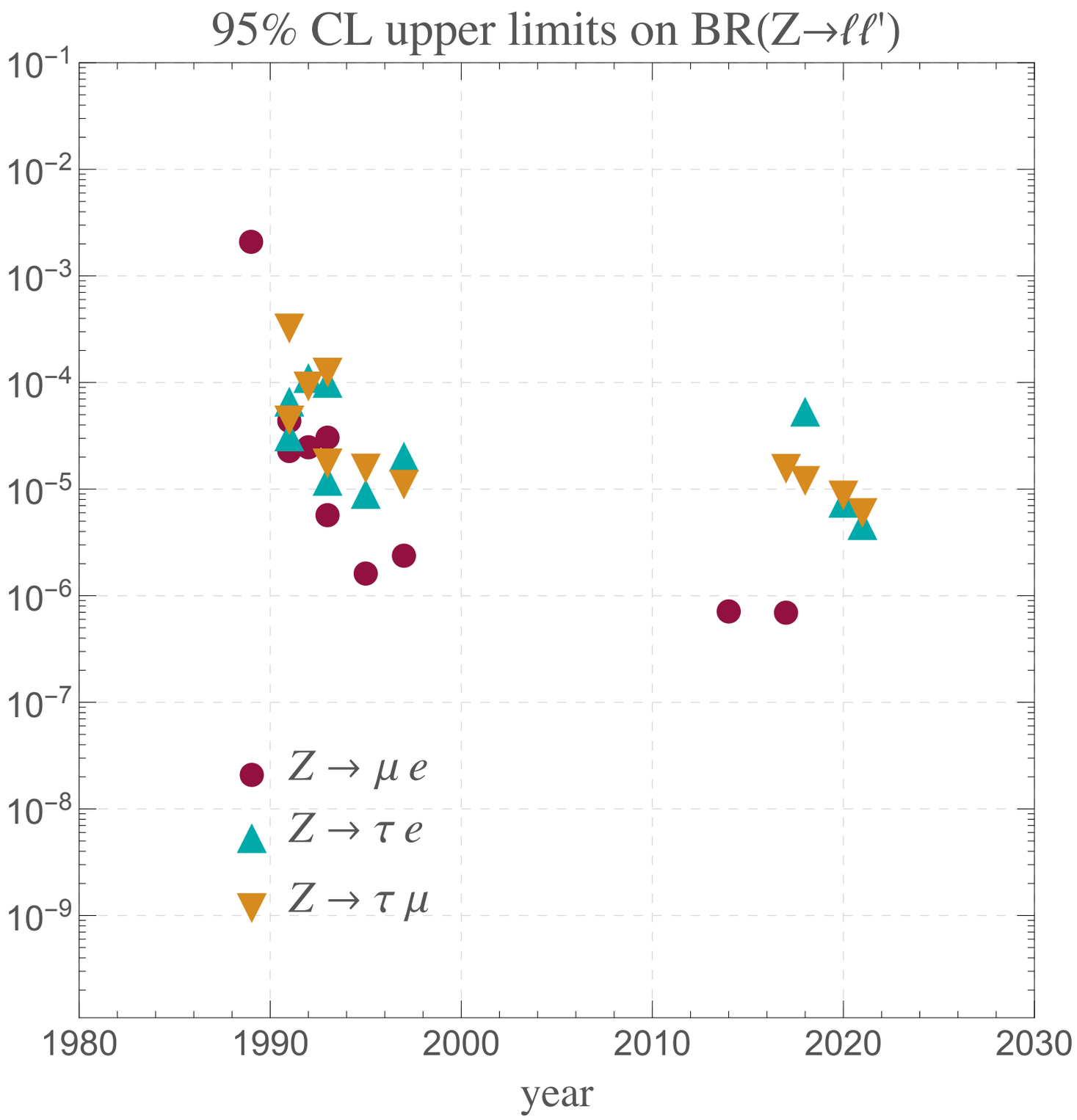


New observables are welcome

— LFV Z Decays —



cLFV at High Energies



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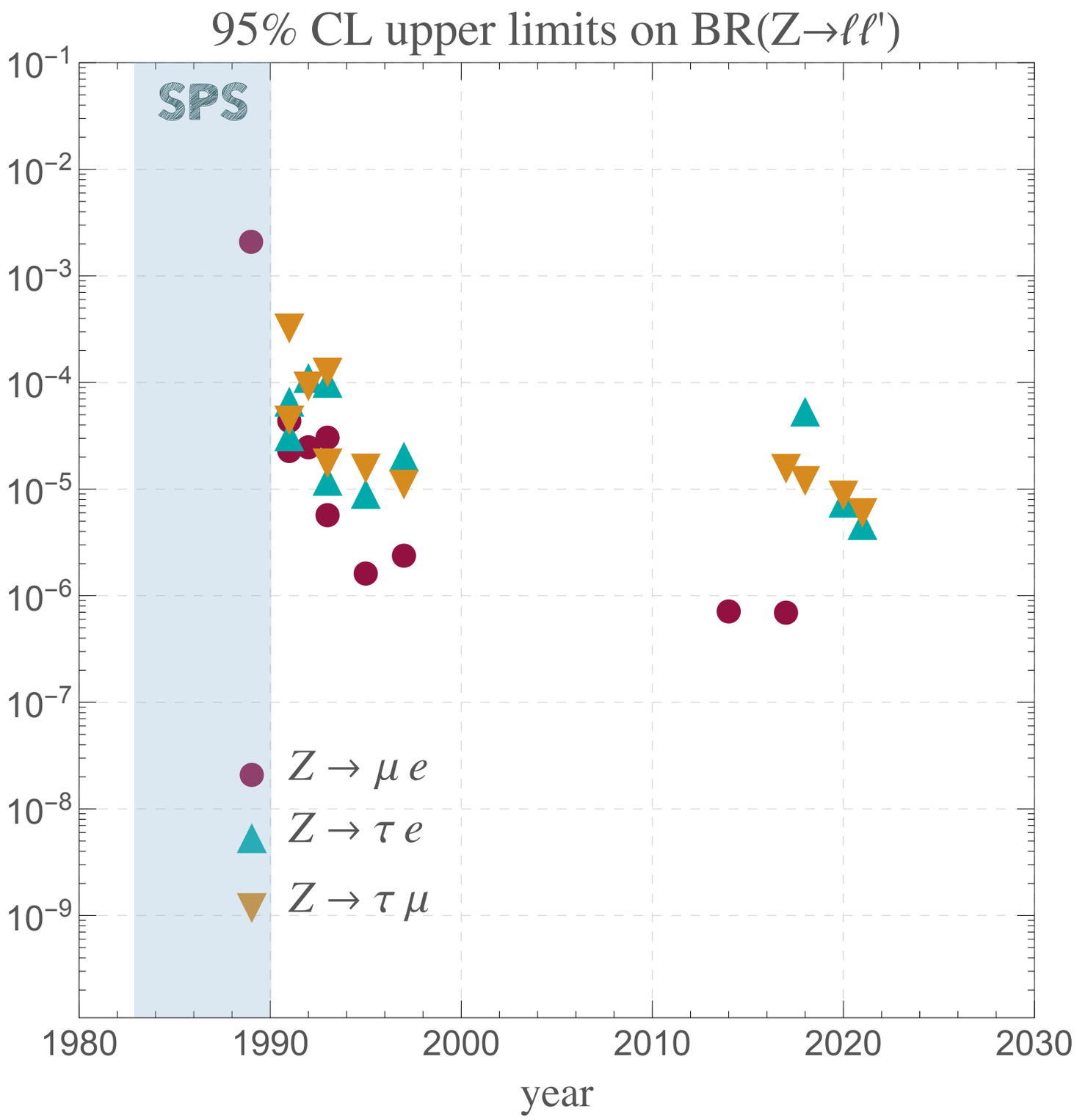

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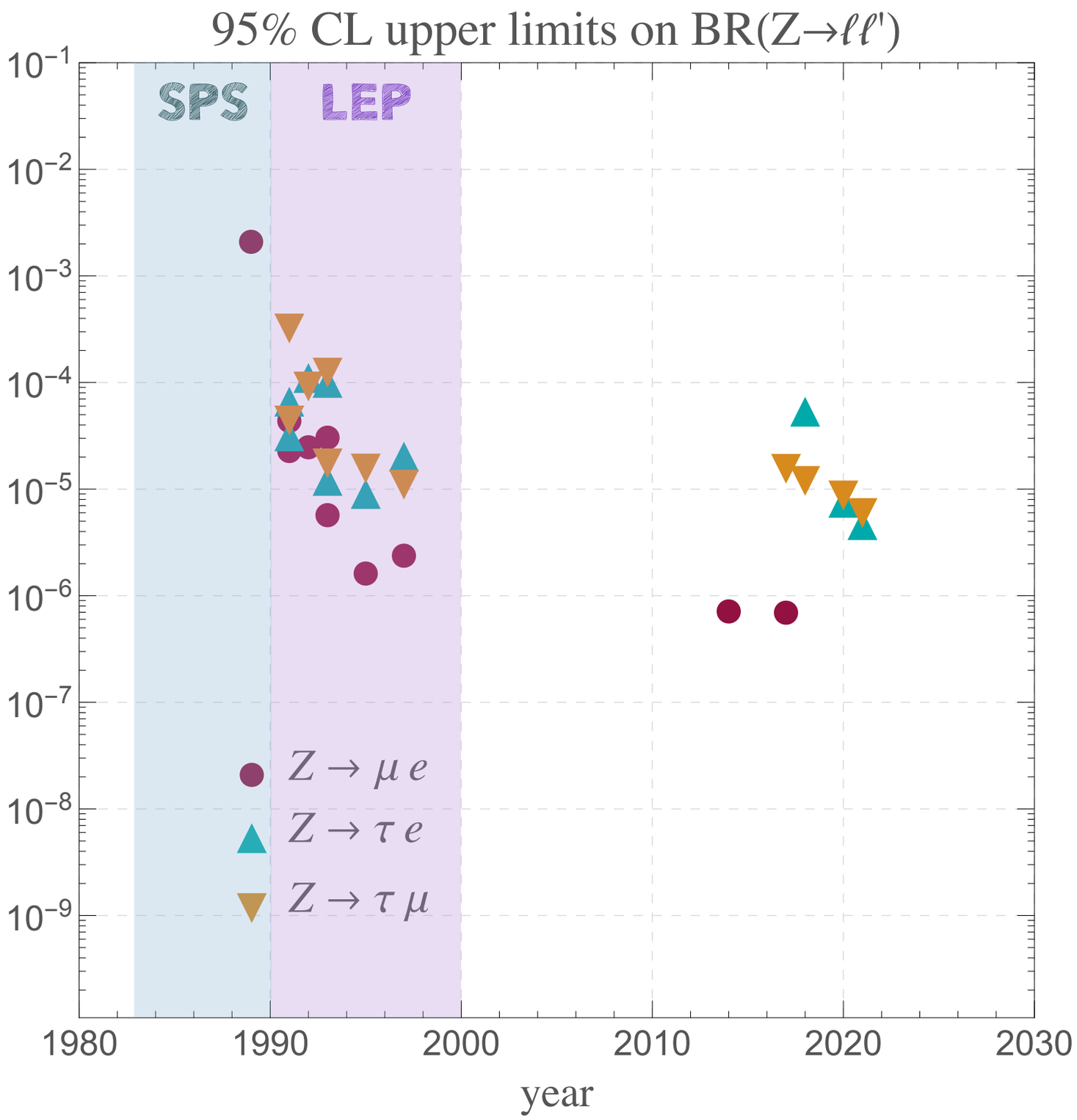

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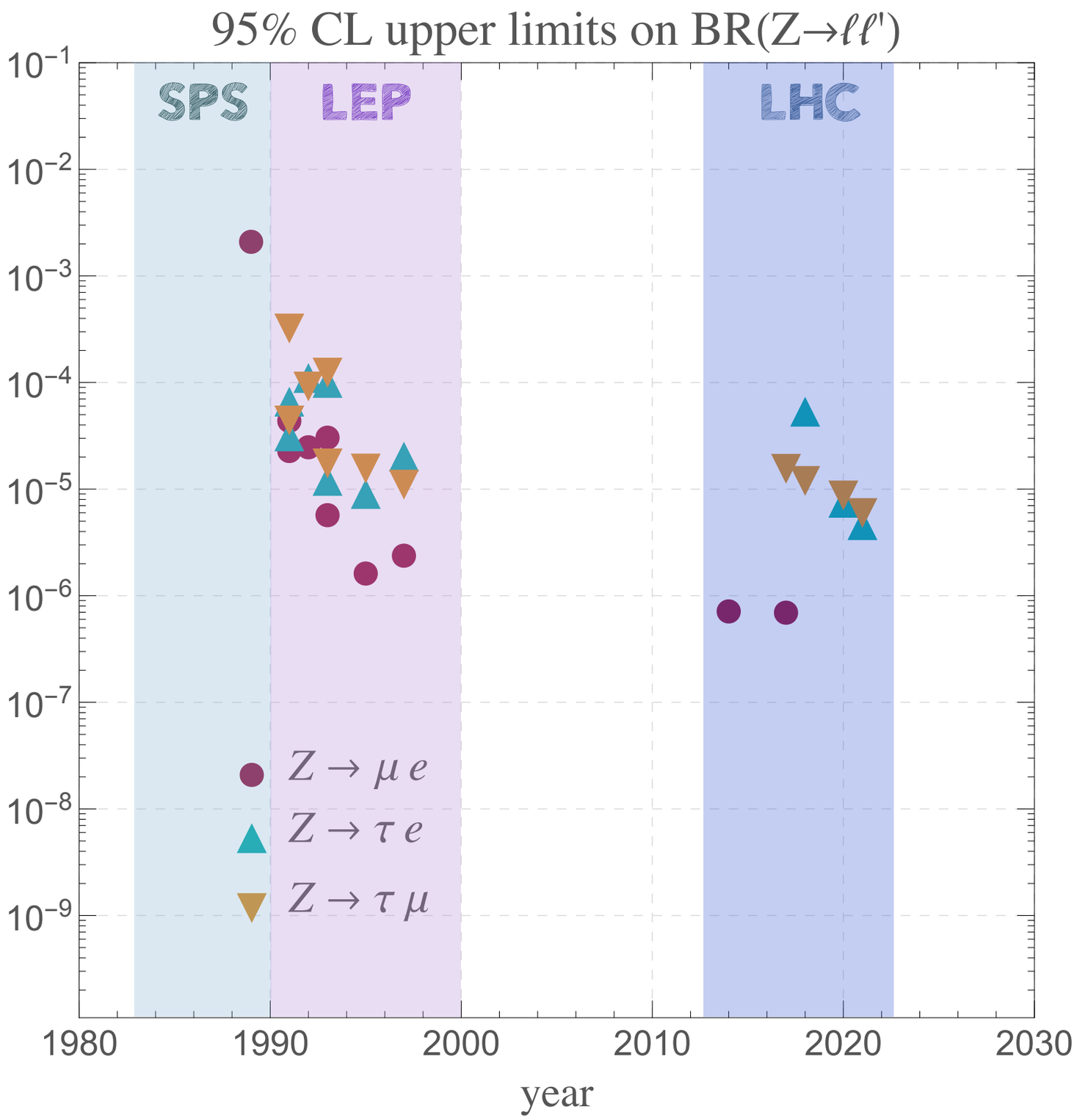

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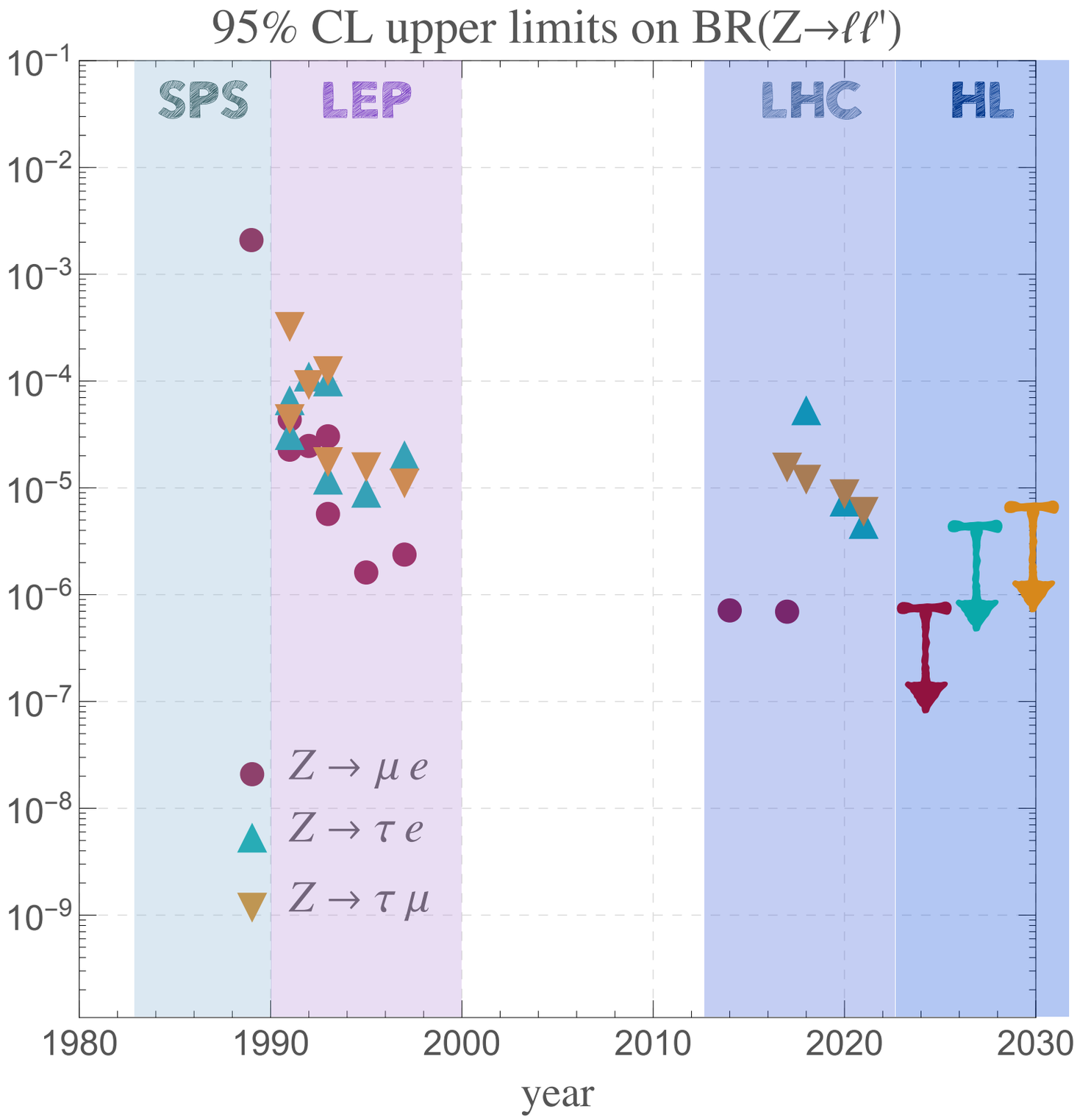

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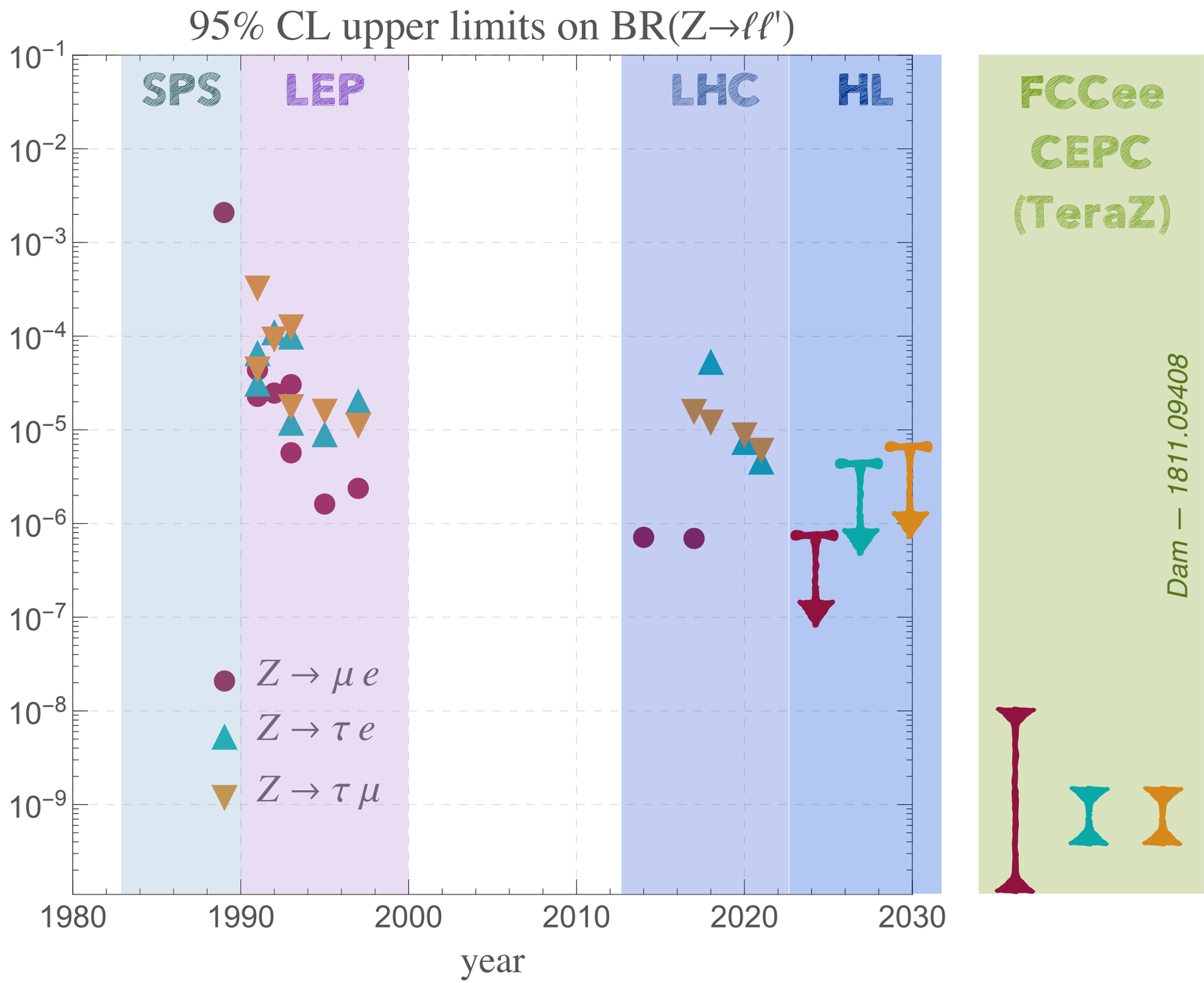

— SMEFT analysis —

- *Motivation I: lack of new light physics* $\Longrightarrow \Lambda \gg m_W$
- *Motivation II: model-independent analysis*

CLFV IN SMEFT (DIM 6)

4-lepton operators		Dipole operators	
$Q_{\ell\ell}$	$(\bar{L}\gamma_\mu L)(\bar{L}\gamma^\mu L)$	Q_{eW}	$(\bar{L}\sigma^{\mu\nu}E)\tau^I\Phi W^I_{\mu\nu}$
Q_{ee}	$(\bar{E}\gamma_\mu E)(\bar{E}\gamma^\mu E)$	Q_{eB}	$(\bar{L}\sigma^{\mu\nu}E)\Phi B_{\mu\nu}$
$Q_{\ell e}$	$(\bar{L}\gamma_\mu L)(\bar{E}\gamma^\mu E)$		
Lepton-Higgs operators			
$Q^{(1)}_{\varphi\ell}$	$i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{L}\gamma^\mu L)$	$Q^{(3)}_{\varphi\ell}$	$i(\Phi^\dagger\tau^I \overleftrightarrow{D}_\mu \Phi)(\bar{L}\tau^I\gamma^\mu L)$
$Q_{\varphi e}$	$i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{E}\gamma^\mu E)$	$Q_{e\varphi 3}$	$(\bar{L}E\Phi)(\Phi^\dagger\Phi)$
2-lepton 2-quark operators			
$Q^{(1)}_{\ell q}$	$(\bar{L}\gamma_\mu L)(\bar{Q}\gamma^\mu Q)$	$Q_{\ell u}$	$(\bar{L}\gamma_\mu L)(\bar{U}\gamma^\mu U)$
$Q^{(3)}_{\ell q}$	$(\bar{L}\gamma_\mu \tau^I L)(\bar{Q}\gamma^\mu \tau^I Q)$	Q_{eu}	$(\bar{E}\gamma_\mu E)(\bar{U}\gamma^\mu U)$
Q_{eq}	$(\bar{E}\gamma^\mu E)(\bar{Q}\gamma_\mu Q)$	$Q_{\ell edq}$	$(\bar{L}^a E)(\bar{D}Q^a)$
$Q_{\ell d}$	$(\bar{L}\gamma_\mu L)(\bar{D}\gamma^\mu D)$	$Q^{(1)}_{\ell equ}$	$(\bar{L}^a E)\epsilon_{ab}(\bar{Q}^b U)$
Q_{ed}	$(\bar{E}\gamma_\mu E)(\bar{D}\gamma^\mu D)$	$Q^{(3)}_{\ell equ}$	$(\bar{L}^a\sigma_{\mu\nu}E)\epsilon_{ab}(\bar{Q}^b\sigma^{\mu\nu}U)$

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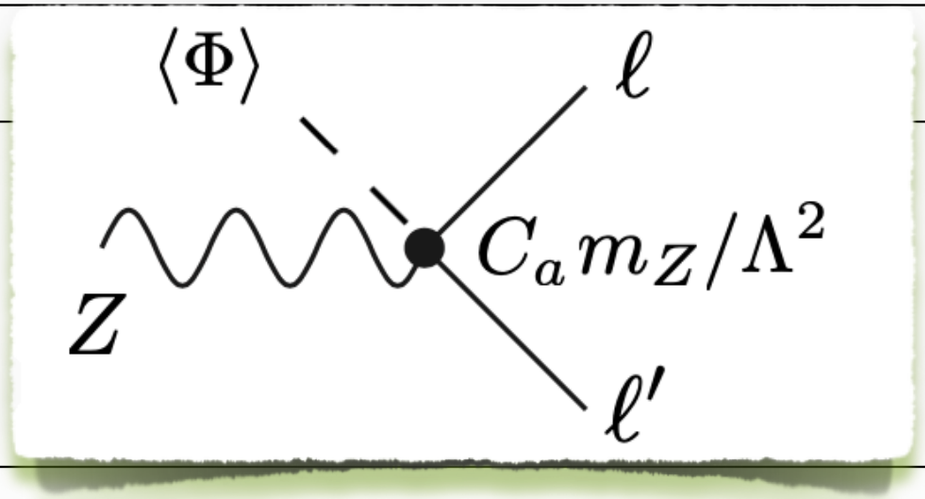

Dipole operators	
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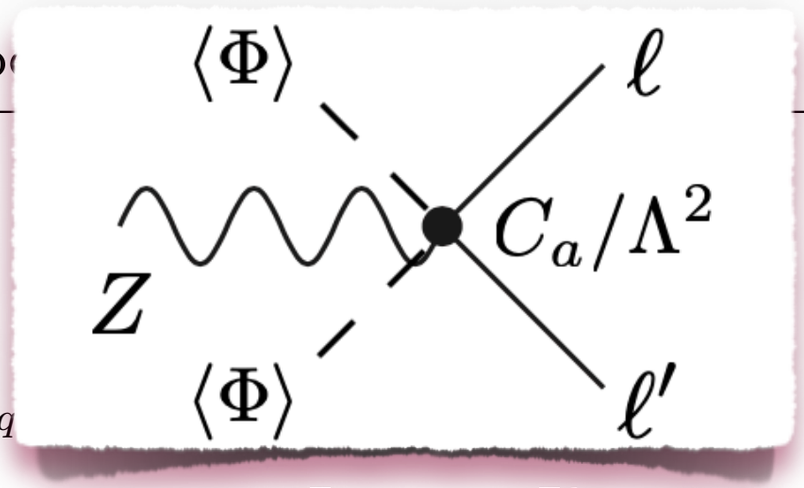
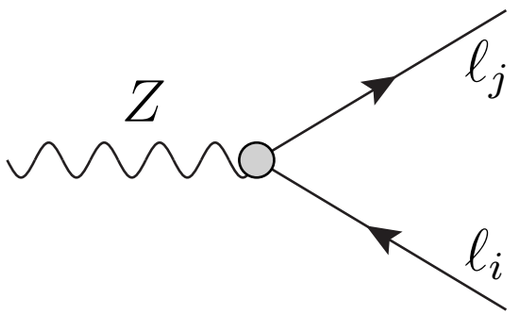
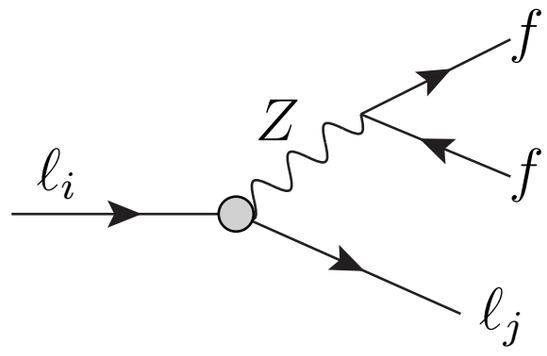
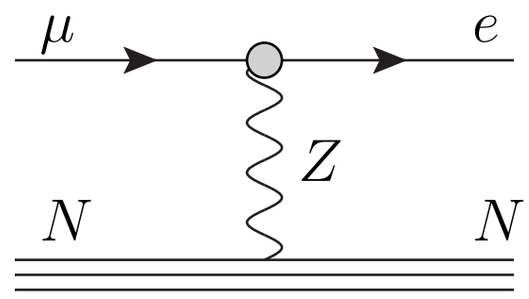


Indirect constraints

- Same operators generate other **low-energy** cLFV observables







- Matching and running play mayor role

— ask me about it—

- Set indirect bounds on LFVZD

— can we improve them at FCCee?—

OUR ANALYSIS

— warning! Wilson coefficients are scale dependent $C \equiv C(\mu)$ *—*

— Single WC at $\mu = M_Z$ *—*

— Single WC at $\mu = \Lambda$ *—*

— Several WC at $\mu = \Lambda$ *—*

Indirect limits on LFVZD

Probing effective operators at $\mu=M_Z$

Observable	Operator	Indirect Limit on LFVZD	Strongest constraint
$\mathrm{BR}(Z \to \mu e)$	$\left(Q_{\varphi\ell}^{(1)} + Q_{\varphi\ell}^{(3)}\right)^{e\mu}$	3.7×10^{-13}	$\mu \to e$, Au
	$Q_{\varphi e}^{e\mu}$	9.4×10^{-15}	$\mu \to e$, Au
	$Q_{eB}^{e\mu}$	1.4×10^{-23}	$\mu \to e\gamma$
	$Q_{eW}^{e\mu}$	1.6×10^{-22}	$\mu \to e\gamma$
$\mathrm{BR}(Z \to \tau e)$	$\left(Q_{\varphi\ell}^{(1)} + Q_{\varphi\ell}^{(3)}\right)^{e\tau}$	6.3×10^{-8}	$\tau \to \rho\, e$
	$Q_{\varphi e}^{e\tau}$	1.2×10^{-15}	$\tau \to e\gamma$
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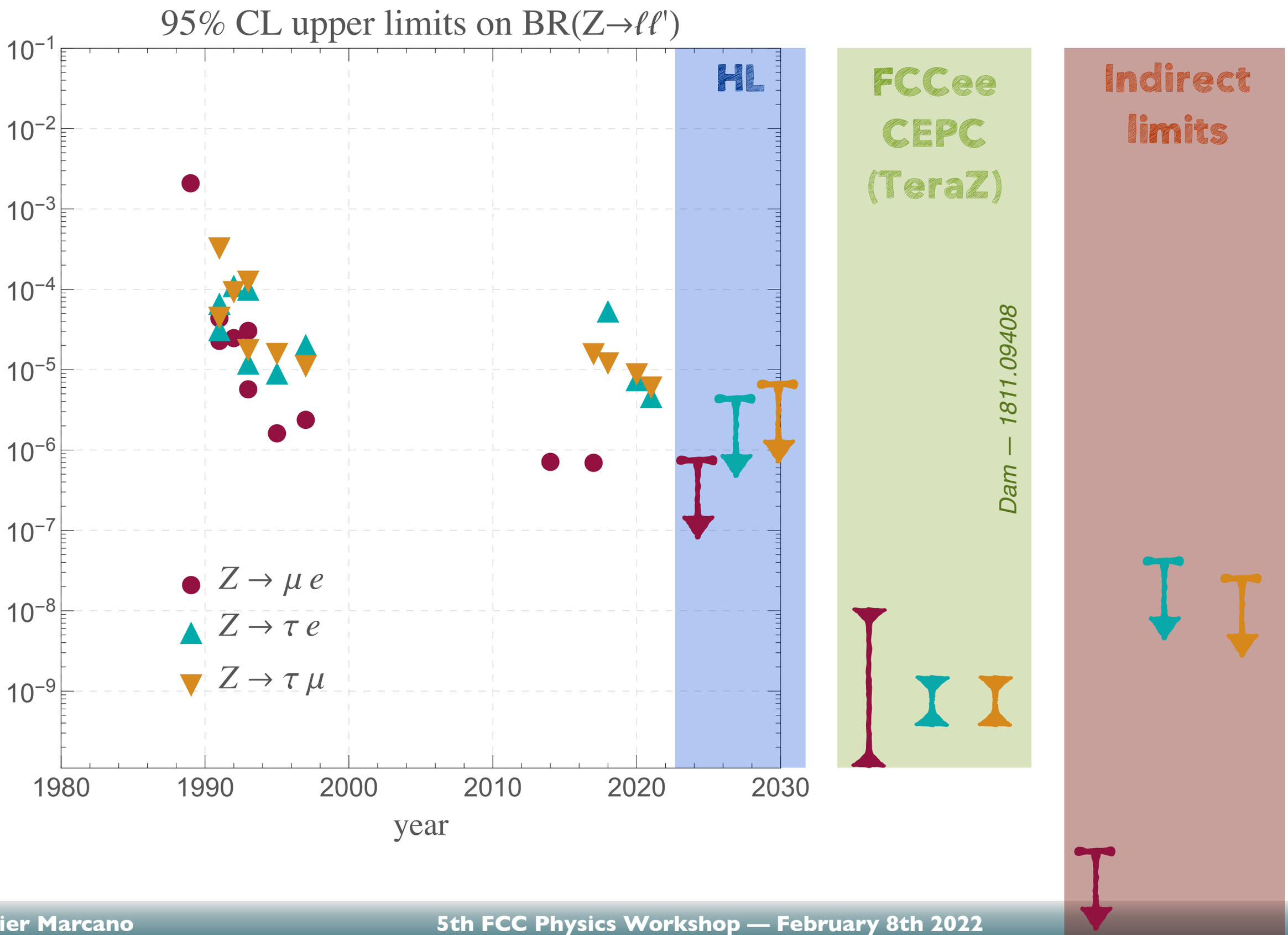
Indirect limits on LFVZD



Hope at Future Lepton Colliders

— Single WC at μ=M_Z —

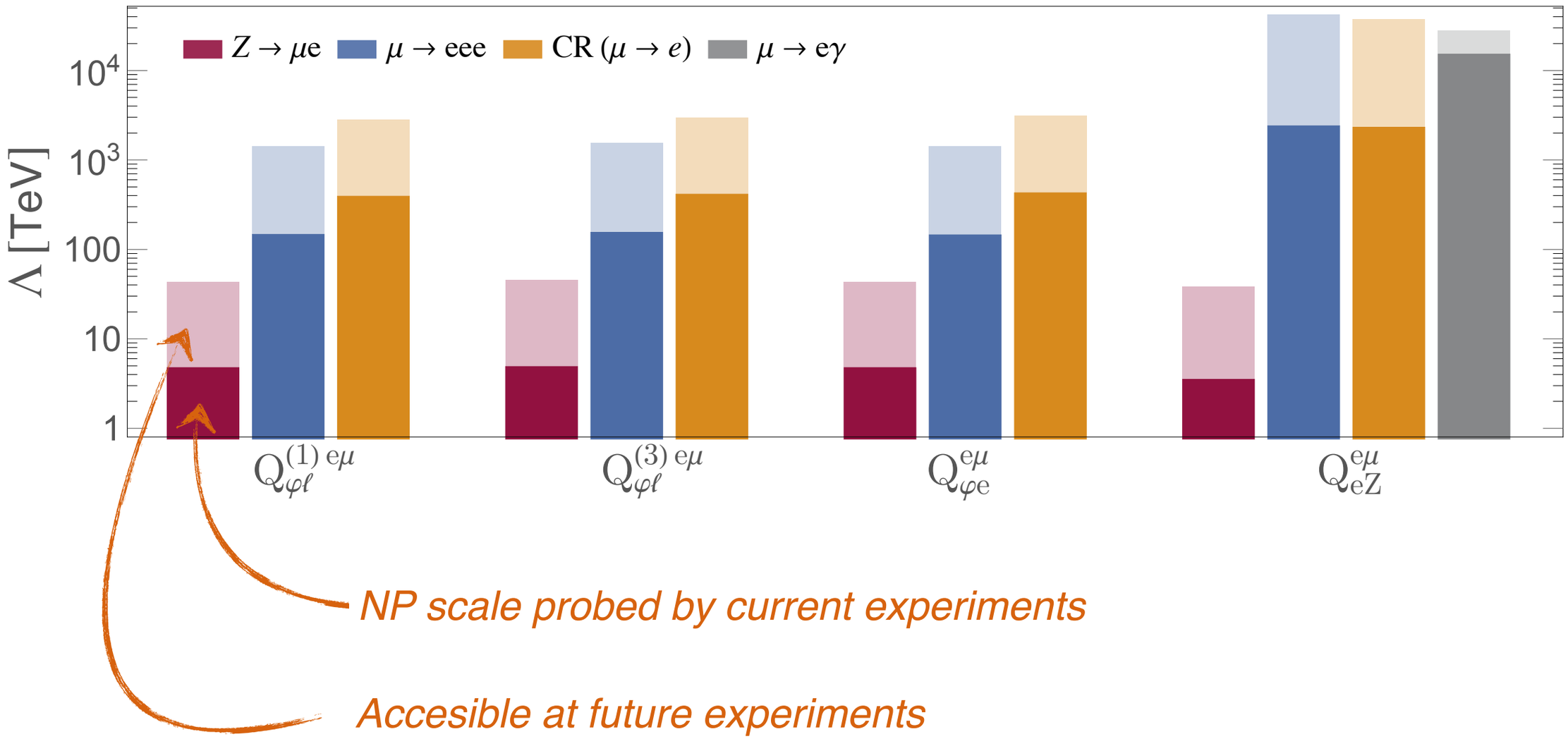
	$Z \to \mu e$	$Z \to \tau \ell$
Dipole	✗	✗
Higgs-lepton	✗	✓

— Single WC at μ= Λ —

— Several WC at μ= Λ —

Sensitivities to New Physics Scale

Probing effective operators at μ=Λ



Sensitivities to New Physics Scale

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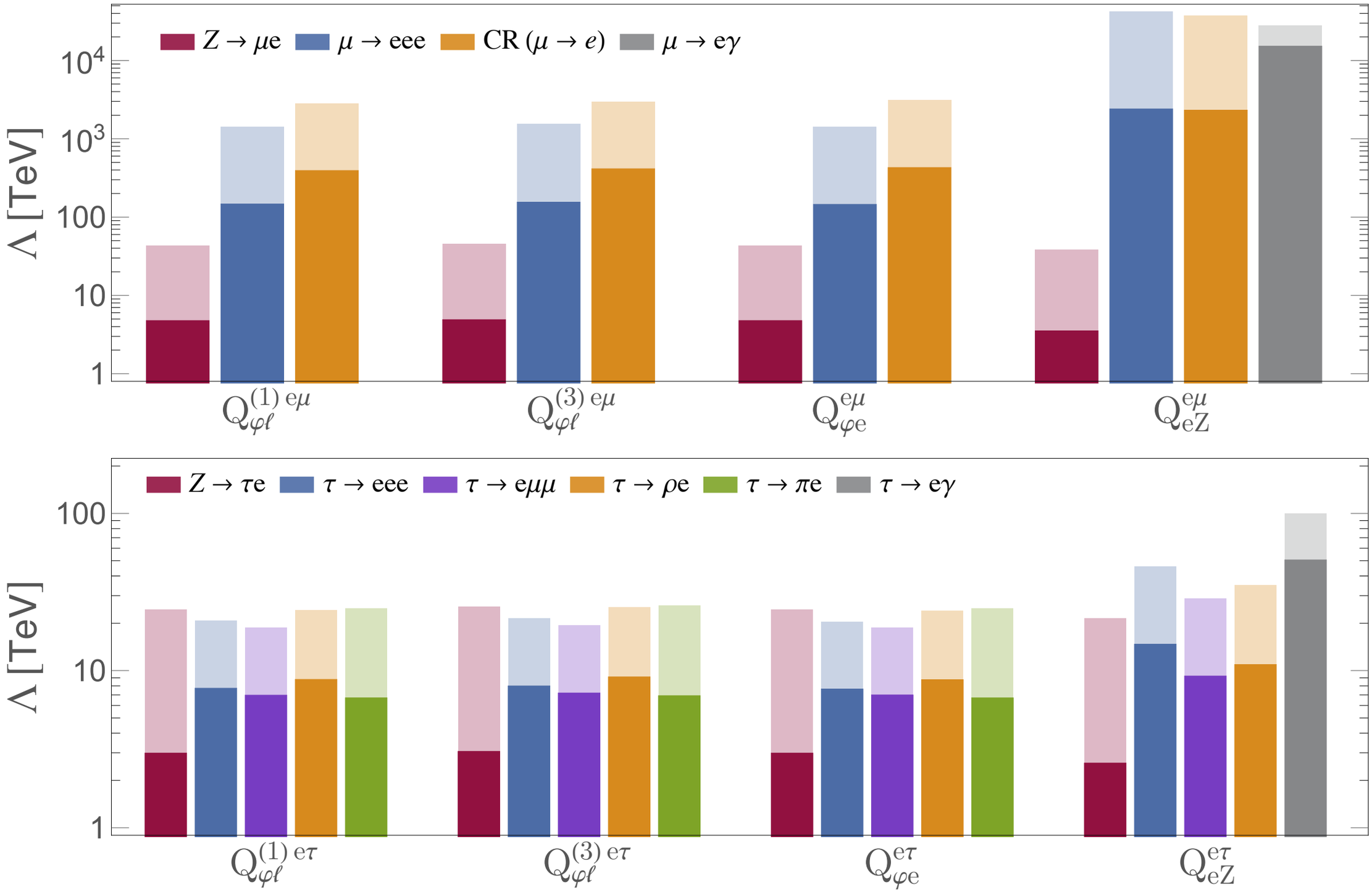

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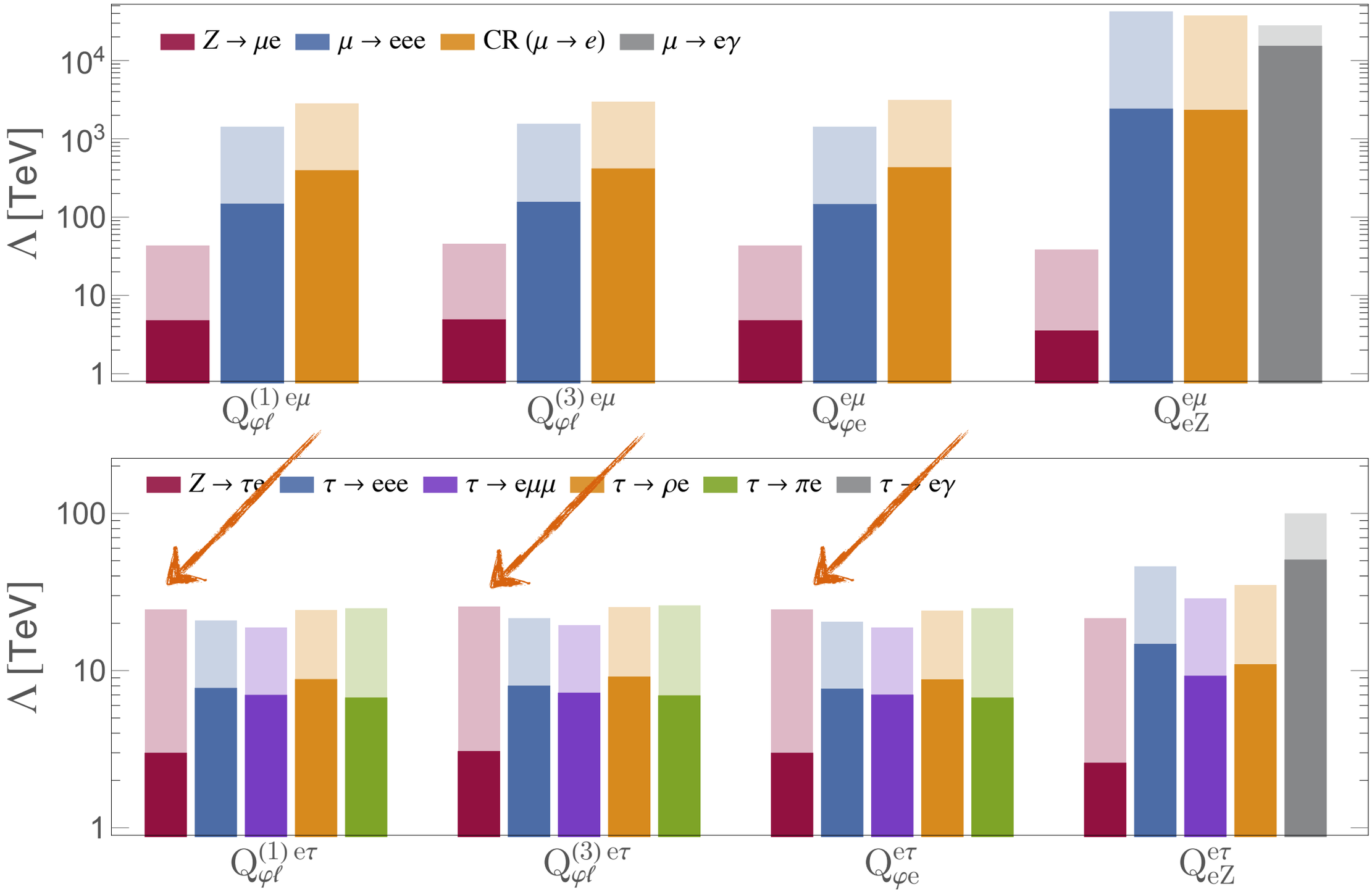

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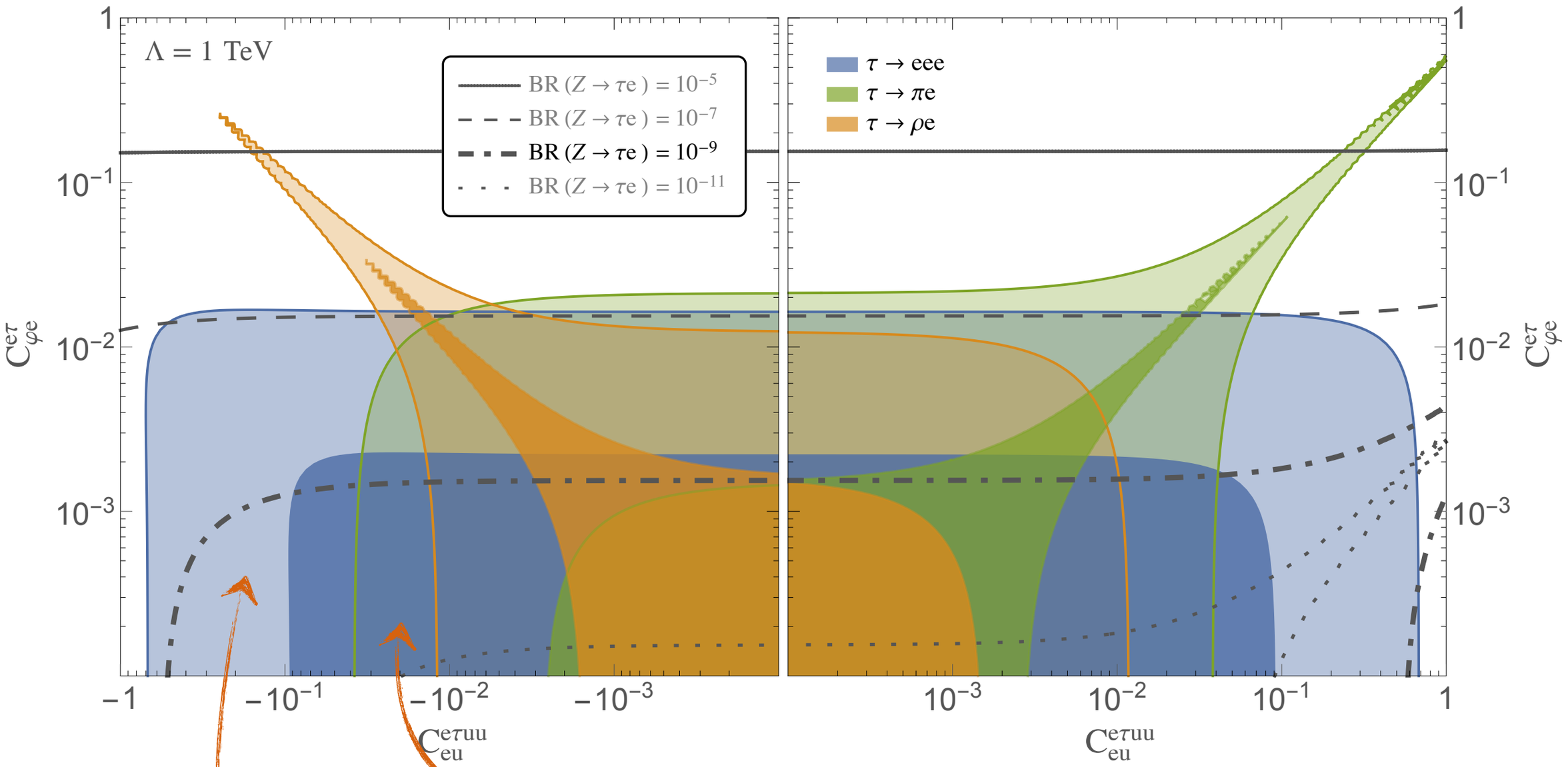
— TeraZ ≥ Belle II —

— Several WC at $\mu=\Lambda$ —

Beyond the Single Operator

Probing two effective operators at $\mu=\Lambda$

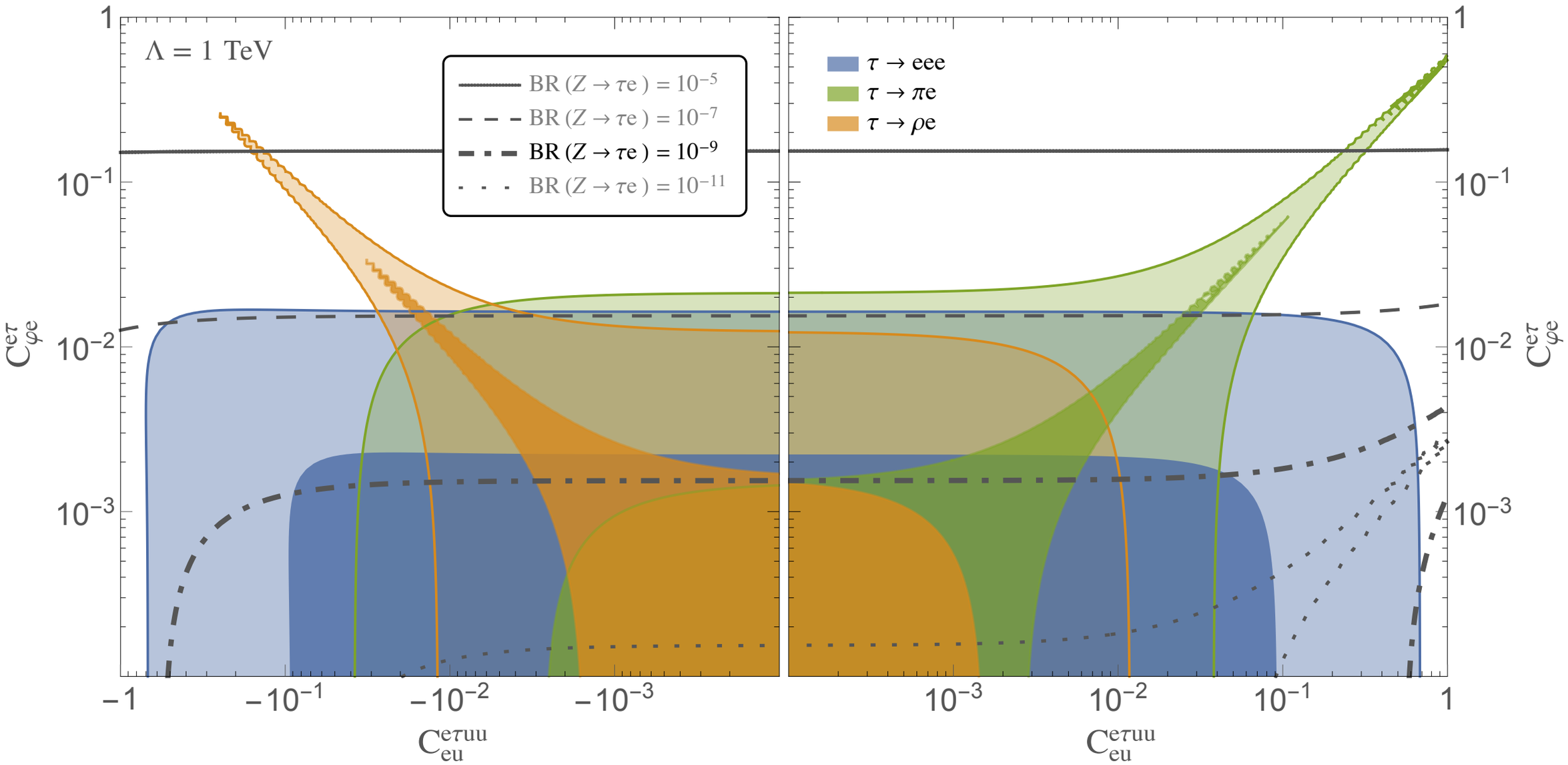


Allowed if future experiments find nothing

Allowed by current experiments

Beyond the Single Operator

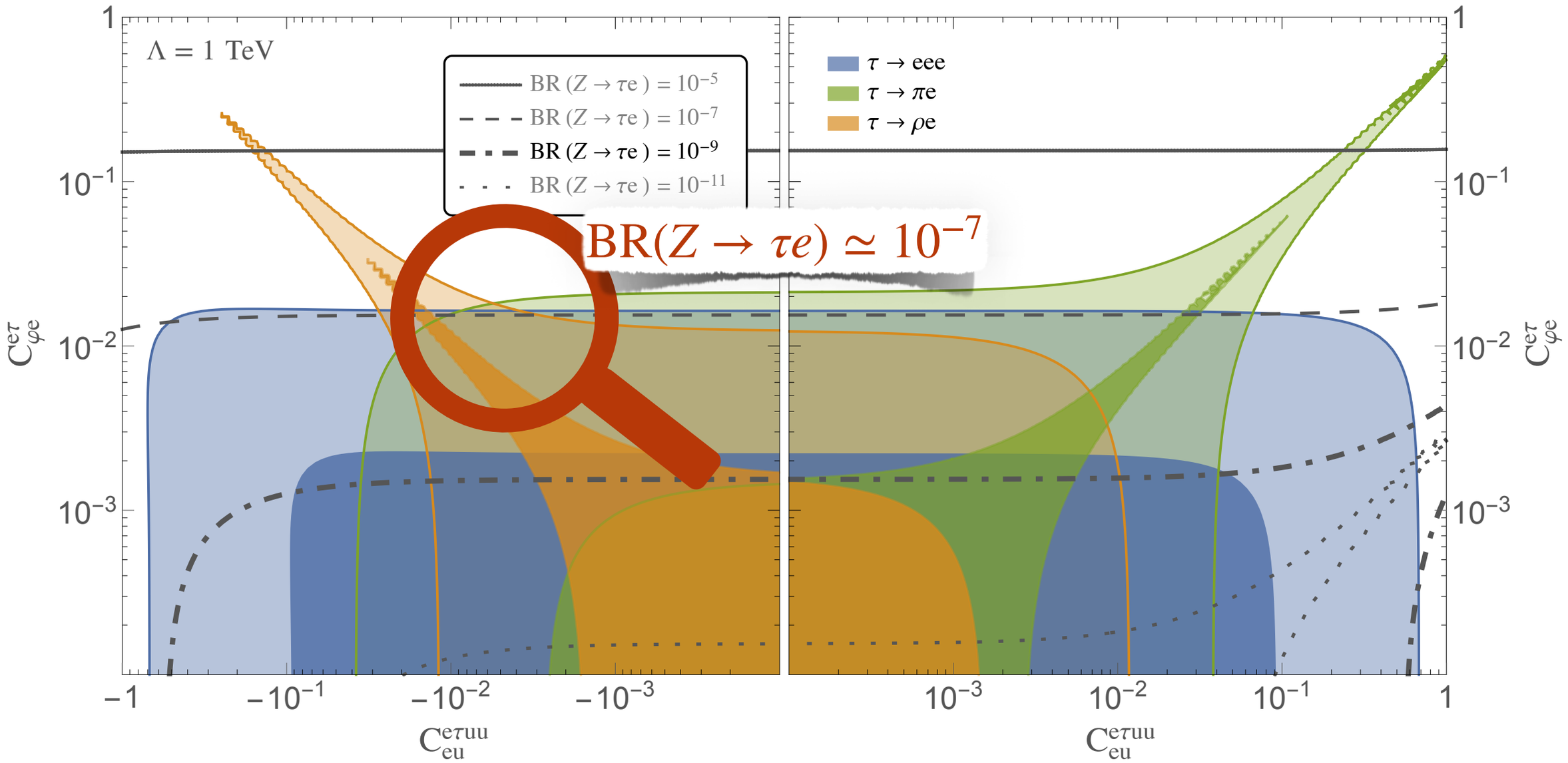
Probing two effective operators at $\mu=\Lambda$



- *Potential 2D flat directions constrained by other observables*
- *Huge amount of fine tuning to avoid all low-energy constraints*
- *More cases in arXiv: 2107.10273 !!!*

Beyond the Single Operator

Probing two effective operators at $\mu=\Lambda$



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— Several WC at $\mu = \Lambda$ —

avoiding extreme fine tuning

	$Z \to \mu e$	$Z \to \tau \ell$
Dipole	✗	✗
Higgs-lepton	✗	✓

$\mathrm{BR}(Z \to \tau \ell) \simeq 10^{-7}$ *still allowed*

Summary

- *TeraZ factory optimal place to boost LFV Z decay searches*

  — improve current limits by 3-4 orders of magnitude —

- *These sensitivities compete with indirect low-energy observables*

- *From our SMEFT analysis:*

	$Z \to \mu e$	$Z \to \tau \ell$
Dipole	✗	✗
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— TeraZ ≥ Belle II —

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- *Don't give up with $Z \to \mu e$!!!*

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— the X marks the spot —

Thank you!

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Back up

SM Effective Field Theory

- *Add higher dimensional operators*

— warning! Wilson coefficients are scale dependent $C \equiv C(\mu)$ —

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \frac{1}{\Lambda}\sum_a C_a^{(5)} Q_a^{(5)} + \frac{1}{\Lambda^2}\sum_a C_a^{(6)} Q_a^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

- $Q_a^{(5)}$ *only 1: Weinberg operator*

— neutrino masses —

- $Q_a^{(6)}$ *59 operators* *[Buchmuller&Wyler'86, Grzadkowski et al' 10]*

— leading order for cLFV (some of them) —

- $Q_a^{(n\geq 7)}$ *just a lot*

— we neglect them —

LFVZD IN SMEFT

- *Following Brignole&Rossi [hep-ph/0404211], Crivellin et al. [1312.0634]*

$$\mathrm{BR}\left(Z \to \ell_i \ell_j\right) = \frac{m_Z}{12\pi\Gamma_Z}\left\{ \left| g_{VR}\delta_{ij} + \delta g^{ij}_{VR} \right|^2 + \left| g_{VL}\delta_{ij} + \delta g^{ij}_{VL} \right|^2 + \frac{m_Z^2}{2}\left(\left| \delta g^{ij}_{TR} \right|^2 + \left| \delta g^{ij}_{TL} \right|^2 \right) \right\},$$

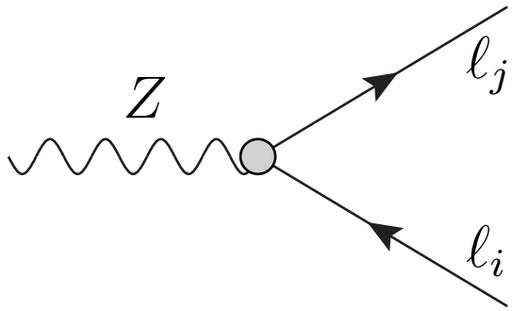
- *Tree-level: 3 incoherent contributions from the 5 operators*

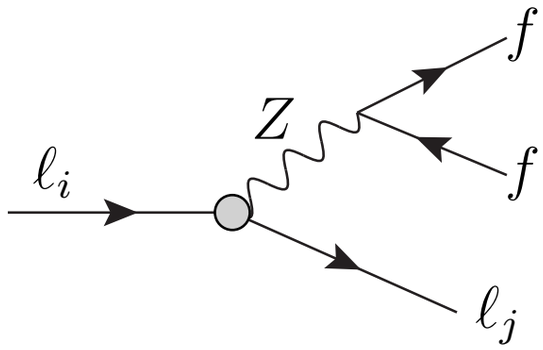
$$g_{VR} = \frac{e s_{\mathrm{w}}}{c_{\mathrm{w}}} \quad , \qquad \delta g^{ij}_{VR} = -\frac{e v^2}{2 s_{\mathrm{w}} c_{\mathrm{w}} \Lambda^2} C^{ij}_{\varphi e}$$

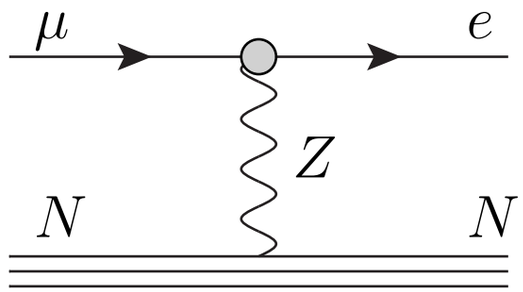
$$g_{VL} = \frac{e}{s_{\mathrm{w}} c_{\mathrm{w}}}\left(-\frac{1}{2} + s^2_{\mathrm{w}}\right) \quad , \qquad \delta g^{ij}_{VL} = -\frac{e v^2}{2 s_{\mathrm{w}} c_{\mathrm{w}} \Lambda^2}\left(C^{(1)\,ij}_{\varphi \ell} + C^{(3)\,ij}_{\varphi \ell} \right)$$

$$\delta g^{ij}_{TR} = \delta g^{ji\,*}_{TL} = -\frac{v}{\sqrt{2}\Lambda^2}\left(s_{\mathrm{w}} C^{ij}_{eB} + c_{\mathrm{w}} C^{ij}_{eW} \right) \equiv C^{ij}_{eZ}$$

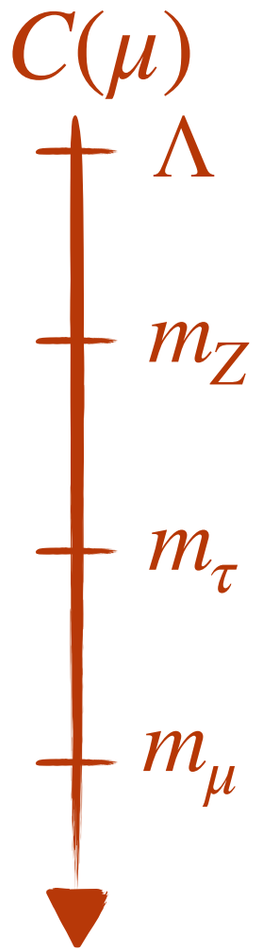
Indirect constraints







- Same operators generate other **low-energy** cLFV observables



match&run

SMEFT RGE

matching

Low-Energy EFT (LEFT) RGE

wilson

compute obs

LFVZD

LFV τ decays

LFV μ decays

flavio

Low-Energy cLFV

LFV obs.	Present bounds (90% CL)		Expected future limits	
$\mathrm{BR}(\mu \to e\gamma)$	4.2×10^{-13}	MEG (2016) [63]	6×10^{-14}	MEG-II [64]
$\mathrm{BR}(\mu \to eee)$	1.0×10^{-12}	SINDRUM (1988) [65]	10^{-16}	Mu3e [66]
$\mathrm{CR}(\mu \to e, \mathrm{Au})$	7.0×10^{-13}	SINDRUM II (2006) [67]	–	–
$\mathrm{CR}(\mu \to e, \mathrm{Al})$	–	–	6×10^{-17}	COMET/Mu2e [68, 69]
$\mathrm{BR}(\tau \to e\gamma)$	3.3×10^{-8}	BaBar (2010) [70]	3×10^{-9}	Belle-II [71]
$\mathrm{BR}(\tau \to eee)$	2.7×10^{-8}	Belle (2010) [72]	5×10^{-10}	Belle-II [71]
$\mathrm{BR}(\tau \to e\mu\mu)$	2.7×10^{-8}	Belle (2010) [72]	5×10^{-10}	Belle-II [71]
$\mathrm{BR}(\tau \to \pi e)$	8.0×10^{-8}	Belle (2007) [73]	4×10^{-10}	Belle-II [71]
$\mathrm{BR}(\tau \to \rho e)$	1.8×10^{-8}	Belle (2011) [74]	3×10^{-10}	Belle-II [71]
$\mathrm{BR}(\tau \to \mu\gamma)$	4.2×10^{-8}	Belle (2021) [75]	10^{-9}	Belle-II [71]
$\mathrm{BR}(\tau \to \mu\mu\mu)$	2.1×10^{-8}	Belle (2010) [72]	4×10^{-10}	Belle-II [71]
$\mathrm{BR}(\tau \to \mu ee)$	1.8×10^{-8}	Belle (2010) [72]	3×10^{-10}	Belle-II [71]
$\mathrm{BR}(\tau \to \pi\mu)$	1.1×10^{-7}	Babar (2006) [76]	5×10^{-10}	Belle-II [71]
$\mathrm{BR}(\tau \to \rho\mu)$	1.2×10^{-8}	Belle (2011) [74]	2×10^{-10}	Belle-II [71]

Refs in arXiv: 2107.10273

SMEFT RGE

- *Full one-loop SMEFT RGE in Jenkins et al. [1308.2627, 1310.4838, 1312.2014]*
- *Most relevant for the LFVZD*

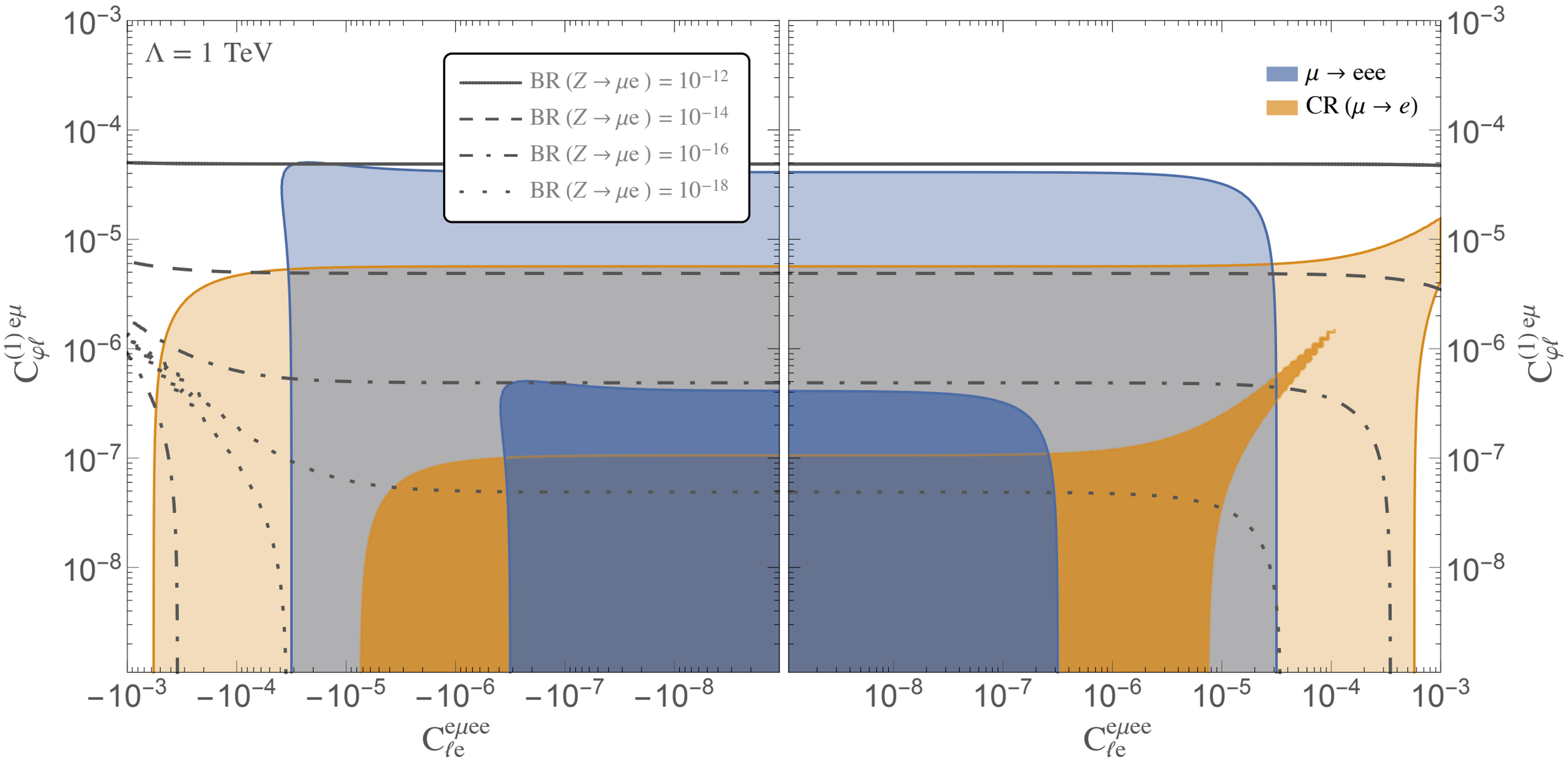
$$\begin{pmatrix} \dot{\boldsymbol{C}}_{\mathbf{1}} \\ \dot{\boldsymbol{C}}_{\mathbf{2}} \\ \dot{\boldsymbol{C}}_{\mathbf{3}} \\ \dot{\boldsymbol{C}}_{\mathbf{4}} \end{pmatrix} \equiv 16\pi^2\mu\frac{d}{d\mu}\begin{pmatrix} \boldsymbol{C}_{\mathbf{1}} \\ \boldsymbol{C}_{\mathbf{2}} \\ \boldsymbol{C}_{\mathbf{3}} \\ \boldsymbol{C}_{\mathbf{4}} \end{pmatrix} = \begin{pmatrix} \gamma_{11} & \gamma_{12} & 0 & 0 \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & 0 \\ 0 & \gamma_{32} & \gamma_{33} & \gamma_{34} \\ 0 & 0 & \gamma_{43} & \gamma_{44} \end{pmatrix} \begin{pmatrix} \boldsymbol{C}_{\mathbf{1}} \\ \boldsymbol{C}_{\mathbf{2}} \\ \boldsymbol{C}_{\mathbf{3}} \\ \boldsymbol{C}_{\mathbf{4}} \end{pmatrix},$$

with

$$\begin{aligned}
\boldsymbol{C_1} \equiv & \left(C_{\varphi\ell}^{(1)}, C_{\varphi\ell}^{(3)}, C_{\varphi e}\right)^T, \\
\boldsymbol{C_2} \equiv & \left(C_{\ell\ell}, C_{\ell e}, C_{ee}, C_{\ell q}^{(1)}, C_{\ell q}^{(3)}, C_{\ell u}, C_{\ell d}, C_{eu}, C_{ed}, C_{qe}\right)^T, \\
\boldsymbol{C_3} \equiv & \left(C_{\varphi q}^{(1)}, C_{\varphi q}^{(3)}, C_{\varphi u}, C_{\varphi d}, C_{qq}^{(1)}, C_{qq}^{(3)}, C_{qu}^{(1)}, C_{qd}^{(1)}, C_{uu}, C_{dd}, C_{ud}^{(1)}\right)^T, \\
\boldsymbol{C_4} \equiv & \left(C_{qu}^{(8)}, C_{qd}^{(8)}, C_{ud}^{(8)}\right)^T,
\end{aligned}$$

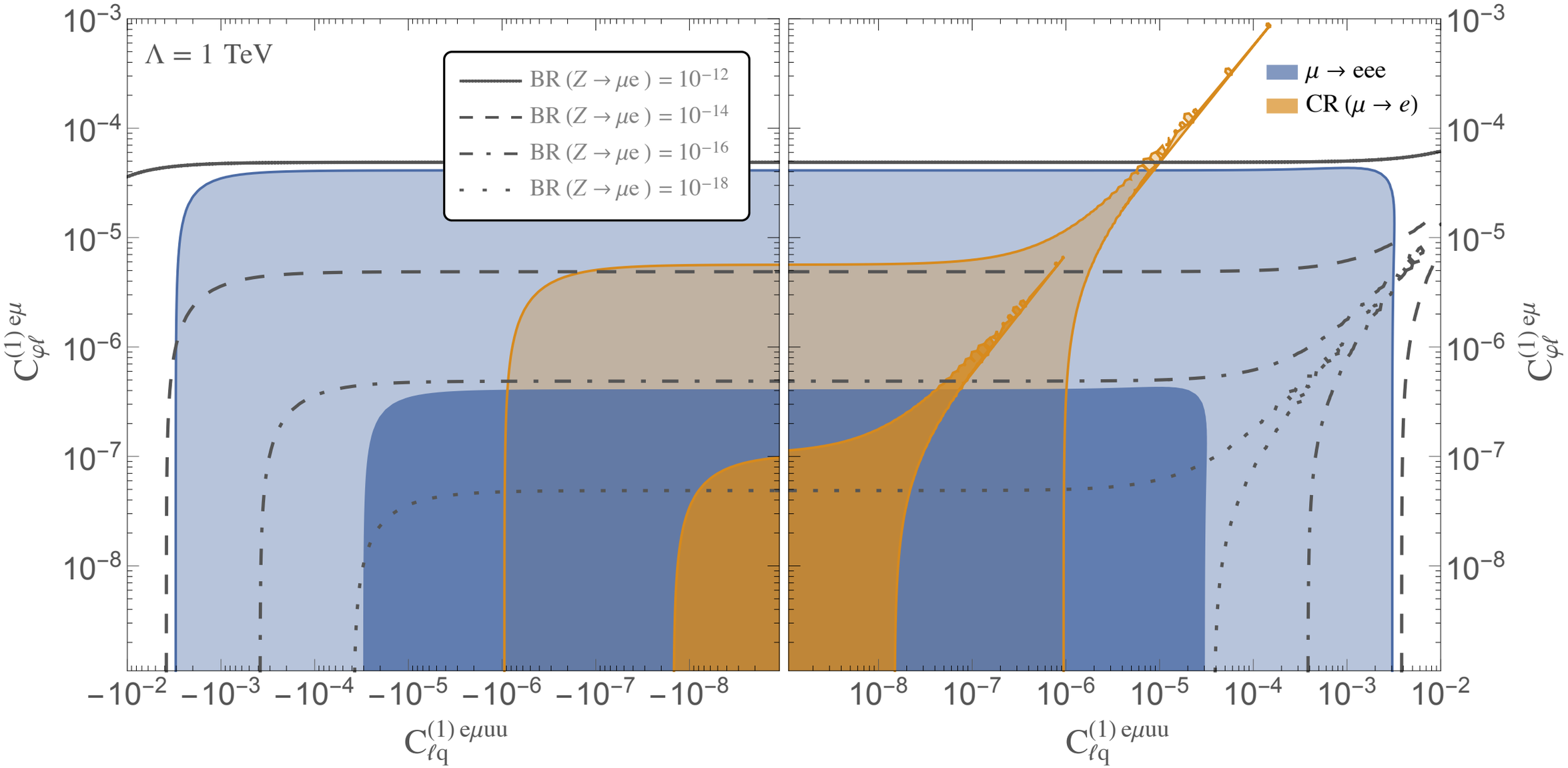
Beyond the Single Operator

Probing two effective operators at $\mu=\Lambda$



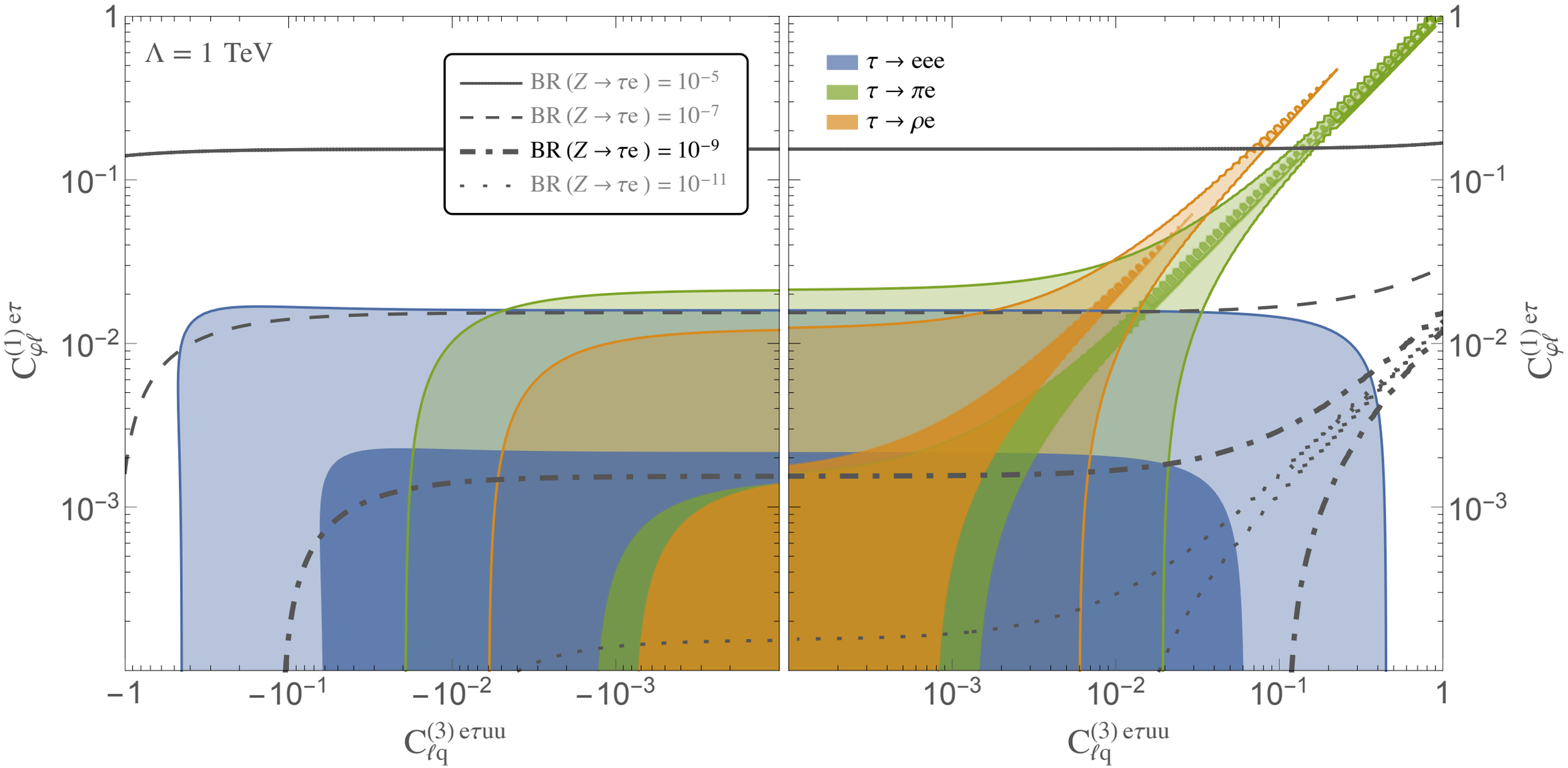
Other operators

Probing two effective operators at μ=Λ



Other operators

Probing two effective operators at $\mu=\Lambda$



Other operators

Probing two effective operators at $\mu=\Lambda$