

## Flavour at FCC(-ee)

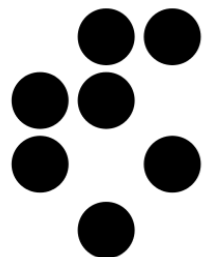
Jernej F. Kamenik

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Main reference: FCC CDR Vol 1. Eur.Phys.J.C 79 (2019) 6, 474 (with updates)



Univerza v *Ljubljani*



Institut "Jožef Stefan"

# Scope of Flavour Physics @ FCC(-ee)

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- Flavour physics reach with  $O(10^{13})$  Z decays ( $10^8$  W,  $10^6$  Higgs, top)
  - rare decays of c- and b-hadrons and CP violation in the heavy-quark sector
  - rare lepton decays
  - rare Z, (W, h, t?) decays
- In the context of ultimate potential of the LHCb upgrade and Belle II experiments.

Working point	Lumi. / IP [ $10^{34} \text{ cm}^{-2} \cdot \text{s}^{-1}$ ]	Total lumi. (2 IPs)	Run time	Physics goal
Z first phase	100	26 $\text{ab}^{-1}$ /year	2	
Z second phase	200	52 $\text{ab}^{-1}$ /year	2	150 $\text{ab}^{-1}$

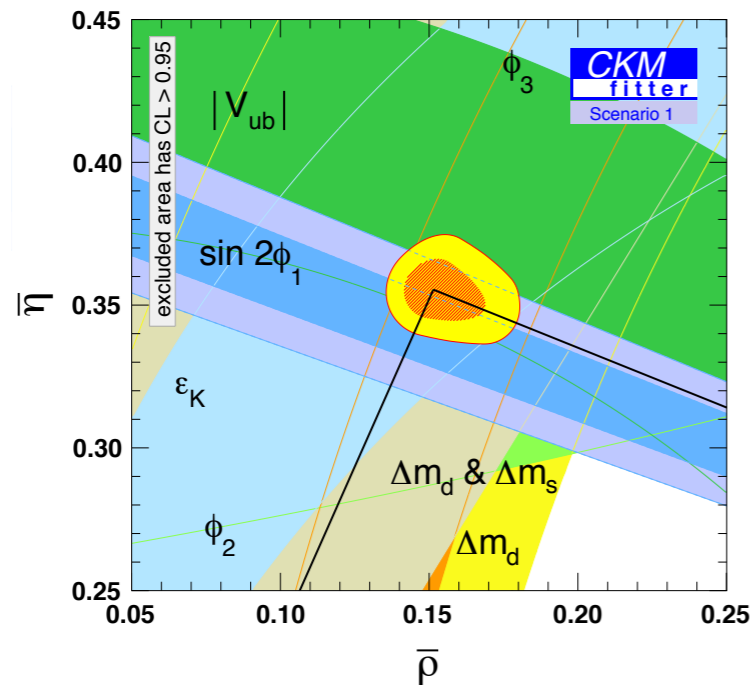
  

Particle production ( $10^9$ )	$B^0$	$B^-$	$B_s^0$	$\Lambda_b$	$c\bar{c}$	$\tau^- \tau^+$
Belle II	27.5	27.5	n/a	n/a	65	45
FCC-ee	400	400	100	100	800	220

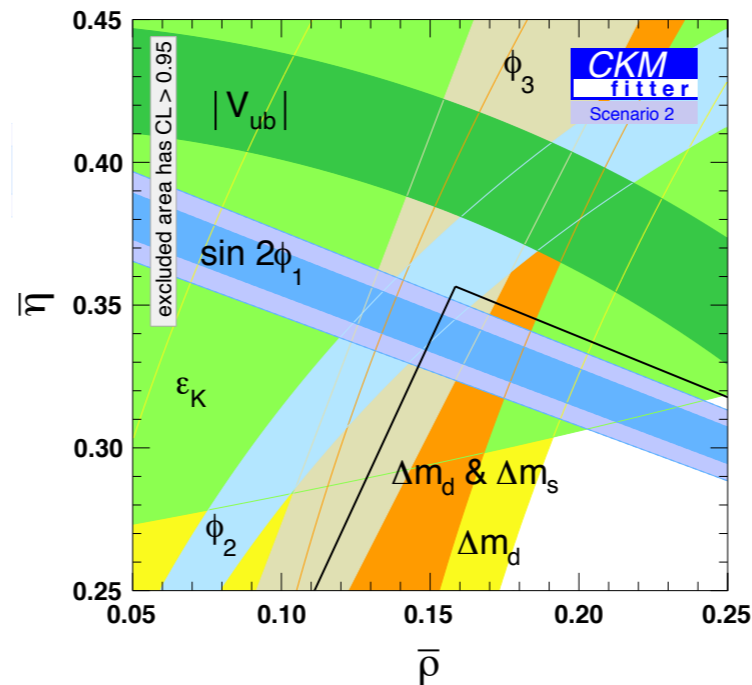
# Flavor physics circa 2030: possible scenarios

WA

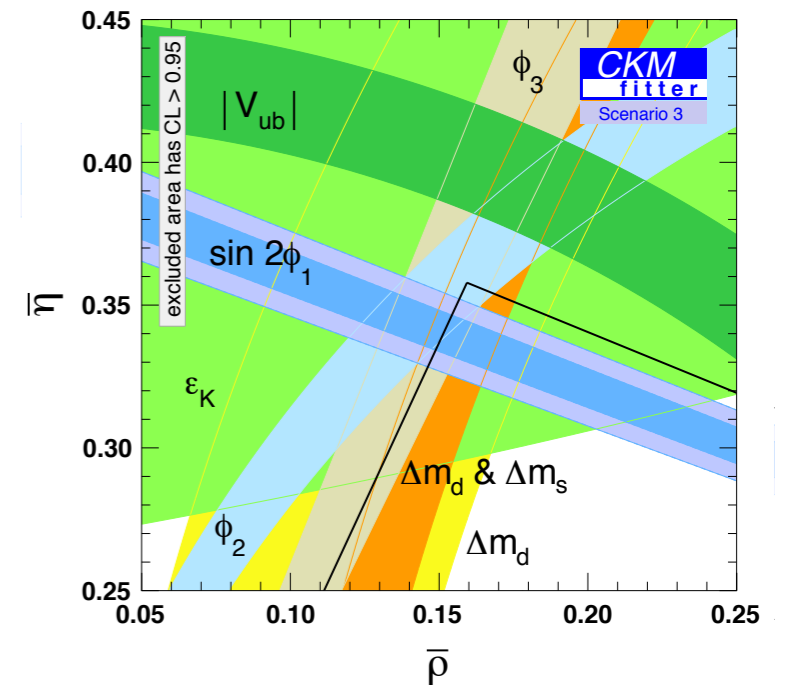
Now



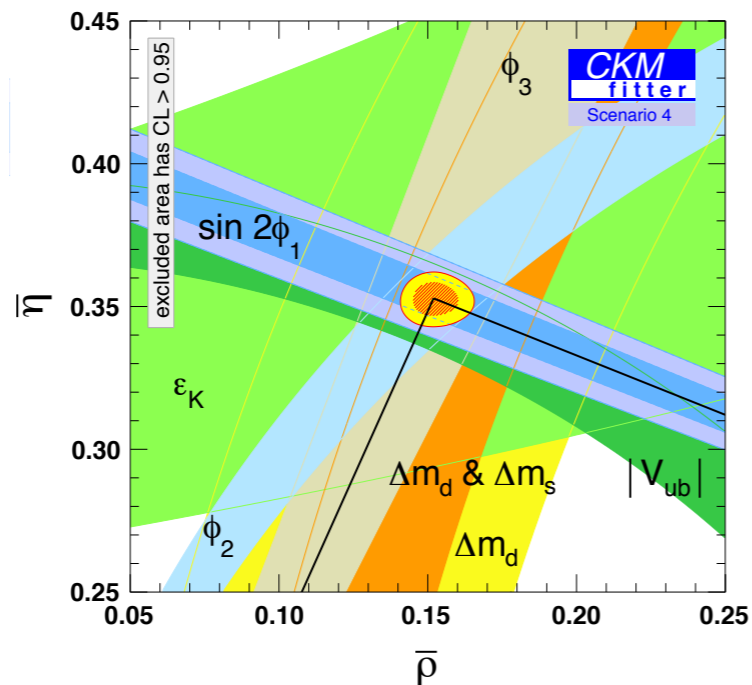
50 ab<sup>-1</sup> Belle II



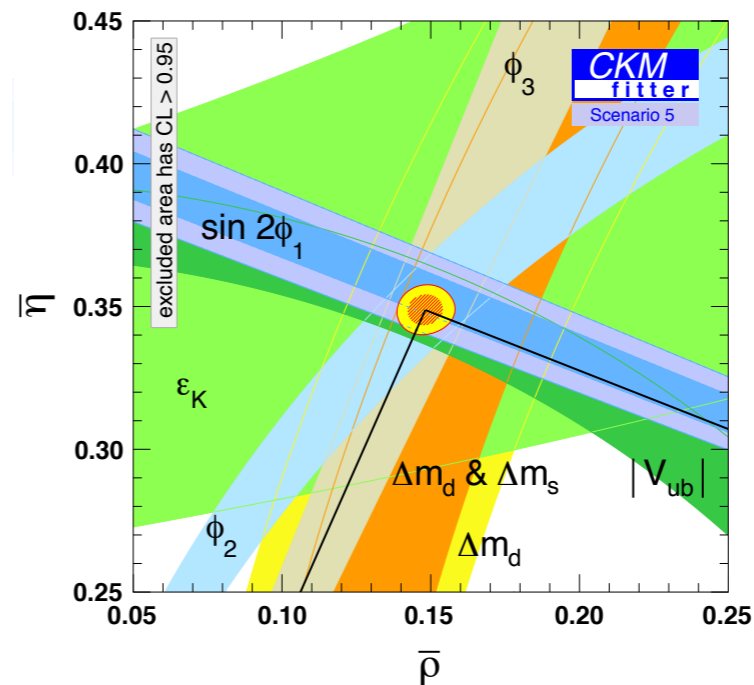
50 ab<sup>-1</sup> Belle II + LHCb



50 ab<sup>-1</sup> Belle II



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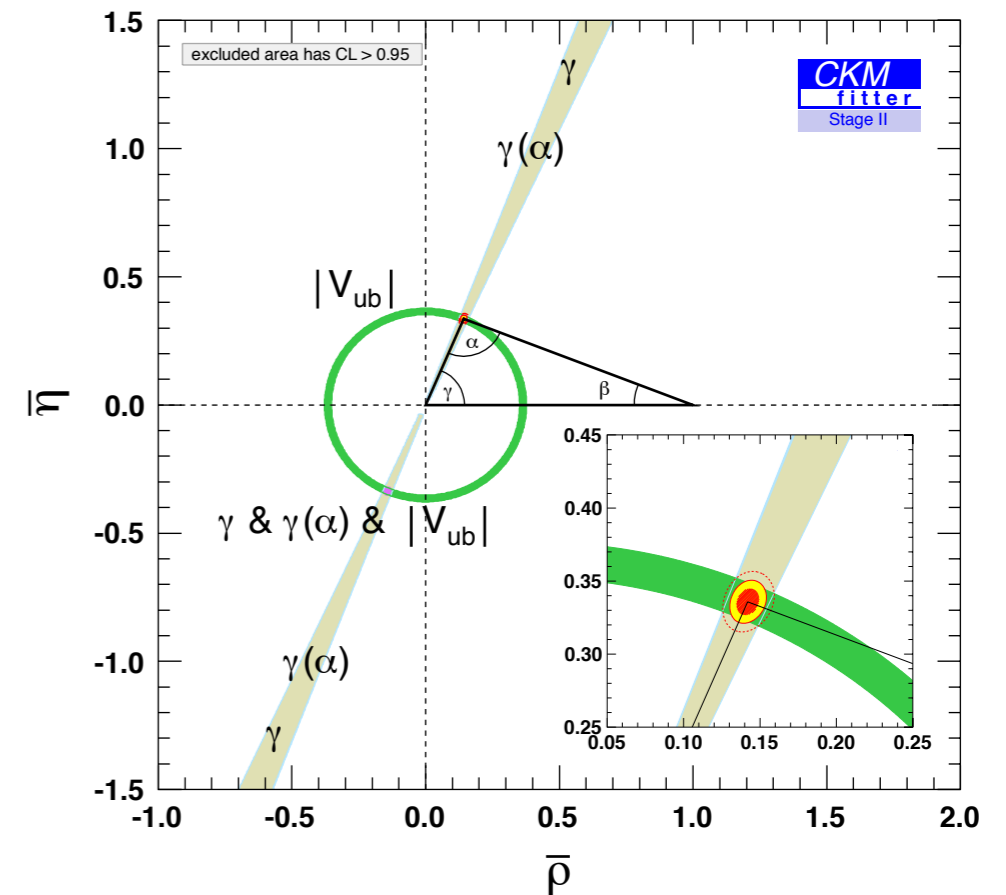
SM-like

+ discoveries  
or bounds  
from high-pT  
LHC searches

# FCC-ee flavour physics benchmarks & explorations

## Part 1: CKM determination

- BSM motivated tree-level obs. fit
  - ▶  $|V_{ub}|(B \rightarrow \pi \ell \nu)$
  - ▶  $\gamma(B \rightarrow DK)$

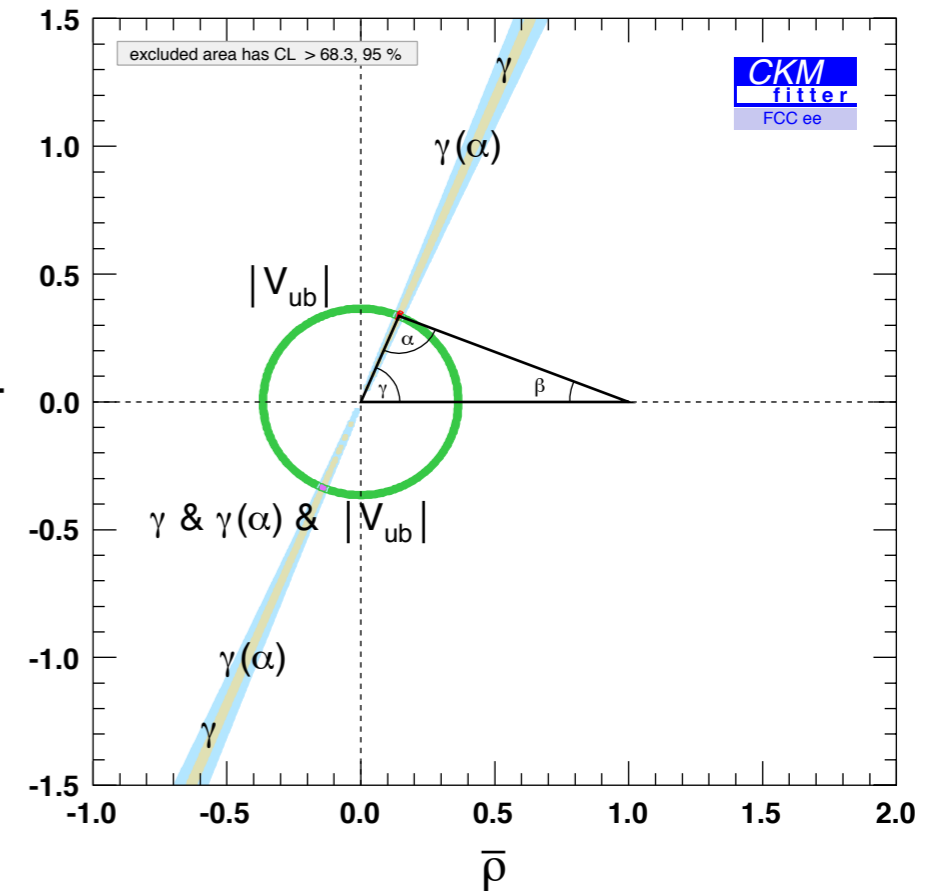


Observable / Experiments	Current W/A	Belle II (50 /ab)	LHCb-U1 (23/fb)	FCC-ee
CKM inputs				
$\gamma$ (uncert., rad)	$1.296^{+0.087}_{-0.101}$	$1.136 \pm 0.026$	$1.136 \pm 0.025$	
$ V_{ub} $ (precision)	5.9%	2.5%	6%	

# FCC-ee flavour physics benchmarks & explorations

## Part 1: CKM determination

- BSM motivated tree-level obs. fit
- improved stat. precision @FCC-ee
- $|V_{ub}|$  will require comparable theory progress (Lattice)



Observable / Experiments	Current W/A	Belle II (50 /ab)	LHCb-U1 (23/fb)	FCC-ee
CKM inputs				
$\gamma$ (uncert., rad)	$1.296^{+0.087}_{-0.101}$	$1.136 \pm 0.026$	$1.136 \pm 0.025$	$1.136 \pm 0.004$
$ V_{ub} $ (precision)	5.9%	2.5%	6%	1%

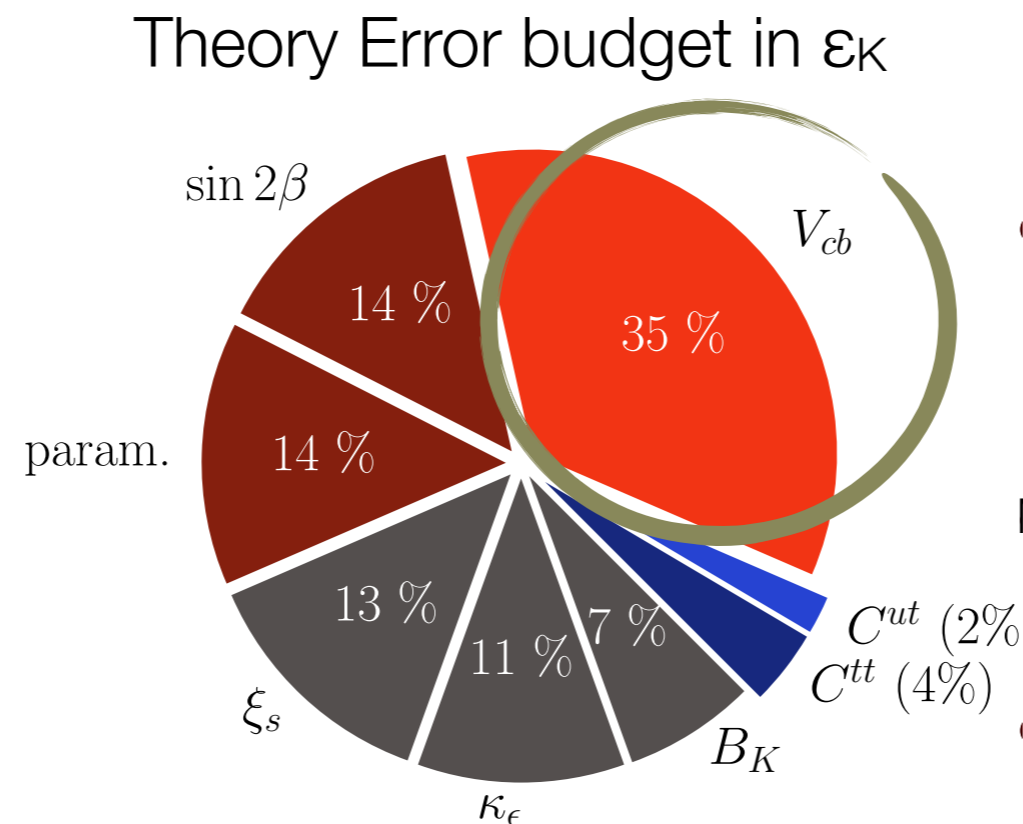
see also  
LHCb, 1808.08865  
Zupan & Brod, 1308.5663

# FCC-ee flavour physics benchmarks & explorations

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## Part 1: CKM determination

- Complementary measurements of  $|V_{cb}|$  (and  $|V_{ub}|$ )
- CKM fit requires knowledge of  $|V_{ub}/V_{cb}|$
- Th. predictions of CPV in K decays rely on  $|V_{cb}|$



M. Gorbahn @ Beauty 2020

# FCC-ee flavour physics benchmarks & explorations

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## Part 1: CKM determination

- Complementary measurements of  $|V_{cb}|$  (and  $|V_{ub}|$ )

▶ using  $B_{u,c} \rightarrow \mu\nu, \tau\nu$

$$Br(B^- \rightarrow \tau^- \bar{\nu}(\gamma))_{SM} = 1.13(1) \times 10^{-4} \left( \frac{f_B}{0.2\text{GeV}} \right)^2 \left( \frac{|V_{ub}|}{4 \times 10^{-3}} \right)^2$$
$$\left[ \frac{\Gamma(B^+ \rightarrow \tau^+ \nu)}{\Gamma(B_c^+ \rightarrow \tau^+ \nu)} \right]_{SM^*} = 0.782 \left| \frac{V_{ub} f_B}{V_{cb} f_{B_c}} \right|^2$$

- ▶ Theoretically cleaner compared to exclusive semileptonic decays

# FCC-ee flavour physics benchmarks & explorations

## Part 1: CKM determination

- Complementary measurements of  $|V_{cb}|$  (and  $|V_{ub}|$ )

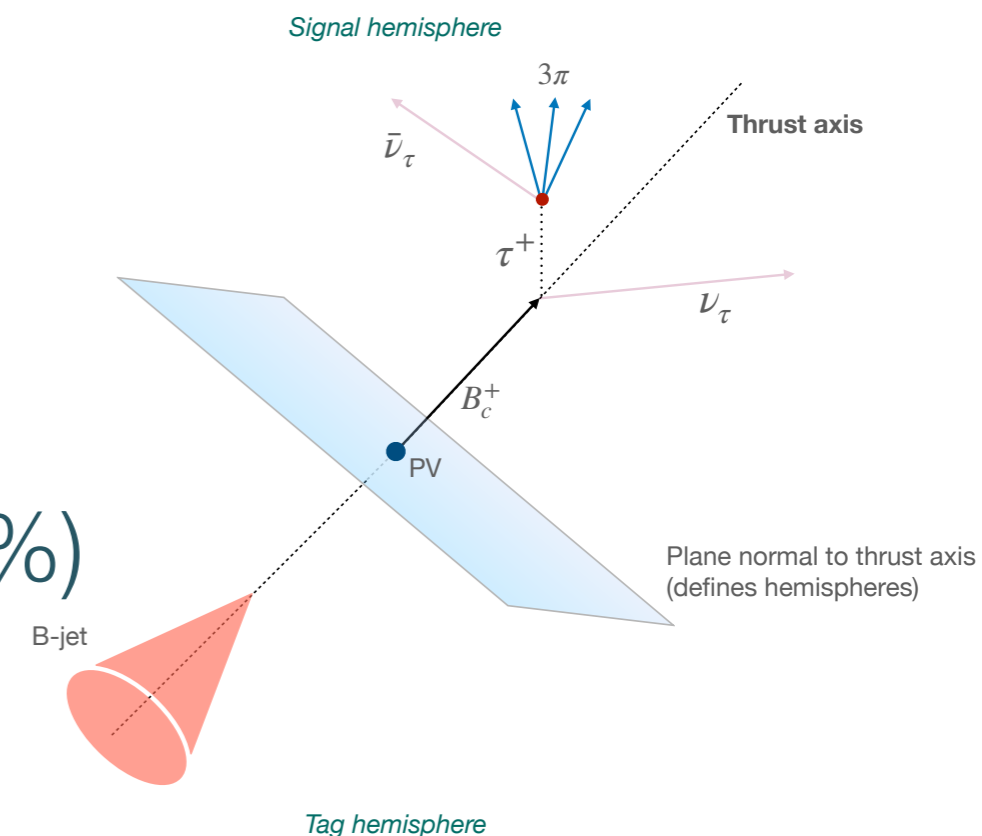
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- ▶ Exp. feasibility studies of  $B_c \rightarrow \tau\nu$ :  
important normalizing mode

$$\mathcal{B}(B_c^+ \rightarrow J/\psi \mu^+ \nu_\mu)$$

- ▶ relative signal yield precision O(few %)





# FCC-ee flavour physics benchmarks & explorations

## Part 1: CKM determination

- Complementary measurements of  $|V_{cb}|$  (and  $|V_{ub}|$ )

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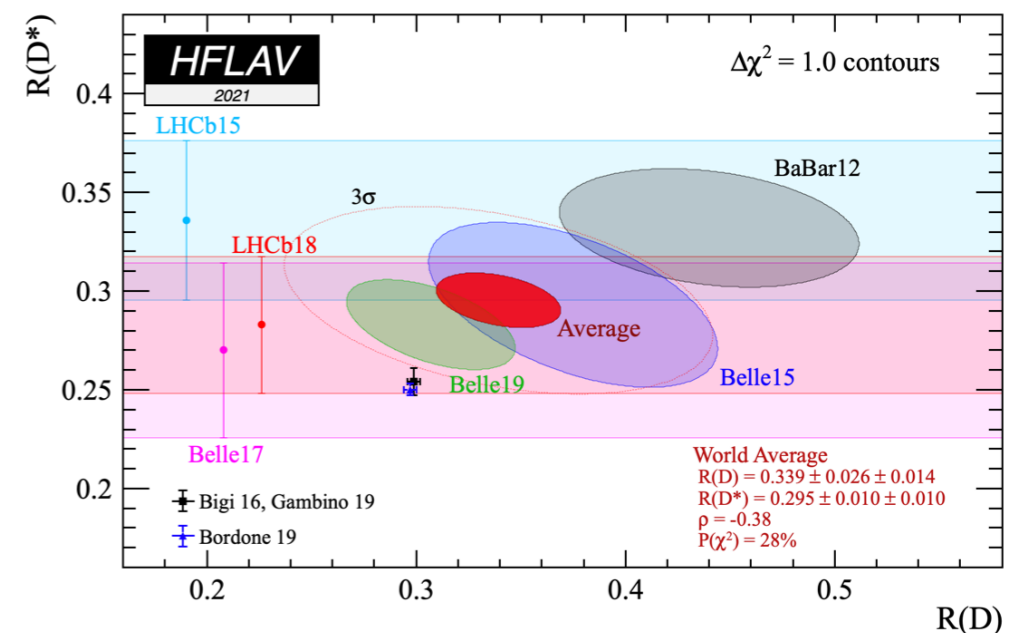
- ▶ alternative test of LFU in c.c.

B decays

Amhis et al., 2105.13330

Zheng et al., 2007.08234

see also LHCb, 1711.05623



$$R(D^{(*)}) \equiv \frac{\Gamma(B \rightarrow D^{(*)} \tau \nu)}{\Gamma(B \rightarrow D^{(*)} \ell \nu)}$$

# FCC-ee flavour physics benchmarks & explorations

## Part 1: CKM determination

- Complementary measurements of  $|V_{cb}|$  (and  $|V_{ub}|$ )

▶ using  $B_{u,c} \rightarrow \mu\nu, \tau\nu$

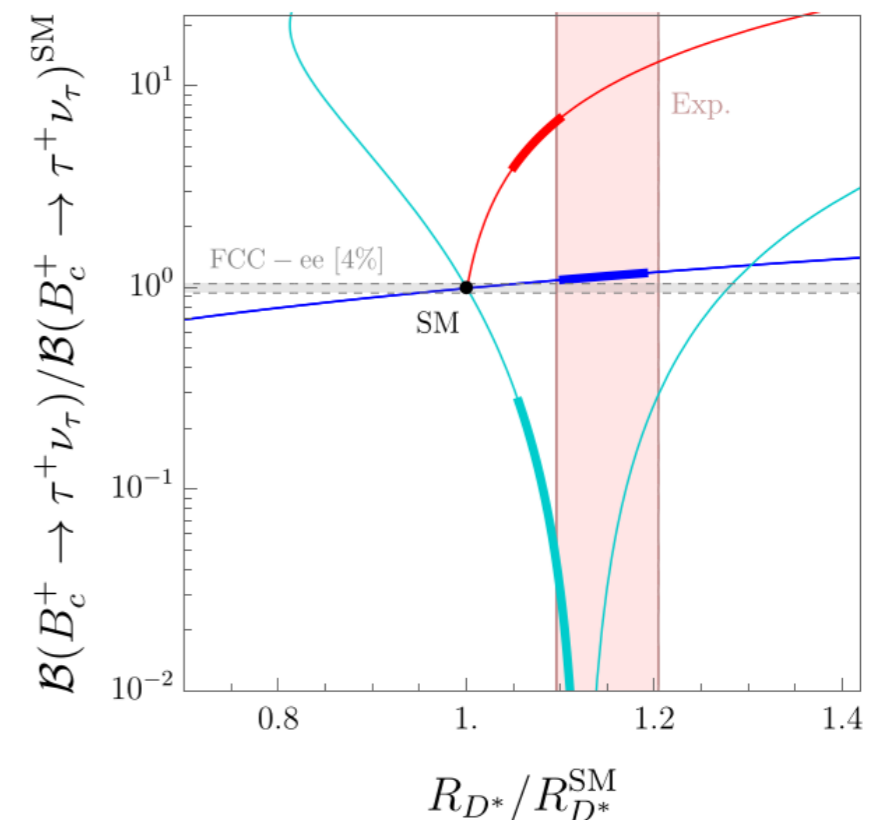
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- ▶ Exp. feasibility studies of  $B_c \rightarrow \tau\nu$ :  
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- ▶ alternative test of LFU in c.c.  
B decays



# FCC-ee flavour physics benchmarks & explorations

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## Part 1: CKM determination

- Complementary measurements of  $|V_{cb}|$  (and  $|V_{ub}|$ )
  - ▶ using  $B_{u,c} \rightarrow \mu\nu, \tau\nu$
  - ▶ using on-shell  $W \rightarrow cb$ 
    - ▶  $\sigma(e^+e^- \rightarrow W^+W^-) \sim 10\text{pb}$  (in energy range of FCC-ee)
    - ▶ With SM value of  $\mathcal{B}(W^+ \rightarrow c\bar{b}) \sim 10^{-3}$  a precision of  $\delta V_{cb}/V_{cb} \sim 0.1\%$  might be within reach...
    - ▶ Relies crucially on efficient c- and b-jet identification

# FCC-ee flavour physics benchmarks & explorations

## Part 2: CPV in $\Delta B=2$

Observable / Experiments	Current W/A	Belle II (50 /ab)	LHCb-U1 (23/fb)	FCC-ee
Mixing-related inputs				
$\sin(2\beta)$	$0.691 \pm 0.017$	$0.691 \pm 0.008$	$0.691 \pm 0.009$	
$\phi_s$ (uncert. rad $10^{-2}$ )	$-1.5 \pm 3.5$	n/a	$-3.65 \pm 0.05$	
$\Delta m_d$ ( $\text{ps}^{-1}$ )	$0.5065 \pm 0.0020$	same	same	
$\Delta m_s$ ( $\text{ps}^{-1}$ )	$17.757 \pm 0.021$	same	same	
$a_{\text{fs}}^d$ ( $10^{-4}$ , precision)	$23 \pm 26$	$-7 \pm 15$	$-7 \pm 15$	
$a_{\text{fs}}^s$ ( $10^{-4}$ , precision)	$-48 \pm 48$	n/a	$0.3 \pm 15$	

- Uncertainties in most  $\Delta B=2$  observables will start to be systematics/theory dominated

- Notable exceptions:  $\phi_s, a_{\text{fs}} = \frac{\Gamma(\bar{B}_q^0 \rightarrow B_q^0 \rightarrow f) - \Gamma(B_q^0 \rightarrow \bar{B}_q^0 \rightarrow \bar{f})}{\Gamma(\bar{B}_q^0 \rightarrow B_q^0 \rightarrow f) + \Gamma(B_q^0 \rightarrow \bar{B}_q^0 \rightarrow \bar{f})}$

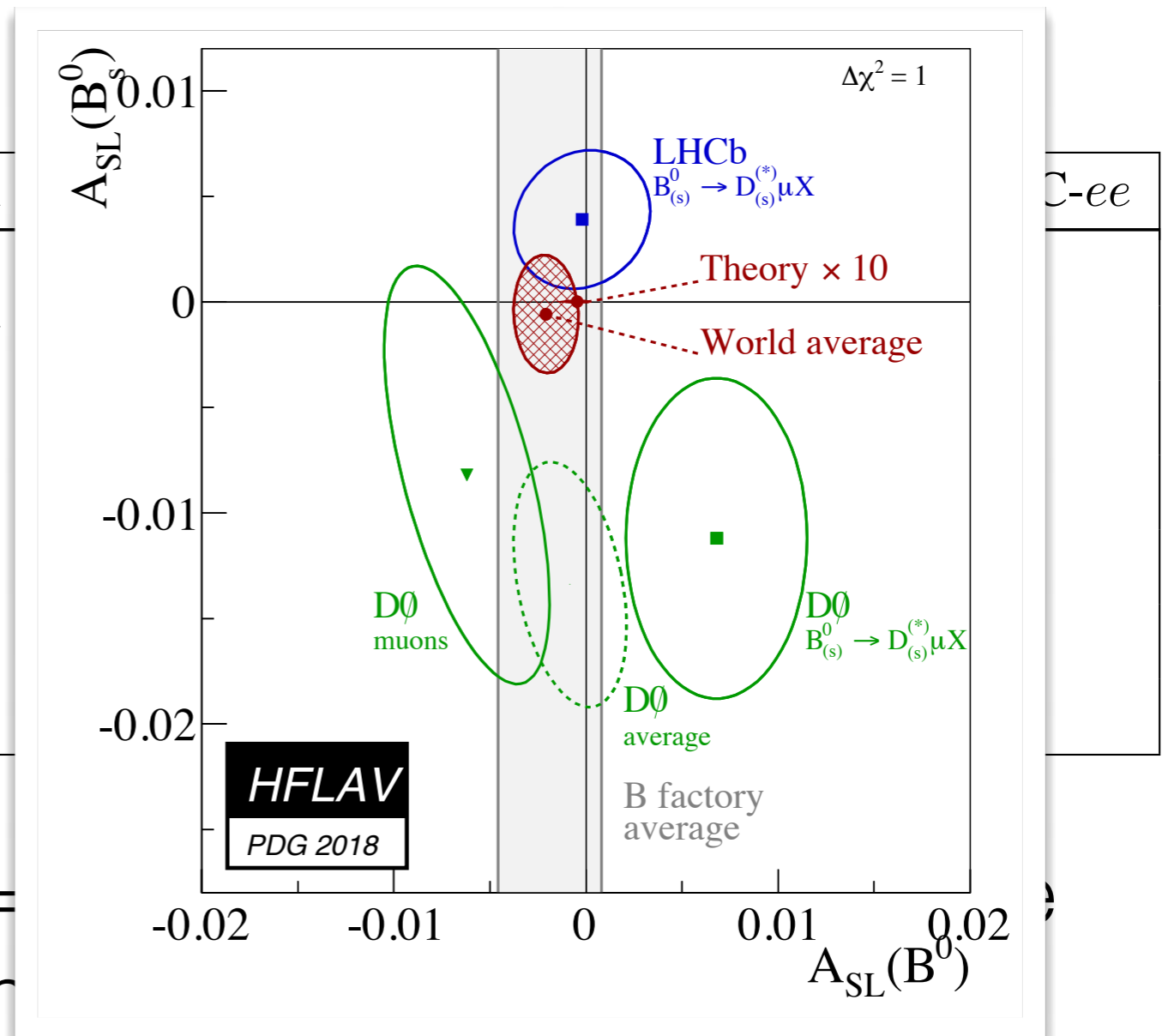
# FCC-ee flavour physics benchmarks & explorations

## Part 2: CPV in $\Delta B=2$

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- Uncertainties in most  $\Delta B=2$  systematics/theory dominant

- Notable exceptions:  $\phi_s, a_{\text{fs}} = \frac{\Gamma(\bar{B}_q^0 \rightarrow B_q^0 \rightarrow f) - \Gamma(B_q^0 \rightarrow \bar{B}_q^0 \rightarrow \bar{f})}{\Gamma(\bar{B}_q^0 \rightarrow B_q^0 \rightarrow f) + \Gamma(B_q^0 \rightarrow \bar{B}_q^0 \rightarrow \bar{f})}$



# FCC-ee flavour physics benchmarks & explorations

## Part 2: CPV in $\Delta B=2$

Observable / Experiments	Current W/A	Belle II (50 /ab)	LHCb-U1 (23/fb)	FCC-ee
Mixing-related inputs				
$\sin(2\beta)$	$0.691 \pm 0.017$	$0.691 \pm 0.008$	$0.691 \pm 0.009$	$0.691 \pm 0.005$
$\phi_s$ (uncert. rad $10^{-2}$ )	$-1.5 \pm 3.5$	n/a	$-3.65 \pm 0.05$	$-3.65 \pm 0.01$
$\Delta m_d$ ( $\text{ps}^{-1}$ )	$0.5065 \pm 0.0020$	same	same	same
$\Delta m_s$ ( $\text{ps}^{-1}$ )	$17.757 \pm 0.021$	same	same	same
$a_{fs}^d$ ( $10^{-4}$ , precision)	$23 \pm 26$	$-7 \pm 15$	$-7 \pm 15$	$-7 \pm 2$
$a_{fs}^s$ ( $10^{-4}$ , precision)	$-48 \pm 48$	n/a	$0.3 \pm 15$	$0.3 \pm 2$

- Significant improvement in both observables @FCC-ee
- Observation of CPV in  $B_d$  mixing possible ( $a_{fs}$ )
- Not included ( $\alpha_s, \gamma_s, \beta_s$ ) from ( $B_s \rightarrow D_s K, B \rightarrow DK, B_s \rightarrow \phi\phi$ )

S. Monteil @ FCC week 2019

Aleksan, Oliver & Perez, 2107.02002, 2107.05311

talks by R. Aleksan, E.Perez

# FCC-ee flavour physics benchmarks & explorations

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Combining 1 & 2: Impact on CPV BSM in  $\Delta B=2$

Model-independent parametrization of BSM in  $\Delta F=2$

$$\langle B_q | \mathcal{H}_{\Delta B=2}^{\text{SM+NP}} | \bar{B}_q \rangle = \langle B_q | \mathcal{H}_{\Delta B=2}^{\text{SM}} | \bar{B}_q \rangle (1 + h_q e^{i\sigma_q})$$

Assumptions:

- ▶ NP only at short distances (UV)
- ▶ CKM unitary (& determined from NP free obs.)

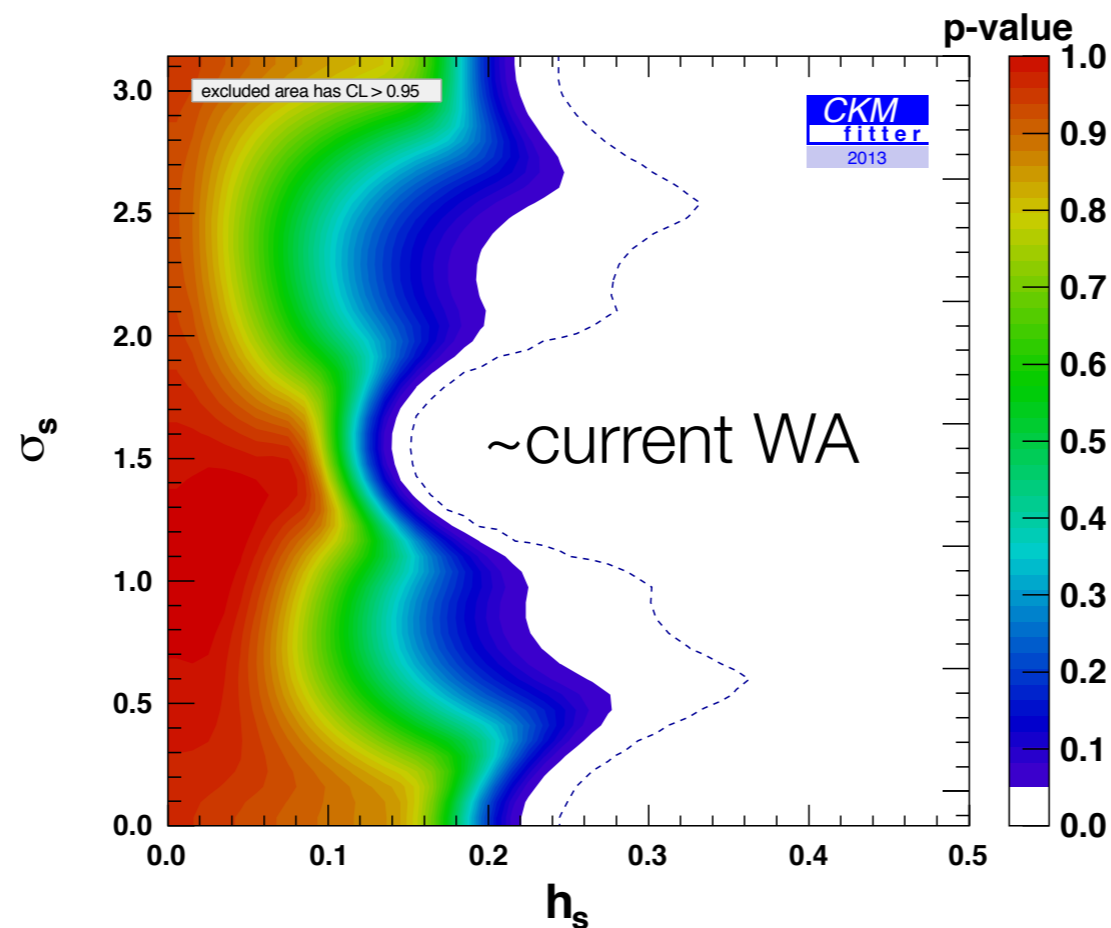
# FCC-ee flavour physics benchmarks & explorations

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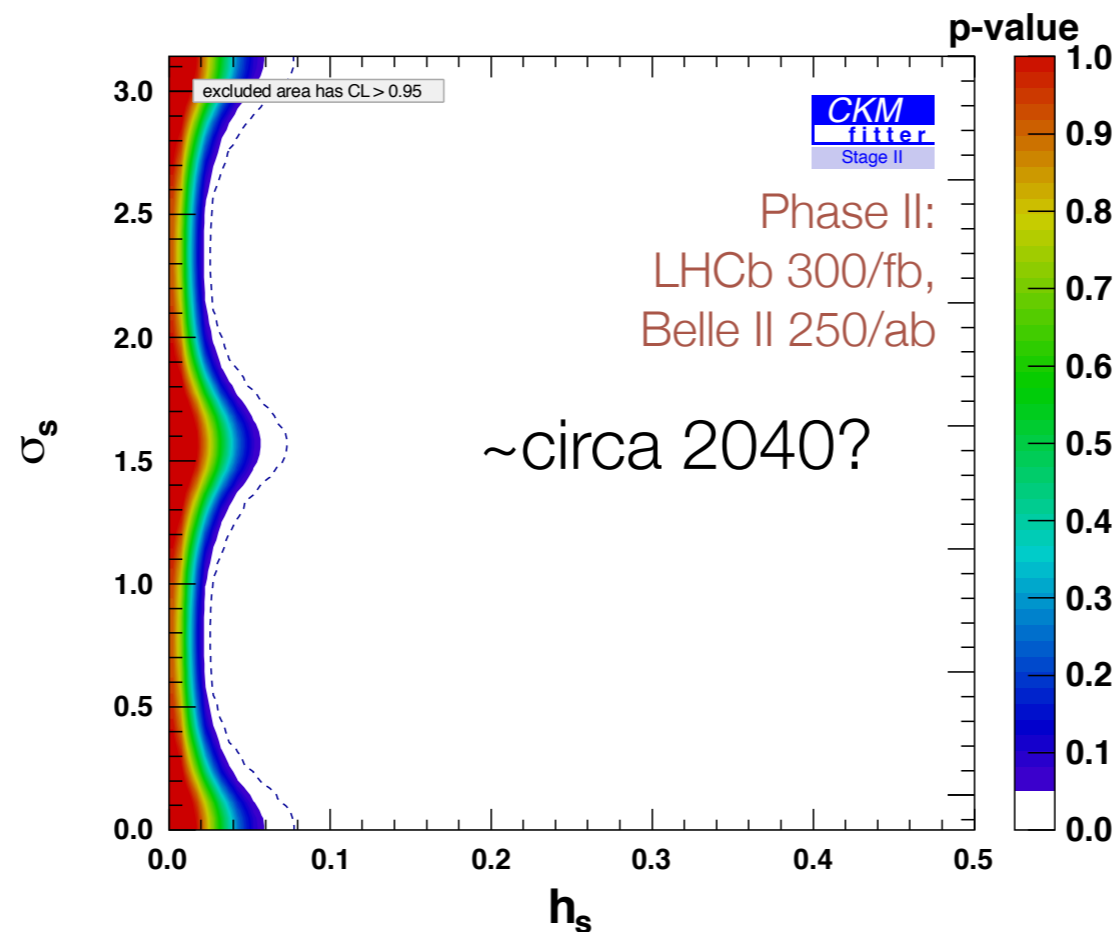


# FCC-ee flavour physics benchmarks & explorations

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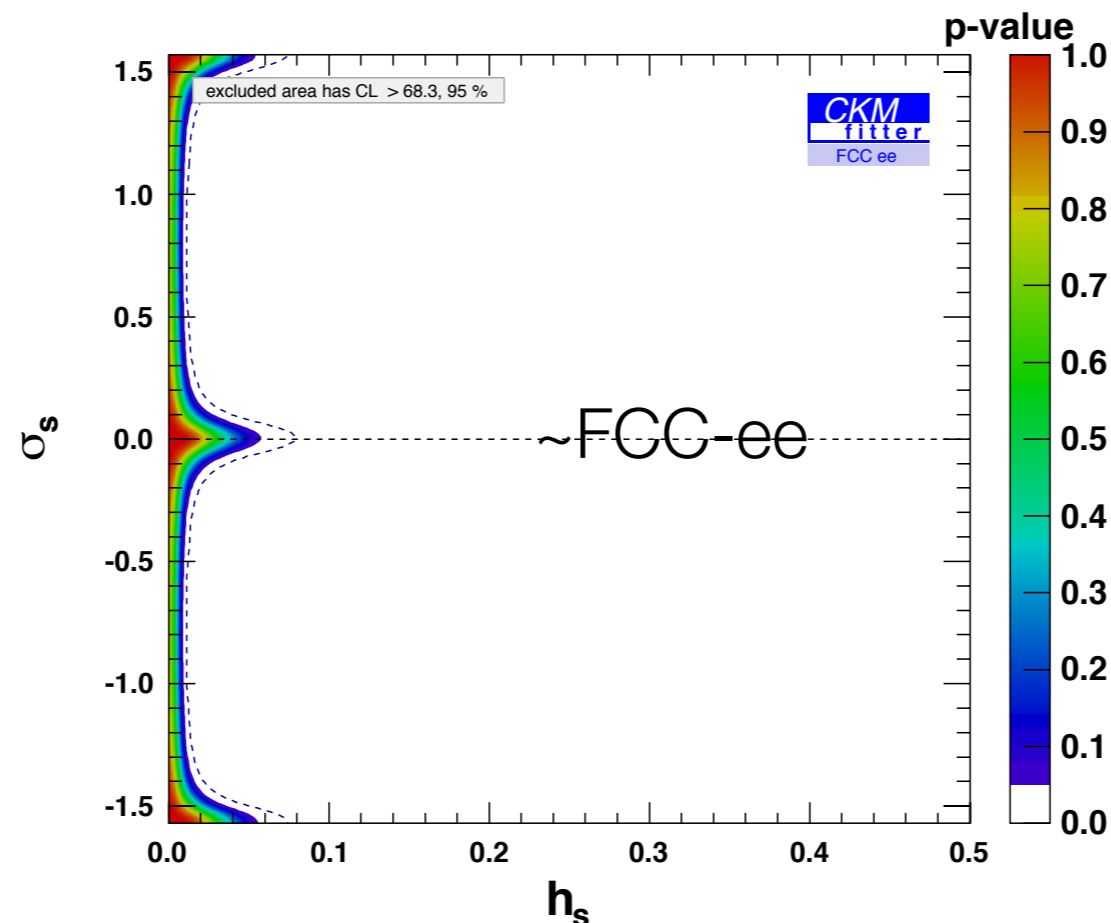
\*TH progress (lattice)

# FCC-ee flavour physics benchmarks & explorations

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Model-independent parametrization of BSM in  $\Delta F=2$

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# FCC-ee flavour physics benchmarks & explorations

Combining 1 & 2: Impact on CPV BSM in  $\Delta B=2$

Model-independent parametrization of BSM in  $\Delta F=2$

$$\langle B_q | \mathcal{H}_{\Delta B=2}^{\text{SM+NP}} | \bar{B}_q \rangle = \langle B_q | \mathcal{H}_{\Delta B=2}^{\text{SM}} | \bar{B}_q \rangle (1 + h_q e^{i\sigma_q})$$

In terms of UV sensitivity

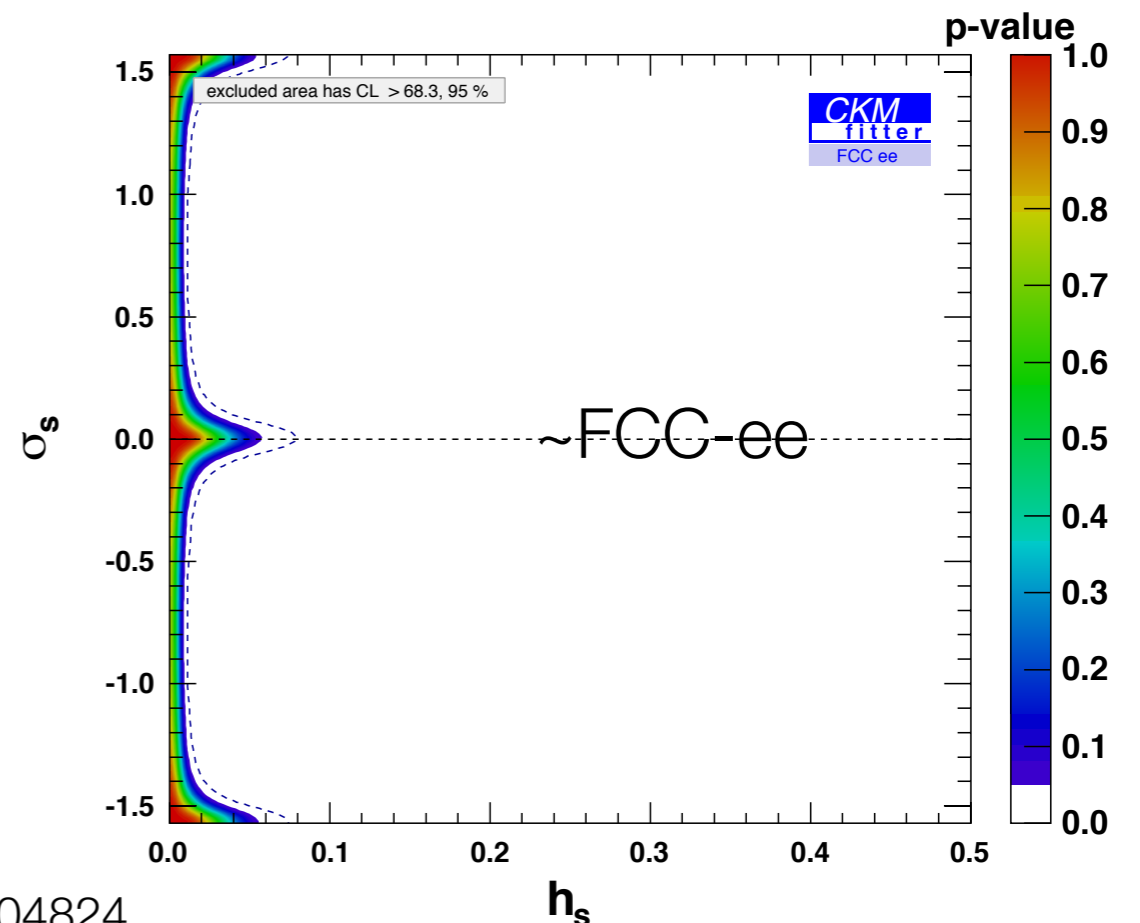
$$h \simeq 1.5 \frac{|C_{ij}|^2}{|\lambda_{ij}^t|^2} \frac{(4\pi)^2}{G_F \Lambda^2} \simeq \frac{|C_{ij}|^2}{|\lambda_{ij}^t|^2} \left( \frac{4.5 \text{ TeV}}{\Lambda} \right)^2,$$

$$\sigma = \arg(C_{ij} \lambda_{ij}^{t*}),$$

For CKM like (MFV) :  $\Lambda > 20 \text{ TeV}$

$$C_{ij} = \lambda_{ij}^t \equiv V_{ti} V_{tj}^*$$

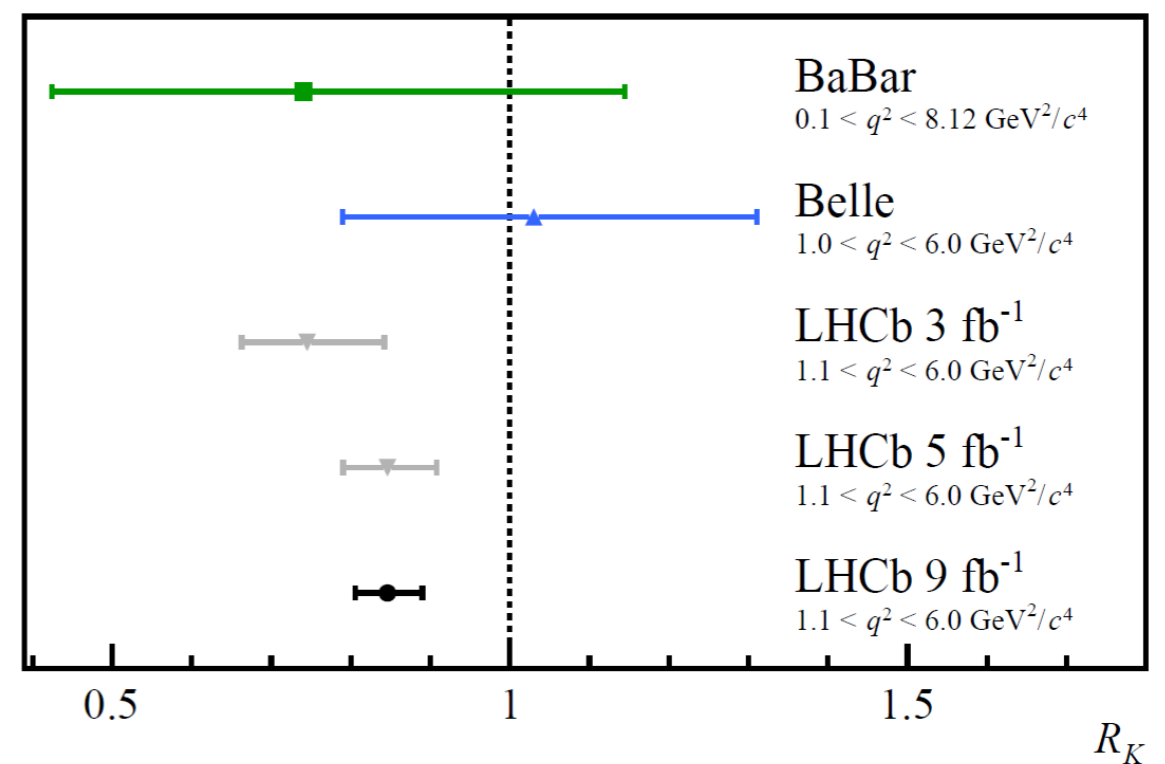
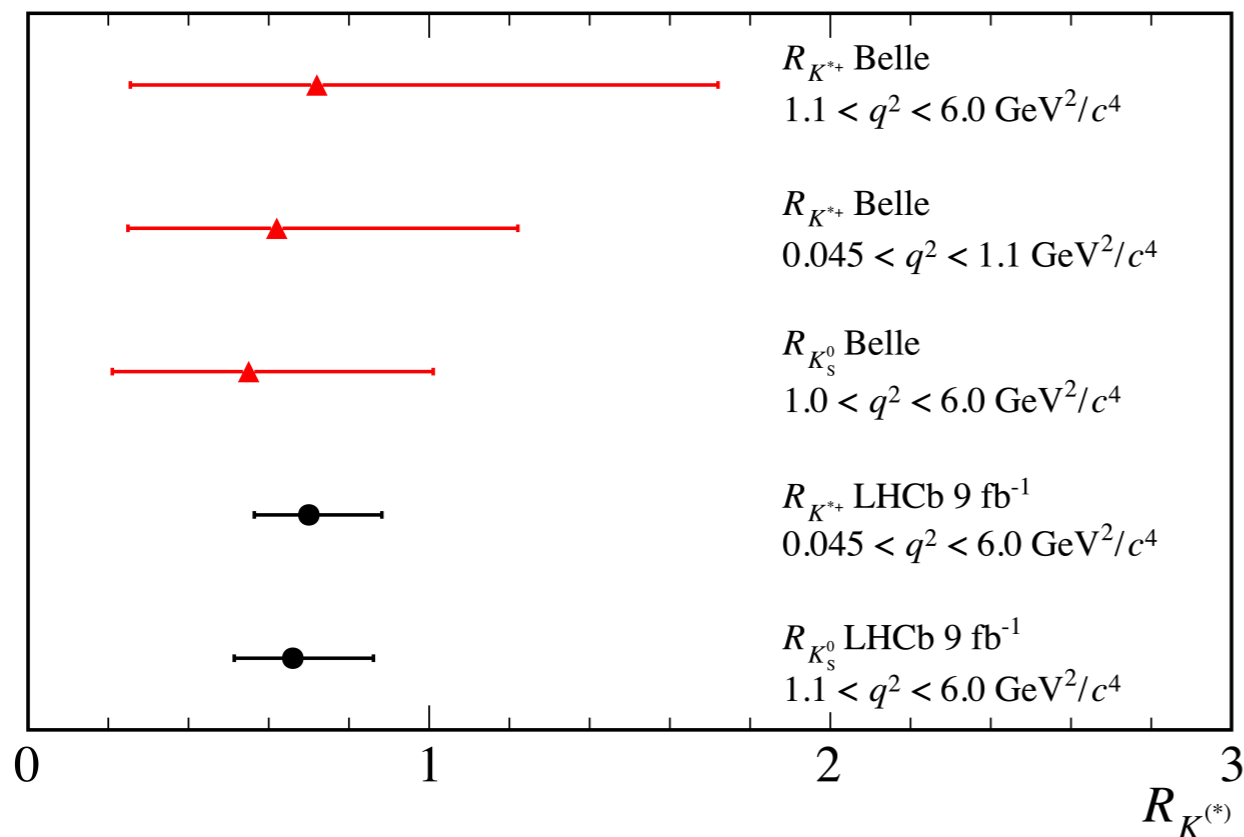
see also Charles et al., 2006.04824



# FCC-ee flavour physics benchmarks & explorations

## Part 3: Rare b-hadron decays to taus

Motivated by current intriguing exp. situation in rare B decays



$$R_H \equiv \frac{\int_{q_{\min}^2}^{q_{\max}^2} \frac{d\mathcal{B}(B \rightarrow H \mu^+ \mu^-)}{dq^2} dq^2}{\int_{q_{\min}^2}^{q_{\max}^2} \frac{d\mathcal{B}(B \rightarrow H e^+ e^-)}{dq^2} dq^2}$$

# FCC-ee flavour physics benchmarks & explorations

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## Part 3: Rare b-hadron decays to taus

Motivated by current intriguing exp. situation in rare B decays

- FCC-ee (unique) probe of SM predictions for  $B \rightarrow K^{(*)} \tau^+ \tau^-$

$$R_{K^+}^{\mu\tau} = 0.87 \pm 0.02 \quad , \quad R_{K^0}^{\mu\tau} = 0.87 \pm 0.02 \quad , \quad 15 \text{ GeV}^2 < q^2 < 22 \text{ GeV}^2 \quad \text{J.F.K. et al., 1705.11106}$$
$$R_{K^{*+}}^{\mu\tau} = 2.44 \pm 0.09 \quad , \quad R_{K^{*0}}^{\mu\tau} = 2.45 \pm 0.08 \quad , \quad 15 \text{ GeV}^2 < q^2 < 19.2 \text{ GeV}^2. \quad \text{Li \& Liu, 2012.00665}$$

+ Complete kinematical reconstruction

- ▶ Access to angular observables, tau polarization
- ▶ Detailed study of backgrounds & detector requirements in progress

see talk by T. Miralles

# FCC-ee flavour physics benchmarks & explorations

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## Part 3: Rare b-hadron decays to taus

Motivated by current intriguing exp. situation in rare B decays

- FCC-ee (unique) probe of SM predictions for  $B \rightarrow K^{(*)} \tau^+ \tau^-$

J.F.K. et al., 1705.11106  
Li & Liu, 2012.00665

- Potentially also complementary leptonic mode  $B_{(s)} \rightarrow \tau^+ \tau^-$

$$\text{BR}(B_s \rightarrow \tau^+ \tau^-) = (7.73 \pm 0.49) \times 10^{-7}$$

$$\text{BR}(B_d \rightarrow \tau^+ \tau^-) = (2.22 \pm 0.19) \times 10^{-8}$$

Bobeth et al., 1311.0903  
see also  
U. Haisch, 1206.1230

(Expected sensitivity at Belle II to BRs of  $O(10^{-4}) \sim O(10^{-5})$ )

see also  
LHCb-CONF-2016-011

# FCC-ee flavour physics benchmarks & explorations

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## Part 3: Rare b-hadron decays to taus, **neutrinos**

Motivated by current intriguing exp. situation in rare B decays

- FCC-ee (unique) probe of SM predictions for  $B \rightarrow K^{(*)} \tau^+ \tau^-$

J.F.K. et al., 1705.11106  
Li & Liu, 2012.00665

- Potentially also complementary leptonic mode  $B_{(s)} \rightarrow \tau^+ \tau^-$

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Bobeth et al., 1311.0903  
see also  
U. Haisch, 1206.1230

(Expected sensitivity at Belle II to BRs of  $O(10^{-4}) \sim O(10^{-5})$ )

see also  
LHCb-CONF-2016-011

- Potential improvement also on  $B \rightarrow K^{(*)} \nu \nu$  (beyond Belle II)

see talk by M. Kenzie

# FCC-ee flavour physics benchmarks & explorations

## Part 4: Flavour violating Z decays

- Lepton flavour violating processes  $Z \rightarrow e\tau, \tau\mu$ 
  - ▶ complementary to lepton flavour violating  $\tau$  decays

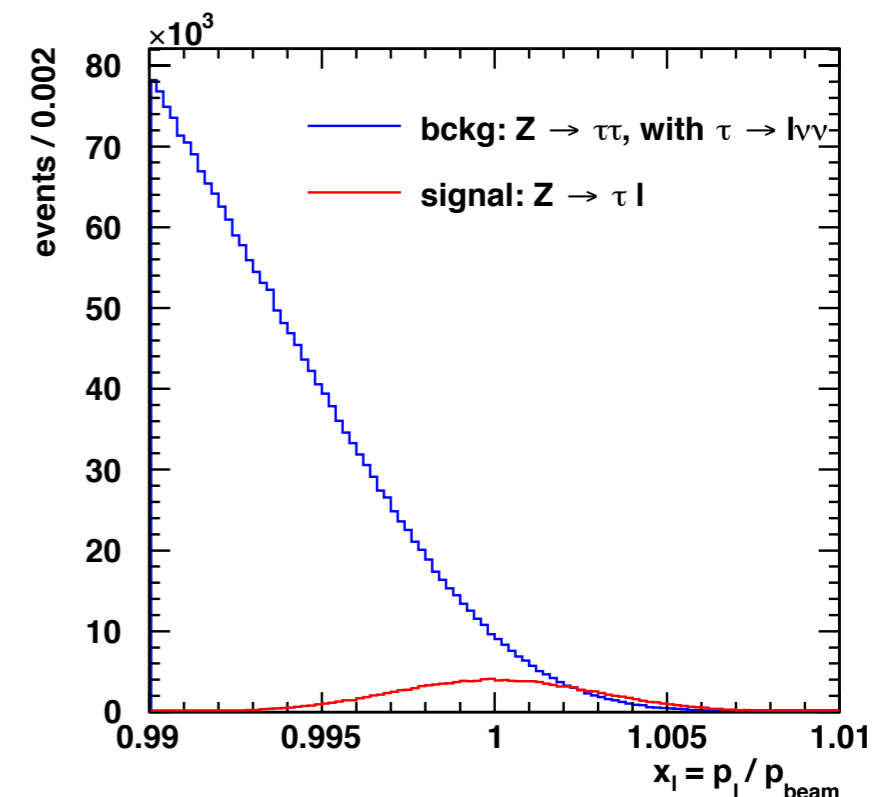
- ▶ FCC-ee feasibility study:

see e.g. Illiana & Masip, hep-ph/0207328  
L. Calibbi, X. Marcano, J. Roy, 2107.10273  
(& talk by X. Marcano)

$$\mathcal{B}(Z \rightarrow \tau^\pm \mu^\mp) < 10^{-9} - 10^{-10}$$

- ▶ would improve current limits by at least 3 orders of magnitude

M. Dam, SciPostPhys.Proc.1,041(2019)  
see also De Romeri et al. JHEP 1504 (2015) 051





# FCC-ee flavour physics benchmarks & explorations

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## Part 4: Flavour violating Z decays

- Lepton flavour violating processes  $Z \rightarrow e\tau, \tau\mu$
- Similar possibility in the quark sector? i.e.  $Z \rightarrow j_b j_{\bar{b}}$

- Motivation: Probing FCNC Z-penguins directly

c.f. Isidori & Giudagnoli, 1302.3909

- In SM  $\mathcal{B}(Z \rightarrow s\bar{b}) \sim 10^{-8}$  - could be probed to 1% level?

Need very efficient b-, s-tagging!

	no PID	$\epsilon_K = 95\%$	$\epsilon_K = 90\%$
$\epsilon_s$	$7 \times 10^{-2}$	$4 \times 10^{-2}$	$3 \times 10^{-2}$
$\epsilon_b$	$3 \times 10^{-4}$	$4 \times 10^{-5}$	$5 \times 10^{-5}$

Preliminary study in context of LC

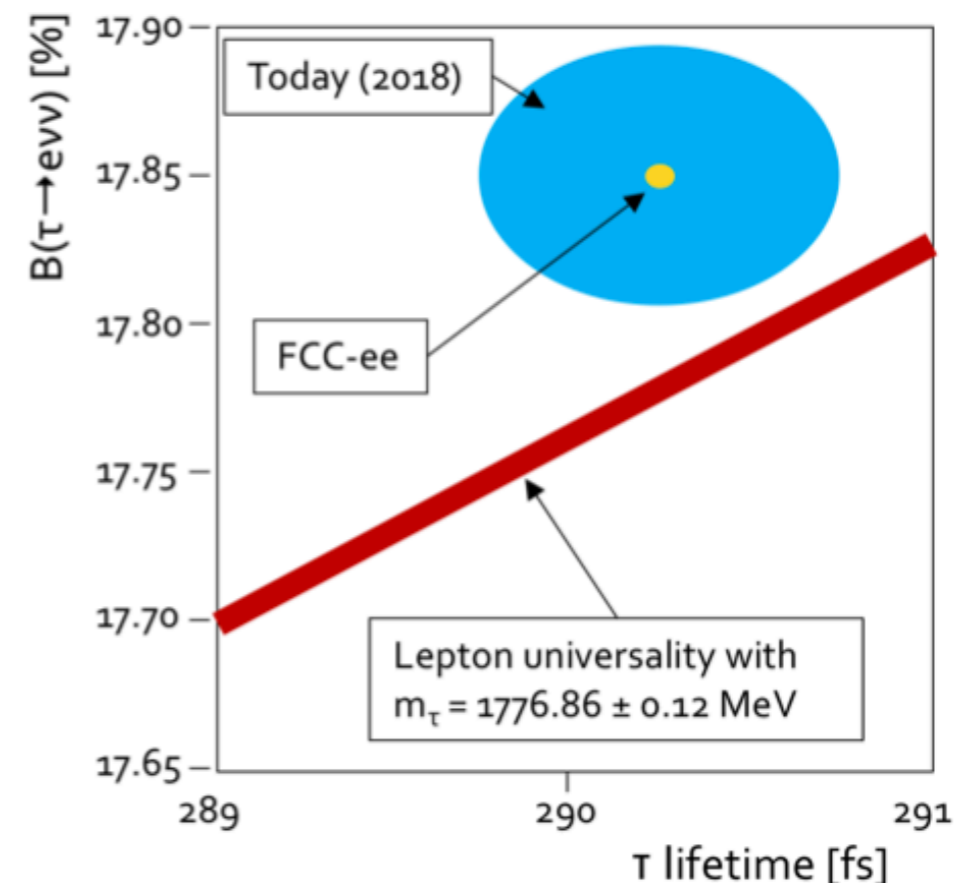
Duarte-Campderros et al., 1811.09636

# FCC-ee flavour physics benchmarks & explorations

## Part 5: tau physics

- Several tau properties and decays could be determined more precisely
- Interesting implications for LFU (in tau decays) & CKM unitarity  
see e.g.  
Allwicher, Isidori, Semilovic, 2109.03833  
M. Dam, SciPostPhys.Proc.1,041(2019)  
Eur. Phys. J. Plus 136, 963 (2021)  
talk by A. Lusiani
- Would possibly require improved calculations

Property	Current WA	FCC-ee stat	FCC-ee syst
Mass [MeV]	1776.86 +/- 0.12	0.004	0.1
Electron BF [%]	17.82 +/- 0.05	0.0001	0.003
Muon BF	17.39 +/- 0.05	0.0001	0.003
Lifetime [fs]	290.3 +/- 0.5	0.005	0.04



# Conclusions

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- FCC-ee could be a powerful and competitive probe of flavour physics post-2030
  - FCC-ee can compete favourably with ultimate precision of LHCb and Belle II
  - There are processes for which FCC-ee is unique
  - Luminosity is key. Many measurements reported here are statistically limited?!

# Conclusions

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- FCC-ee could be a powerful and competitive probe of flavour physics post-2030
- Effort underway to understand exp. precision with which rare decays of c- and b-hadrons and CP violation in heavy-quark sector & LFV processes could be measured
  - Flavour Physics defines shared (vertexing, tracking, calorimetry) and specific (hadronic PID) detector requirements
  - In the coming phase important interplay of physics performance with detector concepts
  - On theory side: go beyond benchmark modes — fill in possible gaps

# Conclusions

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- FCC-ee could be a powerful and competitive probe of flavour physics post-2030
- Effort underway to understand exp. precision with which rare decays of c- and b-hadrons and CP violation in heavy-quark sector & LFV processes could be measured
- Less explored areas include flavour studies using top & Higgs decays, spectroscopy, quarkonium physics, flavor conversion @ high-pT
- Examples: top & Higgs decays to exclusive hadronic final states see e.g. Mangano & Melia, 1410.7475  
dilepton spectra @ FCC-hh Kagan et al., 1406.1722  
see e.g. Garland et al., 2112.05127 talk by S. Jager  
Konig et al., 1505.03870