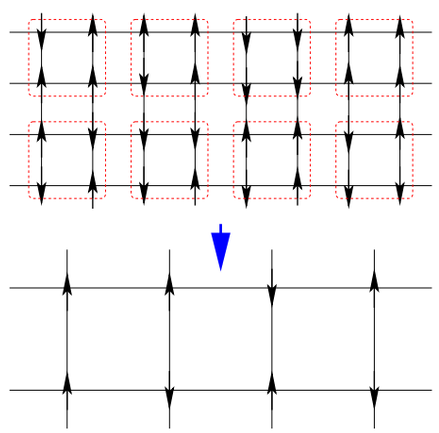
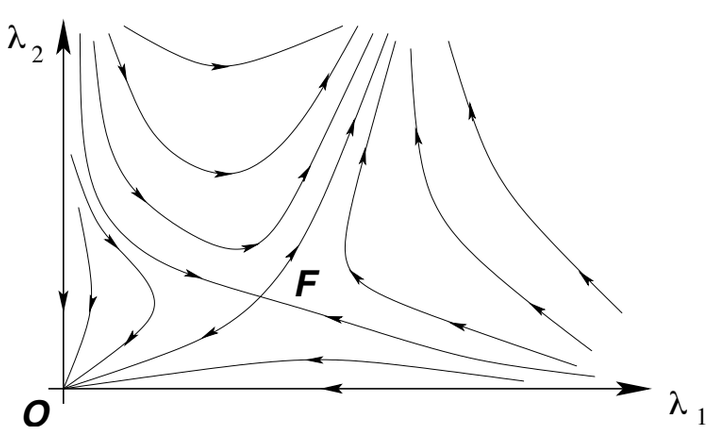
Scale invariance, renormalisation

$\times\, \lambda^{-1}$





Renormalisation deals with the scale dependence of the the physics even if the original theory is scale invariant.

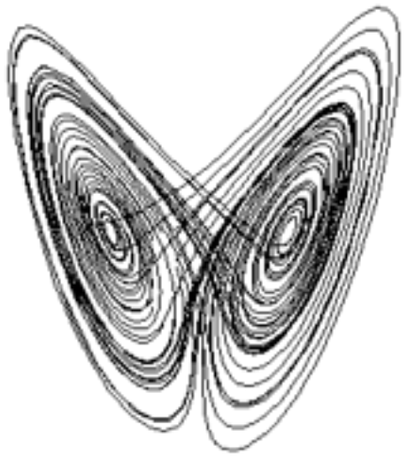
Virtual phenomena can get more complicated or simplify as we move to larger and shorter distances

$$x^\mu \longrightarrow \lambda x^\mu, \qquad \phi(x) \longrightarrow \lambda^{-\Delta}\phi(\lambda^{-1}x),$$

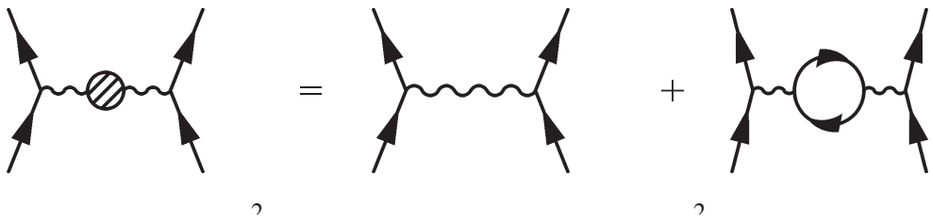
$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{g}{4!}\phi^4, \qquad \mathscr{L} \longrightarrow \lambda^{-4}\mathscr{L}[\phi]$$

$$H \xrightarrow{\mathscr{R}} H^{(1)} \xrightarrow{\mathscr{R}} H^{(2)} \xrightarrow{\mathscr{R}} \ldots \xrightarrow{\mathscr{R}} H_\star.$$

In relativistic QFT we seem to get only fixed points, no limit cycles nor strange attractors



Fixed points beta functions



$$\eta_{\alpha\beta}\left(\bar{v}_e\gamma^\alpha u_e\right)\left\{\frac{e^2}{4\pi q^2}\left[1+\frac{e^2}{12\pi^2}\log\left(\frac{q^2}{\Lambda^2}\right)\right]\right\}\left(\bar{v}_\mu\gamma^\beta u_\mu\right)$$

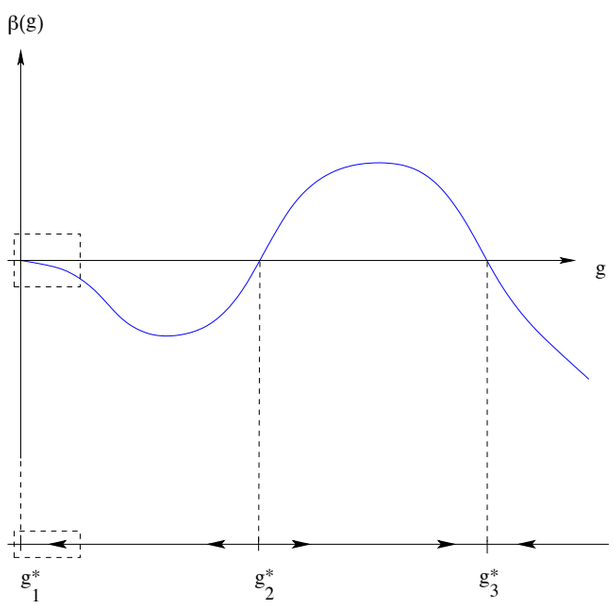
$$e(\mu)^2 = e(\Lambda)^2_{\text{bare}}\left[1+\frac{e(\Lambda)^2_{\text{bare}}}{12\pi^2}\log\left(\frac{\mu^2}{\Lambda^2}\right)\right]$$

$$\beta(g) = \mu\frac{dg}{d\mu}.$$

$$\beta(e)_{\text{QED}} = \frac{e^3}{12\pi^2}.$$

$$\beta(g) = -\frac{g^3}{16\pi^2}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right).$$

At one loop



$$\beta(g^*) = 0.$$

IR free (QED)

$$\beta'(g)|_{g^*} > 0 \quad , \qquad \mu\frac{dg}{d\mu} = \beta'(g-g^*)+\dots$$

$$\mu\uparrow \quad , \qquad g\uparrow$$

UV free (QCD)

$$\beta'(g)|_{g^*} < 0 \quad , \qquad \mu\frac{dg}{d\mu} = \beta'(g-g^*)+\dots$$

$$\mu\uparrow \quad , \qquad g\downarrow$$

There is a dynamically generated scale responsible for most of the mass of the nucleons

$$\langle \mathbf{p}^2\rangle = \Lambda^2_{\text{QCD}}. \qquad \Lambda_{\text{QCD}} \gg m_u, m_d$$

Farewell

- QFT is a vast and complex subject
- SM is a big achievement
- It summarises our knowledge of the fundamental laws of Nature
- But also our ignorance
- Many puzzles and unanswered questions remain
- We may be at the end of a cycle. Perhaps the symmetry paradigm has been exhausted.
- Naturalness, a red herring? Higgs or not Higgs
- Gravity into the picture finally?
- Hopefully we are entering a golden decade

Thank you