Flavour Physics & CP Violation

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Quarks
Leptons
Bosons
up
down
electron
neutrino e
photon
charm
strange
muon
neutrino μ
gluon
Z0 W±
top
beauty
tau
neutrino τ
Higgs

Flavour Structure of the Standard Model

$$\begin{pmatrix} u & \nu_e \\ d & e^- \end{pmatrix} , \quad \begin{pmatrix} c & \nu_\mu \\ s & \mu^- \end{pmatrix} , \quad \begin{pmatrix} t & \nu_\tau \\ b & \tau^- \end{pmatrix}$$

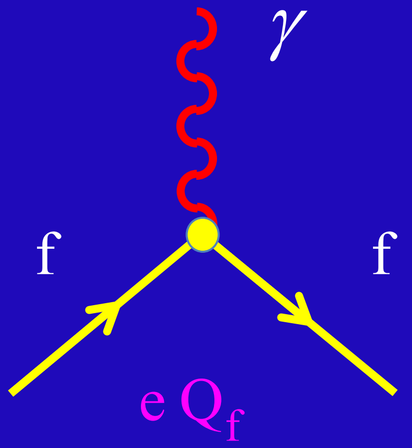
Why 3 ?

- Pattern of masses
- Flavour Mixing
- $\not{CP}$

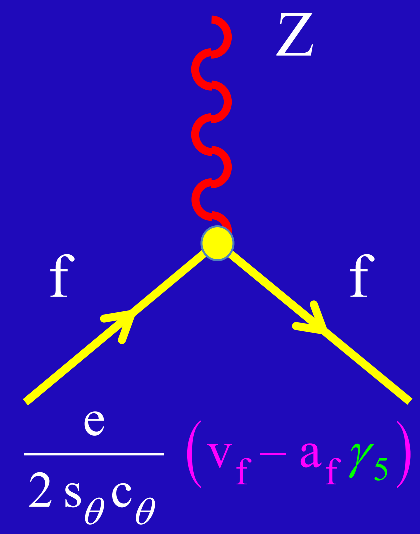
⟷ Related to SSB

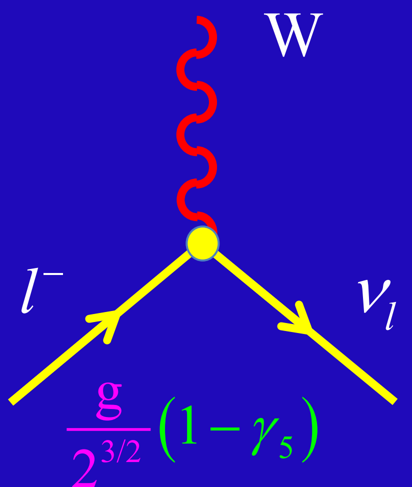
Scalar Sector (Higgs)

- **Kaon Factories :** u , d , s
- **τcF :** c , τ
- **BF, SuperB :** b , c , τ
- **LHC :** t , b
- **LC :** t , …
- **νF :** ν_e , ν_μ , ν_τ

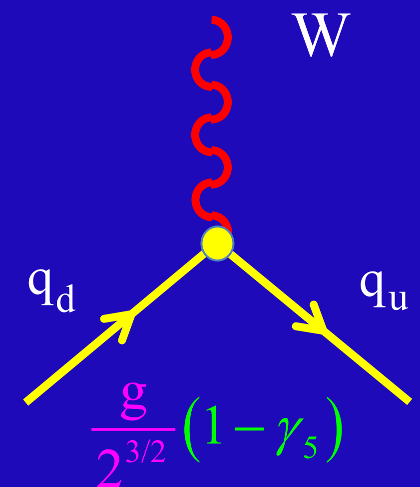


NEUTRAL CURRENTS





CHARGED CURRENTS



Three Families $\begin{bmatrix} \nu_e & u \\ e^- & d' \end{bmatrix} \quad , \quad \begin{bmatrix} \nu_\mu & c \\ \mu^- & s' \end{bmatrix} \quad , \quad \begin{bmatrix} \nu_\tau & t \\ \tau^- & b' \end{bmatrix}$

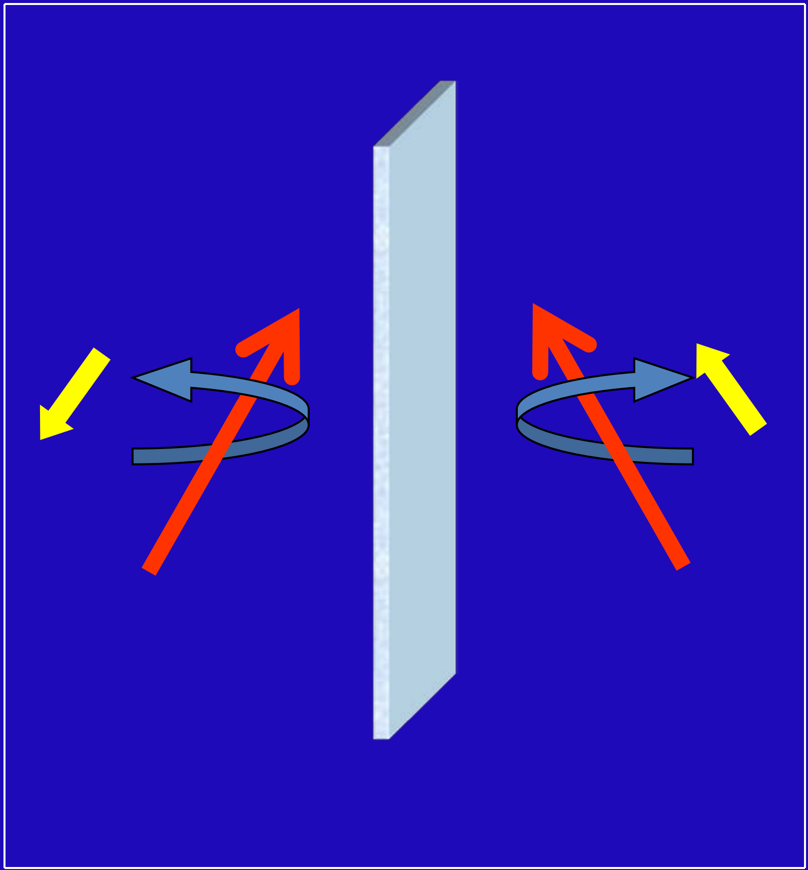
Family Structure

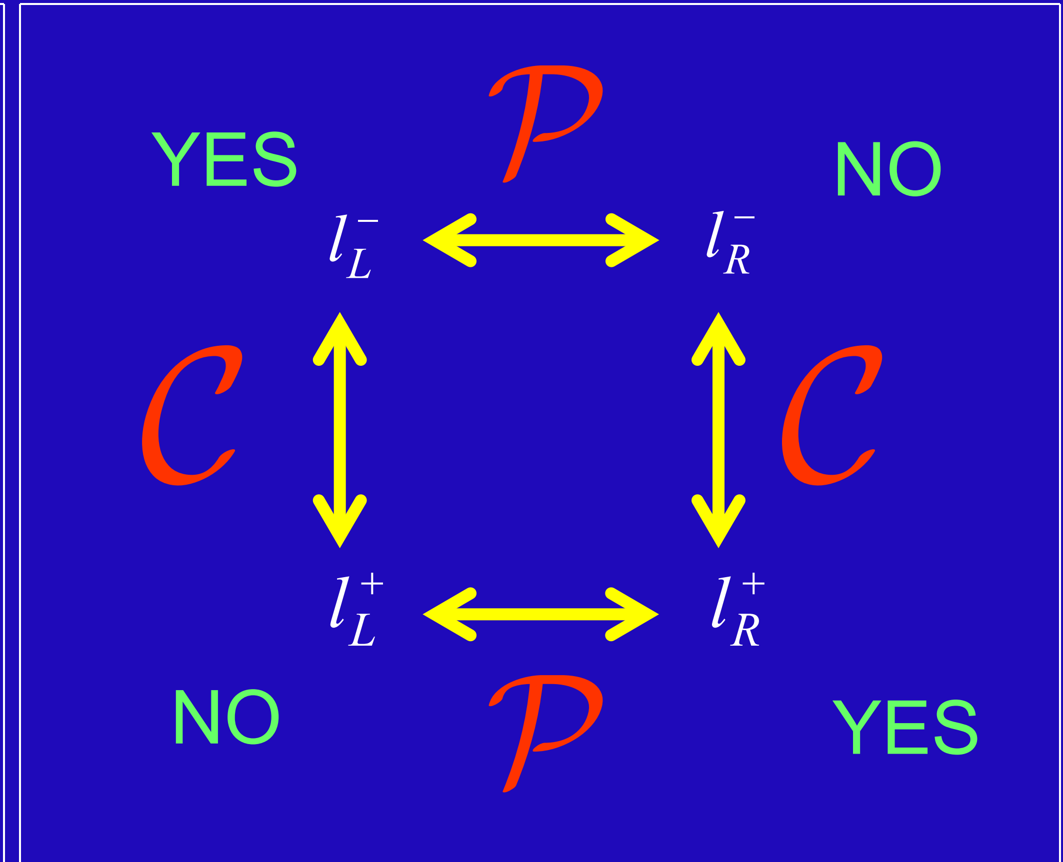
$$\begin{bmatrix} \nu_l & q_u \\ l^- & q_d \end{bmatrix} \equiv \left\{ \begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L , (\nu_l)_R , l_R^- \right\} ; \left\{ \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L , (q_u)_R , (q_d)_R \right\}$$

Charged Currents $W^\pm$ $\begin{cases} \text{Left–handed Fermions only} \\ \text{Flavour Changing: } \nu_l \Leftrightarrow l \; , \; q_u \Leftrightarrow q_d \end{cases}$

Neutral currents $\gamma \, , \, Z$ Flavour Conserving

Universality Family – Independent Couplings





$\not{\mathcal{P}}$ **and** $\not{\mathcal{C}}$ **in Weak Interactions**

$\mathcal{CP}$ **still a good symmetry** **(1 family)**

FERMION MASSESS

Scalar – Fermion Couplings allowed by Gauge Symmetry

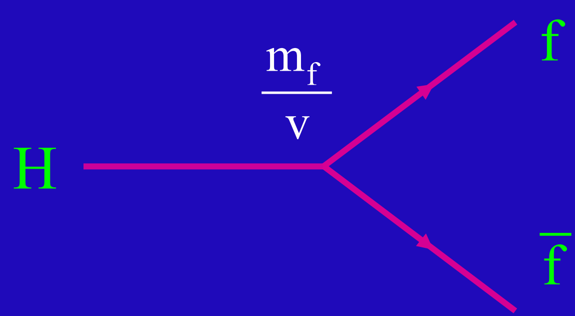
$$\mathcal{L}_Y = -(\bar{q}_u, \bar{q}_d)_L \left[c^{(d)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} (q_d)_R + c^{(u)} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} (q_u)_R \right] - (\bar{\nu}_l, \bar{l})_L \; c^{(l)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l_R + \text{h.c.}$$

SSB

$$\mathcal{L}_Y = -\left(1 + \frac{H}{\mathrm{v}}\right) \left\{ m_{q_d} \; \bar{q}_d \, q_d + m_{q_u} \; \bar{q}_u \, q_u + m_l \; \bar{l} \, l \right\}$$

Fermion Masses are New Free Parameters

$$\left[m_{q_d}, \; m_{q_u}, \; m_l \right] = \left[c^{(d)}, \; c^{(u)}, \; c^{(l)} \right] \frac{\mathrm{v}}{\sqrt{2}}$$



Couplings Fixed: $g_{Hf\bar{f}} = \frac{m_f}{v}$

FERMION GENERATIONS

$N_G = 3$ **Identical Copies** **Masses are the only difference**

$$Q=0 \qquad \begin{pmatrix} \nu'_j & u'_j \\ l'_j & d'_j \end{pmatrix} \qquad Q=+2/3$$

$$Q=-1 \qquad\qquad\qquad Q=-1/3$$

$$\left(j = 1, \cdots, N_G \right)$$

WHY ?

$$\mathcal{L}_Y = -\sum_{jk} \left\{ \left(\bar{u}'_j, \bar{d}'_j\right)_L \left[c_{jk}^{(d)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} d'_{kR} + c_{jk}^{(u)} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} u'_{kR} \right] - \left(\bar{\nu}'_j, \bar{l}'_j\right)_L c_{jk}^{(l)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l'_{kR} \right\} + \text{h.c.}$$

SSB

$$\mathcal{L}_Y = -\left(1 + \frac{H}{v}\right) \left\{ \bar{d}'_L \cdot \mathbf{M}'_d \cdot d'_R + \bar{u}'_L \cdot \mathbf{M}'_u \cdot u'_R + \bar{l}'_L \cdot \mathbf{M}'_l \cdot l'_R + \text{h.c.} \right\}$$

Arbitrary Non-Diagonal Complex Mass Matrices

$$\left[\mathbf{M}'_d , \mathbf{M}'_u , \mathbf{M}'_l \right]_{jk} = \left[c_{jk}^{(d)} , c_{jk}^{(u)} , c_{jk}^{(l)} \right] \frac{v}{\sqrt{2}}$$

DIAGONALIZATION OF MASS MATRICES

$$\mathbf{M}'_d = \mathbf{H}_d \cdot \mathbf{U}_d = \mathbf{S}_d^\dagger \cdot \mathcal{M}_d \cdot \mathbf{S}_d \cdot \mathbf{U}_d$$
$$\mathbf{M}'_u = \mathbf{H}_u \cdot \mathbf{U}_u = \mathbf{S}_u^\dagger \cdot \mathcal{M}_u \cdot \mathrm{S}_u \cdot \mathbf{U}_u$$
$$\mathbf{M}'_l = \mathbf{H}_l \cdot \mathbf{U}_l = \mathbf{S}_l^\dagger \cdot \mathcal{M}_l \cdot \mathbf{S}_l \cdot \mathbf{U}_l$$

$$\mathbf{H}_\mathrm{f} = \mathbf{H}_\mathrm{f}^\dagger$$
$$\mathbf{U}_\mathrm{f} \cdot \mathbf{U}_\mathrm{f}^\dagger = \mathbf{U}_\mathrm{f}^\dagger \cdot \mathbf{U}_\mathrm{f} = 1$$
$$\mathbf{S}_\mathrm{f} \cdot \mathbf{S}_\mathrm{f}^\dagger = \mathbf{S}_\mathrm{f}^\dagger \cdot \mathbf{S}_\mathrm{f} = 1$$

$$\mathcal{L}_Y = -\left(1+\frac{H}{v}\right) \left\{ \bar{\mathrm{d}} \cdot \mathcal{M}_d \cdot \mathrm{d} + \bar{\mathrm{u}} \cdot \mathcal{M}_u \cdot \mathrm{u} + \bar{l} \cdot \mathcal{M}_l \cdot l \right\}$$

$$\mathcal{M}_u = \mathrm{diag}\,(m_u, m_c, m_t) \quad ; \quad \mathcal{M}_d = \mathrm{diag}\,(m_d, m_s, m_b) \quad ; \quad \mathcal{M}_l = \mathrm{diag}\,(m_e, m_\mu, m_\tau)$$

$$\mathrm{d}_L \equiv \mathbf{S}_d \cdot \mathrm{d}'_L \quad ; \quad \mathrm{u}_L \equiv \mathbf{S}_u \cdot \mathrm{u}'_L \quad ; \quad l_L \equiv \mathbf{S}_l \cdot l'_L$$
$$\mathrm{d}_R \equiv \mathbf{S}_d \cdot \mathbf{U}_d \cdot \mathrm{d}'_R \quad ; \quad \mathrm{u}_R \equiv \mathbf{S}_u \cdot \mathbf{U}_u \cdot \mathrm{u}'_R \quad ; \quad l_R \equiv \mathbf{S}_l \cdot \mathbf{U}_l \cdot l'_R$$

Mass Eigenstates ≠ Weak Eigenstates

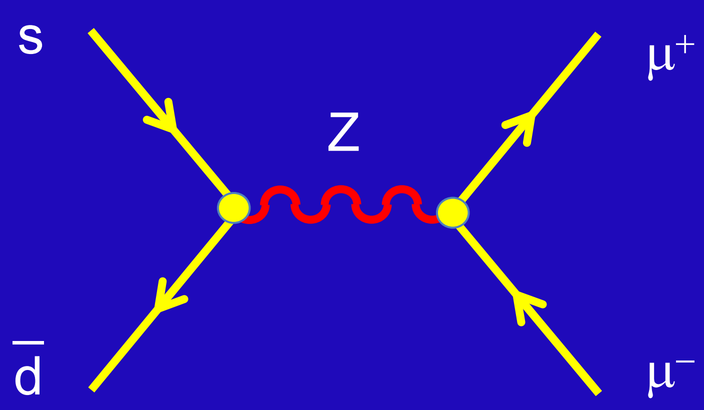
$$\bar{\mathrm{f}}'_L \, \mathrm{f}'_L = \bar{\mathrm{f}}_L \, \mathrm{f}_L \quad ; \quad \bar{\mathrm{f}}'_R \, \mathrm{f}'_R = \bar{\mathrm{f}}_R \, \mathrm{f}_R \quad \Longrightarrow \quad \mathcal{L}'_{\mathrm{NC}} = \mathcal{L}_{\mathrm{NC}}$$

$$\bar{\mathrm{u}}'_L \, \mathrm{d}'_L = \bar{\mathrm{u}}_L \cdot \mathbf{V} \cdot \mathrm{d}_L \quad ; \quad \mathbf{V} \equiv \mathbf{S}_u \cdot \mathbf{S}_d^\dagger \quad \Longrightarrow \quad \mathcal{L}'_{\mathrm{CC}} \neq \mathcal{L}_{\mathrm{CC}}$$

QUARK MIXING

Flavour Conserving Neutral Currents (GIM)

$$\mathcal{L}_{NC}^{Z} = -\frac{e}{2\sin\theta_W\cos\theta_W} Z_\mu \sum_f \bar{f}\gamma^\mu\left[v_f - a_f\gamma_5\right] f$$



NO

$$\mathrm{Br}(K_L\to\mu^+\mu^-) = (6.84\pm 0.11)\times 10^{-9} \quad , \quad \mathrm{Br}(K_S\to\mu^+\mu^-) < 3.2\times 10^{-7}$$

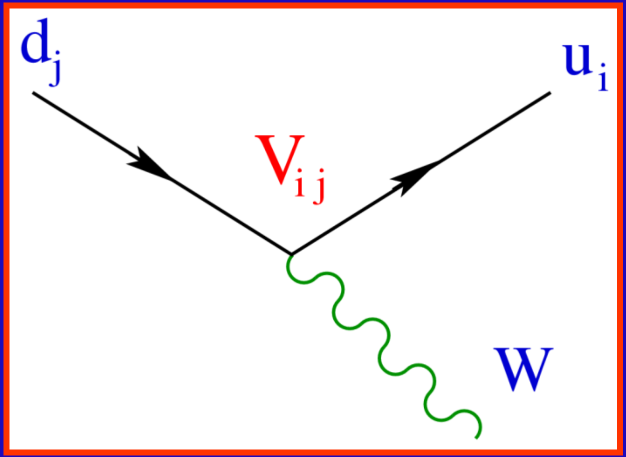
$$K_L \to \pi^{0*} \to (\gamma\gamma)^* \to \mu^+\mu^-$$

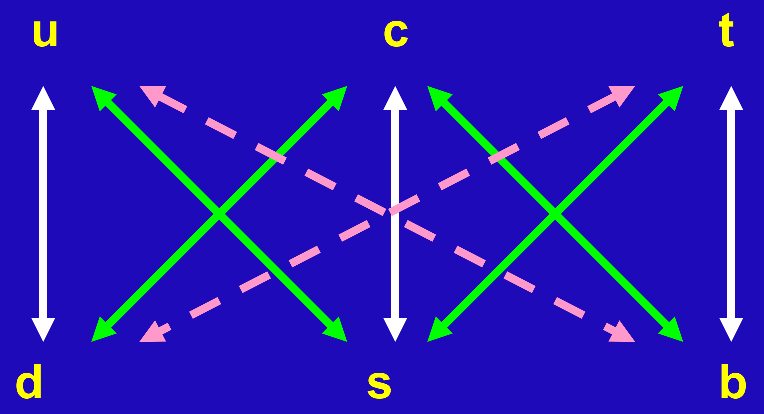
$$K_S \to (\pi^+\pi^-)^* \to (\gamma\gamma)^* \to \mu^+\mu^-$$

Flavour Changing Charged Currents

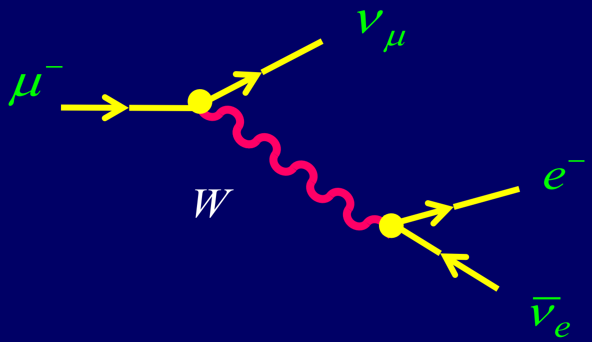
$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}} W_\mu^\dagger \left[\sum_{ij} \bar{u}_i \gamma^\mu (1-\gamma_5) \mathbf{V}_{ij} d_j + \sum_l \bar{\nu}_l \gamma^\mu (1-\gamma_5) l \right] + \text{h.c.}$$

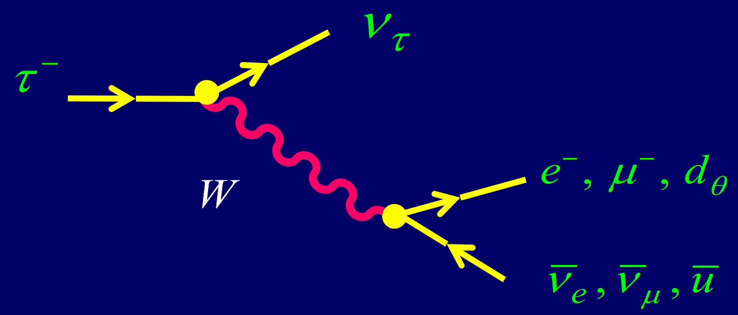
$$\left(\bar{\nu}_{l_j} \equiv \bar{\nu}_i \; \mathbf{V}_{ij}^{(l)} \right)$$





Weak Decays



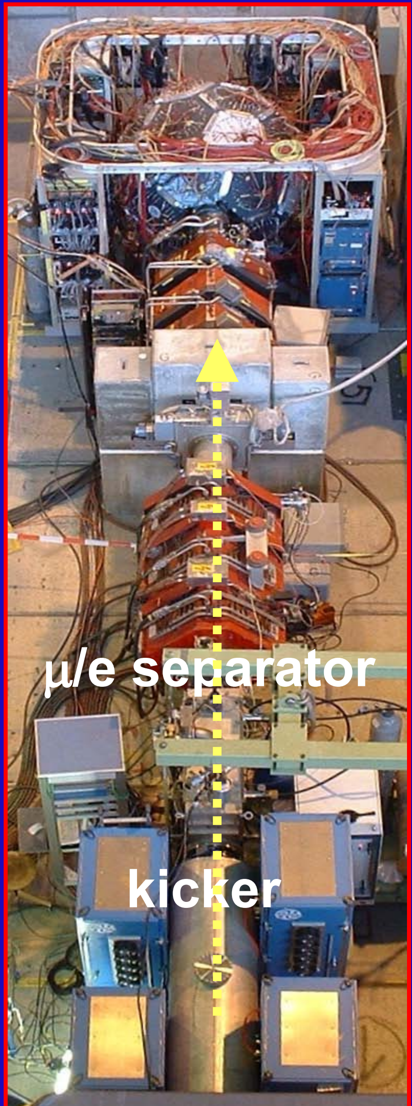


$$T(l \to \nu_l\, l' \bar{\nu}_{l'}) \sim \frac{g_W^2}{M_W^2 - q^2} \xrightarrow{q^2 << M_W^2} \frac{g_W^2}{M_W^2} = 4\sqrt{2}\, G_F$$

$$\frac{1}{\tau_\mu} = \frac{G_F^2\, m_\mu^5}{192\,\pi^3}\, f(m_e^2/m_\mu^2)\, r_{EW} \quad \Rightarrow \quad G_F = \left(1.166\,378\,8 \pm 0.000\,000\,7\right) \times 10^{-5}\ \text{GeV}^{-2}$$

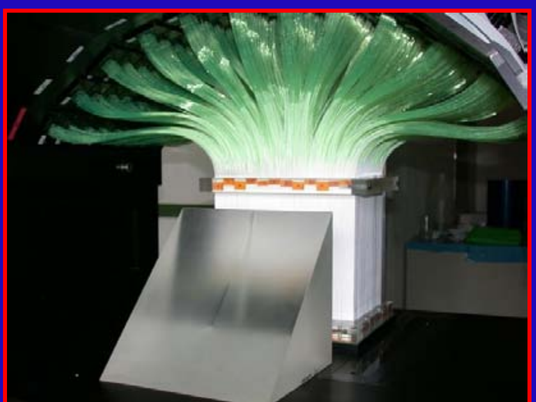
$$r_{EW} = \left[1 + \frac{\alpha(m_\mu)}{2\pi}\left(\frac{25}{4} - \pi^2\right) + C_2\,\frac{\alpha(m_\mu)^2}{\pi^2}\right]\left[1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2} - 2\frac{m_e^2}{M_W^2}\right] = 0.9958 \quad ; \quad f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$$

Muon Lifetime



$$\tau_\mu (\mu s) = \begin{cases} 2.197\,03 \pm 0.000\,04 & \text{PDG '06} \\ 2.197\,013 \pm 0.000\,024 & \text{MuLan '07} \\ 2.197\,083 \pm 0.000\,035 & \text{FAST '08} \\ 2.196\,980\,3 \pm 0.000\,002\,2 & \text{MuLan '10} \end{cases}$$

MULAN



FAST

$$\frac{1}{\tau_\mu} = \frac{G_F^2\, m_\mu^5}{192\, \pi^3} \left(1 + \delta_{\text{QED}}\right)$$

δ_{QED} known to 0.3 ppm (van-Ritbergen & Stuart)

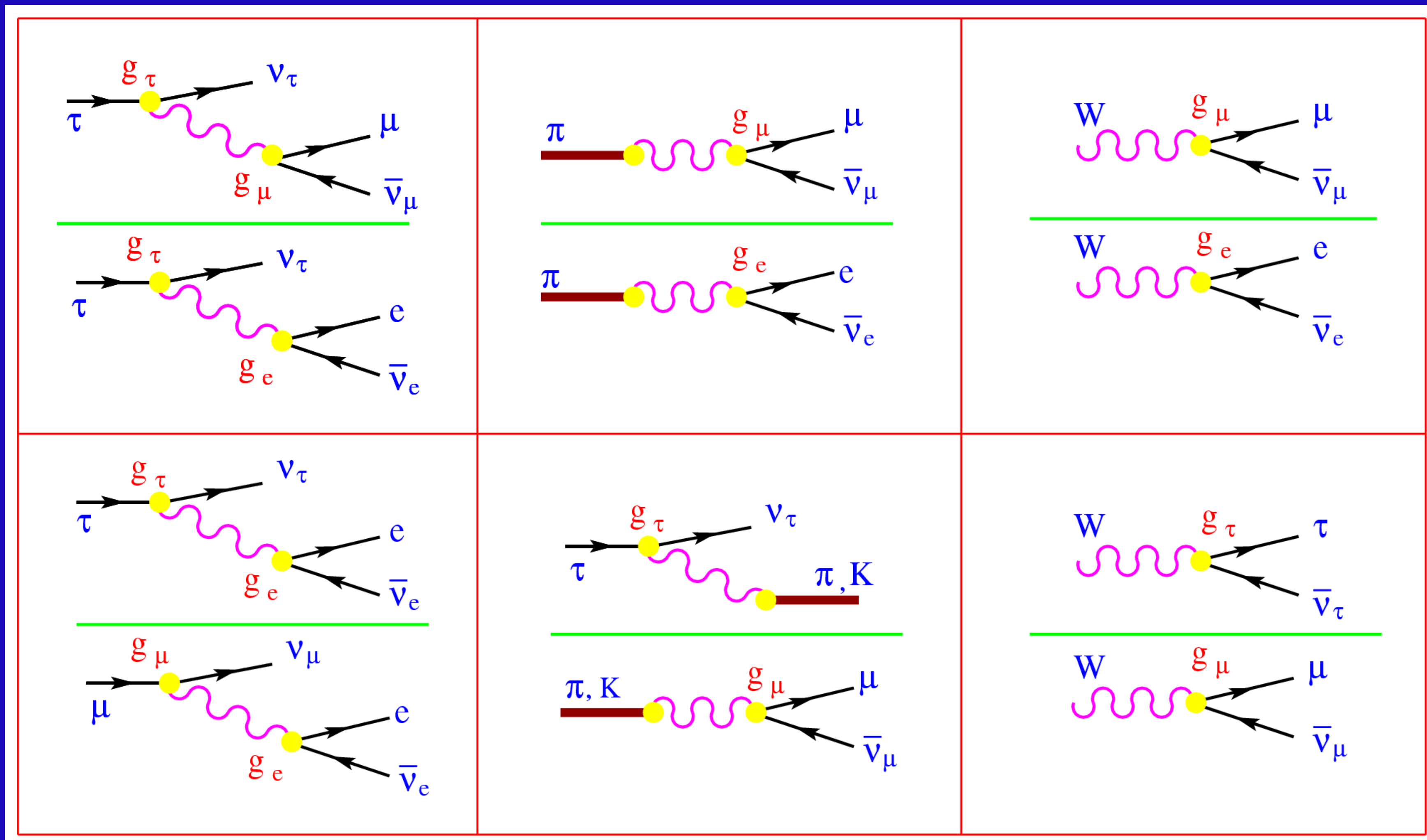
New World Average:

τ_μ = 2.196 980 3 (22) μs → G_F = 1.166 378 8 (7) x 10^{-5} GeV^{-2} (0.6 ppm)

LEPTON UNIVERSALITY

$$\frac{g_\mu}{g_e}$$

$$\frac{g_\tau}{g_\mu}$$



CHARGED CURRENT UNIVERSALITY

$|g_\mu / g_e|$

$B_{\tau\to\mu} / B_{\tau\to e}$	1.0018 ± 0.0014
$B_{\pi\to\mu} / B_{\pi\to e}$	1.0021 ± 0.0016
$B_{K\to\mu} / B_{K\to e}$	0.996 ± 0.006
$B_{K\to\pi\mu} / B_{K\to\pi e}$	1.001 ± 0.002
$B_{W\to\mu} / B_{W\to e}$	0.997 ± 0.010

$|g_\tau / g_\mu|$

$B_{\tau\to e} \;\; \tau_\mu / \tau_\tau$	1.0006 ± 0.0022
$\Gamma_{\tau\to\pi} / \Gamma_{\pi\to\mu}$	0.991 ± 0.004
$\Gamma_{\tau\to K} / \Gamma_{K\to\mu}$	0.982 ± 0.008
$B_{W\to\tau} / B_{W\to\mu}$	1.039 ± 0.013

$|g_\tau / g_e|$

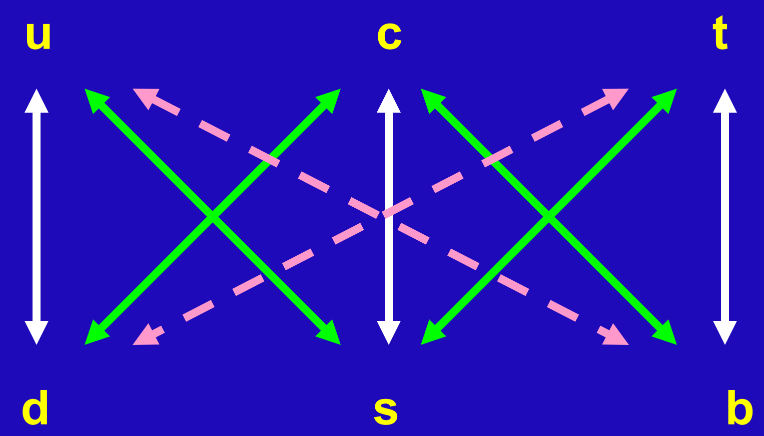
$B_{\tau\to\mu} \;\; \tau_\mu / \tau_\tau$	1.0005 ± 0.0023
$B_{W\to\tau} / B_{W\to e}$	1.036 ± 0.014

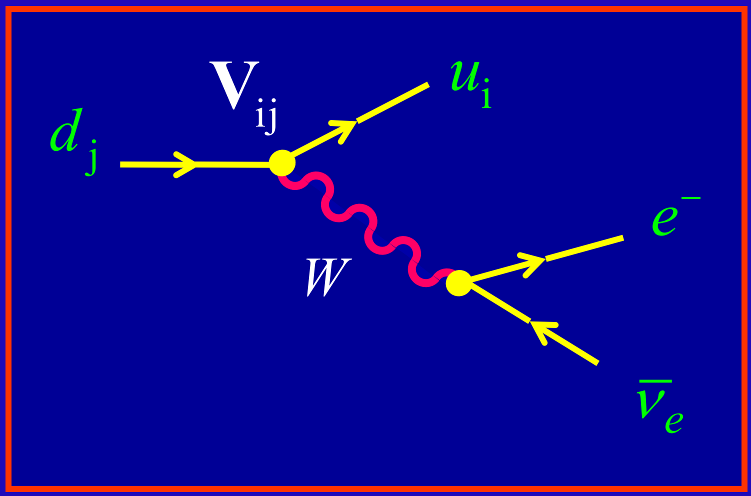
LEPTON FLAVOUR VIOLATION

90 % CL Upper Limits on $Br(l^- \to X^-)$ [BABAR / BELLE]

Decay	U.L.	Decay	U.L.	Decay	U.L.
$\mu^- \to e^-\gamma$	$1.2 \cdot 10^{-11}$	$\mu^- \to e^-e^+e^-$	$1.0 \cdot 10^{-12}$	$\mu^- \to e^-\gamma\gamma$	$7.2 \cdot 10^{-11}$
$\tau^- \to e^-\gamma$	$3.3 \cdot 10^{-8}$	$\tau^- \to e^-e^+e^-$	$2.7 \cdot 10^{-8}$	$\tau^- \to e^-e^+\mu^-$	$1.8 \cdot 10^{-8}$
$\tau^- \to \mu^-\gamma$	$4.4 \cdot 10^{-8}$	$\tau^- \to e^-\mu^+\mu^-$	$2.7 \cdot 10^{-8}$	$\tau^- \to \mu^-e^+\mu^-$	$1.7 \cdot 10^{-8}$
$\tau^- \to e^-e^-\mu^+$	$1.5 \cdot 10^{-8}$	$\tau^- \to \mu^-\mu^+\mu^-$	$2.1 \cdot 10^{-8}$	$\tau^- \to e^-\pi^0$	$8.0 \cdot 10^{-8}$
$\tau^- \to \mu^-\pi^0$	$1.1 \cdot 10^{-7}$	$\tau^- \to e^-\eta'$	$1.6 \cdot 10^{-7}$	$\tau^- \to \mu^-\eta'$	$1.3 \cdot 10^{-7}$
$\tau^- \to e^-\eta$	$9.2 \cdot 10^{-8}$	$\tau^- \to \mu^-\eta$	$6.5 \cdot 10^{-8}$	$\tau^- \to e^-K^{*0}$	$5.9 \cdot 10^{-8}$
$\tau^- \to e^-K_S$	$2.6 \cdot 10^{-8}$	$\tau^- \to \mu^-K_S$	$2.3 \cdot 10^{-8}$	$\tau^- \to \mu^-\rho^0$	$2.6 \cdot 10^{-8}$
$\tau^- \to e^-K^+K^-$	$5.4 \cdot 10^{-8}$	$\tau^- \to e^-K^+\pi^-$	$5.2 \cdot 10^{-8}$	$\tau^- \to e^-\pi^+K^-$	$5.8 \cdot 10^{-8}$
$\tau^- \to \mu^-K^+K^-$	$6.8 \cdot 10^{-8}$	$\tau^- \to \mu^-K^+\pi^-$	$1.0 \cdot 10^{-7}$	$\tau^- \to \mu^-\pi^+K^-$	$1.6 \cdot 10^{-7}$
$\tau^- \to e^-\pi^+\pi^-$	$4.4 \cdot 10^{-8}$	$\tau^- \to \mu^-\pi^+\pi^-$	$3.3 \cdot 10^{-8}$	$\tau^- \to \Lambda\pi^-$	$7.2 \cdot 10^{-8}$
$\tau^- \to e^+K^-K^-$	$6.0 \cdot 10^{-8}$	$\tau^- \to e^+K^-\pi^-$	$6.7 \cdot 10^{-8}$	$\tau^- \to e^+\pi^-\pi^-$	$8.8 \cdot 10^{-8}$
$\tau^- \to \mu^-K^{*0}$	$5.9 \cdot 10^{-8}$	$\tau^- \to e^-\phi$	$3.1 \cdot 10^{-8}$	$\tau^- \to \mu^-\omega$	$8.9 \cdot 10^{-8}$
$\tau^- \to \mu^+K^-K^-$	$9.6 \cdot 10^{-8}$	$\tau^- \to \mu^+K^-\pi^-$	$9.4 \cdot 10^{-8}$	$\tau^- \to \mu^+\pi^-\pi^-$	$3.7 \cdot 10^{-8}$

Flavour Changing Charged Currents





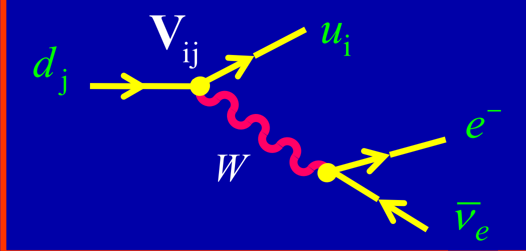
$$\Gamma(d_j \to u_i\, e^-\, \bar{\nu}_e) \propto |V_{ij}|^2$$

We measure decays of hadrons (no free quarks)

Important QCD Uncertainties

$\mathbf{V_{ij}}$ Determination $(0^- \to 0^-)$

$K\to\pi l\nu$, $D\to K l\nu$ ...



$$\langle P'(k')|\bar{u}_i\gamma^\mu d_j|P(k)\rangle = C_{PP'}\left\{(k+k')^\mu\, f_+(q^2) + (k-k')^\mu\, f_-(q^2)\right\}$$

$$\Gamma(P\to P'l\nu) = \frac{G_F^2 M_P^5}{192\pi^3}\,|\mathbf{V_{ij}}|^2\, C_{PP'}^2\, |f_+(0)|^2\, \mathbf{I}\, (1+\delta_{\rm RC})$$

$$\mathbf{I} \approx \int_0^{(M_P-M_{P'})^2} \frac{dq^2}{M_P^8}\,\lambda^{3/2}(q^2,M_P^2,M_{P'}^2)\left|\frac{f_+(q^2)}{f_+(0)}\right|^2$$

$f_-(q^2)$ suppressed $(m_{u_i}-m_{d_j}, m_l)$

- Measure the q^2 distribution ➡ $\mathbf{I}$
- Measure Γ ➡ $f_+(0)\,|\mathbf{V_{ij}}|$
- Get a theoretical prediction for $f_+(0)$ ➡ $|\mathbf{V_{ij}}|$

Theory is always needed: Symmetries

$|V_{ud}|$

$$f_+(0) = 1 + O[(m_u - m_d)^2]$$

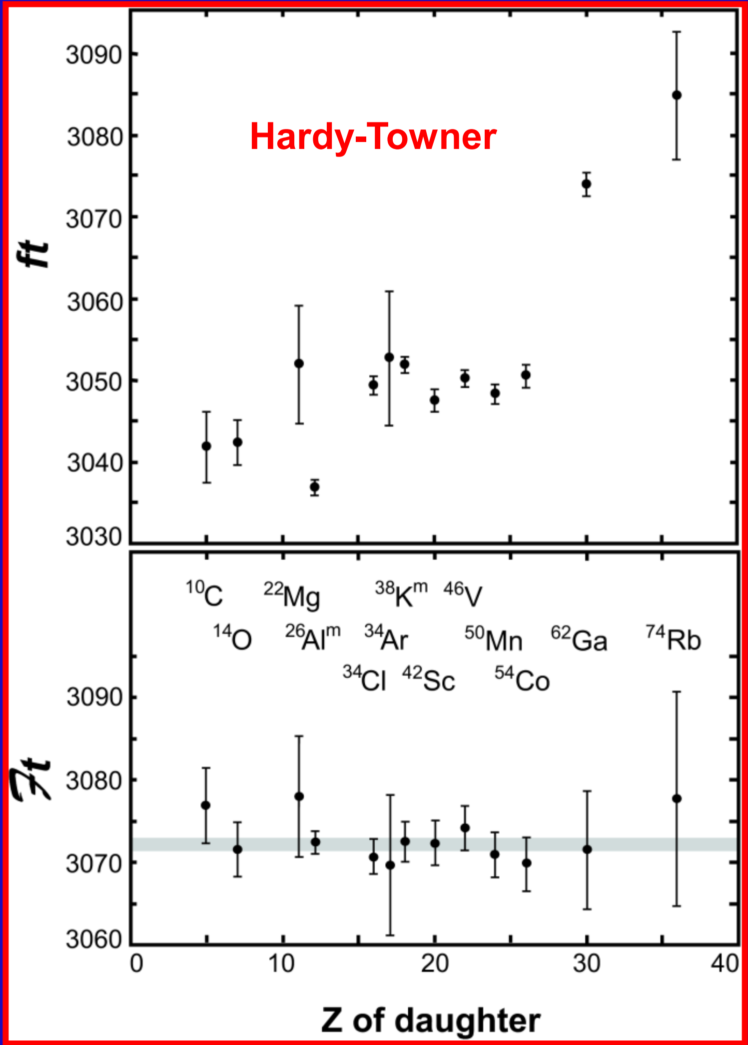
Superallowed Nuclear β Transitions $(0^+ \to 0^+)$

$$|V_{ud}|^2 = \frac{\pi^3 \ln 2}{ft\, G_F^2 m_e^5\, (1+\delta_{RC})} = \frac{(2984.48 \pm 0.05)\ \mathrm{s}}{ft\,(1+\delta_{RC})}$$

(Marciano – Sirlin)

$|V_{ud}| = 0.97425 \pm 0.00022$

$|V_{ud}| = 0.97377 \pm 0.00027$ (PDG 06)

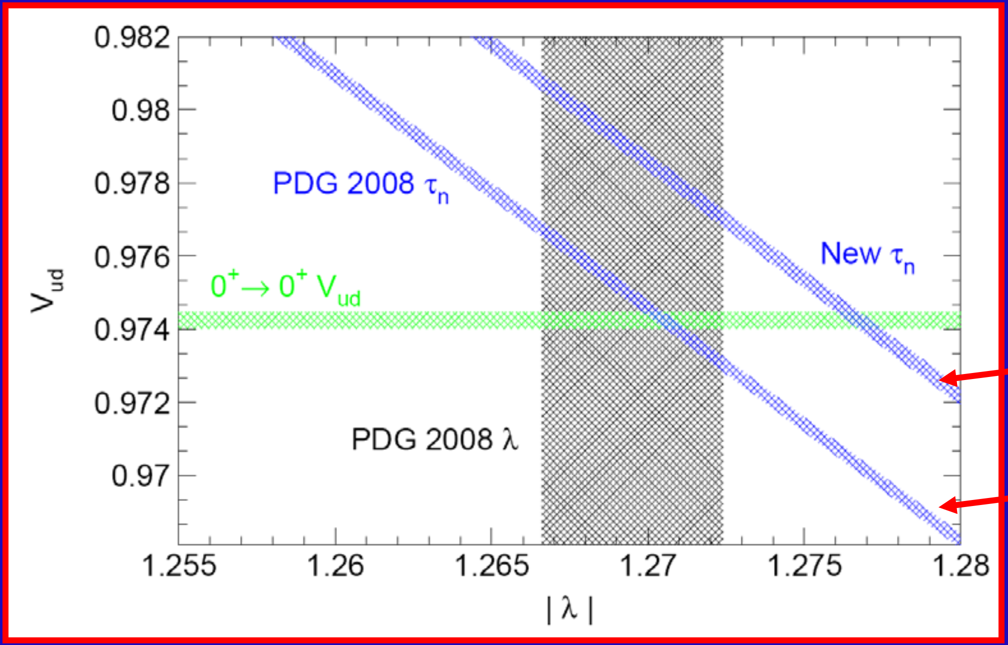


• Neutron Decay:

$$|V_{ud}|^2 = \frac{(4908.7 \pm 1.9)\,\mathrm{s}}{\tau_n (1 + 3\lambda^2)}$$

(Czarnecki – Marciano – Sirlin)

PDG10: $\tau_n = (885.7 \pm 0.8)$ s , $\lambda \equiv g_A / g_V = 1.2694 \pm 0.0028$



$|V_{ud}| = 0.9746 \pm 0.0019$

$\tau_n = (878.5 \pm 0.7 \pm 0.3)$ s

(Serebrov et al, 2005)

PDG10

• Pion Decay:

$$\mathrm{Br}(\pi^+ \to \pi^0 e^+ \nu_e) = (1.036 \pm 0.006) \times 10^{-8}$$

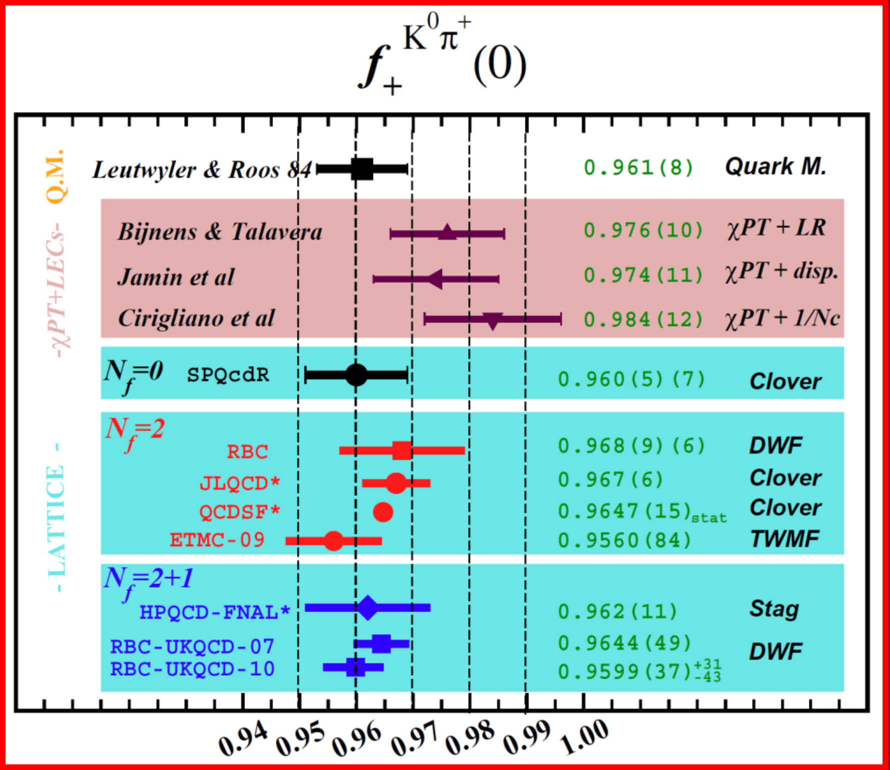
(PIBETA)

$|V_{ud}| = 0.9741 \pm 0.0026$

$K \to \pi l \nu$ Decays

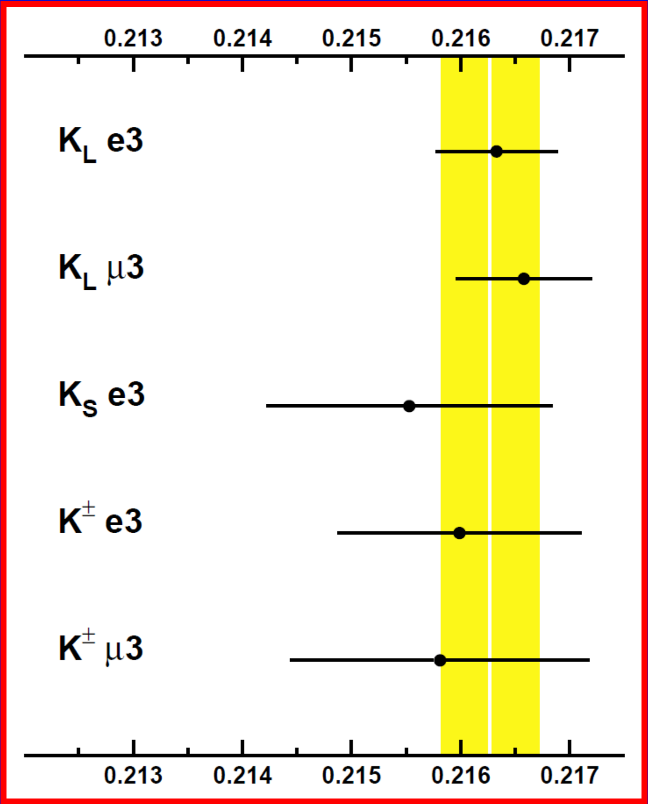
Flavianet, arXiv:1005.2323 [hep-ph]

Large $O(p^6)$ ChPT correction (Bijnens-Talavera)



$O(p^4)$

$O(p^6)$



$$\left|f_+(0)\, \mathbf{V_{us}}\right| = 0.2163 \pm 0.0005$$

$$f_+(0) = 0.959 \pm 0.005 \quad \Rightarrow \quad \left|\mathbf{V_{us}}\right| = \mathbf{0.2255 \pm 0.0013}$$

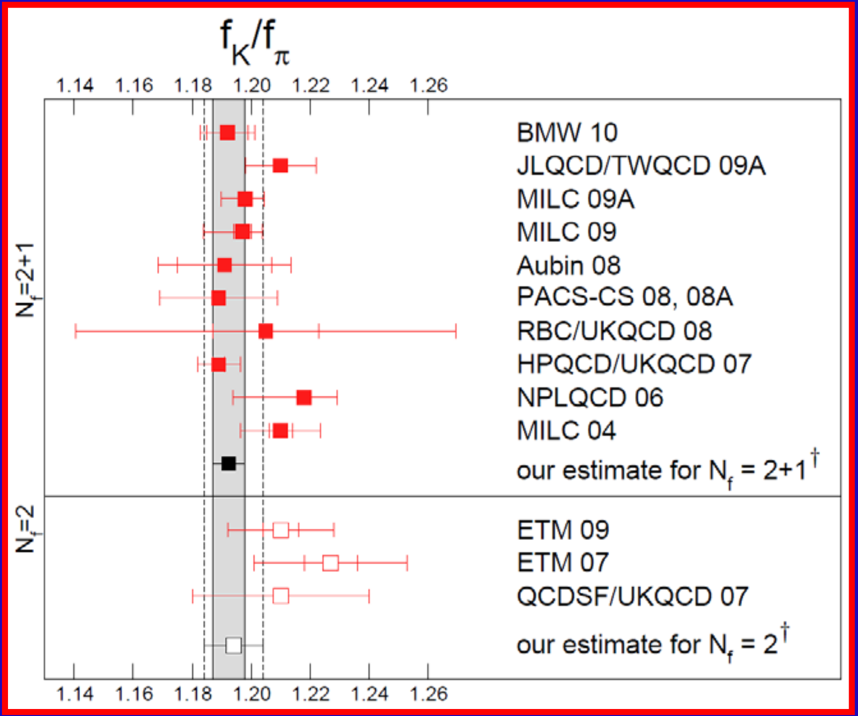
$$f_+(0) = 0.97 \pm 0.01 \quad \Rightarrow \quad \left|\mathbf{V_{us}}\right| = 0.2230 \pm 0.0024$$

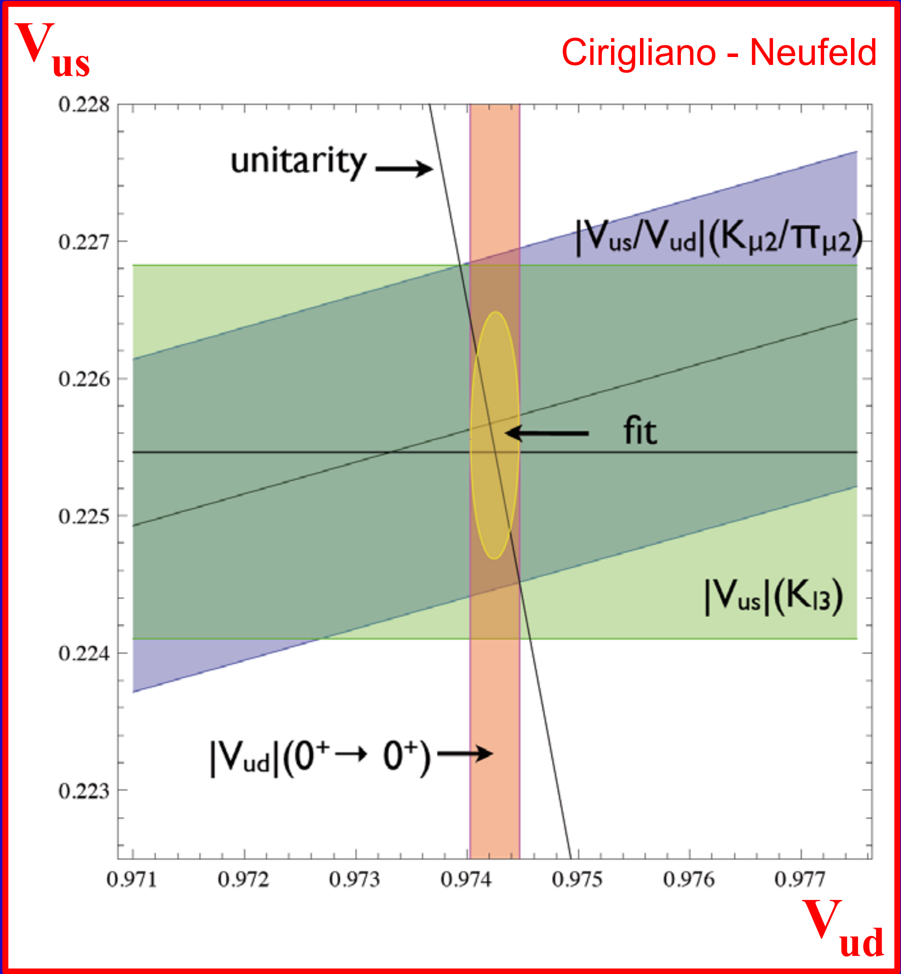
$\Gamma(K^+ \to \mu^+ \nu_\mu) \,/\, \Gamma(\pi^+ \to \mu^+ \nu_\mu)$

(Marciano 04)

$$\frac{f_K}{f_\pi}\frac{|V_{us}|}{|V_{ud}|} = 0.2763 \pm 0.0005 \quad \Rightarrow \quad \frac{|V_{us}|}{|V_{ud}|} = 0.2316 \pm 0.0012$$

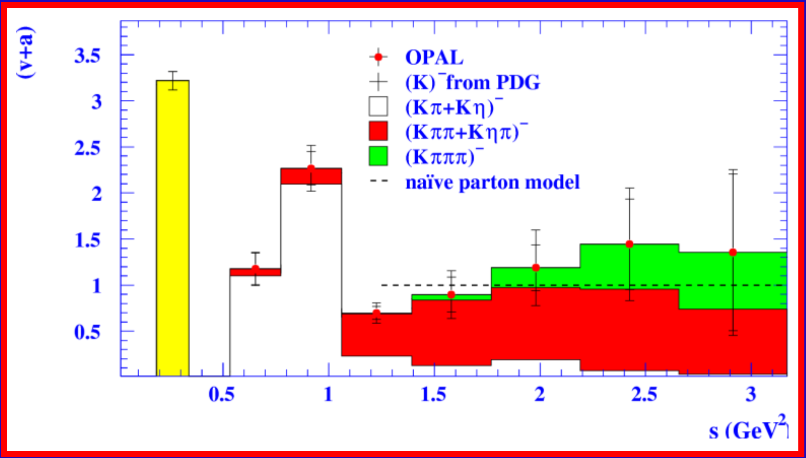
$$\langle 0 | \bar{d}_i \gamma^\mu \gamma_5 u_j | P(k) \rangle = i\, f_P\, k^\mu$$

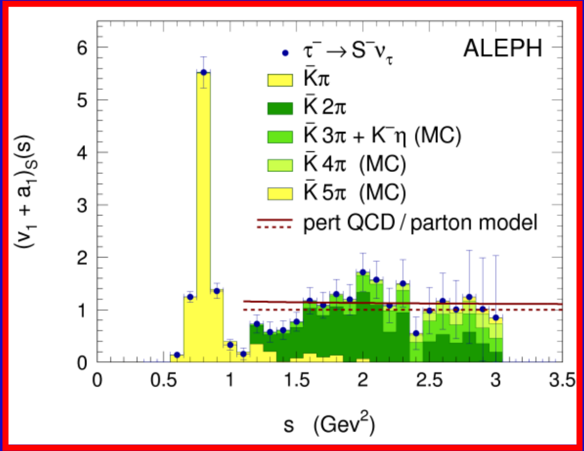




$$f_K / f_\pi = 1.193 \pm 0.006 \qquad \text{(FLAG 2010)}$$

$R_{\tau,S} = \Gamma(\tau^- \to \nu_\tau S^-) / \Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)$





$$\delta R_\tau \equiv \frac{R_{\tau,ud}}{|\mathbf{V_{ud}}|^2} - \frac{R_{\tau,S}}{|\mathbf{V_{us}}|^2} \approx 24 \, \frac{m_s^2(m_\tau^2)}{m_\tau^2} \, \Delta(\alpha_s)$$

$$|\mathbf{V_{us}}|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,ud}}{|\mathbf{V_{ud}}|^2} - \delta R_\tau^{\mathrm{th}}}$$

$$m_s(2\,\mathrm{GeV}) = 96 \pm 10 \;\; \mathrm{MeV}$$

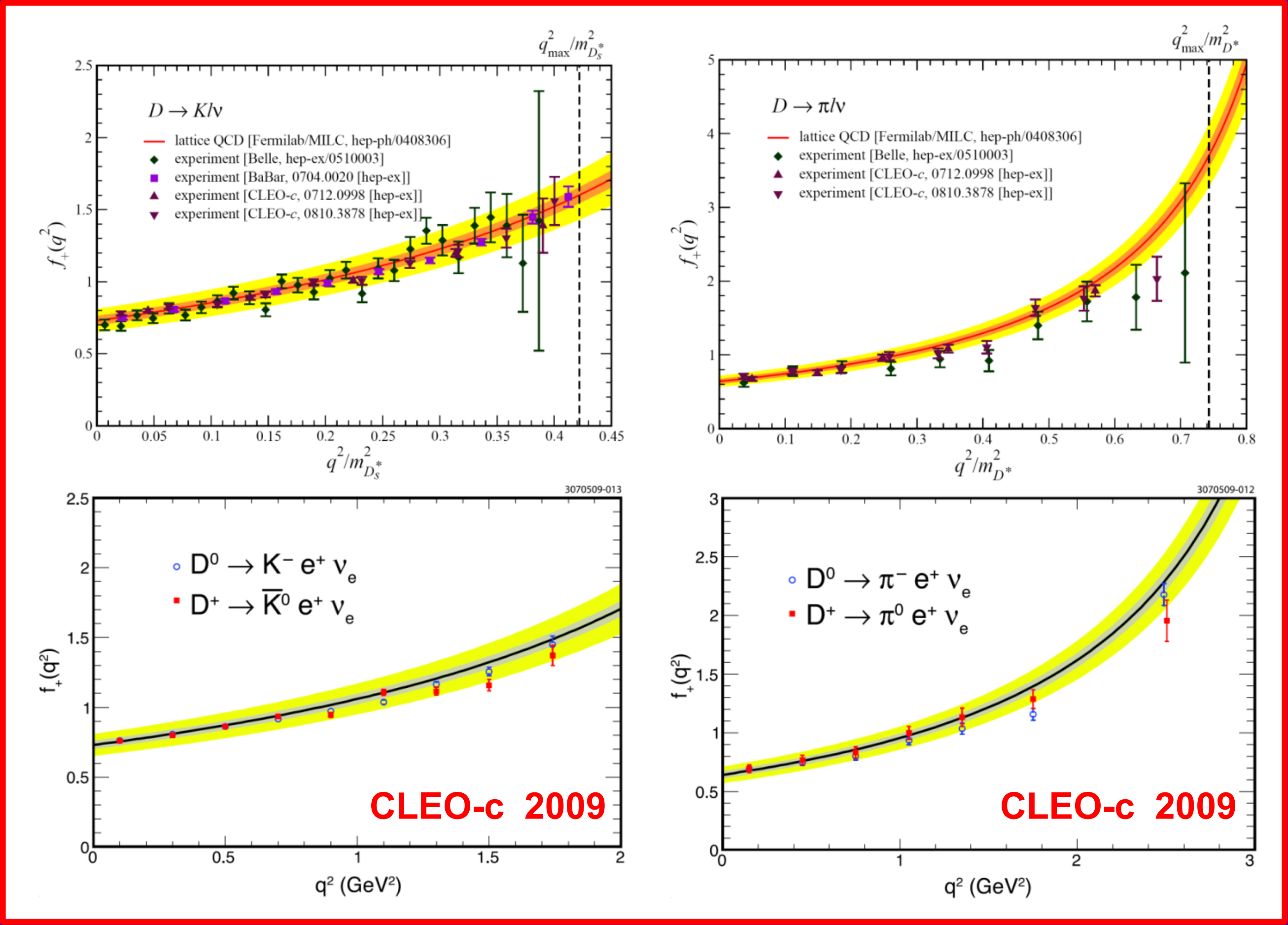
Gámiz-Jamin-Pich-Prades-Schwab

$$|\mathbf{V_{us}}| = 0.2166 \pm 0.0019_{\mathrm{exp}} \pm 0.0005_{\mathrm{th}}$$

Simultaneous m_s & V_{us} fit possible with better data

The τ could give the most precise V_{us} determination

$D \to K/\pi\, l\, \nu$



Lattice input

$$|V_{cs}|_{D\to K l\nu} = 0.98 \pm 0.10$$

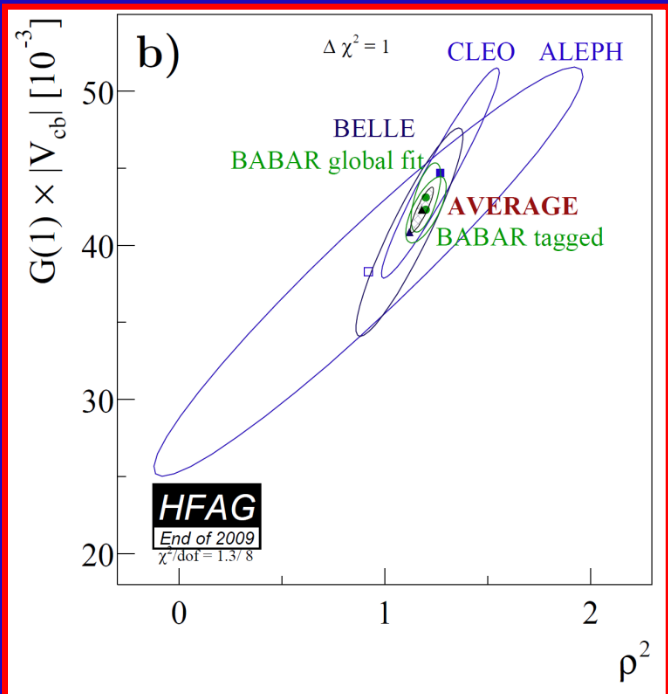
$$|V_{cd}|_{D\to \pi l\nu} = 0.229 \pm 0.025$$

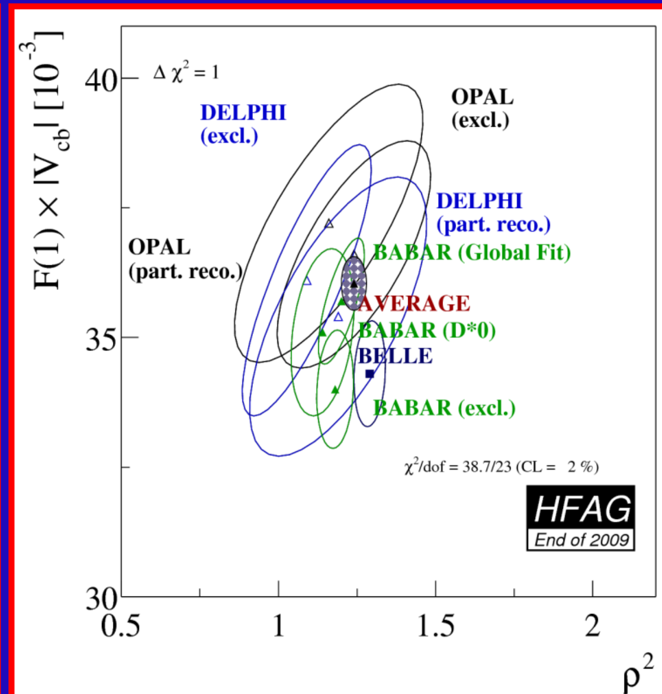
PDG 2010:

$$|V_{cd}|_{\nu c\to \mu d} = 0.230 \pm 0.011 \qquad |V_{cs}|_{D\to K l\nu,\, D_s\to l\nu} = 1.023 \pm 0.036$$

$B \to D\, l\, \nu$ $B \to D^*\, l\, \nu$

QCD Symmetries at $1/M_Q \to 0$





$$G(1)\ |\mathbf{V_{cb}}| = (42.3 \pm 1.5)\times 10^{-3}$$

$$F(1)\ |\mathbf{V_{cb}}| = (36.04 \pm 0.52)\times 10^{-3}$$

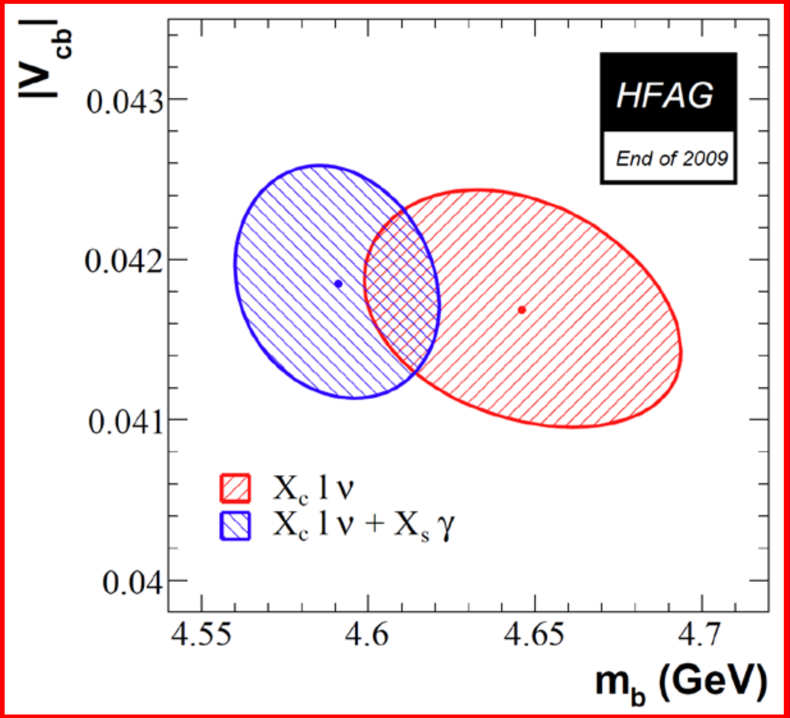
$G(1) = 1.074 \pm 0.018 \pm 0.016$ (FNAL / MILC) ➡ $|\mathbf{V_{cb}}| = (39.2 \pm 1.4_{exp} \pm 0.9_{th})\cdot 10^{-3}$

$F(1) = 0.927 \pm 0.013 \pm 0.020$ (MILC) ➡ $|\mathbf{V_{cb}}| = (38.9 \pm 0.6_{exp} \pm 1.0_{th})\cdot 10^{-3}$

➡ $$\left|\mathbf{V_{cb}}\right|_{excl} = (39.0 \pm 0.9)\cdot 10^{-3}$$

Inclusive B Decays (OPE, HQET)

$$\Gamma(\bar{B} \to X_c \ell \bar{\nu}) = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3} \left\{ f(\rho) + k(\rho) \frac{\mu_\pi^2}{2m_b^2} + g(\rho) \frac{\mu_G^2}{2m_b^2} \right\}$$



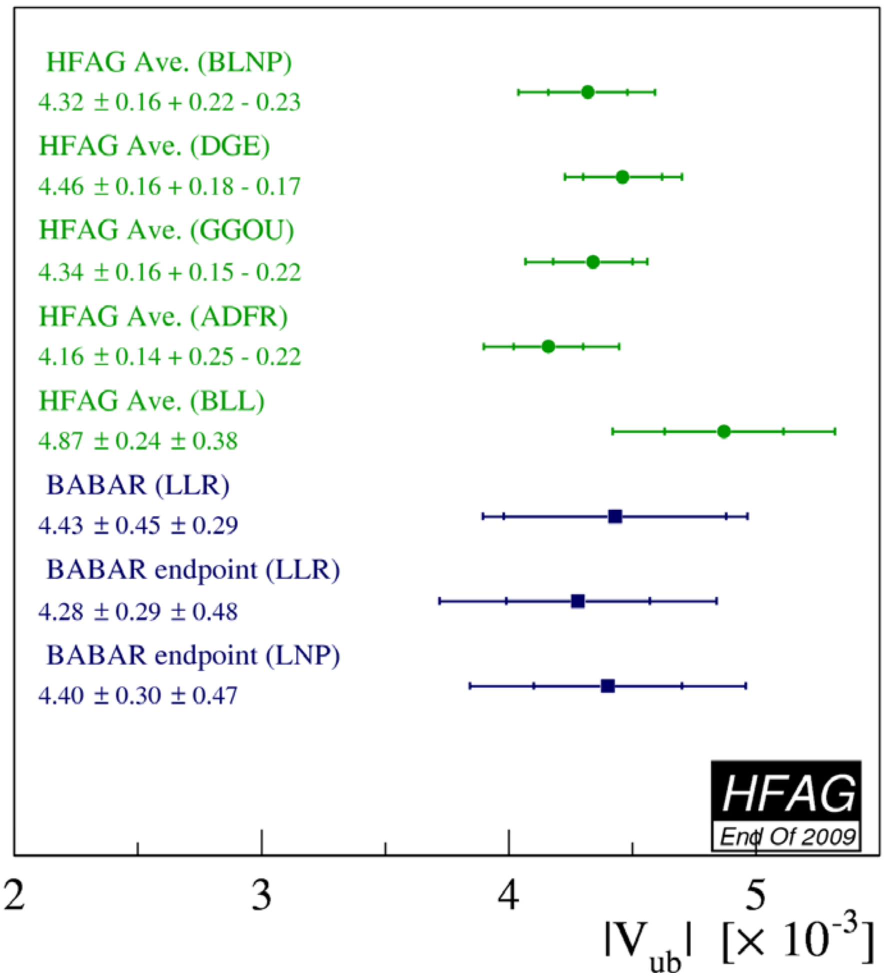
Fits to lepton energy, hadronic invariant mass and photon energy moments

$$\left|V_{cb}\right|_{incl} = (41.8 \pm 0.7) \cdot 10^{-3}$$

2.5 σ discrepancy with exclusive measurement

$$\left|V_{cb}\right| = (40.7 \pm 1.4) \cdot 10^{-3}$$

$B \to X_u\, l\, \nu$



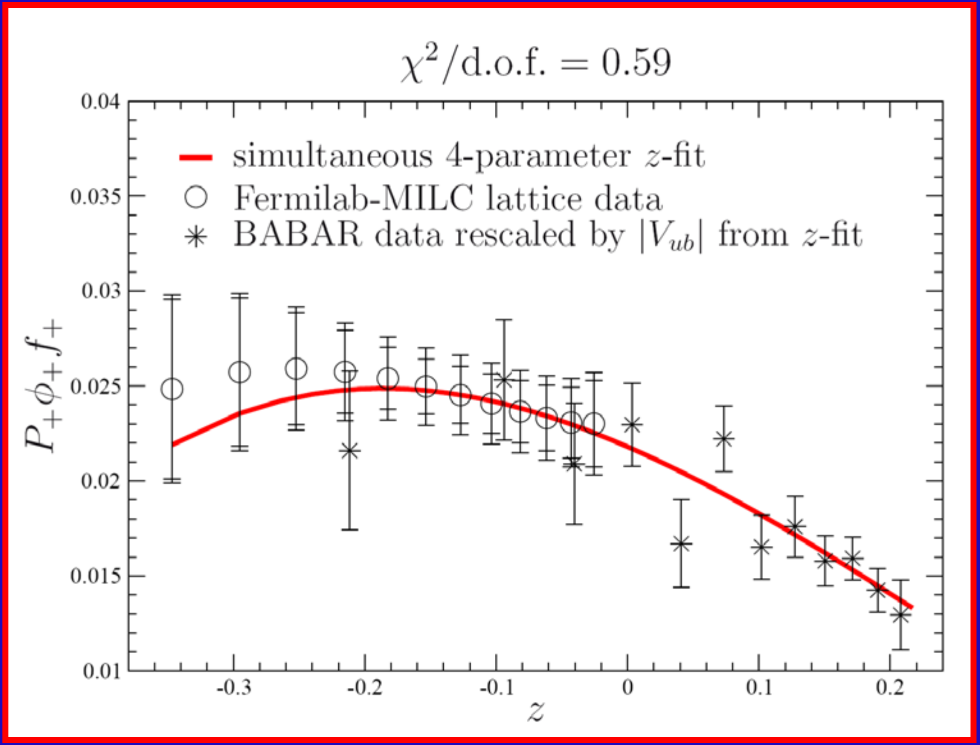
- Large backgrounds from $B \to X_c\, l\, \nu$
- Strong experimental cuts
- Large theoretical uncertainties

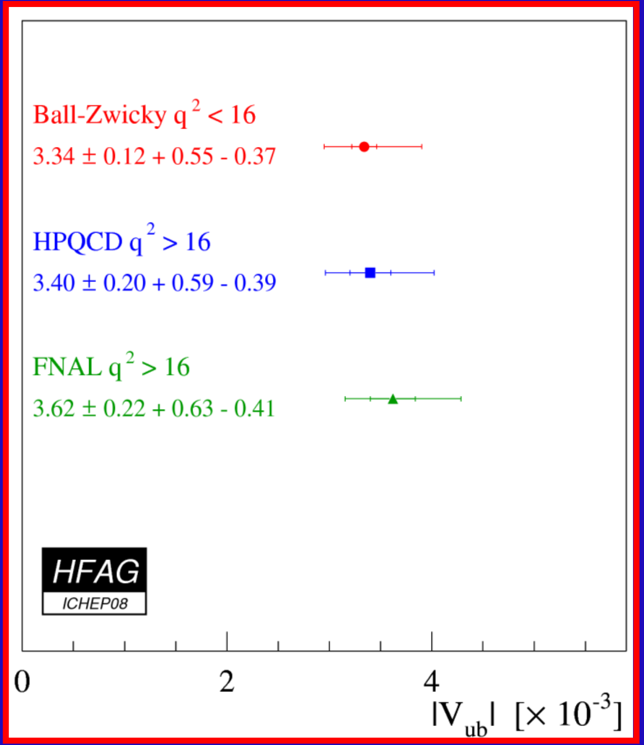
PDG 2010:

$$\left|V_{ub}\right|_{incl} = \left(4.27 \pm 0.38\right) \cdot 10^{-3}$$

$B \to \pi l \nu$

Large theoretical uncertainties





PDG 2010:

$$\left|V_{ub}\right|_{excl} = (3.38 \pm 0.36) \cdot 10^{-3}$$

$$\left|V_{ub}\right| = (3.89 \pm 0.44) \cdot 10^{-3}$$

V_{ij} DETERMINATIONS

CKM entry	Value	Source
$\lvert V_{ud} \rvert$	$\mathbf{0.97425 \pm 0.00022}$ 0.9746 ± 0.0019 0.9741 ± 0.0026	Nuclear β decay $n \to p e^- \bar{\nu}_e$ $\pi^+ \to \pi^0 e^+ \nu_e$
$\lvert V_{us} \rvert$	$\mathbf{0.2255 \pm 0.0013}$ 0.2166 ± 0.0020 0.2256 ± 0.0012	$K \to \pi e^- \bar{\nu}_e$ τ decays $K / \pi \to \mu \nu$, Lattice
$\lvert V_{cd} \rvert$	$\mathbf{0.230 \pm 0.011}$ 0.229 ± 0.025	$\nu d \to c X$ $D \to \pi l \nu$, Lattice
$\lvert V_{cs} \rvert$	$\mathbf{1.023 \pm 0.036}$	$D \to K l \nu$, $D_s \to l \nu$, Lattice
$\lvert V_{cb} \rvert$	0.0390 ± 0.0009 0.0418 ± 0.0007 $\mathbf{0.0407 \pm 0.0014}$	$B \to D^* / D l \bar{\nu}_l$ $b \to c l \bar{\nu}_l$
$\lvert V_{ub} \rvert$	0.00338 ± 0.00038 0.00427 ± 0.00038 $\mathbf{0.00389 \pm 0.00044}$	$B \to \pi l \bar{\nu}_l$ $b \to u l \bar{\nu}_l$
$\lvert V_{tb} \rvert / \sqrt{\sum_q \lvert V_{tq} \rvert^2}$ $\lvert V_{tb} \rvert$	> 0.89 $\mathbf{0.88 \pm 0.07}$	$t \to bW / t \to qW$ $p \bar{p} \to tb + X$

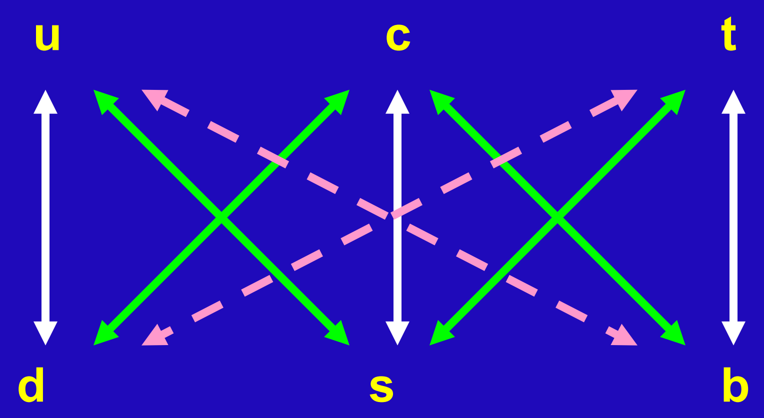
$$\lvert V_{ud} \rvert^2 + \lvert V_{us} \rvert^2 + \lvert V_{ub} \rvert^2 = 1.0000 \pm \mathbf{0.0010}$$

$$\sum_j \left(\lvert V_{uj} \rvert^2 + \lvert V_{cj} \rvert^2 \right) = 2.002 \pm \mathbf{0.027} \quad \text{(LEP)}$$

Hierarchical Structure

$$\mathbf{V} \approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right)$$

$$\lambda \approx \sin\theta_C \approx 0.226 \quad ; \quad A \approx 0.81 \quad ; \quad \sqrt{\rho^2+\eta^2} \approx 0.37$$



QUARK MIXING MATRIX

- **Unitary** $N_G \times N_G$ **Matrix:** N_G^2 **parameters**

$$\mathbf{V} \cdot \mathbf{V}^\dagger = \mathbf{V}^\dagger \cdot \mathbf{V} = \mathbf{1}$$

- $2\, N_G - 1$ **arbitrary phases:**

$$u_i \to e^{i\phi_i}\, u_i \quad ; \quad d_j \to e^{i\theta_j}\, d_j \quad \Rightarrow \quad \mathbf{V}_{ij} \to e^{i(\theta_j - \phi_i)}\, \mathbf{V}_{ij}$$

$\mathbf{V}_{ij}$ **Physical Parameters:**

$\frac{1}{2} N_G\,(N_G - 1)$ **Moduli** ; $\frac{1}{2}(N_G - 1)\,(N_G - 2)$ **phases**

- **$N_f = 2$: 1 angle, 0 phases (Cabibbo)**

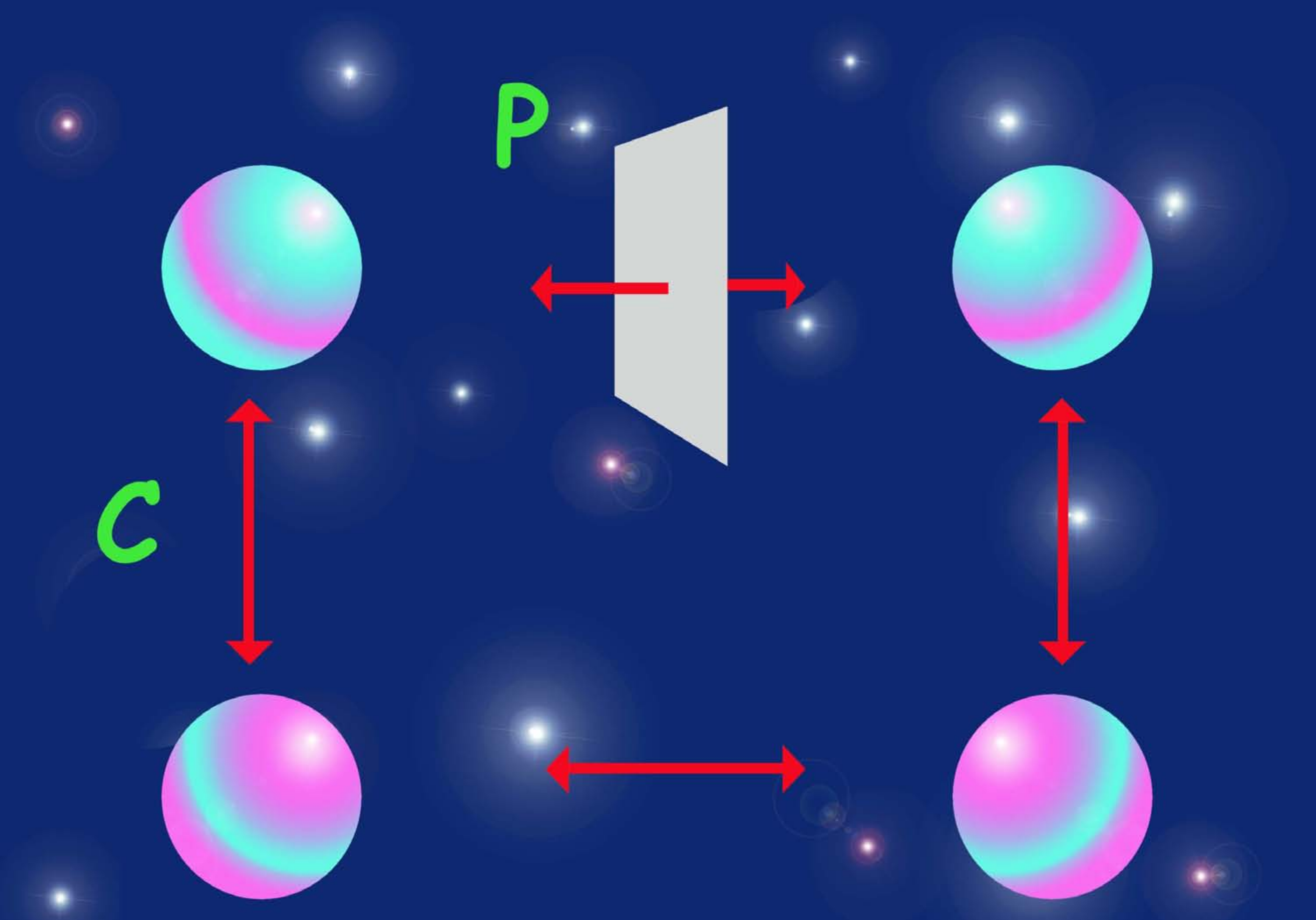
$$V = \begin{bmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{bmatrix} \quad \Rightarrow \quad \text{No } \not{CP}$$

- **$N_f = 3$: 3 angles, 1 phase (CKM)** $\quad c_{ij} \equiv \cos\theta_{ij} \; ; \; s_{ij} \equiv \sin\theta_{ij}$

$$V = \begin{bmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta_{13}} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & c_{23}\, c_{13} \end{bmatrix}$$

$$\approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3\,(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\,\lambda^2 \\ A\lambda^3\,(1-\rho-i\eta) & -A\,\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right)$$

$\lambda \approx \sin\theta_C \approx 0.226 \;\; ; \;\; A \approx 0.81 \;\; ; \;\; \sqrt{\rho^2 + \eta^2} \approx 0.37$ $\qquad \delta_{13} \neq 0 \;\; (\eta \neq 0) \;\Rightarrow\; \not{CP}$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