On the On-Shell Renormalization of Fermion Masses, Fields and Mixing Matrices at 1-loop

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- *Usually in case of mixing* fermion masses and fields are renormalized as

$$m_i^0 \to m_i + \underbrace{\delta m_i}_{\text{diagonal ct}} \qquad \psi_j^0 \to \underbrace{\left(1 + \delta \tilde{Z}_{ji}\right)}_{\text{non-diagonal ct}} \psi_i$$

- *On shell no-mixing conditions* [1]

$$\frac{1}{\not{p} - m_j} \Sigma_{ji}(p^2) u_i = 0, \qquad \bar{u}_j \Sigma_{ji}(p^2) \frac{1}{\not{p} - m_i} = 0, \qquad \boxed{i \neq j}$$

$\Longrightarrow \delta \tilde{Z}_{L,R}$ *and* $\delta \tilde{Z}^\dagger_{L,R}$ in terms of self-energies — **no** $\Sigma^\dagger$!

Introduction II

- ***Overspecification*** of field renormalization constants [2]

$$\Sigma \neq \gamma^0 \Sigma^\dagger \gamma^0 \qquad \Longrightarrow \qquad \left(\tilde{Z}_{L,R}\right)^\dagger \neq \tilde{Z}^\dagger_{L,R}$$

  - (pseudo-)Hermiticity violated by **absorptive parts**
  - *Different* in/out LSZ factors
    - additional set of constants $\bar{Z} \Longrightarrow \boldsymbol{\mathcal{L}} \neq \boldsymbol{\mathcal{L}}^\dagger$, but OK for external legs
    - relax the no-mixing conditions (e.g. $\widetilde{Re}$) $\Longrightarrow$ non-diagonal OS propagator

Introduction III

- CKM counterterm by Denner and Sack [3] proposed already 30 years ago

$$\delta V \sim -\delta \tilde{Z}_L^{A,u} V + V \delta \tilde{Z}_L^{A,d}$$

  - Correctly cancels UV in the Wud vertex ✓
  - Depends on field reno and $\boldsymbol{\partial_\xi \delta V \neq 0}$ [4]
    - Various other approaches that deal with ξ...[2, 4, 5, 6, 7, 8, 9]
    - Many renormalization conditions, some complicated and process dependent

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- In Kniehl and Sirlin [10, 11, 12], half the no-mixing condition and $\partial_\xi \delta V = 0$, but
  - Self-energies *only for the SM*
  - No explicit field renormalization
  - External leg "formalism"; discussion rather removed from the Lagrangian

Introduction IV

Additional problems...

- Usual field renormalizations constants carry factors of $\frac{1}{m_i^2 - m_j^2}$
  - **No** massless limit
  - **No** degenerate mass limit
  - **Numerical** enchancement of certain parameter regions
- $\frac{1}{m_i^2 - m_j^2}$ **carried over** to mixing matrix renormalization
- Mixing matrix and self-energy renormalization are **closely related**

Need consistent self-energy renormalization!

Setup I

- Only interested in **off-diagonal** counterterms!
- Want a hermitian Lagrangian $\Longrightarrow$ **no-mixing** condition only for **incoming** particles

$$\boxed{\frac{1}{\not{p} - m_j} \Sigma_{ji}(p^2) u_i = 0}, \qquad i \neq j$$

  - Outgoing particles *still mix*, but only due to absorptive parts!
- Want to separate ξ $\Longrightarrow$ **off-diagonal mass counterterms**

$$\boxed{\delta m_i \rightarrow \delta m_{\boldsymbol{ji}}}$$

- Want *universal* mass, field, and mixing matrix ct's *in terms of self-energies* $\Longrightarrow$ ...?

Setup II

- Field is renormalized in a *standard* way

$$\psi^0_{L,R} \to Z^{1/2}_{L,R}\psi_{L,R}, \quad \bar{\psi}^0_{L,R} \to \bar{\psi}_{L,R} Z^{1/2\dagger}_{R,L}$$

- Mass now has *left and right* counterterms

$$\bar{\psi}\left(m + \delta m^L P_L + \delta m^R P_R\right)\psi \quad \xrightarrow{Hermiticity} \quad \left(\delta m^L\right)^\dagger = \delta m^R$$

- Self-energy decomposition

$$\Sigma_{ji}(p^2) = \underbrace{\Sigma^L_{ji}(p^2)\not{p}P_L + \Sigma^R_{ji}(p^2)\not{p}P_R + \Sigma^{sL}_{ji}(p^2)P_L + \Sigma^{sR}_{ji}(p^2)P_R + \frac{1}{2}\left(\delta Z^\dagger_{Lji} + \delta Z_{Lji}\right)\not{p}P_L + \frac{1}{2}\left(\delta Z^\dagger_{Rji} + \delta Z_{Rji}\right)\not{p}P_R}_{\text{standard piece}}$$
$$- \left(\boxed{\delta m^L_{\boldsymbol{ji}}} + \frac{1}{2}\delta Z^\dagger_{Rji} m_i + \frac{1}{2} m_j \delta Z_{Lji}\right) P_L$$
$$- \left(\boxed{\delta m^R_{\boldsymbol{ji}}} + \frac{1}{2}\delta Z^\dagger_{Lji} m_i + \frac{1}{2} m_j \delta Z_{Rji}\right) P_R$$

(Hereinafter $i \neq j$!)

No-mixing $\Longrightarrow$ ***relation*** between field and mass renormalization

$$\left(m_i^2 - m_j^2\right) \delta Z_{Lji} \underbrace{-2m_j \delta m_{ji}^L - 2m_i \delta m_{ji}^R}_{\text{new contributions}} = -2\left(m_i^2 \Sigma_{ji}^L(m_i^2) + m_i m_j \Sigma_{ji}^R(m_i^2)\right.$$
$$\underbrace{\left. + m_j \Sigma_{ji}^{sL}(m_i^2) + m_i \Sigma_{ji}^{sR}(m_i^2)\right)}_{\text{standard piece}}$$

$$\left(m_i^2 - m_j^2\right) \delta Z_{Rji} \underbrace{-2m_j \delta m_{ji}^R - 2m_i \delta m_{ji}^L}_{\text{new contributions}} = -2\left(m_i^2 \Sigma_{ji}^R(m_i^2) + m_i m_j \Sigma_{ji}^L(m_i^2)\right.$$
$$\underbrace{\left. + m_j \Sigma_{ji}^{sR}(m_i^2) + m_i \Sigma_{ji}^{sL}(m_i^2)\right)}_{\text{standard piece}}$$

Setup V

- $\left(\delta m^{L}\right)^{\dagger} = \delta m^{R} \implies$ relation to the **anti-hermitian part** of field renormalization

$$\boxed{\left(m_i^2 - m_j^2\right)}\delta Z^{\boldsymbol{A}}_{Lji} - 2m_j\delta m^{L}_{ji} - 2m_i\delta m^{R}_{ji} = -\left(m_i^2\Sigma^{L}_{ji}(m_i^2) + m_i m_j\Sigma^{R}_{ji}(m_i^2) + m_j\Sigma^{sL}_{ji}(m_i^2) + m_i\Sigma^{sR}_{ji}(m_i^2)\right) + \boxed{H.C.}$$

- Analogous equation for δZ^{A}_{Rji}
- 2 equations and 3 unknowns $\implies \mathbf{3 > 2}$!
- Very *distinct mass structure* in front of field renormalization

Let us explore the properties of $\boxed{m_i^2 - m_j^2}$

From Nielsen Identities [13, 14, 15]

$$\partial_\xi \Sigma_{ji}(p^2) = \Lambda_{jj'}\Sigma_{j'i}(p^2) + \Sigma_{ji'}(p^2)\bar{\Lambda}_{i'i}$$

$$\xrightarrow{1-\text{loop}} \partial_\xi \Sigma_{ji}(p^2) = \Lambda_{ji}\left(\not{p} - m_i\right) + \left(\not{p} - m_j\right)\bar{\Lambda}_{ji}$$

- $\Lambda' s$ have Dirac structure and describe ξ-dependence

$$\boxed{\left(m_i^2 - m_j^2\right)} \partial_\xi \delta Z^A_{Lji} - 2m_j \partial_\xi \delta m^L_{ji} - 2m_i \partial_\xi \delta m^R_{ji} = - \boxed{\left(m_i^2 - m_j^2\right)} \underbrace{\left(m_i \bar{\Lambda}^R_{ji}(m_i^2) + \bar{\Lambda}^{sL}_{ji}(m_i^2)\right)}_{\xi} + H.C.$$

$\Longrightarrow$ $\boxed{m_i^2 - m_j^2}$ **mass structure carried by ξ-dependent terms!**

Exploration: UV divergences I

- 1-loop contributions to self-energies in terms of PV functions [16]
- For boson contributions we have

$$\Sigma^{L,R}(p^2) = f_{L,R} B_1(p^2, m^2_{\psi\text{loop}}, m^2_{\text{bos.}}) \quad \overset{\text{UV}}{\Longrightarrow} \quad f_{L,R} \cdot \boxed{-\frac{1}{\epsilon_{\text{UV}}}}$$

$$\Sigma^{sL,(sR)}(p^2) = m_{\psi\text{loop}} f_s^{(\dagger)} B_0(p^2, m^2_{\psi\text{loop}}, m^2_{\text{bos.}}) \quad \overset{\text{UV}}{\Longrightarrow} \quad m_{\psi\text{loop}} f_s^{(\dagger)} \cdot \boxed{\frac{2}{\epsilon_{\text{UV}}}}$$

- For fermion tadpoles we have

$$\Sigma^{sL,(sR)}(p^2) = f_T^{(\dagger)} m_{\psi\text{loop}} A_0(m^2_{\psi\text{loop}}) \quad \overset{\text{UV}}{\Longrightarrow} \quad f_T^{(\dagger)} m_{\psi\text{loop}} \cdot \boxed{\frac{2m^2_{\psi\text{loop}}}{\epsilon_{\text{UV}}}}$$

- f's are appropriate couplings and $f^\dagger_{L,R} = f_{L,R}$

- We have

$$\left[\left(m_i^2 - m_j^2\right)\delta Z^A_{Lji} - 2m_j\delta m^L_{ji} - 2m_i\delta m^R_{ji}\right]_{\text{div.}} = -\frac{1}{\epsilon_{\text{UV}}}\Big(-f_L\boxed{(m_i^2+m_j^2)} - f_R\boxed{2m_im_j} + 4m_{\psi\text{loop}}f_s\boxed{m_j} + 4m_{\psi\text{loop}}f_s^\dagger\boxed{m_i} + 4m^3_{\psi\text{loop}}f_T\boxed{m_j} + 4m^3_{\psi\text{loop}}f_T^\dagger\boxed{m_i}\Big)$$

- **No UV** divergences with $m_i^2 - m_j^2$!
- Only $(m_i^2+m_j^2)$, $2m_im_j$, m_i and m_j on the r.h.s
  - Rewriting **mass ct's** on l.h.s.:

    $$\Longrightarrow\quad -2\boxed{m_j}\delta m^L_{ji} - 2\boxed{m_i}\delta m^R_{ji} - 2\boxed{(m_i^2+m_j^2)}\delta m^-_{ji} - 2\cdot\boxed{2m_im_j}\delta m^+_{ji}$$

  - $\delta m^{L,R}_{ji}$ are dimension*ful*, $\delta m^{+,-}_{ji}$ are dimension*less*

$\Longrightarrow$ **UV structure mimics mass counterterms!**

Exploration: Summary

- **Gauge dependence** always comes with $\boxed{m_i^2 - m_j^2}$ factors
  - ξ-*independent* terms may also have this structure
  - All the other mass structures are ξ-independent
  - Applies to finite *and* UV divergent terms

- **No UV** divergences with $\boxed{m_i^2 - m_j^2}$ factors
- UV divergences come with $m_i^2 + m_j^2$, $2m_i m_j$, m_j, and m_i mass structures

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- $\boxed{m_i^2 - m_j^2}$ accompanies **field** ct's
- $m_i^2 + m_j^2$, $2m_i m_j$, m_j, and m_i accompanies **mass** ct's

Definitions: Field renormalization

Define the **anti-hermitian part** of field renormalization as the **coefficient of** $\boxed{m_i^2 - m_j^2}$ $(i \neq j)$

$$\delta Z^A_{Lji} \equiv -\left[m_i^2 \Sigma^L_{ji}(m_i^2) + m_i m_j \Sigma^R_{ji}(m_i^2) \right.$$
$$\left. + m_j \Sigma^{sL}_{ji}(m_i^2) + m_i \Sigma^{sR}_{ji}(m_i^2) + H.C.\right]_{\boldsymbol{(m_i^2 - m_j^2)}}$$

$$\delta Z^A_{Rji} \equiv -\left[m_i^2 \Sigma^R_{ji}(m_i^2) + m_i m_j \Sigma^L_{ji}(m_i^2) \right.$$
$$\left. + m_j \Sigma^{sR}_{ji}(m_i^2) + m_i \Sigma^{sL}_{ji}(m_i^2) + H.C.\right]_{\boldsymbol{(m_i^2 - m_j^2)}}$$

- Only ***finite*** logarithmic terms
- Contains all possible **gauge dependence**
- **Universal** definition in terms of self-energies and restrictions
- **No** $\frac{1}{m_i^2 - m_j^2}$ factors by definition!
- The hermitian part is unchanged w.r.t. the usual approach

Definitions: Mass renormalization

Now SOLVE for $\delta m^{L,R}$!

$$\delta m^L_{ji} = \frac{1}{2}\left(m_i \Sigma^R_{ji}(m_i^2) + \Sigma^{sL}_{ji}(m_i^2) + m_j \Sigma^{L\dagger}_{ji}(m_j^2) + \Sigma^{sR\dagger}_{ji}(m_j^2)\right) + \frac{1}{2}\left(m_i \delta Z^A_{Rji} - m_j \delta Z^A_{Lji}\right)$$

$$\delta m^R_{ji} = \frac{1}{2}\left(m_i \Sigma^L_{ji}(m_i^2) + \Sigma^{sR}_{ji}(m_i^2) + m_j \Sigma^{R\dagger}_{ji}(m_j^2) + \Sigma^{sL\dagger}_{ji}(m_j^2)\right) + \underbrace{\frac{1}{2}\left(m_i \delta Z^A_{Lji} - m_j \delta Z^A_{Rji}\right)}_{\text{cancels gauge-dependence}}$$

- **Gauge dependence canceled** by field renormalization
- **Universal** expression in terms of self-energies
- For $\delta Z^A = 0$ can ***extend the real part to the diagonal***, then $\mathrm{Re}\left(\delta m^L_{ii}\right) = \mathrm{Re}\left(\delta m^R_{ii}\right)$ and also matches the results in [2]
- Also **no** $\frac{1}{m_i^2 - m_j^2}$ factors!
- **No** $\widetilde{\mathrm{Re}}$ or Re for Dirac particles
  - *Exceptions* for Majorana particles

- The *bare* Majorana condition $\nu^0 = \nu^{0c}$ at 1-loop *additionally* implies

$$\nu^0 = \nu_L^0 + \nu_L^{0c} \quad \Longrightarrow \quad Z_L \nu_L + Z_R \nu_L^c \quad \Longrightarrow \qquad \delta Z_L = \delta Z_R^\star$$

- *But this **does not hold** due to absorptive parts even when **one no-mixing condition is dropped***

$$\boxed{\delta Z_L \neq \delta Z_R^\star}$$

- The *Majorana condition* is not compatible with in/out no-mixing already at 1-loop
  - **Must drop** absorptive parts — $\widetilde{Re}$

How to use the mass structures if particles are massless?

- Say we have 4 particles, two of which are massless at tree level, i.e. $m = \mathrm{diag}\,(0, 0, m_3, m_4)$
- Then at 1-loop

$$m_{ji} + \delta m_{ji} \sim \left(\begin{array}{c|c} \begin{array}{c}\text{Fully massless, } i,j<3 \\ m_{j,i}=0\end{array} & \begin{array}{c}\text{Partially massive, } j<3, i>2 \\ m_i \neq 0\end{array} \\ \hline \begin{array}{c}\text{Partially massive, } i<3, j>2 \\ m_j \neq 0\end{array} & \begin{array}{c}\text{Fully massive, } i,j>2 \\ m_{j,i} \neq 0\end{array} \end{array}\right)$$

How to use the mass structures if particles are massless?

- Say we have 4 particles, two of which are massless at tree level, i.e. $m = \mathrm{diag}\,(0, 0, m_3, m_4)$
- Then at 1-loop

$$m_{ji} + \delta m_{ji} \sim \left(\begin{array}{c|c} ? & ? \\ \hline ? & \begin{array}{c} \delta Z, \delta m, m_{j,i} \neq 0 \\ \checkmark \text{ out of the box } \checkmark \end{array} \end{array}\right)$$

Massless Particles and Radiative Masses III

How to use the mass structures if particles are massless?

- Say we have 4 particles, two of which are massless at tree level, i.e. $m = \mathrm{diag}\,(0, 0, m_3, m_4)$
- Then at 1-loop

$$m_{ji} + \delta m_{ji} \sim \left(\begin{array}{c|c} \begin{array}{c}\delta m, m_{j,i} = 0 \\ \Sigma^{sL,sR}(0)\end{array} & \delta Z, \delta m, m_i \neq 0 \\ \hline \delta Z, \delta m, m_j \neq 0 & \begin{array}{c}\delta Z, \delta m, m_{j,i} \neq 0 \\ \checkmark \text{ out of the box } \checkmark\end{array} \end{array}\right)$$

$$\delta m^L_{ji} = \frac{1}{2}\Big(m_i \Sigma^R_{ji}(m_i^2) + \Sigma^{sL}_{ji}(m_i^2) + m_j \Sigma^{L\dagger}_{ji}(m_j^2) + \Sigma^{sR\dagger}_{ji}(m_j^2)\Big) + \frac{1}{2}\boxed{m_i \delta Z^A_{Rji} - m_j \delta Z^A_{Lji}}$$

How to use mass structures if particles are massless?

- Say we have 4 particles, two of which are massless at tree level, i.e. $m = \mathrm{diag}\,(0, 0, m_3, m_4)$
- Then at 1-loop

$$m_{ji} + \delta m_{ji} \sim \left(\begin{array}{c|c} \begin{array}{c} \delta m, m_{j,i} = 0 \\ \Sigma^{sL,sR}(0) \Leftrightarrow \not{m}\ \text{limit} \end{array} & \begin{array}{c} \Leftarrow \\ \delta Z, \delta m, m_i \neq 0 \\ \not{m_j}\ \text{limit} \end{array} \\ \hline \begin{array}{c} \Uparrow \\ \delta Z, \delta m, m_j \neq 0 \\ \not{m_i}\ \text{limit} \end{array} & \begin{array}{c} \Leftarrow\Uparrow \\ \delta Z, \delta m, m_{j,i} \neq 0 \\ \checkmark\ \text{out of the box}\ \checkmark \end{array} \end{array} \right)$$

$$\delta Z^A_{Lji} = -\left[m_i^2 \Sigma^L_{ji}(m_i^2) + m_i m_j \Sigma^R_{ji}(m_i^2) + m_j \Sigma^{sL}_{ji}(m_i^2) + m_i \Sigma^{sR}_{ji}(m_i^2) + H.C.\right]_{(m_i^2 - m_j^2)}$$

Massless Particles and Radiative Masses V

How to use mass structures if particles are massless?

- Say we have 4 particles, two of which are massless at tree level, i.e. $m = \text{diag}\left(0, 0, m_3, m_4\right)$
- Then at 1-loop

$$m_{ji} + \delta m_{ji} \sim \left(\begin{array}{c|c} \begin{matrix}\delta m, m_{j,i} = 0 \\ \Sigma^{sL,sR}\left(0\right)\end{matrix} & \dots \\ \hline \dots & \dots \end{array}\right)$$

- Tree level *massless* particles are *indistinguishable — mixing matrix not fully determined!*
- $\Sigma^{sL,sR}\left(0\right)$ can be diagonalized with leftover freedom from the tree-level!

$\implies$ *Radiative mass!* [17]

Explicit SM up-quark field renormalization

To give an explicit example:

- SM up quark field renormalization is

$$\delta Z^{A,u}_{Lji} = -\frac{V_{jk}V^{\star}_{ik}}{2^D\pi^{D-2}v^2}\left[(m^d_k)^2 - (m^u_i)^2 + \xi_W m^2_W\right] B_0((m^u_i)^2,(m^d_k)^2,\xi_W m^2_W)$$
$$+ \frac{V_{jk}V^{\star}_{ik}}{2^D\pi^{D-2}v^2}\left[(m^d_k)^2 - (m^u_j)^2 + \xi_W m^2_W\right] B_0((m^u_j)^2,(m^d_k)^2,\xi_W m^2_W)^{\star}$$
$$\delta Z^{A,u}_{Rji} = 0$$

- **No** $\frac{1}{m_i^2 - m_j^2}$ factors $\longrightarrow$ massless and degenerate mass limits!
- **Finite** and ξ-dependent
- Note complex conjugation on B_0!
- **Vanishing** $\delta Z^{A,u}_{Rji}$ is *in contrast* with the usual approach

The *Need* to Renormalize Mixing Matrices? I

	Usual approach(es)	Proposed scheme
On-shell propagator	Overspecified δZ *or* non-diag.	"diag." **in** *or* **out**
Field ct; hermitian part	$\xi, \epsilon_{\mathrm{UV}}$	$\xi, \epsilon_{\mathrm{UV}}$
diagonal mass ct	ϵ_{UV}	ϵ_{UV}
off-diagonal mass ct	$\times$	$\boldsymbol{\epsilon_{\mathrm{UV}}}$
Field ct; **anti**-hermitian part	$\xi, \boldsymbol{\epsilon_{\mathrm{UV}}}$	$\xi,$ ~~$\boldsymbol{\epsilon_{\mathrm{UV}}}$~~
Wud vertex	$\boldsymbol{\epsilon_{\mathrm{UV}}}$	~~$\boldsymbol{\epsilon_{\mathrm{UV}}}$~~
CKM ct	$\delta V \sim -\delta \tilde{Z}_L^{A,u} V + V \delta \tilde{Z}_L^{A,d}$ $\boldsymbol{-\epsilon_{\mathrm{UV}}}$ and sometimes ξ	$\boldsymbol{\delta V = 0}$ ~~$\xi, \epsilon_{\mathrm{UV}}$~~

- UV divergences stay in the mass term and do *not migrate* to other terms
- Usual **CKM** ct only needed to **cancel the migration**!
  - That initially included ξ-dependent terms...

Is it consistent to **not** renormalize mixing matrices?

Mixing matrices appear due to diagonalization of mass!

Scenario 1	Scenario 2
– Diagonalize the mass — V^0	– Do not diagonalize — $\cancel{V^0}$
– Renormalize — $V^0 \to V + \delta V$	– Renormalize — $\cancel{V^0}$ means $\cancel{\delta V}$
– Rotate back — $V + \delta V \to \cancel{V} + \delta V'$	– Diagonalize — $V + \cancel{\delta V}$

- ***Both*** scenarios must be ***valid***, so

$$\begin{cases} V + \delta V = V + \cancel{\delta V} \\ \cancel{V} + \delta V' = \cancel{V^0} + \cancel{\delta V} \end{cases}$$

- ✓ **holds if** $\boxed{\delta V = 0}$ ✓
- × **otherwise** *inconsistent* ×

The *Need* to Renormalize Mixing Matrices? III

- Mixing matrices are *basis artefacts* — **renormalization is inconsistent!**
  - *Mixing is physical and mass fully accounts for it*
- The argument is *extremely simple* and so is valid
  - For fermions and bosons
  - And at all orders
- Our scheme gives an **explicit example of non-renormalization**!

Conclusions

We defined a new fermion renormalization scheme that

- ✓ Is *universal*
- ✓ Relies on (incoming) no-mixing conditions and *mass structures*
- ✓ Does *not rely on dropping the absorptive parts*
- ✓ Has ξ–*independent* mass counterterms
- ✓ Has *finite* anti-hermitian part of field counterterms
- ✓ Can incorporate massless particles and radiative mass generation
- ✓ Avoids *migrating* UV divergences and keeps the Lagrangian Hermitian

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Finally and perhaps most importantly

- ✓ **There is no need to renormalize mixing matrices!**

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