Benchmark of e-cooling simulations with RF-Track

Agnieszka Borucka, Davide Gamba, Andrea Latina

agnieszka.elzbieta.borucka@cern.ch

September 8, 2021

Simulation codes

- RF-Track

  A tracking code developed by Andrea Latina (CERN)
  `https://gitlab.cern.ch/rf-track/rf-track-2.0`

- Betacool

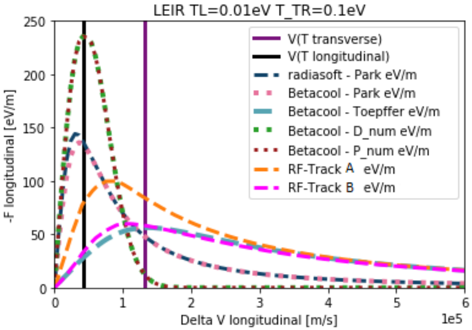
  A standard and widely used code for electron cooling simulation. Developed by JINR but no longer supported.
  `https://gitlab.cern.ch/e-beam/betacool/-/tree/master`

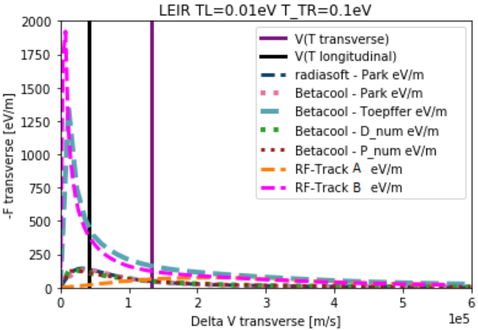
Models

Available models for magnetized cases:

- RF-Track
  - Model based on book of Nersisyan Hrachya and Toepffer Christian and Zwicknagel Günter[2] – three versions (A, B, C)
- Betacool[1]
  - Parkhomchuk – semi-empirical formula in the friction force in magnetized electron beam
  - Toepffer – binary collision model assuming the ion velocity stays constant in a collision with an electron
  - Debrenev-Skrinsky-Meskov – model the interaction based on the maximum impact parameter, can have some problems with intagration for small ion velocity

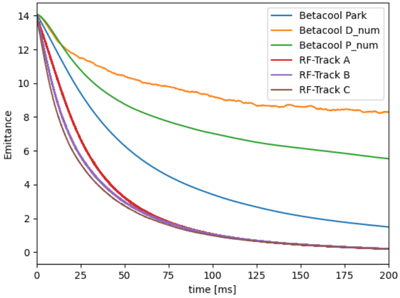
LEIR – model comparision

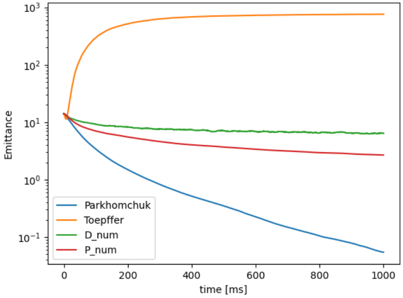




- Ions: A:208, Q=+54, K_0=862.68 MeV, $V_{||}$=2.8e7 m/s
- Electrons: uniform distribution, I=0.6A, $T_\perp$=0.1 eV, $T_{||}$=0.01 eV
- Cooler: L=2.5m, B=0.075T, r=25mm

Comparison of emittance evolution in time for aviable models.





Discarded:

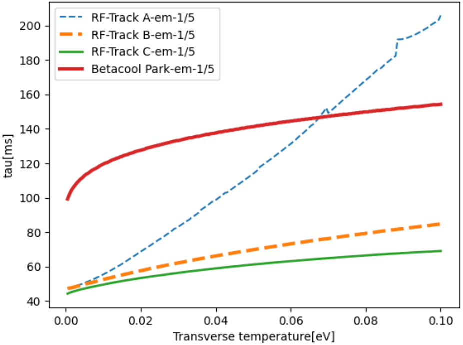
Toepffer model:
Unexpected behavior.
Reasons not understood.

Debrenev-Skrinsky model:
Long computing time

- Ions: A:208, Q=+54, K_0=862.68 MeV, #10000
- Electrons: uniform distribution, I=0.6A, $T_\perp$=0.01eV, $T_\parallel$=0.001 eV
- Cooler: L=2.5m, B=0.075T, r=25mm, $\beta_{x/y}$=5m, $\alpha = 0$, D=0m, D'=0

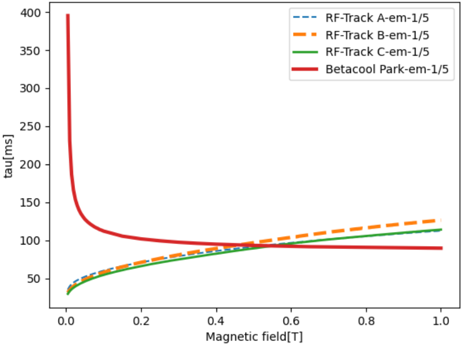
NOTE: No additional effects like IBS or space charge.

LEIR – scan of cooling time $\left(\tau = \frac{\epsilon_0}{5}\right)$



- **Scan over transverse temperature:**
  - Different behaviour of RF-Track A in respect to all other models
  - The main behaviour of cooling time is similar for RF-Track B and C and Betacool but there is a significant difference in timescale

LEIR – scan of cooling time $\left(\tau = \frac{\epsilon_0}{5}\right)$



- **Scan over magnetic field:**
  - Opposite behaviour of cooling time for RF-Track and Betacool
  - Differences visible in force for different versions of RF-Track are not significant for cooling time

<u>NOTE:Impact of cooler magnetic field on ion beam is neglected.</u>

RF-Track by default takes it into account but it was disabled for a fair comparison with Betacool.

Models – RF-Track

The difference between RF-Track versions is in their magnetized components of force:

RF-Track A – $F_{magnetised} = L_M \iiint \left[\frac{U_{B\perp}^2}{U_B^5} \left(\vec{U}_{B\parallel}^5 + \frac{\vec{U}_{B\perp}}{2} \left(1 - \frac{U_{B\parallel}^2}{U_{B\perp}^2} \right) f(v_e) dv_e \right) \right]$

RF-Track B – $F_{magnetised} = L_M \int \left[\frac{U_{B\perp}^2}{U_B^5} \left(\vec{U}_{B\parallel}^5 + \frac{\vec{U}_{B\perp}}{2} \left(1 - \frac{U_{B\parallel}^2}{U_{B\perp}^2} \right) \right) \right] f(v_{e\parallel}) dv_{e\parallel}$

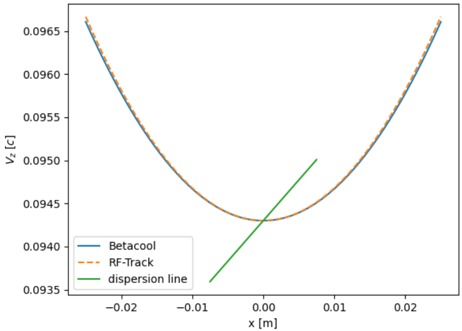
RF-Track C – $F_{magnetised} = \int \left[L_A \frac{\vec{U}_B}{U_B^3} + L_{M2} \frac{U_{B\perp}^2}{U_B^5} \left(\vec{U}_{B\parallel}^5 + \frac{\vec{U}_{B\perp}}{2} \left(1 - \frac{U_{B\parallel}^2}{U_{B\perp}^2} \right) \right) \right] f(v_{e\parallel}) dv_{e\parallel}$

$f(v_e)$ – the distribution of electrons velocities

Based on obtained results the RF-Track C was considered the most stable and complex version and chosen as the default one.

RF-Track A was discarded due to some instabilities. The choice between RF-Track B and C was not straightforward.

Electron beam profile



In following simulations additional properties were considered:

- dispersion in e-cooler (here: 1m)
- electron space charge

Dispersion line: vertical for D=0m and horizontal for D$\rightarrow\infty$

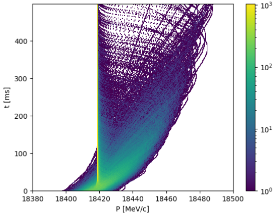
Electron energy follows the dependence:

$$\frac{\Delta E(r)}{E_0} = \frac{I_e r_e}{ec}\frac{\gamma+1}{\beta_0^3\gamma^2}\left(\frac{r}{r_0}\right)^2 \approx 1.2\times10^{-4}\frac{I_e[A]}{\beta_0^3}\left(\frac{r}{r_0}\right)^2 \text{ [4]}$$

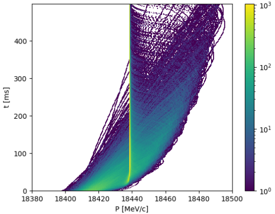
where r_0 is the distance from the beam centre and E_0 is the energy in the centre

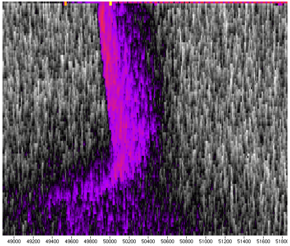
Dispersion and space charge

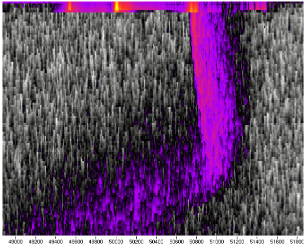
$V_e = V_{beam}$



$V_e = 1.001V_{beam}$







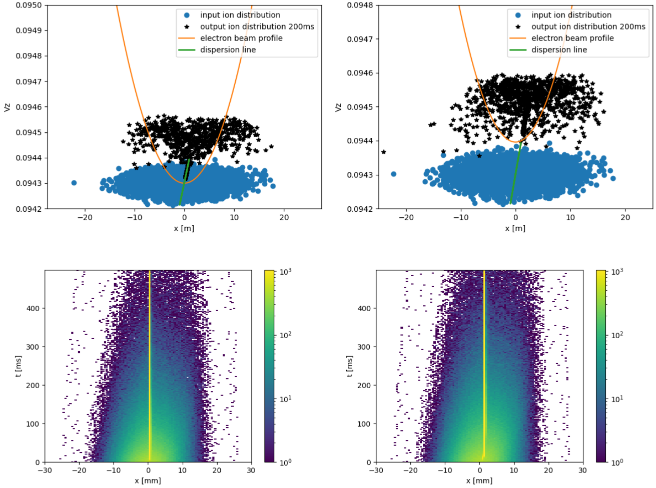
Evolution of momentum distribution of ion beam in a simulation of cooling with dispersion and space charge of electrons for two different mean electron velocities. Dispersion: D_x=1m, D_y=0m

Rough comparison with measurements from AD. – Parameters are not the same.

The simulation predicts the sharp edge visible in Schottky waterfalls of the electron beam in AD.

Dispersion and space charge

$V_e = V_{beam}$ | $V_e = 1.001\,V_{beam}$



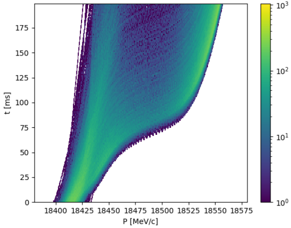
Comparison of input ion beam $V_z(x)$ distribution with the beam after 200ms of cooling simulation.

The beam tends to the velocity and horizontal position corresponding to the intersection of dispersion line and line describing electron velocity.

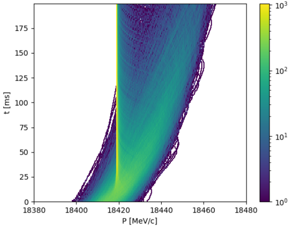
Due to no heating effects being considered, the ions' distributions have an un-physical dense core.

Horizontal shift of electron beam.

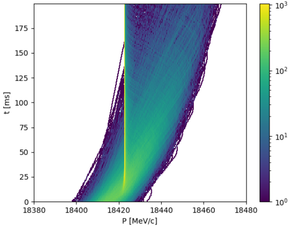
$shift = -3mm$



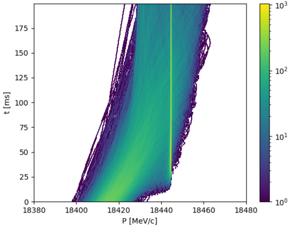
$shift = 0mm$



$shift = 3mm$



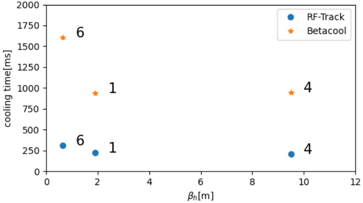
$shift = 9mm$



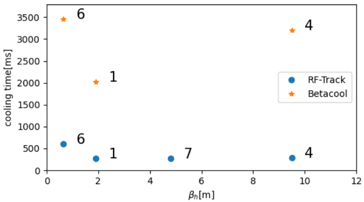
For different horizontal shifts of the electron beam in respect to the ion beam the sharp edge of momentum moves.

Comparison with experimental data

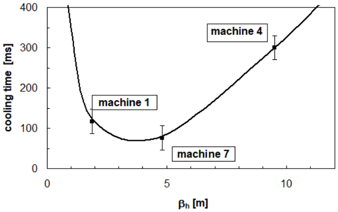
$T_\perp$=0.1eV, $T_\parallel$=0.01eV



$T_\perp$=0.1eV, $T_\parallel$=0.1eV



- Ions: A:208, Q=+54, K_0=862.68 MeV, p_{spread}=0.025%, ϵ_0=10mm.rad
- Electrons: uniform distribution, I=0.35A
- Cooler: L=1.5m, B=0.06T, r=25mm
- Cooling time: $\epsilon(\tau) = \frac{\epsilon_0}{10}$



Machine	1	4	6	7
β_H[m]	1.9	9.5	0.65	4.8
β_V[m]	6.4	10.5	5.5	5.0
D[m]	3.6	0	0	5.0

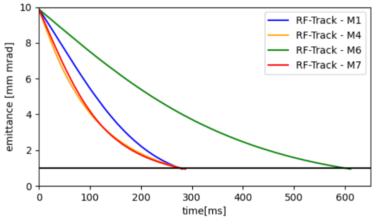
- One parameter changes both timescale and the behaviour significantly.

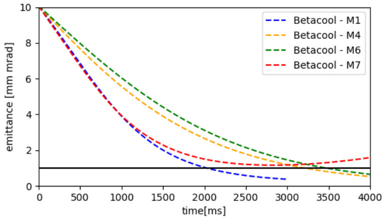
J. Bosser et al. Experimental investigation of electron cooling and stacking of lead ions in a low energy accumulation ring.

Comparison with experimental data

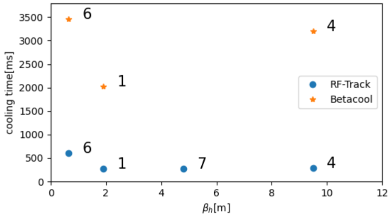
$T_{\perp}$=0.1eV, $T_{\parallel}$=0.1eV

Cooling time: $\epsilon(\tau) = \frac{\epsilon_0}{10}$





Black line indicates the value of emittance $\frac{\epsilon_0}{10}$



- Cooling time definition influences the relation between results for different machines.

Summary

Within this study it was done:

- Comparison of friction force in a wide range of parameters between aviable models – See backup for details.
- Study of the dependence of e-cooling time on a wide range of parameters.
- Study of the behaviour of e-cooling with dispersion and electron space charge.
- Qualitative comparison of experimental data with simulation.

Analysis of results for some extreme cases allows seeing limits for different codes. In the case of RF-Track, it also allowed to solve some minor details and improve its robustness.
The comparison between possible versions of RF-Track helped to choose the most stable and accurate version – RF-Track C.

Future plans

- ☐ Document all results in a summary document – ongoing.
- ☐ Add and study the influence of IBS and space charge effects.
- ☐ Simulate e-cooling with realistic parameters for all CERN coolers/energies.
- ☐ Look for possible experiments that could clearly indicate which model is more accurate, e.g. scan over b-field in e-cooler.

Bibliography

1. Physics guide of BETACOOL codeVersion 1.1
https://www.agsrhichome.bnl.gov/AP/ap_notes/ap_note_262.pdf

2. Nersisyan, Hrachya and Toepffer, Christian and Zwicknagel, Günter, *Interactions between charged particles in a magnetic field: a theoretical approach to ion stopping in magnetized plasmas*, Springer, Berlin, 2007

3. RF-Track documantation
https://gitlab.cern.ch/rf-track/rf-track-2.0/-/tree/master/doc

4. Helmut Poth. Electron cooling: theory, experiment, application.Physics Reports(Review Section of PhysicsLetters), 196:135297, 199

THANK YOU

Backup

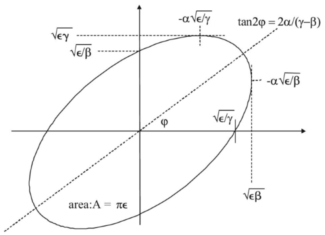
Electron cooling

Size of the beam depends on emittance ϵ which increases when the energy decreases.

$$\beta_{rel}\gamma_{rel}\epsilon = const$$

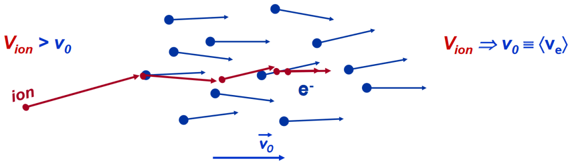
Electron cooling is used to shrink the beam in terms of emittance and momentum spread.

Electrons moving with the same average velocity as ion beam can absorb kinetic energy of ions.



Wiedemann H.

https://doi.org/10.1007/978-3-319-18317-6_8



https://indico.cern.ch/event/297045/contributions/1658342/

Two gases with different temperatures.
$\rightarrow \frac{1}{2}k_B T = \frac{1}{2}mv^2$
As electrons are constantly renewed, ion beam tends to the temperature of electrons reducing the velocity spread by a factor $\sqrt{\frac{m_e}{M_i}}$.

Models – Notation

Notation:

JSPEC - Park – JSPEC2 by JLab, Parkchomchuk model
radiasoft - Park – JSPEC by radiasoft, Parkchomchuk model
radiasoft - Meskov – JSPEC by radiasoft, Meskov model

Betacool (- Park) – Betacool, Parkchomchuk model
Betacool - Topff/Toepffer – Betacool, Toepffer model
Betacool - P_num – Betacool, numerical approach Debrenev-Skrinsky model, Pestrikov-integral
Betacool - P_as – Betacool, asymptotic approach of Debrenev-Skrinsky model, Pestrikov-integral
Betacool - D_num – Betacool, numerical approach Debrenev-Skrinsky model, Debrenev-integral
Betacool - D_as – Betacool, asymptotic approach of Debrenev-Skrinsky model, Debrenev-integral

RF-Track A – RF-Track, $F_{magnetised}$ with triple integral
RF-Track B (newRF-Track) – RF-Track, $F_{magnetised}$ with single integral
RF-Track C – RF-Track, $F_{magnetised}$ with two components

Modeling – important parameters

One of the simplest and widely used methods to model an electron cooling process is model based on semi-empirical Parkhomchuk formula.

$$\vec{F} = -4\frac{n_e}{m_e}\frac{(Qe^2)^2}{(4\pi\epsilon_0)^2}\ln\left(\frac{b_{max}+b_{min}+r_c}{b_{min}+r_c}\right)\frac{\vec{v}_{ion}}{(v_{ion}^2+v_{eff}^2)^{3/2}}$$

$$b_{min} = \frac{Qe^2/4\pi\epsilon_0}{m_e v_{ion}^2}, \quad b_{max} = \frac{v_{ion}}{min(\omega_{pe}, 1/T_{cool})}, \quad v_{eff}^2 = v_{e,\parallel}^2 + v_{e,\perp}^2$$

Each model depends on similar parameters but the dependenve can be different in each case. Parameters considered as the most important are temperatures of electrons and magnetic field.

Models – RF-Track

Model based on book of Nersisyan Hrachya and Toepffer Christian and Zwicknagel Günter[2]

$$\vec{F} = -\frac{4\pi n_e K^2}{\mu}\left\{\underbrace{\iiint\left[L_F\frac{\vec{U}}{U^3}\right]f(\vec{v}_e)\,d\vec{v}_e}_{\mathcal{F}_{\text{Unmagnetized}}} + \underbrace{\int\left[L_M\frac{U_{B\perp}^2}{U_B^5}\left(\vec{U}_{B\|} + \frac{\vec{U}_{B\perp}}{2}\left(1-\frac{U_{B\|}^2}{U_{B\perp}^2}\right)\right)\right]f(v_{e\|})\,dv_{e\|}}_{\mathcal{F}_{\text{Magnetized}}}\right\}$$

$f(v_e)$ – the distribution of electrons velocities

There are three versions of RF-Track currently testing. The difference between them is in $F_{magnetised}$ component of the force.

Notation:

RF-Track A – $F_{magnetised}$ with triple integral (over $f(\vec{v}_e)$)
RF-Track B – $F_{magnetised}$ with single integral (over $f(v_{e\|})$)
RF-Track C – $F_{magnetised} = \int\left[L_A\frac{\vec{U}_B}{U_B^3} + L_{M2}\frac{U_{B\perp}^2}{U_B^5}\left(\vec{U}_{B\|}^5 + \frac{\vec{U}_{B\perp}}{2}\left(1-\frac{U_{B\|}^2}{U_{B\perp}^2}\right)\right)\right]f(v_{e\|})dv_{e\|}$

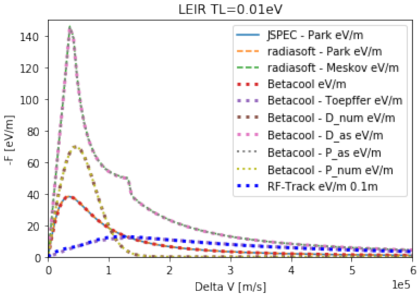
NOTE

For this analysis special conditions were set.

The cooling force in RF-Track was obtained by tracking a single ion.
For longitudinal friction force scan, the transverse velocity difference is equal to 0.

Base on the the definition of RF-Track force: $\Delta V_{transverse} = 0 \rightarrow F_{magnetized} = 0$

It is not exactly known if in Betacool is any additional ϵ on the velocity which allows this component to appear. This can mean that the comparison is not exactly correct.



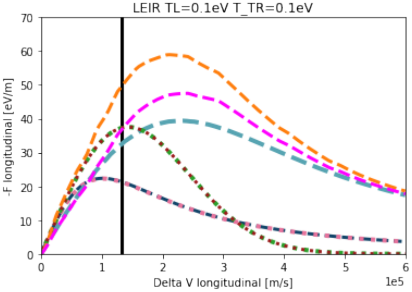
Parameters:

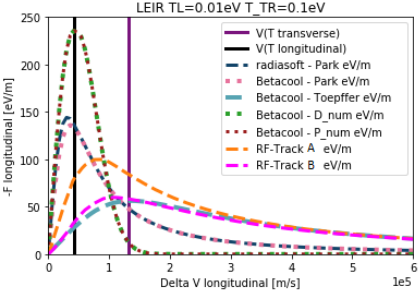
- Ions: A:208, Q=+54, K_0=862.68 MeV, $V_{\parallel}$=2.8e7 m/s
- Electrons: uniform distribution, I=0.6A, T_{tr}=0.1eV, T_l=0.01eV
- Cooler: L=2.5m, B=0.075T, r=25mm

- Different models give significantly different results for the same input parameters
- Meskov model (Radiasoft) and Debrenev-Skrinsky asymptotic approach (Betacool) are discarded from the following analysis because of their additional features

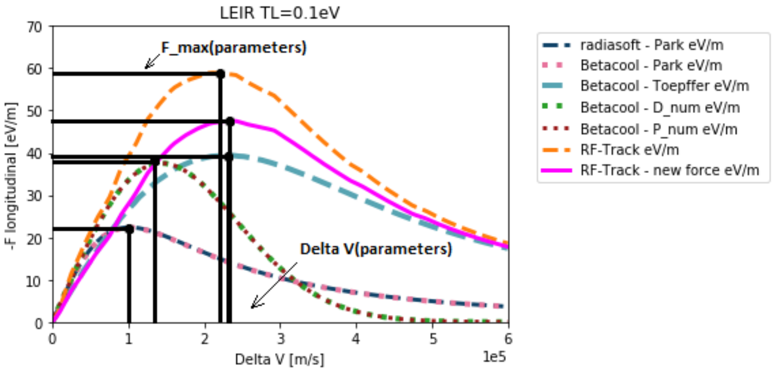
LEIR – model comparision

The modification of single parameter can completely change the results.





- Ions: A:208, Q=+54, K_0=862.68 MeV, $V_{\parallel}$=2.8e7 m/s
- Electrons: uniform distribution, I=0.6A, $T_{\perp}$=0.1 eV, $T_{\parallel}$=0.01 eV
- Cooler: L=2.5m, B=0.075T, r=25mm

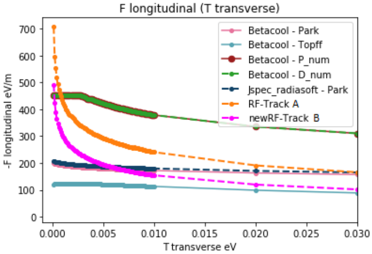


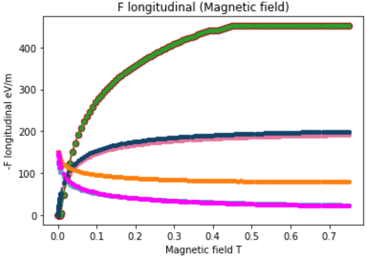
Value of maximum force or ΔV for it for particular set of parameters.

$\downarrow$

Value of maximum force or ΔV for it for scanning over one choosen parameter.

LEIR – scan of longitudinal force

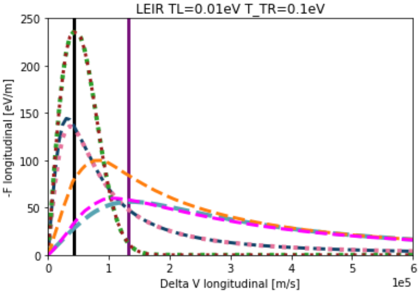


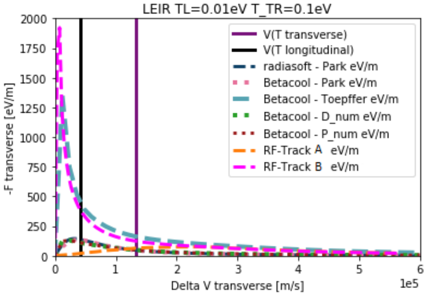


- scan over transverse temperature:
  - Derbenev-Skrinsky – cut for small temperatures.
  - Toepffer and RF-Track B converge. Differences for very low $T_\perp$.
  - At small temperatures, the thermal effect becomes important.
- scan over magnetic field:
  - Two different behaviours: for one group the maximum force raises with magnetic field for other it decreases.
  - Toepffer model(Betacool) agrees with RF-Track B.
  - Debrenev-Skrinsky – for impact parameter smaller than the radius of the electron rotation – influence of the magnetic field is neglected.
  - $\Delta V_\perp = 0$ – magnetized component of the force is very small.

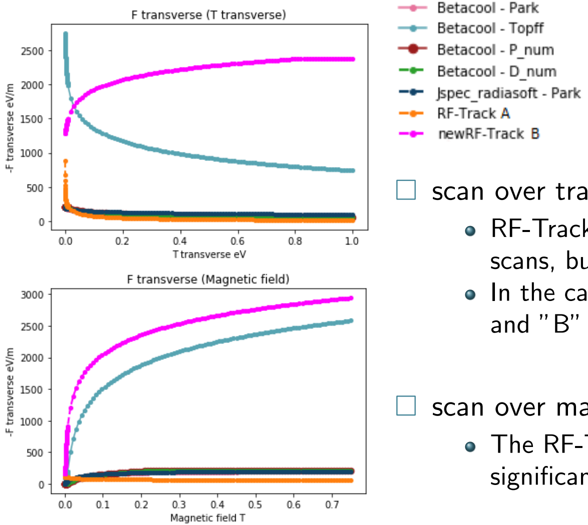
LEIR – model comparision – transverse friction force

Topffer and RF-Track B have additional features for low ΔV but their behaviour for different temperatures can be different.





- Ions: A:208, Q=+54, K_0=862.68 MeV, $V_{\|}$=2.8e7 m/s
- Electrons: uniform distribution, I=0.6A, $T_{\perp}$=0.1 eV, $T_{\|}$=0.01 eV
- Cooler: L=2.5m, B=0.075T, r=25mm



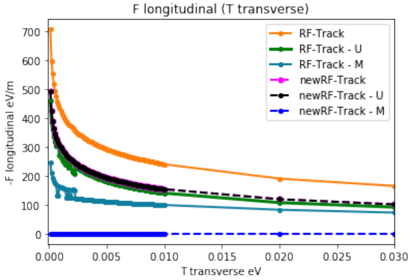
- scan over transverse temperature:
  - RF-Track B fits Toepffer model quite well for most of scans, but there are still some crucial differences.
  - In the case of RF-Track, the two implementations "A" and "B" produce significantly different results.
- scan over magnetic field:
  - The RF-Track B and Toepffer model of Betacool have significantly different behaviour from others.

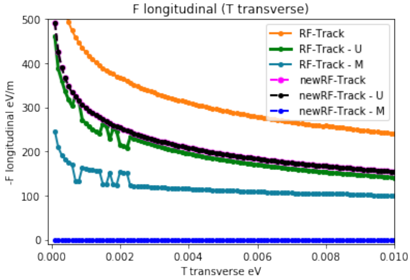
As RF-Track's force is a sum of two components(unmagnetized and magnetized force) the impact of each of them was checked.
The behaviour of the newRF-Track is tipically similar to Toepffer's of Betacool. The most significant difference is in scan of transverse friction force over transverse temperature so this scan was choosen for component's impact analysis.

Notation

U – unmagnetized force
M – magnetized force

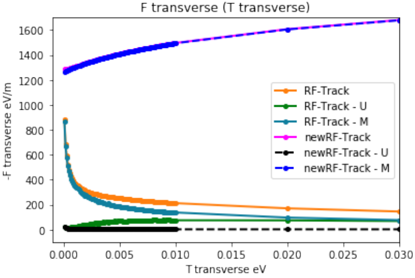
Longitudinal friction force

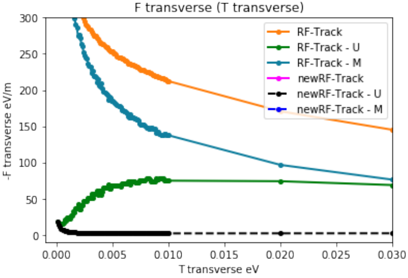




Vlues of each component correspond to ΔV for maximum value of final force.

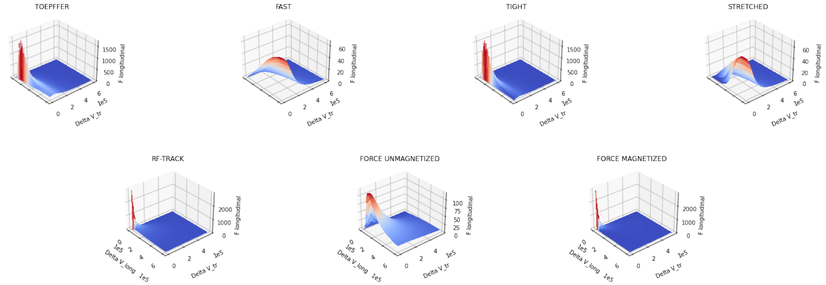
Transverse friction force





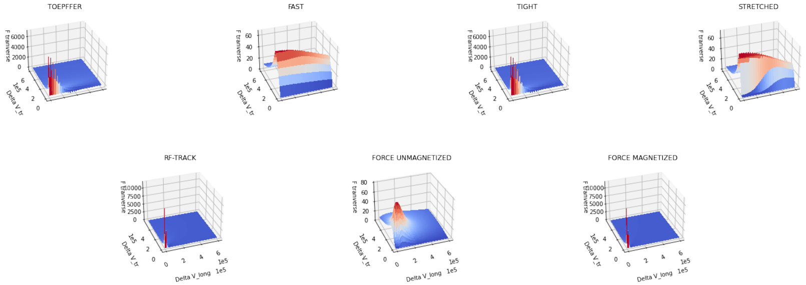
Vlues of each component correspond to ΔV for maximum value of final force.

LEIR – SCAN / RF-Track vs Toepffler



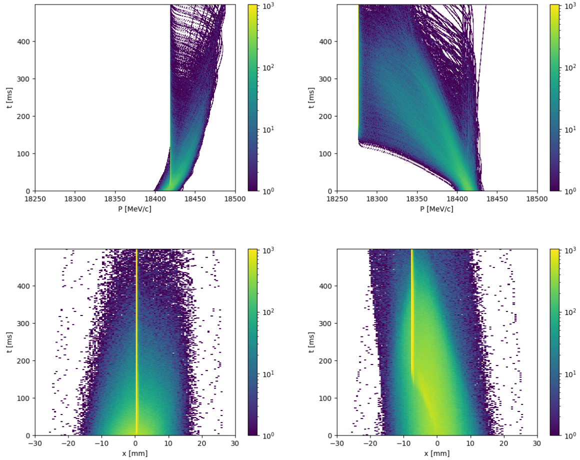
$\Delta V_{||}$ for Toepffler 0-1e5m/s.

Transverse friction force



$\Delta V_{||}$ for Toepffler 0-1e5m/s.

$V_e = V_{beam}$ $\quad$ $V_e = 0.99 V_{beam}$

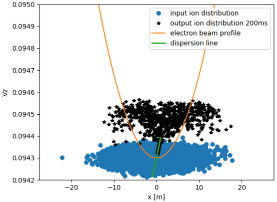


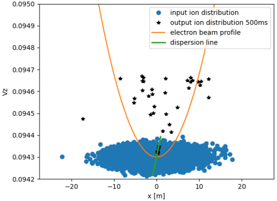
Evolution of momentum distrubition and horizontal distribution of ion beam in simulation of cooling with dispersion and space charge of electrons for two different mean electron velocities.

Dispersion: D_x=1m, D_y=0m

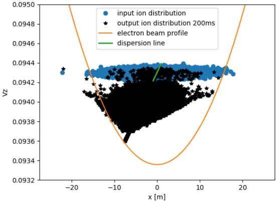
Due to no heatting effects the ions' horizontal distribution has un-physical dense core.

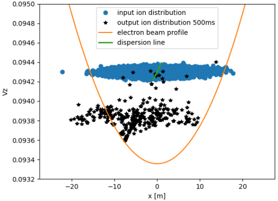
$V_e = V_{beam}$





$V_e = 0.99 V_{beam}$

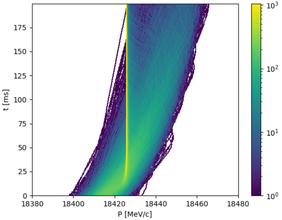




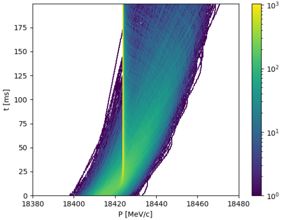
Comparison of input ion beam $V_z(x)$ distribution with the beam after 200ms and 500ms of cooling simulation.

The beam tends to the velocity and horizontal position corresponding to the intersection of dispersion line and line describing electron velocity.

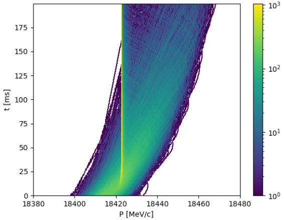
$D = 0m$

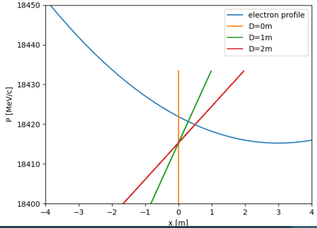


$D = 1m$



$D = 2m$





3mm horizontal shift

The beam tends to the velocity and horizontal position corresponding to the intersection of dispersion line and line describing electron velocity.