The $g_T(x)$ contribution to single spin asymmetry in SIDIS

Abhiram Kaushik
University of Zagreb

Benić, Hatta, Li, AK Phys. Rev. D 104 (2021) 9, 094027

DIS 2022, Santiago de Compostela, Spain, May 2-6, 2022
Single-Spin Asymmetries

- Collisions involving transversely polarised hadrons - left-right asymmetry in particle production
- Observed in various $pp$ and $ep$ processes since the 70s

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

![Graph showing $A_N$ versus $x_F$ for $\pi^+$, $\pi^-$, and $\pi^0$ data.](image)
Single-Spin Asymmetries

- QCD is time reversal invariant
- SSAs are T-odd:

\[ A_N \propto S \cdot (P \times k) \]

An phase (interference) is required!

- Quest for theoretical description of SSAs is a quest for sources of a complex phase
Theoretical description of SSAs

- In a collinear factorization framework, SSAs have been described in terms of **three-parton correlation functions**, i.e. **twist-3 parton distribution functions (ETQS)** functions and **twist-3 fragmentation functions**
  

- Eg., above hard part can be convolved with ETQS distribution. Complex phase arises when internal propagator goes on-shell.
  
  \[
  \frac{1}{(k^2 + i\epsilon)} = \text{PV}(1/k^2) + i\pi\delta(k^2)
  \]

- **Twist-3 distributions not very well known.**
In an earlier work, my collaborators and D.J. Yang had shown that the twist-3 quark distribution $g_T(x)$ can lead to SSA in SIDIS at the two-loop level.

Imaginary phase arises from when certain internal propagators are cut.

$g_T$ is a chiral-even transverse-spin dependent contribution to the quark-quark correlator

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)\psi(\lambda n)|PS\rangle = \frac{M_N}{2} \gamma_5 \gamma_T g_T(x) + ...$$
SSAs at two loops

- Schematically,
  \[ d\sigma^\uparrow - d\sigma^\downarrow \propto \frac{d\Delta\sigma}{dP_{hT}} \sim g_T(x) \otimes H^{(2)} \otimes D_1(z) \]

- Wandzura-Wilczek relation:
  \[ g_T(x) = \int_x^1 \frac{dx'}{x'} \Delta q(x') + \text{(genuine twist-3)} \]

- In effect,
  \[ \frac{d\Delta\sigma}{dP_{hT}} \sim g_T(x) \otimes H^{(2)} \otimes D_1(z) \sim \Delta q(x) \otimes H^{(2)} \otimes D_1(z) \]

- SSA completely determined in terms of well understood twist-2 distributions!
In this work we,

1. extend the analysis to include gluon-initiated contribution from the twist-3 distribution $G_{3T}$, which is the gluonic counterpart of $g_T$.
   
   - In analogy with the quark case,
     \[
     \frac{d\Delta\sigma}{dP_{hT}} \sim G_{3T}(x) \otimes H_g^{(2)} \otimes D_1(z) \sim \Delta G(x) \otimes H_g^{(2)} \otimes D_1(z)
     \]

2. present numerical estimates for asymmetry in SIDIS through these mechanisms at COMPASS and EIC.
**Why $g_T$ at two loops?**

- $g_T$ appears in the hadronic tensor through the collinear expansion of the two-parton correlator $M^{(0)}$.

\[ M^{(0)} \sim \rho f(x) + M_N \rho \gamma_5 (S \cdot n) \Delta q(x) + M_N S_T \gamma_5 g_T(x) + \ldots \]

- $g_T$ appears in correlator with $\gamma_5$. Traces involving $\gamma_5$ produce a factor of $i \Rightarrow g_T$ receives no contributions from hard part $S^{(0)}$ at the Born level.


- Can receive non-zero contributions beyond Born level.

Benić, Hatta, Li, Yang Phys. Rev. D 100 (2019) 9, 094027
Gluon initiated contribution $G_{3T}$

Two-gluon correlator in a polarised proton,

$$M^{(0)\alpha\beta} \sim \langle PS_T | F^\alpha W F^\beta | PS_T \rangle \sim xG(x)g_{T}^{\alpha\beta} + iM_N x \Delta G(x)(S \cdot n)\epsilon^{n\alpha\beta} + iM_N x G_{3T}(x)\epsilon^{n\alpha\beta}s_\perp$$

Hatta, Tanaka, Yoshida, JHEP 02, 003 (2013)

- $G_{3T}$ - transverse spin dependent contribution to the two-gluon correlator.
- WW approximation

$$G_{3T}(x) = \frac{1}{2} \int_x^1 \frac{dx'}{x} \Delta G(x') + \text{(genuine twist three)}$$
**Calculation of hard part**

Asymmetry can be written schematically as,

\[
A_{UT}^\sin(\alpha\phi_h + \beta\phi_S) \sim \frac{\alpha_s^2}{\alpha_s} \left( xg_T(x) \text{ or } xG_3T(x) \right) \otimes H^{(2)} \otimes D_1(z) \sim \alpha_s \frac{x\Delta q \text{ or } x\Delta G}{q(x)}
\]

All contributions to \( H^{(2)} \) have a generic 'AMA' structure:

- Each blob represents 2-2 scattering
- Phase arises from cutting internal lines, i.e., regions of loop momentum \( l_2 \) where the two lines go on-shell.
- Potential divergence when \( l_2 \) gluon is collinear to proton cancels out between \( S^{(0)}_{\mu\nu} \) and \( \frac{dS^{(0)}_{\mu\nu}}{dk_T^\alpha} \)
Calculation of hard part

\[
\frac{d^6 \Delta \sigma}{dx_B dQ^2 dz_f dq_T^2 d\phi d\chi} = \frac{\alpha_{em}^2 \alpha_S^2 M_N}{16\pi^2 x_B^2 S_{ep} Q^2} \sum_k A_k S_k \int \frac{dx}{x} \int \frac{dz}{z} \\
\times \delta \left( \frac{q_T^2}{Q^2} - \left( 1 - \frac{1}{x} \right) \left( 1 - \frac{1}{z} \right) \right) \\
\times \sum_f e_f^2 \left[ D_f(z) x^2 \frac{\partial g_T f (x)}{\partial x} \Delta \hat{\sigma}^{qq}_{Dk} + D_f(z) x g_T f (x) \Delta \hat{\sigma}^{qq}_{k} + (qg \text{ channel}) + (gq \text{ channel}) \right]
\]
Numerical results

$$A_{UT}^{\sin(\alpha \phi_h + \beta \phi_S)} = \frac{2 \int_0^{2\pi} d\phi_h d\phi_S \sin(\alpha \phi_h + \beta \phi_S) \left[ d\sigma(\phi_h, \phi_S) - d\sigma(\phi_h, \phi_S + \pi) \right]}{\int_0^{2\pi} d\phi_h d\phi_S \left[ d\sigma(\phi_h, \phi_S) + d\sigma(\phi_h, \phi_S + \pi) \right],}$$

- Five independent moments including Sivers $A_{UT}^{\sin(\phi_h - \phi_S)}$ and Collins $A_{UT}^{\sin(\phi_h + \phi_S)}$
- Sivers and Collins asymmetry NOT from Sivers and Collins functions.
- $g_T(x)$ and $G_3T$ from helicity distributions using the WW approximation.
  
  $$g_T(x) = \int_x^1 \frac{dx'}{x'} \Delta q(x'), \quad G_{3T}(x) = \int_x^1 \frac{dx'}{x'} \Delta G(x')$$

- We used the latest fits of helicity distributions from NNPDF and JAM.
  

$$A_{UT}^{\sin(\alpha \phi_h + \beta \phi_S)} \sim \frac{\alpha_s^2}{\alpha_s} \left( xg_T(x) \text{ or } xG_{3T}(x) \right) \otimes H^{(2)} \otimes D_1(z) \sim \alpha_s x\Delta q \text{ or } x\Delta G$$

Abhiram Kaushik (Univ. of Zagreb)
Sivers asymmetry at EIC

- Sivers asymmetry up to 2% with JAM at moderate-to-large $x$ and low $Q^2$.
- Decreases at low-$x$. 

Abhiram Kaushik (Univ. of Zagreb)
Channel breakdown

- **cancellation** between $qq$ and $qg$ channels
- $qg$ channel kinematically suppressed at large $z_f$ — sign change in asymmetry with NNPDF
- JAM has smaller $g \rightarrow \pi^+$ FF $\implies$ less cancellations
- Negligible contribution from gluon-initiated (gq) channel ($x^2$ suppression).
All $A_{UT}$ moments at EIC

- $\sin(\phi_h - \phi_S)$ (Sivers), $\sin(\phi_S)$ and $\sin(2\phi_h - \phi_S)$ moments are at percent level. Collins negligible.
• **Sivers at percent level for** $\pi^+$ **with JAM. Collins negligible.**

• **Only available datapoint from COMPASS at** $P_{hT} \approx 1.5$ **shows positive Sivers asymmetry ($\sim 2.5\%$) but with large errors.**

Kane, Pumplin and Repko (1978) presented the first parametric estimate of SSA in pQCD as

$$A_N \sim \alpha_s \frac{m_q}{P_{hT}}$$

expected to vanish since $$m_q \to 0$$

but in this mechanism,

$$A_N \sim \alpha_s \frac{xM_N}{P_{hT}}$$
Conclusions

Presented numerical results for $g_T$ contribution to SSA at two loops.

- Asymmetry can be calculated entirely in terms of well-known twist-2 distributions.
- Upto 2% Sivers asymmetry at EIC at low-$Q^2$ with $P_{hT} > 1$.
- Asymmetry suppressed at small-$x$.
- Two-loop mechanism needs to be accounted for when constraining other mechanisms (ETQS, twist-3 fragmentation etc.).

Work in progress...

- Calculating similar contributions for SSA in $pp$
- Open-charm in SIDIS - isolates the $G_{3T}$ contribution.
Appendix
**$g_T$ contribution to SSA**

**Hadronic tensor in SIDIS**

\[ W_{\mu\nu} = \int_z \frac{dz}{z^2} D(z) w_{\mu\nu}, \]

\[ w_{\mu\nu} = \int_k M^{(0)}(k) S^{(0)}_{\mu\nu}(k) + \int_{k_1} \int_{k_2} M^{(1)}_{\sigma}(k_1, k_2) S^{(1)}_{\mu\nu}(k_1, k_2) \]

- Need to include twist-3 distributions \( \Rightarrow \) consider hadronic tensor up to three parton correlator.
- \( M^{(0)}_{ij} \sim \langle PS_T | \bar{\psi}_j \psi_i | PS_T \rangle \)
- \( M^{(1)}_{ij\sigma} \sim \langle PS_T | \bar{\psi}_j gA^\sigma \psi_i | PS_T \rangle \)
\( g_T \) contribution to SSA

- Hard part - expand in collinear limit: \( k^\mu = xP^\mu + k_T^\mu \)

\[
S_{\mu\nu}^{(0)}(k) = S_{\mu\nu}^{(0)}(xP) + k_T^\alpha \frac{dS_{\mu\nu}^{(0)}}{dk_T^\alpha}(xP)
\]

- Soft part

\[
M^{(0)} \sim pf(x) + M_N p\gamma_5 (S \cdot n) \Delta q(x) + M_N S_T \gamma_5 g_T(x)
\]

+ transversity and higher twist terms...

- \( g_T \) appears in correlator with \( \gamma_5 \). Traces involving \( \gamma_5 \) produce a factor of \( i \).

- Hence \( g_T \) recieves no contributions from \( S^{(0)} \) at the Born level.


- Can receive non-zero contributions beyond Born level.

Benič, Hatta, Li, Yang Phys. Rev. D 100 (2019) 9, 094027
\textbf{g}_T \text{ contribution to SSA}

All order and gauge invariant result:

\[ w_{\mu \nu} = \frac{M_N}{2} \int dx g_T(x) \text{Tr} \left[ \gamma_5 \gamma_T S_{\mu \nu}^{(0)}(xP) \right] \]

\[ - \frac{M_N}{4} \int dx \tilde{g}(x) \text{Tr} \left[ \gamma_5 \gamma_T \left( \frac{\partial S_{\mu \nu}^{(0)}(k)}{\partial k^\alpha_T} \right) \bigg|_{k=xP} \right] \]

\[ + \frac{iM_N}{4} \int dx_1 dx_2 \text{Tr} \left[ \left( \gamma_5 \gamma_T \left( \frac{G_F(x_1, x_2)}{x_1 - x_2} \right) + i\gamma_5 \gamma_T \tilde{G}_F(x_1, x_2) \right) S_{\mu \nu}^{(1)}(x_1 P, x_2 P) \right] \]

- \( \tilde{g}(x) \) is another "kinematic" twist-3 distribution - first moment of worm-gear TMD \( g_{1T} \) (talk by Shohini Bhattacharya)

\[ g_T(x) + \frac{\tilde{g}(x)}{2x} = \int dx' \frac{G_F(x, x') + \tilde{G}_F(x, x')}{x - x'} \]


Neglecting explicit twist-3 contributions (WW approximation),

\[ w_{\mu \nu} \approx \frac{M_N}{2} \int dx g_T(x) S_{\mu \nu}^{\alpha} \left( \frac{\partial}{\partial k^\alpha_T} \text{Tr} \left[ \gamma_5 k S_{\mu \nu}^{(0)}(k) \right] \right)_{k=xP} \]
Energy dependence

\[ A_{UT} = \sin(\phi_h - \phi_S) \]

- Percent level Sivers contribution (JAM) at highest EIC energy.