# The $g_T(x)$ contribution to single spin asymmetry in SIDIS

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Benić, Hatta, Li, AK Phys. Rev. D 104 (2021) 9, 094027

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#### **Single-Spin Asymmetries**

- Collisions involving transversely polarised hadrons left-right asymmetry in particle production
- Observed in various pp and ep processes since the 70s

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

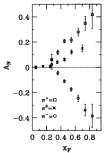
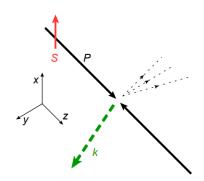


Fig. 4.  $A_N$  versus  $x_F$  for  $\pi^+$ ,  $\pi^-$  and  $\pi^0$  data.



# **Single-Spin Asymmetries**

- QCD is time reversal invariant
- SSAs are T-odd:

$$A_N \propto S \cdot (P \times k)$$

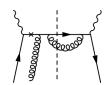
# An phase (interference) is required!

 Quest for theoretical description of SSAs is a quest for sources of a complex phase

# Theoretical descrption of SSAs

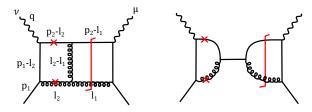
 In a collinear factorization framework, SSAs have been described in terms of three-parton correlation functions, i.e twist-3 parton distribution functions (ETQS) functions and twist-3 fragmentation functions

Efremov, Teryaev, Sov. J. Nucl. Phys. 36, 140 (1982) Qiu, Sterman, Phys. Rev. D 59, 014004 (1999) Yuan, Zhou Phys. Rev. Lett. 103 (2009) 052001 Kang, Yuan, Zhou Phys. Lett. B 691 (2010) 243 Kanazawa, Koike Phys. Rev. D 88 (2013) 074022



- Eg., above hard part can be convolved with ETQS distribution. Complex phase arises when internal propagator goes on-shell.  $1/(k^2 + i\epsilon) = \text{PV}(1/k^2) + i\pi\delta(k^2)$
- Twist-3 distributions not very well known.

# Theoretical descrption of SSAs SSA from $g_T(x)$ :



• In an earlier work, my collaborators and D.J. Yang had shown that the twist-3 quark distribution  $g_T(x)$  can lead to SSA in SIDIS at the two-loop level

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- Imaginary phase arises from when certain certain internal propagators are cut.
- $g_T$  is a chiral-even transverse-spin dependent contribution to the quark-quark correlator

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)\psi(\lambda n)|PS\rangle = \frac{M_N}{2} \gamma_5 \$_T g_T(x) + \dots$$

# SSAs at two loops

Schematically,

$$d\sigma^{\uparrow} - d\sigma^{\downarrow} \propto \frac{d\Delta\sigma}{dP_{hT}} \sim g_T(x) \otimes H^{(2)} \otimes D_1(z)$$

Wandzura-Wilczek relation:

$$g_T(x) = \int_x^1 \frac{dx'}{x'} \Delta q(x') + \text{(genuine twist-3)}$$

In effect,

$$\frac{d\Delta\sigma}{dP_{hT}} \sim g_T(x) \otimes H^{(2)} \otimes D_1(z) \sim \Delta q(x) \otimes H^{(2)} \otimes D_1(z)$$

 SSA completely determined in terms of well understood twist-2 distributions!

#### This work

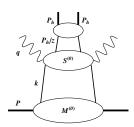
In this work we,

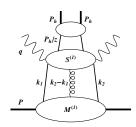
- 1. extend the analysis to include gluon-initiated contribution from the twist-3 distribution  $\mathcal{G}_{3T}$ , which is the gluonic counterpart of  $g_T$ .
  - In analogy with the quark case,

$$\frac{d\Delta\sigma}{dP_{hT}} \sim \mathcal{G}_{3T}(x) \otimes H_g^{(2)} \otimes D_1(z) \sim \Delta G(x) \otimes H_g^{(2)} \otimes D_1(z)$$

2. present numerical estimates for asymmetry in SIDIS through these mechanisms at COMPASS and EIC.

# Why $g_T$ at two loops?





•  $g_T$  appears in the hadronic tensor through the collinear expansion of the two-parton correlator  $M^{(0)}$ .

$$M^{(0)} \sim pf(x) + M_N p\gamma_5(S \cdot n)\Delta q(x) + M_N p\gamma_5 g_T(x) + ...$$

•  $g_T$  appears in correlator with  $\gamma_5$ . Traces involving  $\gamma_5$  produce a factor of  $i \implies g_T$  recieves no contributions from hard part  $S^{(0)}$  at the Born level

Eguchi, Koike, Tanaka Nucl. Phys. B 763, 198, (2007)

Can receive non-zero contributions beyond Born level.

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# Gluon initiated contribution $\mathcal{G}_{3T}$

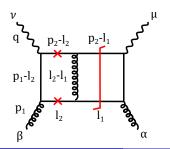
Two-gluon correlator in a polarised proton,

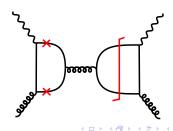
$$\begin{split} M^{(0)\alpha\beta} \sim \langle PS_T | F^{n\alpha} \ W \ F^{n\beta} | PS_T \rangle &\sim & xG(x)g_T^{\alpha\beta} + iM_Nx\Delta G(x)(S \cdot n)\epsilon^{nr\alpha\beta} \\ &+ & iM_Nx\mathcal{G}_{3T}(x)\epsilon^{n\alpha\beta S_\perp} \end{split}$$

Ji, Phys. Lett. B 289, 137 (1992) Hatta, Tanaka, Yoshida, JHEP 02, 003 (2013)

- G<sub>3T</sub> transverse spin dependent contribution to the two-gluon correlator.
- WW approximation

$$G_{3T}(x) = \frac{1}{2} \int_{x}^{1} \frac{dx'}{x'} \Delta G(x') + \text{(genuine twist three)}$$



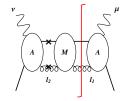


# **Calculation of hard part**

Asymmetry can be written schematically as,

$$A_{UT}^{\sin(\alpha\phi_h+\beta\phi_S)} \sim \frac{\alpha_s^2 \left(xg_T(x) \text{ or } x\mathcal{G}_{3T}(x)\right) \otimes H^{(2)} \otimes D_1(z)}{\alpha_s \ q(x) \otimes H^{(1)} \otimes D_1(z)} \sim \alpha_s \frac{x\Delta q \text{ or } x\Delta G}{q(x)}$$

All contributions to  $H^{(2)}$  have a generic 'AMA' structure:



- Each blob represents 2-2 scattering
- Phase arises from cutting internal lines, i.e., regions of loop momentum l<sub>2</sub> where the two lines go on-shell.
- Potential divergence when  $I_2$  gluon is collinear to proton cancels out between  $S^{(0)}_{\mu\nu}$  and  $\frac{dS^{(0)}_{\mu\nu}}{dk^{\alpha}_{\perp}}$

# Calculation of hard part

$$\begin{split} \frac{d^6 \Delta \sigma}{dx_B dQ^2 dz_f dq_T^2 d\phi d\chi} &= \frac{\alpha_{\rm em}^2 \alpha_5^2 M_N}{16\pi^2 x_B^2 S_{ep}^2 Q^2} \sum_k \mathcal{A}_k \mathcal{S}_k \int \frac{dx}{x} \int \frac{dz}{z} \\ &\times \delta \left( \frac{q_T^2}{Q^2} - \left( 1 - \frac{1}{\hat{x}} \right) \left( 1 - \frac{1}{\hat{z}} \right) \right) \end{split}$$

$$\times \sum_{f} e_{f}^{2} \left[ D_{f}(z) x^{2} \frac{\partial g_{Tf}(x)}{\partial x} \Delta \hat{\sigma}_{Dk}^{qq} + D_{f}(z) x g_{Tf}(x) \Delta \hat{\sigma}_{k}^{qq} + (\text{qg channel}) + (\text{gq channel}) \right]$$

 $+\hat{x}^{2}\left(16-9\hat{z}-4\hat{z}^{2}+N_{c}^{2}\left(-4+3\hat{z}+4\hat{z}^{2}\right)\right)\right\} +2\left(1-\hat{z}\right)\left(3+8\hat{x}^{2}+\hat{z}-\hat{x}\left(11+2\hat{z}\right)\right)\log(1-\hat{z})$ ,

#### **Numerical results**

$$A_{UT}^{\sin(\alpha\phi_h+\beta\phi_S)} = \frac{2\int_0^{2\pi} d\phi_h d\phi_S \sin(\alpha\phi_h+\beta\phi_S) \left[d\sigma(\phi_h,\phi_S) - d\sigma(\phi_h,\phi_S+\pi)\right]}{\int_0^{2\pi} d\phi_h d\phi_S \left[d\sigma(\phi_h,\phi_S) + d\sigma(\phi_h,\phi_S+\pi)\right]}$$

- Five independent moments including Sivers  $A_{UT}^{\sin(\phi_h-\phi_S)}$  and Collins  $A_{UT}^{\sin(\phi_h+\phi_S)}$
- Sivers and Collins asymmetry NOT from Sivers and Collins functions.
- $g_T(x)$  and  $\mathcal{G}_{3T}$  from helicity distributions using the WW approximation.

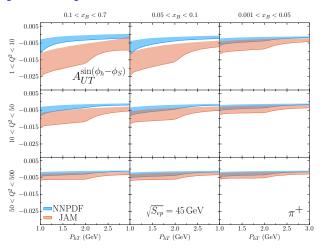
$$g_{T}(x) = \int_{x}^{1} \frac{dx'}{x'} \Delta q(x'), \qquad \mathcal{G}_{3T}(x) = \int_{x}^{1} \frac{dx'}{x'} \Delta G(x')$$

 We used the latest fits of helicity distributions from NNPDF and JAM. Nocera, Ball, Forte, Ridolfi, and Rojo, Nucl. Phys. B 887, 276 (2014) Ethier, Sato, and Melnitchouk, Phys. Rev. Lett. 119, 132001 (2017)

$$A_{UT}^{\sin(\alpha\phi_h+\beta\phi_5)} \sim \frac{\alpha_s^2 \left(xg_T(x) \text{ or } x\mathcal{G}_{3T}(x)\right) \otimes H^{(2)} \otimes D_1(z)}{\alpha_s \ q(x) \otimes H^{(1)} \otimes D_1(z)} \sim \alpha_s \frac{x\Delta q \text{ or } x\Delta G}{q(x)}$$

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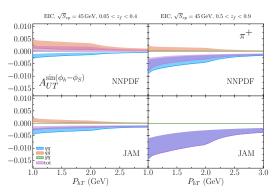
#### Sivers asymmetry at EIC



- Sivers asymmetry up to 2% with JAM at moderate-to-large x and low  $Q^2$ .
- Decreases at low-x.

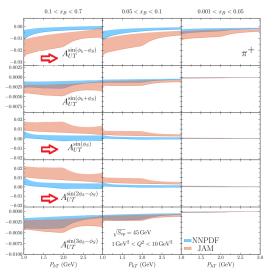


#### Channel breakdown



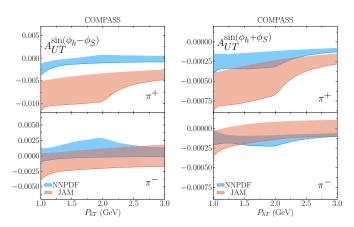
- cancellation between qq and qg channels
- qg channel kinematically suppressed at large  $z_f$  sign change in asymmetry with NNPDF
- JAM has smaller  $g \to \pi^+$  FF  $\implies$  less cancellations
- Negligible contribution from gluon-initiated (gq) channel ( $x^2$  suppression).

#### All $A_{IJT}$ moments at EIC



•  $\sin(\phi_h - \phi_S)$  (Sivers),  $\sin(\phi_S)$  and  $\sin(2\phi_h - \phi_S)$  moments are at percent level. Collins negligible.

#### Sivers and Collins at COMPASS



- Sivers at percent level for  $\pi^+$  with JAM. Collins negligible.
- Only available datapoint from COMPASS at  $P_{hT} \approx 1.5$  shows positive Sivers asymmetry ( $\sim 2.5\%$ ) but with large errors.

Adolph et al., Phys. Lett. B744, 250 (2015)

#### Comparison with the KPR estimate



 Kane, Pumplin and Repko (1978) presented the first parametric estimate of SSA in pQCD as

$$A_N \sim lpha_s rac{m_q}{P_{hT}}$$

- expected to vanish since  $m_q o 0$
- but in this mechanism,

$$A_N \sim lpha_s rac{ imes M_N}{P_{hT}}$$



#### **Conclusions**

Presented numerical results for  $g_T$  contribution to SSA at two loops.

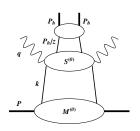
- Asymmetry can be calculated entirely in terms of well-known twist-2 distributions.
- Upto 2% Sivers asymmetry at EIC at low- $Q^2$  with  $P_{hT} > 1$ .
- Asymmetry suppressed at small-x.
- Two-loop mechanism needs to be accounted for when constraining other mechanisms (ETQS, twist-3 fragmentation etc.).

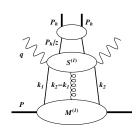
#### Work in progress...

- Calculating similar contributions for SSA in pp
- Open-charm in SIDIS isolates the  $\mathcal{G}_{3T}$  contribution.

 ${\sf Appendix}$ 

# $g_T$ contribution to SSA Hadronic tensor in SIDIS





•  $W^{\mu\nu}=\int_{z}\frac{dz}{z^{2}}D(z)w^{\mu\nu}$ ,

$$w_{\mu\nu} = \int_{k} M^{(0)}(k) S_{\mu\nu}^{(0)}(k) + \int_{k_{1}} \int_{k_{2}} M_{\sigma}^{(1)}(k_{1}, k_{2}) S_{\mu\nu}^{(1)\sigma}(k_{1}, k_{2})$$

- Need to include twist-3 distributions 

  consider hadronic tensor upto three parton correlator.
- $M_{ij}^{(0)} \sim \langle PS_T | \bar{\psi}_j \psi_i | PS_T \rangle$
- $M_{ii}^{(1)\sigma} \sim \langle PS_T | \bar{\psi}_i g A^{\sigma} \psi_i | PS_T \rangle$



#### $g_T$ contribution to SSA

• Hard part - expand in collinear limit:  $k^\mu = x P^\mu + k_T^\mu$ 

$$S_{\mu\nu}^{(0)}(k) = S_{\mu\nu}^{(0)}(xP) + k_T^{\alpha} \frac{dS_{\mu\nu}^{(0)}}{dk_T^{\alpha}}(xP)$$

Soft part

$$M^{(0)} \sim pf(x) + M_N p\gamma_5(S \cdot n)\Delta q(x) + M_N p\gamma_5 g\tau(x)$$
  
+transversity and higher twist terms...

- $g_T$  appears in correlator with  $\gamma_5$ . Traces involving  $\gamma_5$  produce a factor of i.
- Hence  $g_T$  recieves no contributions from  $S^{(0)}$  at the Born level.

Eguchi, Koike, Tanaka Nucl. Phys. B 763, 198, (2007)

Can receive non-zero contributions beyond Born level.

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#### $g_T$ contribution to SSA

All order and gauge invariant result:

$$\begin{split} w_{\mu\nu} &= \frac{M_N}{2} \int dx g_T(x) \mathrm{Tr} \left[ \gamma_5 \$_T S_{\mu\nu}^{(0)}(xP) \right] \\ &- \frac{M_N}{4} \int dx \tilde{g}(x) \mathrm{Tr} \left[ \gamma_5 \rlap{/}P S_T^{\alpha} \left. \frac{\partial S_{\mu\nu}^{(0)}(k)}{\partial k_T^{\alpha}} \right|_{k=xP} \right] \\ &+ \frac{iM_N}{4} \int dx_1 dx_2 \mathrm{Tr} \left[ \left( \rlap{/}P \epsilon^{\alpha P n S_T} \frac{G_F(x_1, x_2)}{x_1 - x_2} + i \gamma_5 \rlap{/}P S_T^{\alpha} \frac{\tilde{G}_F(x_1, x_2)}{x_1 - x_2} \right) S_{\mu\nu\alpha}^{(1)}(x_1 P, x_2 P) \right] \end{split}$$

•  $\tilde{g}(x)$  is another "kinematic" twist-3 distribution - first moment of worm-gear TMD  $g_{1T}$  (talk by Shohini Bhattacharya)

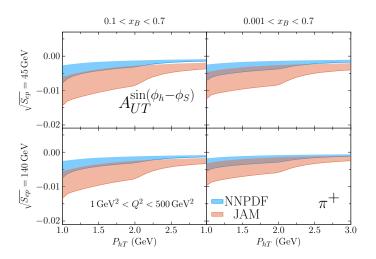
$$g_{T}(x) + \frac{\tilde{g}(x)}{2x} = \int dx' \frac{G_{F}(x,x') + \tilde{G}_{F}(x,x')}{x - x'}$$

Eguchi, Koike, Tanaka, Nucl. Phys. B 763, 198 (2007)

Neglecting explicit twist-3 contributions (WW approximation),

$$w_{\mu
u} pprox rac{M_N}{2} \int dx g_{\mathcal{T}}(x) S_{\mathcal{T}}^{lpha} \left( rac{\partial}{\partial k_{\mathcal{T}}^{lpha}} \mathrm{Tr}[\gamma_5 k S_{\mu
u}^{(0)}(k)] 
ight)_{k=xF}$$

#### **Energy dependence**



Percent level Sivers contribution (JAM) at highest EIC energy.