

# Power corrections to TMD factorization

**Alexey Vladimirov**

for  
**I.Scimemi**

based on [2109.09771] & [2204.03856]



U N I V E R S I D A D  
**COMPLUTENSE**  
M A D R I D

# DIS2022

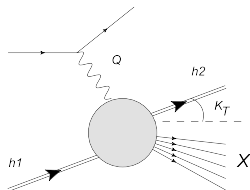
**XXIX International Workshop on Deep-  
Inelastic Scattering and Related Subjects**

Santiago de Compostela, 2-6 May 2022

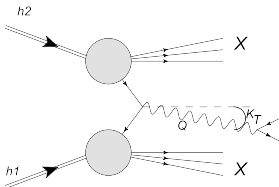
## Transverse momentum dependent factorization

$$\frac{d\sigma}{dq_T} \simeq \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{-i(bq_T)} |C_V(Q)|^2 F_1(x_1, b; Q, Q^2) F_2(x_2, b; Q, Q^2)$$

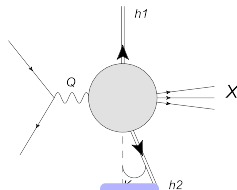
**LP term is studied VERY WELL!**



**SIDIS**



**Drell-Yan**



**SIA**

$q$  is momentum of initiating EW-boson

$$q^2 = \pm Q^2$$

$q_T^\mu$  transverse component

$$\begin{cases} Q^2 \gg \Lambda_{QCD}^2 \\ Q^2 \gg q_T^2 \end{cases}$$



## Transverse momentum dependent factorization

$$\begin{aligned}
 \frac{d\sigma}{dq_T} &\simeq \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{-i(bq_T)} \left\{ |C_V(Q)|^2 F_1(x_1, b; Q, Q^2) F_2(x_2, b; Q, Q^2) \right. && \leftarrow \text{LP} \\
 &+ \left( \frac{q_T}{Q} \text{ or } \frac{\Lambda}{Q} \right) [C_2(Q) \otimes F_3(x, b; Q, Q^2) F_4(x, b; Q, Q^2)](x_1, x_2) && \leftarrow \text{NLP} \\
 &+ \left( \frac{q_T^2}{Q^2} \text{ or } \dots \right) [C_3(Q) \otimes F_5(x, b; Q, Q^2) F_6(x, b; Q, Q^2)](x_1, x_2) && \leftarrow \text{NNLP} \\
 &+ \dots
 \end{aligned}$$

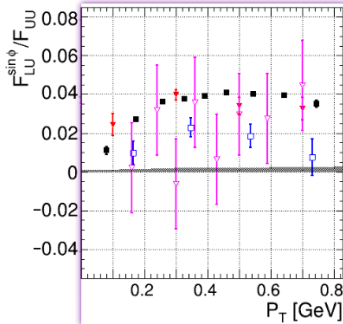
The talk is very brief review of main results

### Outline

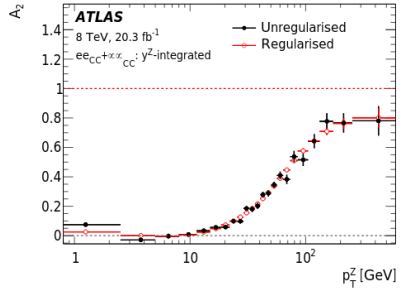
- ▶ TMD operator expansion
- ▶ TMD-twist
- ▶ TMD factorization at NLP/NLO
- ▶ Evolution of twist-three TMDs

## Motivation

### ► Sub-leading power observables

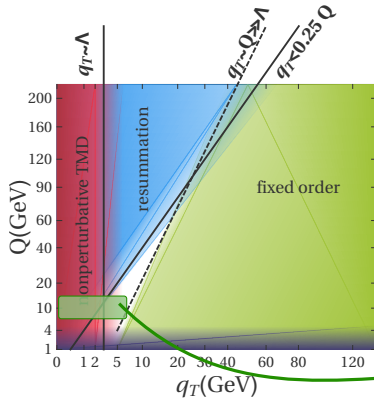


[CLAS, 2101.03544]



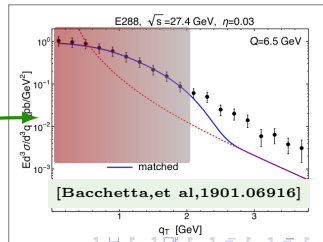
## Motivation

- ▶ Sub-leading power observables
- ▶ Increase of applicability domain



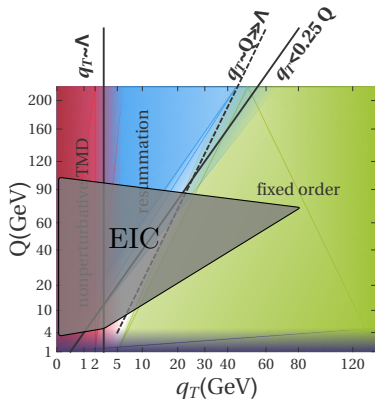
LP TMD factorization has limited region of application.

For SIDIS it cuts **the most part** of the data



## Motivation

- ▶ Sub-leading power observables
- ▶ Increase of applicability domain
- ▶ Theoretical consistency



Phase space of EIC is centered  
directly in  
the transition region

COMPASS, JLab  
have large contribution of power corrections



# Sources of power corrections

\*(exact)=known at all powers

$$\frac{d\sigma}{dP.S.} = \sigma_{PS} L_{\mu\nu} W^{\mu\nu}$$

Phase space PC (exact)  
e.g. SIDIS  $\sigma_{PS} = \frac{\pi}{\sqrt{1 + \gamma^2 \frac{\mathbf{p}_{h\perp}^2}{z^2 Q^2}}}$

Leptonic tensor (exact)  
e.g. un.DY with fid.cuts  
 $L^{\mu\nu} \sim (l^\mu l'^\nu + l'^\mu l^\nu - g^{\mu\nu} (ll')) \mathcal{P}$   
•  $l, l'$  with transverse parts  
•  $\mathcal{P}$  fiducial part

Hadronic tensor (e.g. DY)  
 $W^{\mu\nu} = \int \frac{d^4 y e^{i(yq)}}{(2\pi)^4} \langle p_1 p_2 | J^\mu(y) | X \rangle \langle X | J^\nu | p_1 p_2 \rangle$

Factorized in powers of  
 $\frac{q_T}{q^+}, \frac{q_T}{q^-}, \frac{\Lambda}{q^+}, \frac{\Lambda}{q^-}$

Power corrections due to frame choice (exact)

$$p_1^+ \gg p_1^-, \quad p_2^- \gg p_2^+ \\ \text{e.g. SIDIS } q_T^2 = \frac{p_\perp^2}{z^2} \frac{1 + \gamma^2}{1 - \gamma^2 \frac{p_\perp^2}{z^2 Q^2}}$$



## New method **TMD operator expansion**

### features

- ▶ Operator-level formulation
- ▶ Systematicness of OPE
- ▶ Position space [a lot of simplification for beyond leading twist]
- ▶ Has common parts with small- $x$  and SCET computations



**TMD operator expansion**  
 is conceptually similar to ordinary OPE  
**The only difference** is counting rule for  $y$

$$\begin{aligned}
 W_{\text{DY}}^{\mu\nu} &= \int \frac{d^4 y}{(2\pi)^4} e^{-i(yq)} \sum_X \langle p_1, p_2 | J^{\mu\dagger}(y) | X \rangle \langle X | J^\nu(0) | p_1, p_2 \rangle, \\
 W_{\text{SIDIS}}^{\mu\nu} &= \int \frac{d^4 y}{(2\pi)^4} e^{i(yq)} \sum_X \langle p_1 | J^{\mu\dagger}(y) | p_2, X \rangle \langle p_2, X | J^\nu(0) | p_1 \rangle, \\
 W_{\text{SIA}}^{\mu\nu} &= \int \frac{d^4 y}{(2\pi)^4} e^{i(yq)} \sum_X \langle 0 | J^{\mu\dagger}(y) | p_1, p_2, X \rangle \langle p_1, p_2, X | J^\nu(0) | 0 \rangle. \\
 W^{\mu\nu}(q) &\rightarrow J^\mu(y) J^\nu(0) \implies \mathcal{J}_{\text{eff}}^{\mu\nu}(y) \rightarrow \text{fact.theorem}
 \end{aligned}$$

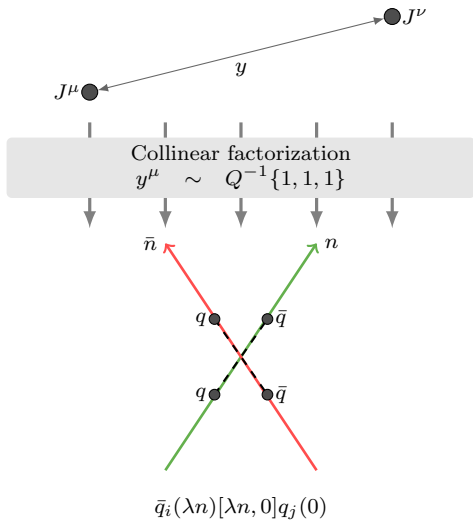
$$(q \cdot y) \sim 1 \quad \Rightarrow \quad \{y^+, y^-, y_T\} \sim \left\{ \frac{1}{q^-}, \frac{1}{q^+}, \frac{1}{q_T} \right\} \sim \frac{1}{Q} \{1, 1, \lambda^{-1}\}$$

To be accounted in operator expansion

$$z_T^\mu \partial_\mu q \sim \text{NLP}, \quad y_T^\mu \partial_\mu q \sim \text{LP}$$



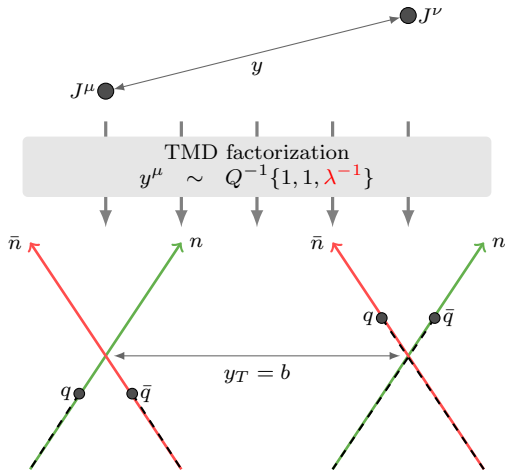
Geometric structure of  
TMD operator expansion



**Two**  
light-cone operators  
 $\Downarrow$   
**Two**  
parton distribution function  
PDFs & FFs



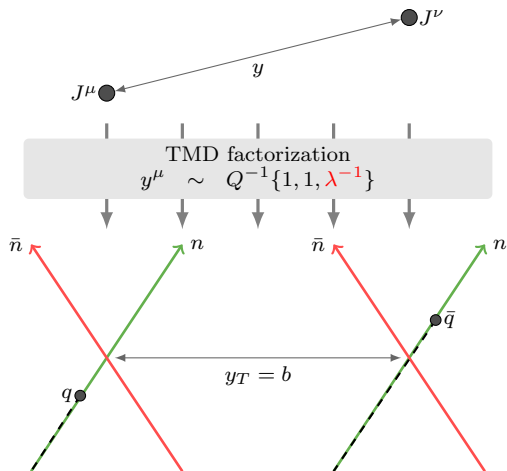
Geometric structure of  
TMD operator expansion



**Four**  
light-cone operators  
 $\Downarrow$   
**Two**  
TMD distributions  
TMDPDFs & TMDFFs



Geometric structure of  
TMD operator expansion



**Four**  
light-cone operators  
↓  
**Two**  
TMD distributions  
TMDPDFs & TMDFFs

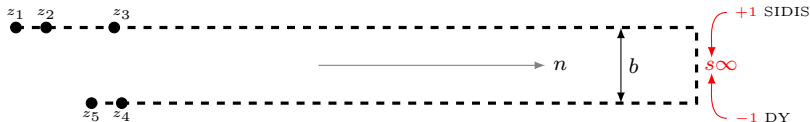
$$\bar{q}_i(\lambda n + b)[\lambda n + b, \pm\infty n + b] [\pm\infty n, 0] q_j(0)$$



## TMD operators and their divergences

Any TMD operator is the product of two *semi-compact* operators

$$\mathcal{O}_{NM}(\{z_1, \dots, z_n\}, b) = U_N(\{z_1, \dots\}, b) U_M(\{\dots, z_n\}, 0)$$



$$\mathcal{O}_{NM}^{\text{bare}}(\{z_1, \dots, z_n\}, b) = R(b^2) Z_{U_N}(\{z_1, \dots\}) \otimes Z_{U_M}(\{\dots, z_n\}) \otimes \mathcal{O}_{NM}(\mu, \zeta)$$

- UV divergence for  $U_N$
- UV divergence for  $U_M$
- Rapidity divergence

Three independent divergences  
Three renormalization constants  
Three anomalous dimensions

TMD-twist = (N,M)  
two integer numbers



LP = TMD-twist-(1,1)

$$\Phi_{11}(x, b) = \int dz e^{-izp+x} \langle p, s | \underbrace{\bar{q}[zn+b, \infty]}_{\text{tw-1}} \gamma^+ \underbrace{[\infty, 0]q|p, \rangle}_{\text{tw-1}} \rangle$$

NLP = TMD-twist-(2,1) & (1,2)

$$\Phi_{21}(x_1, x_2, x_3, b) = \int dz e^{-ip+\sum_i x_i z_i} \langle p, s | \underbrace{\bar{q}[z_1 n+b, z_2 n+b] F_{\mu+}[\dots, \infty]}_{\text{tw-2}} \gamma^+ \underbrace{[\infty, z_3 n]q|p, \rangle}_{\text{tw-1}} \rangle$$

$$\Phi_{12}(x_1, x_2, x_3, b) = \int dz e^{-ip+\sum_i x_i z_i} \langle p, s | \underbrace{\bar{q}[z_1 n+b, \infty]}_{\text{tw-1}} \gamma^+ \underbrace{[\infty, z_2 n] F_{\mu+}[z_2 n, z_3 n]q|p, \rangle}_{\text{tw-2}} \rangle$$

NNLP = TMD-twist-(3,1) & (2,2) & (1,3)

...



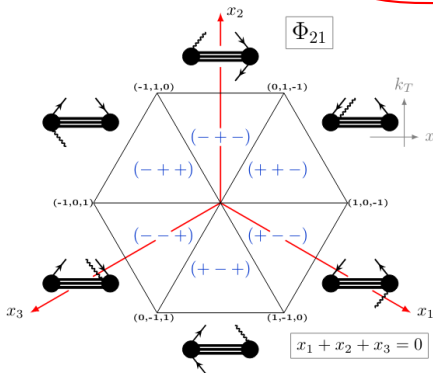
## To momentum-fraction space

$$\tilde{\Phi}_{11}^{[\Gamma]}(z_1, z_2, b) = p^+ \int_{-1}^1 dx e^{ix(z_1 - z_2)p^+} \Phi_{11}^{[\Gamma]}(x, b),$$

$$\tilde{\Phi}_{\mu,21}^{[\Gamma]}(z_1, z_2, z_3, b) = (p^+)^2 \int [dx] e^{-i(x_1 z_1 + x_2 z_2 + x_3 z_3)p^+} \Phi_{\mu,21}^{[\Gamma]}(x_1, x_2, x_3, b),$$

$$\tilde{\Phi}_{\mu,12}^{[\Gamma]}(z_1, z_2, z_3, b) = (p^+)^2 \int [dx] e^{-i(x_1 z_1 + x_2 z_2 + x_3 z_3)p^+} \Phi_{\mu,12}^{[\Gamma]}(x_1, x_2, x_3, b),$$

$$\int [dx] = \int_{-1}^1 dx_1 \int_{-1}^1 dx_2 \int_{-1}^1 dx_3 \delta(x_1 + x_2 + x_3)$$



Support domain  $|x_i| < 1$   
momentum-fractions  
could be **positive or negative**

- important for divergences-cancellation
- agreement with collinear evolution
- evolution mixture



## Evolution equations have involved structure

$$\begin{aligned}\mu^2 \frac{d}{d\mu^2} \Phi_{\mu,21}^{[\Gamma]} &= \left( \frac{\Gamma_{\text{cusp}}}{2} \ln \left( \frac{\mu^2}{\zeta} \right) + \Upsilon_{x_1 x_2 x_3} + 2\pi i s \Theta_{x_1 x_2 x_3} \right) \Phi_{\mu,21}^{[\Gamma]} \\ &\quad + \mathbb{P}_{x_2 x_1}^A \otimes \Phi_{\nu,21}^{[\gamma^\nu \gamma^\mu \Gamma]} + \mathbb{P}_{x_2 x_1}^B \otimes \Phi_{\nu,21}^{[\gamma^\mu \gamma^\nu \Gamma]}, \\ \mu^2 \frac{d}{d\mu^2} \Phi_{\mu,12}^{[\Gamma]} &= \left( \frac{\Gamma_{\text{cusp}}}{2} \ln \left( \frac{\mu^2}{\zeta} \right) + \Upsilon_{x_3 x_2 x_1} + 2\pi i s \Theta_{x_3 x_2 x_1} \right) \Phi_{\mu,12}^{[\Gamma]} \\ &\quad + \mathbb{P}_{x_2 x_3}^A \otimes \Phi_{\nu,12}^{[\Gamma \gamma^\mu \gamma^\nu]} + \mathbb{P}_{x_2 x_3}^B \otimes \Phi_{\nu,12}^{[\Gamma \gamma^\nu \gamma^\mu]},\end{aligned}$$

[2204.03856]

$$\begin{aligned}\zeta \frac{d}{d\zeta} \Phi_{\mu,12}^{[\Gamma]}(x_1, x_2, x_3, b; \mu, \zeta) &= -\mathcal{D}(b, \mu) \Phi_{\mu,12}^{[\Gamma]}(x_1, x_2, x_3, b; \mu, \zeta), \\ \zeta \frac{d}{d\zeta} \Phi_{\mu,21}^{[\Gamma]}(x_1, x_2, x_3, b; \mu, \zeta) &= -\mathcal{D}(b, \mu) \Phi_{\mu,21}^{[\Gamma]}(x_1, x_2, x_3, b; \mu, \zeta).\end{aligned}$$

Complex evolution  
for  
complex functions



Physical functions  
are  
real combinations



T-parity-definite & real combinations

$$\begin{aligned}\Phi_{\mu,\oplus}^{[\Gamma]}(x_1, x_2, x_3, b) &= \frac{\Phi_{\mu,21}^{[\Gamma]}(x_1, x_2, x_3, b) + \Phi_{\mu,12}^{[\Gamma]}(-x_3, -x_2, -x_1, b)}{2}, \\ \Phi_{\mu,\ominus}^{[\Gamma]}(x_1, x_2, x_3, b) &= i \frac{\Phi_{\mu,21}^{[\Gamma]}(x_1, x_2, x_3, b) - \Phi_{\mu,12}^{[\Gamma]}(-x_3, -x_2, -x_1, b)}{2}\end{aligned}$$

**Real functions, with real evolution**

**Price:** evolution mixes  $\oplus$  and  $\ominus$  sectors



- ▶ Three  $\Gamma$ -structures  $\{\gamma^+, \gamma^+ \gamma^5, i\sigma^{\alpha+} \gamma^5\}$
- ▶ In the tensor case, one can sort  $F_{\mu+} \sigma^{\alpha+}$ -tensors into  $J = 0, 1, 2$  cases.
- ▶ **32 distributions** ( $\oplus$  and  $\ominus$ )
- ▶ **16 T-odd** and **16 T-even**

### Example

$$\begin{aligned} \Phi_{\bullet}^{\mu[\gamma^+]}(x_{1,2,3}, b) &= \epsilon^{\mu\nu} s_{T\nu} M f_{\bullet T}(x_{1,2,3}, b) + i b^\mu M^2 f_{\bullet}^{\perp}(x_{1,2,3}, b) \\ &\quad + i \lambda \epsilon^{\mu\nu} b_\nu M^2 f_{\bullet L}^{\perp}(x_{1,2,3}, b) + b^2 M^3 \epsilon_T^{\mu\nu} \left( \frac{g_{T,\nu\rho}}{2} - \frac{b_\nu b_\rho}{b^2} \right) s_T^\rho f_{\bullet T}^{\perp}(x_{1,2,3}, b) \end{aligned}$$

$$f_{\oplus, T; \textcolor{red}{DY}} = f_{\oplus, T; \textcolor{red}{SIDIS}}, \quad f_{\oplus; \textcolor{red}{DY}}^{\perp} = -f_{\oplus; \textcolor{red}{SIDIS}}^{\perp}$$

	U	L	$T_{J=0}$	$T_{J=1}$	$T_{J=2}$
U	$f_{\bullet}^{\perp}$	$g_{\bullet}^{\perp}$		$h_{\bullet}$	$h_{\bullet}^{\perp}$
L	$f_{\bullet L}^{\perp}$	$g_{\bullet L}^{\perp}$	$h_{\bullet L}$		$h_{\bullet L}^{\perp}$
T	$f_{\bullet T}, f_{\bullet T}^{\perp}$	$g_{\bullet T}, g_{\bullet T}^{\perp}$	$h_{\bullet T}^{D\perp}$	$h_{\bullet T}^{A\perp}$	$h_{\bullet T}^{S\perp}, h_{\bullet T}^{T\perp}$



Evolution equations split into two cases:

**Evolution with kernels  $\mathbb{P}^A$  or  $\mathbb{P}^B$**

Example  $\mathbb{P}^A$

$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} H_{\oplus}^A \\ H_{\ominus}^A \end{pmatrix} = \left( \frac{\Gamma_{\text{cusp}}}{2} \ln \left( \frac{\mu^2}{\zeta} \right) + \Upsilon_{x_1 x_2 x_3} \right) \begin{pmatrix} H_{\oplus}^A \\ H_{\ominus}^A \end{pmatrix} + \begin{pmatrix} 2\mathbb{P}_{x_2 x_1}^A & 2\pi s \Theta_{x_1 x_2 x_3} \\ -2\pi s \Theta_{x_1 x_2 x_3} & 2\mathbb{P}_{x_2 x_1}^A \end{pmatrix} \begin{pmatrix} H_{\oplus}^A \\ H_{\ominus}^A \end{pmatrix},$$

$$\begin{pmatrix} f_{\oplus}^{\perp} + g_{\oplus}^{\perp} \\ f_{\ominus}^{\perp} - g_{\oplus}^{\perp} \end{pmatrix}, \quad \begin{pmatrix} f_{\oplus,L}^{\perp} + g_{\oplus,L}^{\perp} \\ f_{\ominus,L}^{\perp} - g_{\oplus,L}^{\perp} \end{pmatrix}, \quad \begin{pmatrix} f_{\oplus,T} + g_{\ominus,T} \\ f_{\ominus,T} - g_{\oplus,T} \end{pmatrix}, \quad \begin{pmatrix} f_{\oplus,T}^{\perp} + g_{\oplus,T}^{\perp} \\ f_{\ominus,T}^{\perp} - g_{\oplus,T}^{\perp} \end{pmatrix},$$

$$\begin{pmatrix} h_{\oplus} \\ h_{\ominus} \end{pmatrix}, \quad \begin{pmatrix} h_{\oplus,L} \\ h_{\ominus,L} \end{pmatrix}, \quad \begin{pmatrix} h_{\oplus,T}^{A\perp} \\ h_{\ominus,T}^{A\perp} \end{pmatrix}, \quad \begin{pmatrix} h_{\oplus,T}^{D\perp} \\ h_{\ominus,T}^{D\perp} \end{pmatrix}.$$

- ▶ Real functions = real evolution
- ▶ Mixes T-odd and T-even distributions
- ▶ Mixing is proportional to  $s$ , so T-parity is preserved, and distributions are universal
- ▶ Structure simplifies at large- $N_c$



## TMD factorization at NLP

Effective operator for any process (DY, SIDIS, SIA)

$$\begin{aligned}
 \mathcal{J}_{\text{eff}}^{\mu\nu}(q) = & \int \frac{d^2b}{(2\pi)^2} e^{-i(qb)} \left\{ \int dx d\tilde{x} \delta\left(x - \frac{q^+}{p_1^+}\right) \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) |C_1|^2 \mathcal{J}_{1111}^{\mu\nu}(x, \tilde{x}, b) \right. \\
 & + \int [dx] d\tilde{x} \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) \\
 & \quad \times \left( \delta\left(x_1 - \frac{q_1^+}{p_1^+}\right) C_1^* C_2(x_{2,3}) \mathcal{J}_{1211}^{\mu\nu}(x, \tilde{x}, b) + \delta\left(x_3 + \frac{q_1^+}{p_1^+}\right) C_2^*(x_{1,2}) C_1 \mathcal{J}_{2111}^{\mu\nu}(x, \tilde{x}, b) \right) \\
 & + \int dx [d\tilde{x}] \delta\left(x - \frac{q^+}{p_1^+}\right) \\
 & \quad \times \left( C_1^* C_2(\tilde{x}_{2,3}) \delta\left(\tilde{x}_1 - \frac{q^-}{p_2^-}\right) \mathcal{J}_{1112}^{\mu\nu}(x, \tilde{x}, b) + C_2^*(\tilde{x}_{1,2}) C_1 \delta\left(\tilde{x}_3 + \frac{q^-}{p_2^-}\right) \mathcal{J}_{1121}^{\mu\nu}(x, \tilde{x}, b) \right) \\
 & \left. + \dots \right\}
 \end{aligned} \tag{6.17}$$



## TMD factorization at NLP

Effective operator for any process (DY, SIDIS, SIA)

$$\mathcal{J}_{\text{eff}}^{\mu\nu}(q) = \int \frac{d^2b}{(2\pi)^2} e^{-i(qb)} \left\{ \int dx d\tilde{x} \delta\left(x - \frac{q^+}{p_1^+}\right) \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) |C_1|^2 \mathcal{J}_{1111}^{\mu\nu}(x, \tilde{x}, b) \right. \\ \left. + \int [dx] d\tilde{x} \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) \right\} \quad (6.17)$$

$$\mathcal{J}_{1111}^{\mu\nu}(x, \tilde{x}, b) = \frac{\gamma_{T,ij}^\mu \gamma_{T,kl}^\nu}{N_c} \left( \mathcal{O}_{11,\bar{n}}^{li}(x, b) \bar{\mathcal{O}}_{11,n}^{jk}(\tilde{x}, b) + \bar{\mathcal{O}}_{11,\bar{n}}^{jk}(x, b) \mathcal{O}_{11,n}^{li}(\tilde{x}, b) \right) \\ + i \frac{n^\mu \gamma_{T,ij}^\rho \gamma_{T,kl}^\nu + n^\nu \gamma_{T,ij}^\mu \gamma_{T,kl}^\rho}{q^+ N_c} \left( \partial_\rho \mathcal{O}_{11,\bar{n}}^{li}(x, b) \bar{\mathcal{O}}_{11,n}^{jk}(\tilde{x}, b) + \partial_\rho \bar{\mathcal{O}}_{11,\bar{n}}^{jk}(x, b) \mathcal{O}_{11,n}^{li}(\tilde{x}, b) \right) \\ + i \frac{\bar{n}^\mu \gamma_{T,ij}^\rho \gamma_{T,kl}^\nu + \bar{n}^\nu \gamma_{T,ij}^\mu \gamma_{T,kl}^\rho}{q^- N_c} \left( \mathcal{O}_{11,\bar{n}}^{li}(x, b) \partial_\rho \bar{\mathcal{O}}_{11,n}^{jk}(\tilde{x}, b) + \bar{\mathcal{O}}_{11,\bar{n}}^{jk}(x, b) \partial_\rho \mathcal{O}_{11,n}^{li}(\tilde{x}, b) \right),$$

- Operators of  $(1, 1) \times (1, 1)$  (ordinary TMDs)
- Contains LP and NLP (total derivatives)
- Restores EM gauge invariance up to  $\lambda^3$

$$q_\mu J_{1111}^{\mu\nu} \sim (p_1^- q_T + p_2^+ q_T) J_{1111}$$



## TMD factorization at NLP

Effective operator for any process (DY, SIDIS, SIA)

$$\begin{aligned}
 \mathcal{J}_{\text{eff}}^{\mu\nu}(q) = & \int \frac{d^2b}{(2\pi)^2} e^{-i(qb)} \left\{ \int dx d\tilde{x} \delta\left(x - \frac{q^+}{p_1^+}\right) \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) |C_1|^2 \mathcal{J}_{1111}^{\mu\nu}(x, \tilde{x}, b) \right. \\
 & + \int [dx] d\tilde{x} \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) \\
 & \times \left( \delta\left(x_1 - \frac{q_1^+}{p_1^+}\right) C_1^* C_2(x_{2,3}) \mathcal{J}_{1211}^{\mu\nu}(x, \tilde{x}, b) + \delta\left(x_3 + \frac{q_1^+}{p_1^+}\right) C_2^*(x_{1,2}) C_1 \mathcal{J}_{2111}^{\mu\nu}(x, \tilde{x}, b) \right) \\
 & \left. + \int dx [d\tilde{x}] \delta\left(x - \frac{q^+}{p_1^+}\right) \right\}
 \end{aligned} \tag{6.17}$$

$$\begin{aligned}
 \mathcal{J}_{1211}^{\mu\nu}(x, \tilde{x}, b) = & \frac{ig}{x_2} \left( \frac{\bar{n}^\nu}{q^-} - \frac{n^\nu}{q^+} \right) \frac{\gamma_{T,ij}^\mu \delta_{kl}}{N_c} \left( \mathcal{O}_{12,\bar{n}}^{jk}(x, b) \bar{\mathcal{O}}_{11,n}^{jk}(\tilde{x}, b) - \bar{\mathcal{O}}_{12,\bar{n}}^{jk}(x, b) \mathcal{O}_{11,n}^{li}(\tilde{x}, b) \right)
 \end{aligned}$$

- Operators of  $(1, 2) \times (1, 1)$
- EM gauge invariant only up to NNLP

$$q_\mu J_{1211}^{\mu\nu} \sim (p_1^- + p_2^+) J_{1211}$$



## TMD factorization at NLP

Effective operator for any process (DY, SIDIS, SIA)

$$\begin{aligned}
 \mathcal{J}_{\text{eff}}^{\mu\nu}(q) = & \int \frac{d^2b}{(2\pi)^2} e^{-i(qb)} \left\{ \int dx d\tilde{x} \delta\left(x - \frac{q^+}{p_1^+}\right) \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) |C_1|^2 \mathcal{J}_{1111}^{\mu\nu}(x, \tilde{x}, b) \right. \\
 & + \int [dx] d\tilde{x} \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) \\
 & \times \left( \delta\left(x_1 - \frac{q_1^+}{p_1^+}\right) C_1^* C_2(x_{2,3}) \mathcal{J}_{1211}^{\mu\nu}(x, \tilde{x}, b) + \delta\left(x_3 + \frac{q_1^+}{p_1^+}\right) C_2^*(x_{1,2}) C_1 \mathcal{J}_{2111}^{\mu\nu}(x, \tilde{x}, b) \right) \\
 & + \int dx [d\tilde{x}] \delta\left(x - \frac{q^+}{p_1^+}\right)
 \end{aligned} \tag{6.17}$$

$$C_1 = 1 + a_s C_F \left( -\mathbf{L}_Q^2 + 3\mathbf{L}_Q - 8 + \frac{\pi^2}{6} \right) + O(a_s), \quad (x, \tilde{x}, b)$$

$$\begin{aligned}
 C_2(x_{1,2}) = & 1 + a_s \left[ C_F \left( -\mathbf{L}_Q^2 + \mathbf{L}_Q - 3 + \frac{\pi^2}{6} \right) + C_A \frac{x_1 + x_2}{x_1} \ln \left( \frac{x_1 + x_2}{x_2} \right) \right. \\
 & \left. + \left( C_F - \frac{C_A}{2} \right) \frac{x_1 + x_2}{x_2} \ln \left( \frac{x_1 + x_2}{x_1} \right) \left( 2\mathbf{L}_Q - \ln \left( \frac{x_1 + x_2}{x_1} \right) - 4 \right) \right]
 \end{aligned}$$

- ▶ Coefficient functions up to NLO
- ▶  $C_1$  is know up to N<sup>3</sup>LO
- ▶  $C_1$  is same for LP, NLP, ... parts of operator  $\mathcal{J}_{1111}^{\mu\nu}$



# Conclusion

Theory of power corrections to the TMD factorization essentially progressed during last years.

[Balitsky,Tarasov,17], [Balitsky,19-21], [Ebert, et al, 21], [Moos,AV,20]

**Still far from practice.**

TMD operator expansion – an efficient approach to TMD factorization beyond LP

- ▶ Operator level
- ▶ Position space
- ▶ Strict & intuitive rules for operator sorting (TMD-twist)
- ▶ All processes

TMD factorization at NLP is derived

- ▶ Coefficient function at NLO
- ▶ Evolution at NLO
- ▶ Rapidity evolution of NLP is the same as for LP



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**Thank you for attention!**

