# Two-loop vector singlet coefficient function for DVCS

J. Schoenleber

University of Regensburg

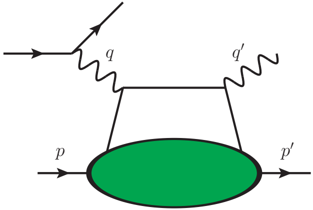
**Work in progress V. Braun, Y. Ji, J.S.**

May 2, 2022

- Generalized parton distributions (GPDs) reveal interesting facts about nucleon structure, such as 3d-tomography and orbital angular momentum in terms of its constituents.
- Deeply virtual Compton scattering (DVCS) is the most prominent process to extract GPDs from data. Measured at JLAB and upcoming EIC.
- Radiative corrections to DVCS are known to be substantial. NNLO is required for good precision of the GPD extraction from data.
- NNLO analysis of DVCS has been done [Kumericki:2007sa], but only in the conformal scheme (CS) $\leftarrow$ GPD model must be constructed in CS and the translation of CS to $\overline{\text{MS}}$ is time consuming and at NNLO it requires the calculation of two-loop (singlet) conformal anomaly (difficult).

DVCS

$$\gamma^*(q)\ N(p) \longrightarrow \gamma(q')\ N(p')$$




Leading order approximation

The usual kinematical parameters

$$P = \frac{p+p'}{2}, \quad t = (p'-p)^2, \quad Q^2 = -q^2, \quad M^2 = p^2 = p'^2, \quad x_B = \frac{Q^2}{2p\cdot q},$$
$$\xi = \frac{(p-p')\cdot q}{2P\cdot q} \approx \frac{p^+ - p'^+}{p^+ + p'^+} \approx \frac{x_B}{2-x_B}$$

(light-cone coordinates with respect to $\bar{n} \propto P,\ n \propto q',\ n\cdot\bar{n} = 1,\ v = v^+\bar{n} + v^- n + v_\perp$).

- The hadronic part of the DVCS scattering amplitude is parametrized in terms of the Compton form factors (CFFs). Leading twist: $\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}$.

- $\mathcal{H}$ gives generally dominant contribution to observables, e.g. $\sigma_{\mathrm{DVCS}} \propto \xi^2 |\mathcal{H}|^2$ at small $\xi$.

- CFFs factorize in terms of GPDs, e.g.

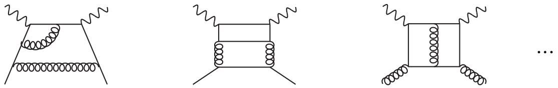
$$\mathcal{H} = \sum_{q=u,d,s} \frac{1}{\xi} \int_{-1}^{1} dx\, C_q(x/\xi, Q, \mu) H_q(x, \xi, t, \mu) + \frac{1}{\xi^2} \int_{-1}^{1} dx\, C_g(x/\xi, Q, \mu) H_g(x, \xi, t, \mu)$$

- Expansion in $\alpha_s$

$$C_q = C_q^{(0)} + \frac{\alpha_s}{4\pi} C_q^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_q^{(2)} + O(\alpha_s^3),$$
$$C_g = \frac{\alpha_s}{4\pi} C_g^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_g^{(2)} + O(\alpha_s^3).$$

The contribution from $C_q^{(2)}$ and $C_g^{(2)}$ are subject of this talk.

- Example diagrams



- Many diagrams are trivially related by crossing symmetry. Need to calculate $\sim 70$ diagrams which are not trivially related.

- Steps are as follows:
  $\Rightarrow$ Graph generation (qgraf)
  $\Rightarrow$ Apply Feynman rules and trace projection (FORM)
  $\Rightarrow$ Integration-by-parts reduction (FIRE) to 12 (scalar) master integrals
  $\Rightarrow$ Calculation of master integrals using method of differential equations. Fortunately there have been no new master integrals other than the ones appearing in the non-singlet case, they have been calculated in [Gao:2021iqq].

- Non-singlet CF in $\overline{\text{MS}}$ has been calculated [Braun:2020yib] using conformal symmetry. We redid the calculation by computation of Feynman diagrams and confirmed the result.

- Need to calculate also "Infrared subtractions" (relevant starting at two-loop), which involve convolution of CF (including $\epsilon^1$ terms) with one-loop Z-factor ($\Leftarrow$ get from evolution kernel [Braun:2019qtp]), e.g.

$$\int_{-1}^{1} \frac{dx}{\xi} C_q(x/\xi, Q, \mu) H_{q,\text{parton}}(x, \xi, t, \mu) \supset \alpha_s^2 \int_{-1}^{1} \frac{dx}{\xi}\, \epsilon C_q^{(1,1)}(x/\xi) \frac{1}{\epsilon} H_{q,\text{parton}}^{(1,-1)}(x, \xi)$$

  gives a *finite* contribution to the CF. Convolutions calculated with HyperInt program.

- All infrared singularities have to cancel such that the CF is finite

$$\underbrace{\mathcal{H}_{\text{parton}}}_{\text{IR divergent}} = \int_{-1}^{1} \frac{dx}{\xi} \underbrace{C_q(x/\xi, Q, \mu)}_{\text{finite}} \underbrace{H_{q,\text{parton}}(x, \xi, t, \mu)}_{\text{IR divergent}} + \int_{-1}^{1} \frac{dx}{\xi^2} \underbrace{C_g(x/\xi, Q, \mu)}_{\text{finite}} \underbrace{H_{g,\text{parton}}(x, \xi, t, \mu)}_{\text{IR divergent}},$$

  where LHS and RHS are considered renormalized.

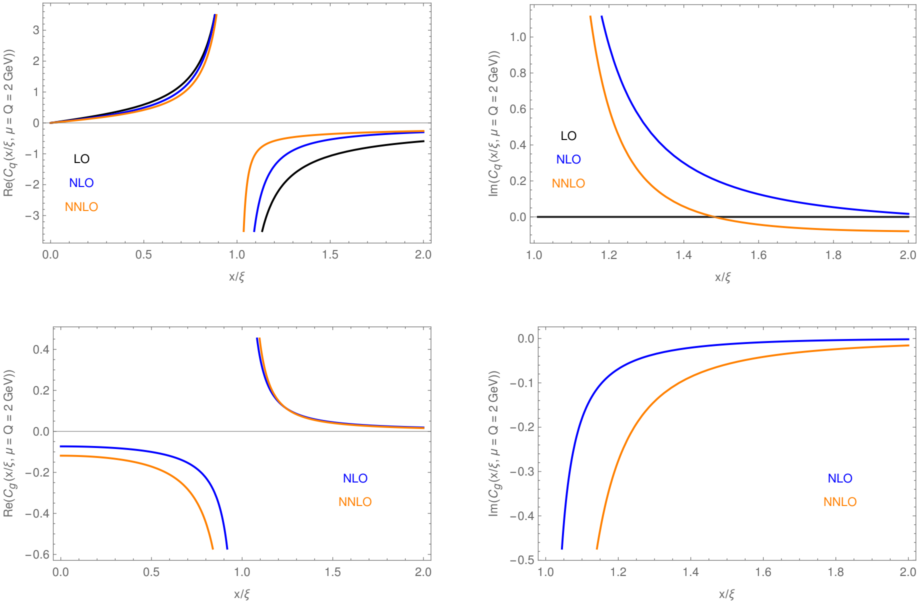
- There is mixing between quark and gluon terms $\Rightarrow$ a highly non-trivial check!

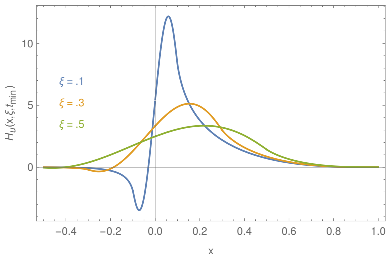
$H_{i,j,\ldots}$ are harmonic polylogarithms and $\bar{z} = 1 - z$.

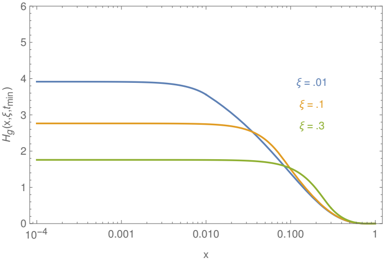
$$C_g^{(2)}(x/\xi) = \left(\sum_q e_q^2\right) T_F \left[C_F \mathsf{T}^{(C_F)}(z) + C_A \mathsf{T}^{(C_A)}(z)\right] \Big|_{z=\frac{1}{2}(1-x/\xi)},$$

$$\begin{aligned}
\mathsf{T}^{(C_F)}(z) = \frac{1}{z^2\bar{z}^2}\Bigg\{ & z^2\Big( - H_{2,1}(z) + 2H_{2,2}(z) - \frac{23}{2}H_{3,0}(z) - \frac{17}{2}H_{2,0,0}(z) - 11H_{2,1,0}(z) - \frac{23}{2}H_{0,0,0,0}(z) \\
& - 9H_{1,0,0,0}(z) - 9H_{1,1,0,0}(z)\Big) + \frac{1}{2}\left(-3z^2 + 16z + \pi^2(1 - 2z(z+1)) - 13\right) H_{1,1}(z) \\
& + 2\left(z^4 - 2z^3 + z\right)\left(H_{1,2}(z) - H_{2,0}(z)\right) - \frac{1}{4}\left(\left(6 + 7\pi^2\right) z + 20\right) zH_{0,0}(z) + \frac{5}{2}(z+1)zH_{0,0,0}(z) \\
& + \left[-\frac{1}{4}\bar{z}(z(8z-3)+3) - \frac{1}{6}\pi^2(z(7z+4)-2)\right] H_{1,0}(z) + \frac{1}{4}[z^2(-8(z-2)z-5) + 2z - 5]H_{1,1,0}(z) \\
& + \bar{z}^2\left(H_{1,0,0}(z) - \frac{5}{2}H_{1,1,2}(z) + \frac{1}{2}H_{1,2,1}(z) - \frac{5}{2}H_{1,1,1,1}(z) - 2H_{1,3}(z)\right) \\
& + \frac{5}{2}\bar{z}(z-2)H_{1,1,1}(z) + (2 - z(7z+4))\left(H_{1,2,0}(z) + H_{1,1,1,0}(z)\right) - 9z^2H_4\left(-\frac{\bar{z}}{z}\right) + 2z^2H_4(z) \\
& - \frac{1}{12}zH_0(z)\left[24z(z+\zeta(3)) - 6(23z+9) + \pi^2(z(4(z-2)z+3)+5)\right] \\
& + \frac{1}{24}\bar{z}H_1(z)\left[12(z(4z - \zeta(3) + 15) + \zeta(3) - 28) + \pi^2(3 - z(8\bar{z}z+9))\right] \\
& - \frac{1}{12}\left[z\left(24z + 22\pi^2 - 39\right) + 24\right] zH_2(z) + \frac{1}{4}[z(8(z-2)z+5)+8]zH_3(z) + 6z^4\zeta(3) \\
& + \frac{1}{3}z^3\left(\pi^2 - 36\zeta(3)\right) - \frac{1}{360}z^2\left[90(\zeta(3)+36) + 195\pi^2 + 103\pi^4\right] + z\left(6\zeta(3) + 9 + \frac{\pi^2}{3}\right)\Bigg\}
\end{aligned}$$

# Plots of CF


Re(C_q(x/ξ, μ = Q = 2 GeV))
LO
NLO
NNLO
x/ξ
Im(C_q(x/ξ, μ = Q = 2 GeV))
LO
NLO
NNLO
x/ξ
Re(C_g(x/ξ, μ = Q = 2 GeV))
NLO
NNLO
x/ξ
Im(C_g(x/ξ, μ = Q = 2 GeV))
NLO
NNLO
x/ξ




- We used the model by Goloskokov and Kroll (GK) [Goloskokov:2006hr] in order to estimate the size of the NNLO correction to $\mathcal{H}$.

- GK model is obtained from the double distribution (DD) ansatz (D-term is neglected)

$$F(\beta,\alpha,t) = e^{(b+\alpha' \ln(1-|\beta|))(t-t_{\min})} f(\beta) h(\beta,\alpha)$$

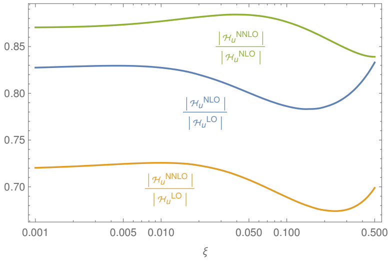
$$H(x,\xi,t) = \int_{\{|\alpha|+|\beta|\leq 1\}} d\alpha d\beta \; F(\beta,\alpha,t)\delta(x-\beta-\xi\alpha)$$

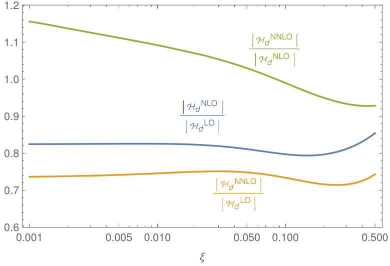
- We refited the PDF parameters to HERA20PDF NNLO data.

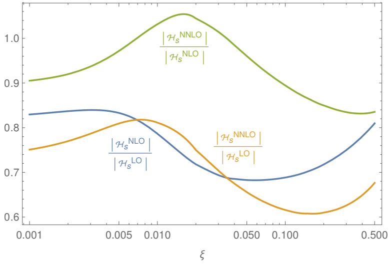
## Size of correction to CFF $\mathcal{H}$ for each parton

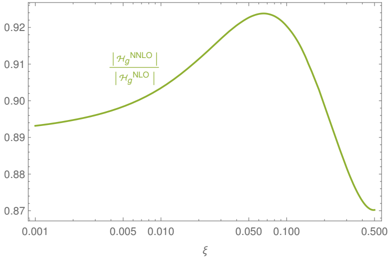
$\mu = Q = 2$ GeV, $t = t_{\text{min}}$

PRELIMINARY



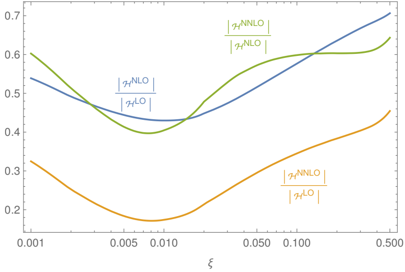



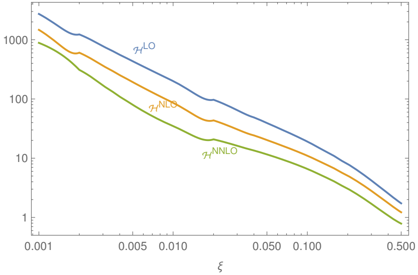



# Size of correction to CFF $\mathcal{H}$ total

$\mu = Q = 2$ GeV, $t = t_{\text{min}}$

PRELIMINARY




- Large correction when adding up the CFFs corresponding to each parton! This happens already at NLO.
- Contribution of quarks is positive while imaginary part of gluon contribution is negative $\rightarrow$ cancellation, e.g. at $\xi = 10^{-3}$:

$$\underbrace{268 + 1344i}_{=\mathcal{H}_u} + \underbrace{77 + 338i}_{=\mathcal{H}_d} + \underbrace{71 + 258i}_{=\mathcal{H}_s} + \underbrace{170 - 1275i}_{=\mathcal{H}_g} = 585 + 664i$$

- We have calculated the two-loop vector singlet CF for DVCS using computer algebra methods for calculating the Feynman diagrams.
- Size of NNLO radiative corrections strongly depends on the GPD model.
- Including three loop evolution is needed to complete NNLO program $\rightarrow$ three loop singlet evolution is not known yet, but will be available soon.
- Possible near future extensions: calculate two-loop axial singlet CF, calculate $\sim n_f$ contribution of three-loop to estimate size of N3LO correction, produce public computer code for NNLO predictions of leading twist DVCS.