Observable for gluon Orbital Angular Momentum

Shohini Bhattacharya
BNL
4 May 2022

In Collaboration with:
Renaud Boussarie (CPHT, CNRS)
Yoshitaka Hatta (BNL)

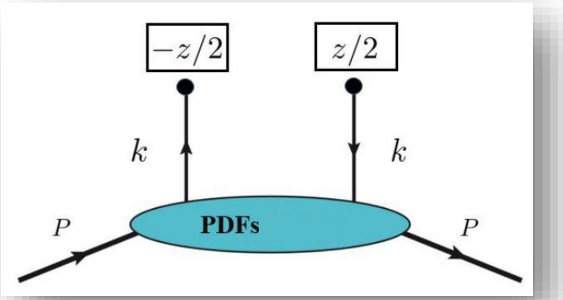
Based on:

PRL 128, 182002 (arXiv: 2201.08709)

Outline

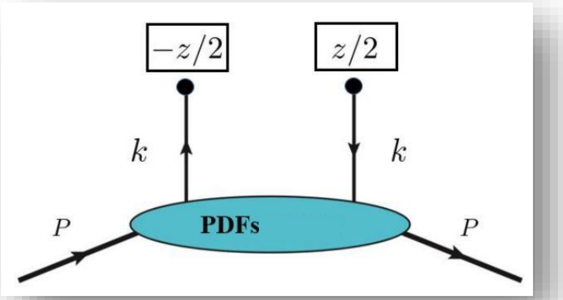
- **Wigner distribution & gluon OAM**
- **Exclusive dijet production as a probe of gluon OAM**
- **Summary & Outlook**

Wigner distribution - The “mother distribution”

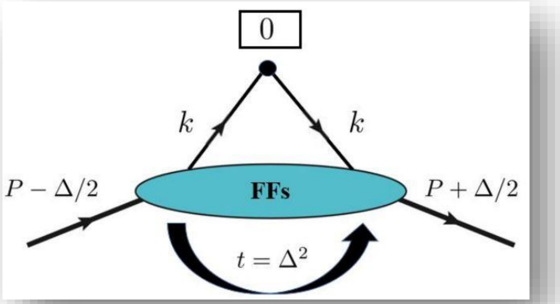


PDFs (x)

Wigner distribution - The “mother distribution”



PDFs (x) **FFs** (Δ)



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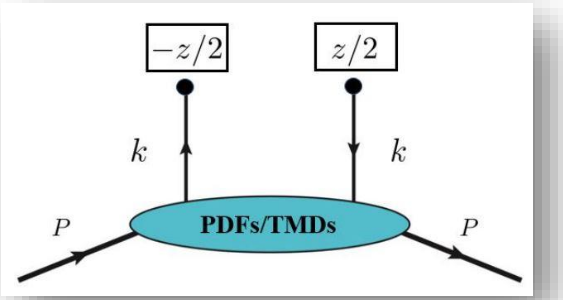


TMDs $(x, \vec{k}_\perp)$

$\int d^2\vec{k}_\perp$

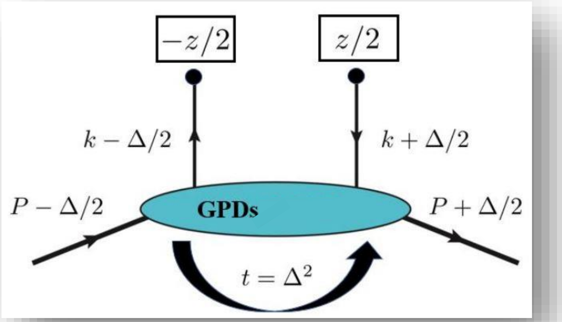
GPDs (x, Δ)

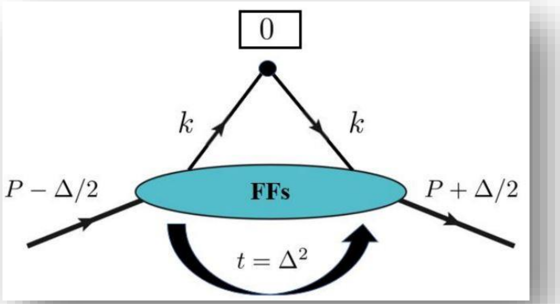
$\Delta = 0$

$\int dx$

PDFs (x)

FFs (Δ)





Wigner distribution - The "mother distribution"

(Meissner, Metz, Schlegel, 2009)

GTMDs $(x, \vec{k}_\perp, \Delta)$

$\Delta = 0$

$\int d^2\vec{k}_\perp$

TMDs $(x, \vec{k}_\perp)$

GPDs (x, Δ)

$\int d^2\vec{k}_\perp$

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$\int dx$

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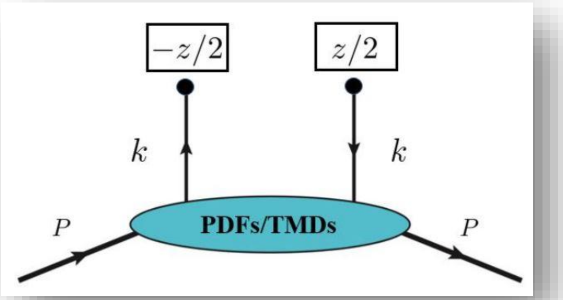
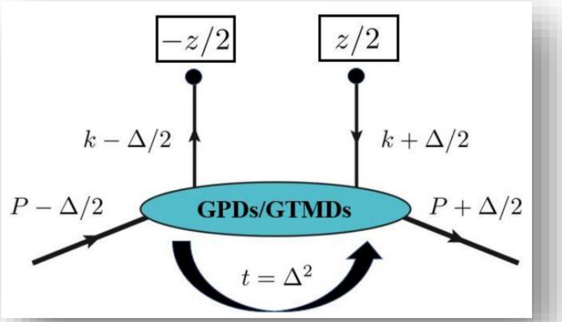
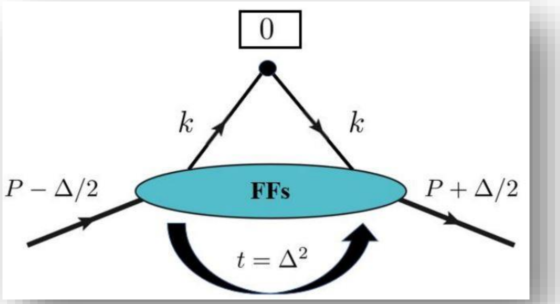






Wigner distribution - The “mother distribution”

Wigner Distribution $(x, \vec{k}_\perp, \vec{b}_\perp)$ **(Belitsky, Ji, Yuan, 2003)**

2-D Fourier Transform $(\vec{\Delta}_\perp)$ ↑ $\xi = 0$

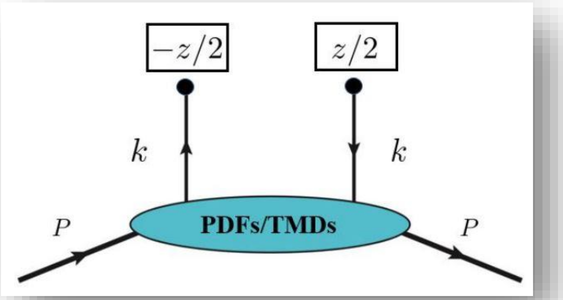
(Meissner, Metz, Schlegel, 2009) **GTMDs** $(x, \vec{k}_\perp, \Delta)$

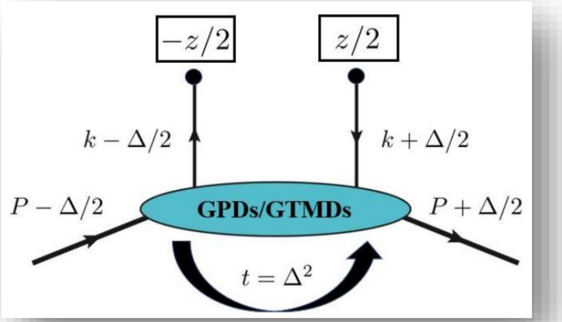
$\Delta = 0$ ↙ ↘ $\int d^2\vec{k}_\perp$

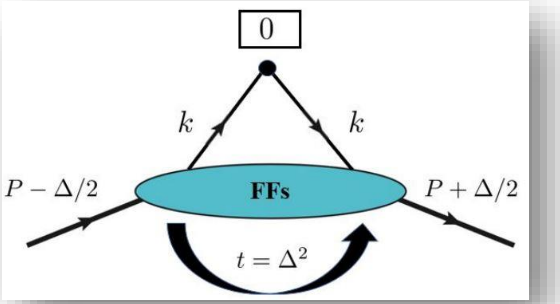
TMDs $(x, \vec{k}_\perp)$ **GPDs** (x, Δ)

$\int d^2\vec{k}_\perp$ ↘ $\Delta = 0$ ↙ $\int dx$ ↘

PDFs (x) **FFs** (Δ)



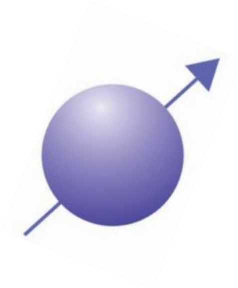




Applications of Wigner distributions

Jaffe-Manohar spin decomposition

- **An incomplete story:**



$$\frac{1}{2}$$

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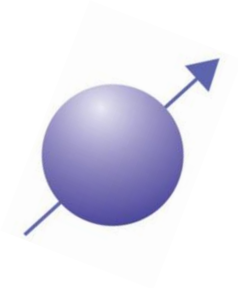

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma$$

Best known

Quark helicity $\sim 30\%$

Applications of Wigner distributions

Jaffe-Manohar spin decomposition

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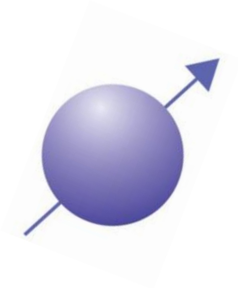


$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G$$

Best known

How well do we know?

Quark helicity $\sim 30\%$

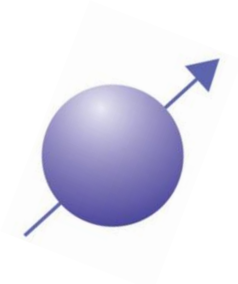
Gluon helicity $\sim 40\%$

(See Yiyu's talk, Tue.)

Applications of Wigner distributions

Jaffe-Manohar spin decomposition

- **An incomplete story:**



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g$$

Best known — **How well do we know?** — **??**

Quark helicity $\sim 30\%$ — **Gluon helicity** $\sim 40\%$ — **OAM of quarks & gluons**

Applications of Wigner distributions

Jaffe-Manohar spin decomposition

- An incomplete story:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g$$

An intuitive definition

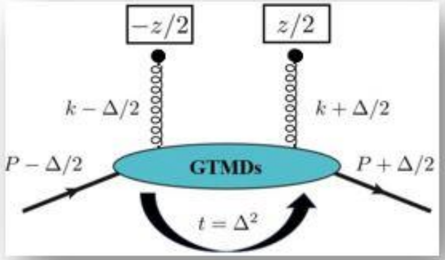
NRQM: $\langle \mathcal{O} \rangle = \int dx \int dk\ \mathcal{O}(x,k)\, W(x,k)$

- **OAM as a moment of Wigner distribution : (Lorcé, Pasquini, 2011 / Hatta, 2011 / Ji, Xiong, Yuan, 2012)**

$$L_z^{q,g} = \int dx \int d^2k_\perp d^2b_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z\, W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp)$$

Applications of Wigner distributions

Parameterization of a GTMD correlator (unpolarized gluons):

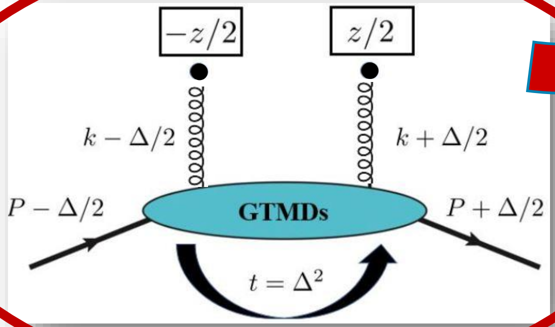


$$= \frac{1}{2M}\,\bar{u}(p',\lambda')\left[F^g_{1,1} + \frac{i\sigma^{i+}k^i_\perp}{P^+} F^g_{1,2} + \frac{i\,\sigma^{i+}\Delta^i_\perp}{P^+} F^g_{1,3} + \frac{i\sigma^{ij}k^i_\perp \Delta^j_\perp}{M^2} F^g_{1,4} \right] u(p,\lambda)$$

SB, Metz, Ojha, Tsai, Zhou, 1802.10550

- **OAM as a moment of Wigner distribution/GTMD: (Lorcé, Pasquini, 2011 / Hatta, 2011 / Ji, Xiong, Yuan, 2012)**

$$L_z^{q,g} = \int dx \int d^2k_\perp d^2b_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z \left\{ \int e^{i\vec{b}_\perp \cdot \vec{\Delta}_\perp} \right\}$$



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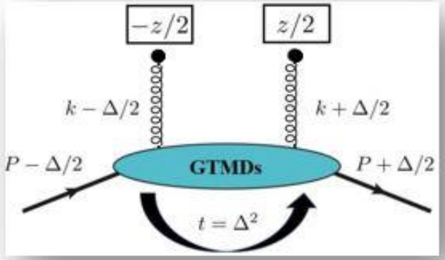

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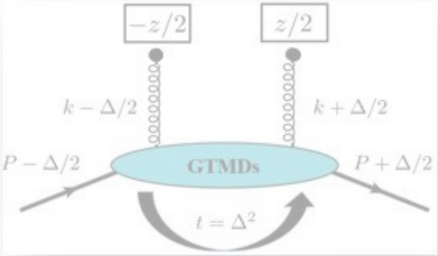
- **OAM as a moment of Wigner distribution/GTMD: (Lorcé, Pasquini, 2011 / Hatta, 2011 / Ji, Xiong, Yuan, 2012)**

$$\boxed{L^{q,g}_z = -\int dx \int d^2\vec{k}_\perp \frac{\vec{k}^2_\perp}{M^2} F^{q,g}_{1,4}(x, \vec{k}^2_\perp)}$$

Relation between GTMD $F^{q,g}_{1,4}$ & OAM

Applications of Wigner distributions

Parameterization of a GTMD correlator (unpolarized gluons):



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SB, Metz, Ojha,

Big question: Is this measurable?

- OAM as a n ... g, Yuan, 2012)

$$L_z^{q,g} = -\int dx \int d^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} F_{1,4}^{q,g}(x, \vec{k}_\perp^2)$$

Relation between GTMD $F_{1,4}^{q,g}$ & OAM

Developments

arXiv: 1601.01585 (2016)

Probing the Small-x Gluon Tomography in Correlated Hard Diffractive Dijet Production in DIS

Yoshitaka Hatta,[1] Bo-Wen Xiao,[2] and Feng Yuan[3]

Developments

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Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider

Xiangdong Ji,[1,2] Feng Yuan,[3] and Yong Zhao[1,3]

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Gluon orbital angular momentum at small-x

Yoshitaka Hatta,[1] Yuya Nakagawa,[1] Bowen Xiao,[2] Feng Yuan,[3] and Yong Zhao[3,4,5]

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Probing the Weizsäcker-Williams gluon Wigner distribution in pp collisions

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Probing the gluon Sivers function with an unpolarized target:
GTMD distributions and the Odderons

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The CMS Collaboration

See Michael's talk

Angular correlations in exclusive dijet photoproduction in
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See Ya-jin's talk, Thurs.

Probing the gluon tomography in photoproduction of di-pions

Yoshikazu Hagiwara, Cheng Zhang, Jian Zhou, and Ya-jin Zhou

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Gluon orbital angular momentum at sı

Yoshitak

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gluons

n Zhou[2]

We took a fresh look at this 2016 paper

arXiv: 1807.08697 (2018)

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Summary of the 2016 paper

arXiv: 1612.02438 (2016)

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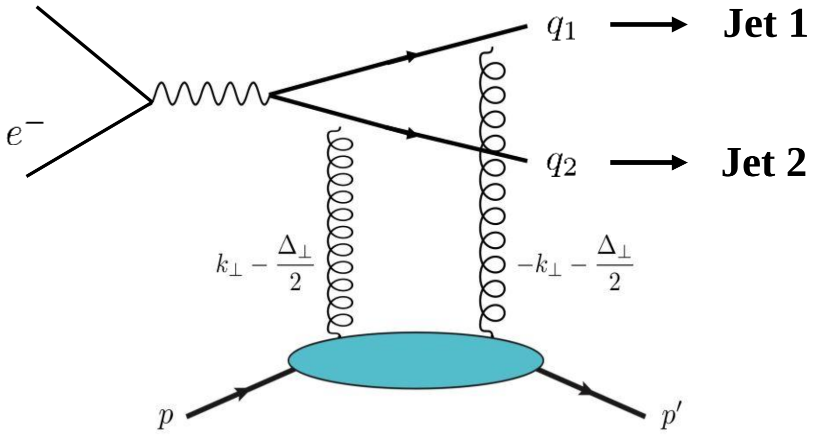
Xiangdong Ji,[1,2] Feng Yuan,[3] and Yong Zhao[1,3]



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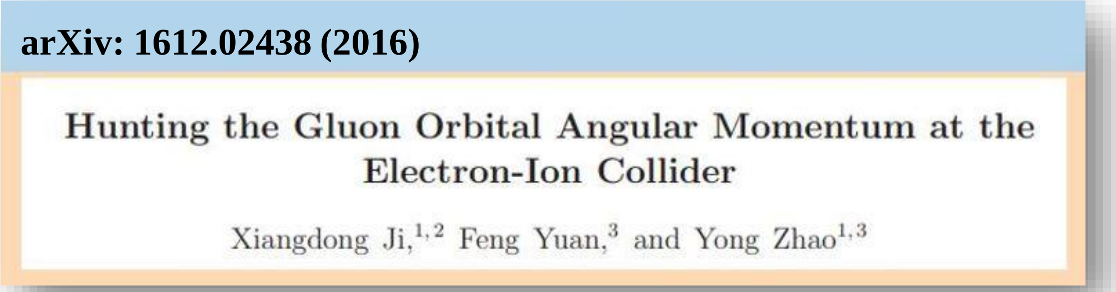
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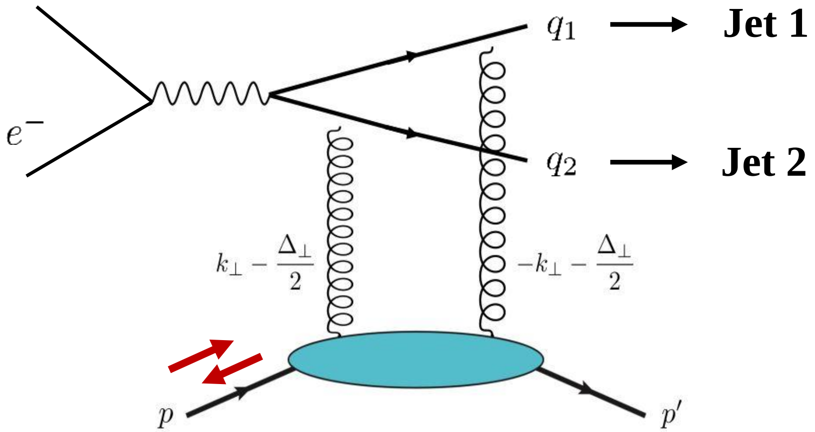

Longitudinal single spin asymmetry (SSA):

$$\frac{d\Delta\sigma}{dy\,dQ^2 d\Omega} = \sigma_0 h_p \frac{2(\bar{z}-z)(q_\perp \times \Delta_\perp)}{q_\perp^2+\mu^2}\Big[16\beta(1-y)\mathfrak{Im}[F_g^* + 4\xi^2\bar{\beta}F_g'^*][\mathcal{L}_g + 8\xi^2\bar{\beta}\mathcal{L}_g'] + (1+(1-y)^2)\,\mathfrak{Im}[F_g^* + 2\xi^2(1-2\beta)F_g'^*][\mathcal{L}_g + 2\bar{\beta}(1/z\bar{z}-2)(\mathcal{L}_g + 4\xi^2(1-2\beta)\mathcal{L}_g')]\Big]$$

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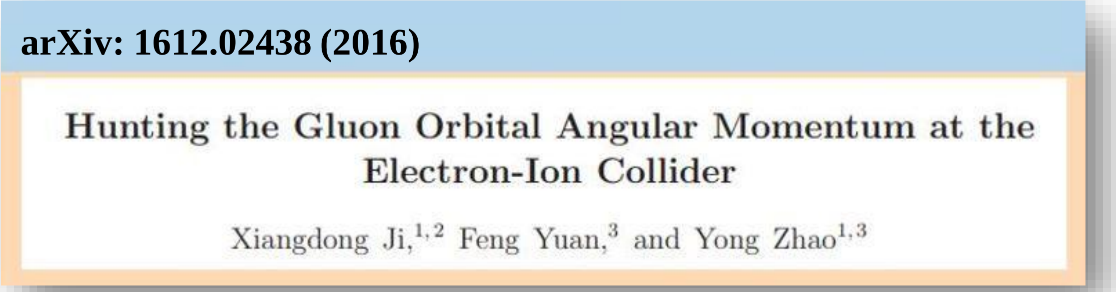
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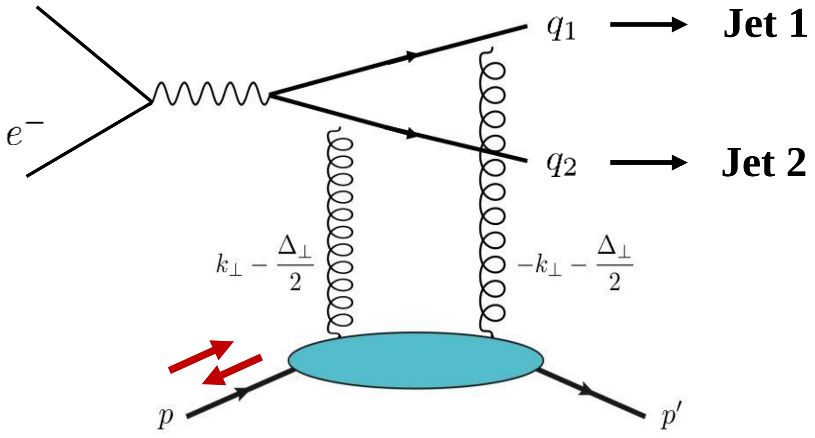

Schematic structure of SSA (oversimplified):

$$\frac{d\sigma}{dy dQ^2 d\Omega} \sim \sigma_0 h_p \sin(\phi_{q_\perp} - \phi_{\Delta_\perp})\,(\bar{z} - z)\left[\mathfrak{Im}\Big(F_g^*(\xi)\mathcal{L}_g(\xi)\Big)\right]$$

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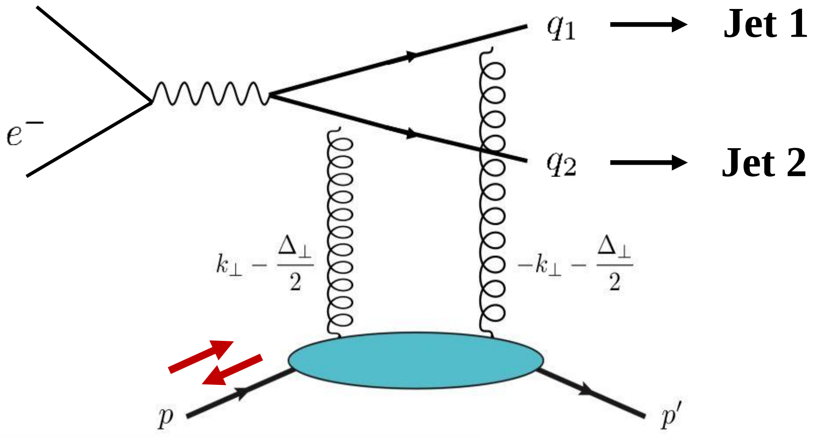

Signature of OAM is sinusoidal angular modulation

Moment of GPD

$$q_\perp = q_{1\perp} - q_{2\perp}$$

$$\boxed{\frac{d\sigma}{dydQ^2d\Omega} \sim \sigma_0 h_p \sin(\phi_{q_\perp} - \phi_{\Delta_\perp})(\bar{z} - z)\left[\Im\mathfrak{m}\left(F_g^*(\xi)\mathcal{L}_g(\xi)\right)\right]}$$

Moment of OAM

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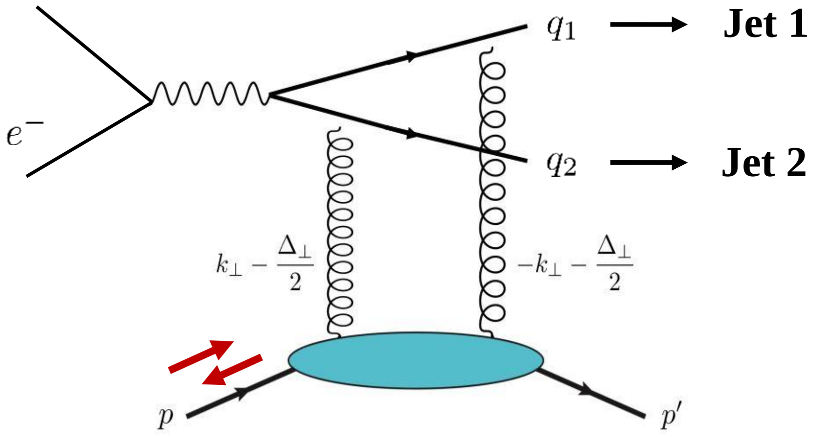


Issues with SSA:

$$q_\perp = q_{1\perp} - q_{2\perp}$$

$$\boxed{\frac{d\sigma}{dy dQ^2 d\Omega} \sim \sigma_0 h_p \sin(\phi_{q_\perp} - \phi_{\Delta_\perp})\,(\bar{z} - z)\left[\mathfrak{Im}\Big(F_g^*(\xi)\mathcal{L}_g(\xi)\Big)\right]}$$

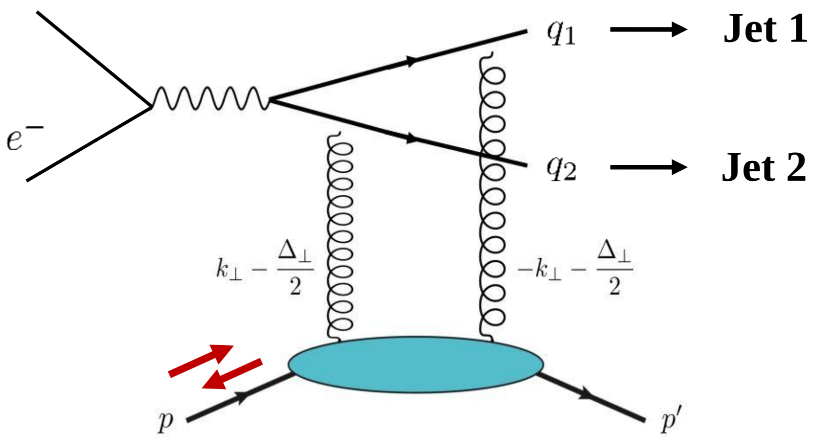
Probing gluon OAM through exclusive dijet production

Summary of the 2016 paper

arXiv: 1612.02438 (2016)

Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider

Xiangdong Ji,[1,2] Feng Yuan,[3] and Yong Zhao[1,3]



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SSA vanishes for symmetric jet configurations $z = \bar{z} = \frac{1}{2}$

Probing gluon OAM through exclusive dijet production

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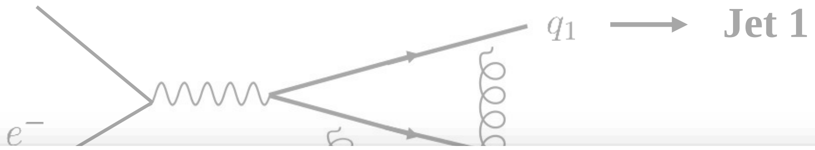

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Hunting the Gluon Orbital Angular Momentum
Electron-Ion Collider

Xiangdong Ji,[1,2] Feng Yuan,[3] and Yong Zhao[1,3]

"Compton Form Factor":

$$\boxed{\mathcal{L}_g(\xi) = \int dx \frac{x^2 \xi L_g(x,\xi)}{(x^2 - \xi^2 + i\xi\epsilon)^3}}$$

Third pole at $x = \pm\xi$ ⟶ potentially dangerous for collinear factorization

(See Cui, Hu, Ma, 1804.05293)

Issues with SSA:

$$q_\perp = q_{1\perp} - q_{2\perp}$$

$$\boxed{\frac{d\sigma}{dy dQ^2 d\Omega} \sim \sigma_0 h_p \sin(\phi_{q_\perp} - \phi_{\Delta_\perp}) (\bar{z} - z) \left[\Im\mathrm{m}\left(F_g^*(\xi)\mathcal{L}_g(\xi)\right) \right]}$$

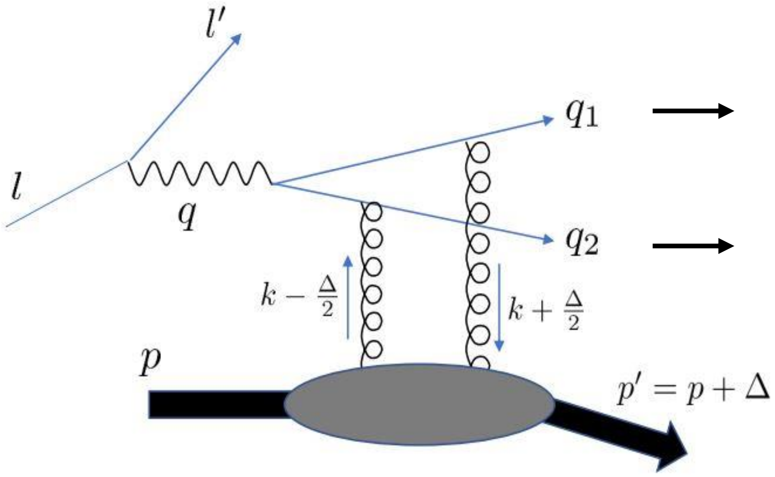
SSA vanishes for symmetric jet configurations $z = \bar{z} = \dfrac{1}{2}$

Probing gluon OAM through exclusive dijet production

Our work

Signature of the gluon orbital angular momentum

Shohini Bhattacharya,[1,*] Renaud Boussarie,[2,†] and Yoshitaka Hatta[1,3,‡]



$\longrightarrow$ **Jet 1**

$\longrightarrow$ **Jet 2**

Probing gluon OAM through exclusive dijet production

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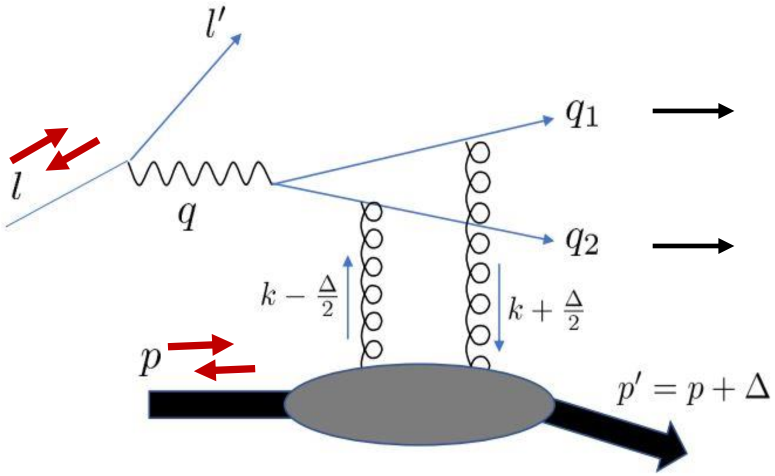


$q_1 \longrightarrow$ **Jet 1**

$q_2 \longrightarrow$ **Jet 2**

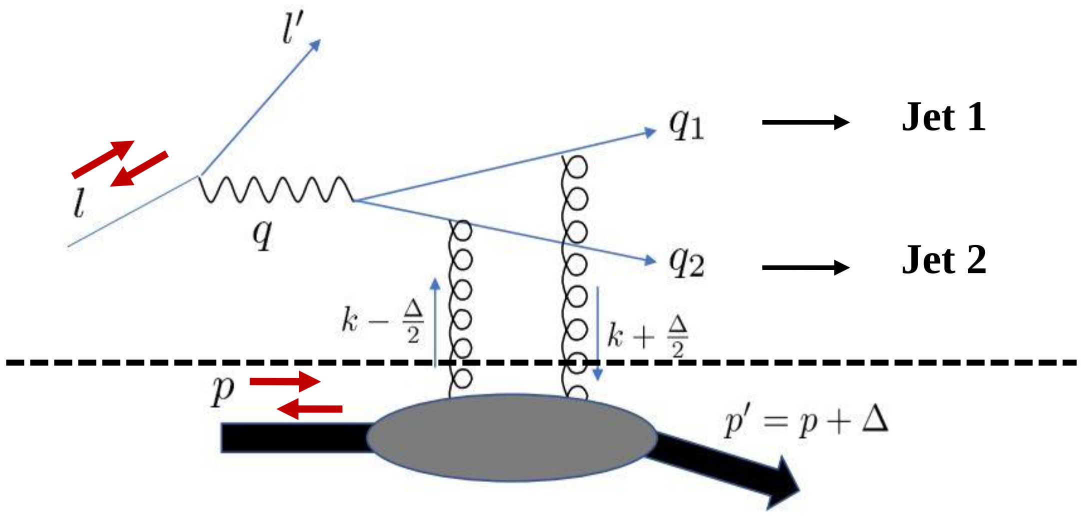
Distinct feature in our work

Double spin asymmetry (DSA):-

Both electron & incoming proton are longitudinally polarized

Probing gluon OAM through exclusive dijet production

Scattering amplitude



- **6 leading-order Feynman diagrams**
- **Scattering amplitude:**

$$\boxed{A \propto \int dx \int d^2k_\perp \, \mathcal{H}(x, \xi, q_\perp, k_\perp, \Delta_\perp) \, x f_g(x, \xi, k_\perp, \Delta_\perp)}$$

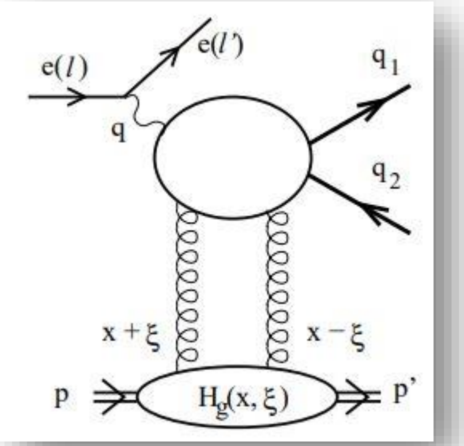
Hard part   Soft part

Probing gluon OAM through exclusive dijet production

Scattering amplitude

Twist expansion:

- **Twist-2 amplitude:** **Proportional to gluon GPD**



Braun, Ivanov, 0505263

$$A_T^2 = \frac{ig_s^2 e_{em} e_q}{N_c} \frac{1}{q_\perp^2 + \mu^2} \left(\bar{u}(q_1) \not{\epsilon}_\perp v(q_2)\right) \int dx \frac{1}{(x+\xi-i\varepsilon)(x-\xi+i\varepsilon)} \times \left(1 + \frac{2\xi^2(1-2\beta)}{(x+\xi-i\varepsilon)(x-\xi+i\varepsilon)}\right) \boxed{\int d^2k_\perp x f_g(x,\xi,k_\perp,\Delta_\perp)}$$

$$A_L^2 = \frac{ig_s^2 e_{em} e_q}{N_c} \frac{1}{(q_\perp^2 + \mu^2)^2} 4\xi z\bar{z} QW \left(\bar{u}(q_1)\gamma^- v(q_2)\right) \int dx \frac{1}{(x+\xi-i\varepsilon)(x-\xi+i\varepsilon)} \times \left(1 + \frac{4\xi^2\bar{\beta}}{(x+\xi-i\varepsilon)(x-\xi+i\varepsilon)}\right) \boxed{\int d^2k_\perp x f_g(x,\xi,k_\perp,\Delta_\perp)}$$

Probing gluon OAM through exclusive dijet production

Scattering amplitude

Twist expansion:

- **Twist-3 amplitude:** **Proportional to gluon OAM**

$$A_T^3 = -\frac{ig_s^2 e_{em} e_q}{N_c}\frac{2(\bar{z}-z)}{(q_\perp^2+\mu^2)^2}\bar{u}(q_1)\epsilon_\perp\cdot\gamma_\perp v(q_2)\int dx \frac{x}{(x^2-\xi^2+i\xi\varepsilon)^2}\left(2\xi+\frac{(2\xi)^3(1-2\beta)}{(x^2-\xi^2+i\xi\varepsilon)}\right)\boxed{\int d^2k_\perp q_\perp\cdot \boldsymbol{k}_\perp\, x f_g(x,\xi,k_\perp,\Delta_\perp)}$$
$$-\frac{ig_s^2 e_{em} e_q}{N_c}\frac{2(2\xi)^2 z\bar{z}W}{(q_\perp^2+\mu^2)^2}\bar{u}(q_1)\gamma^- v(q_2)\int dx\frac{x}{(x^2-\xi^2+i\xi\varepsilon)^2}\boxed{\int d^2k_\perp \epsilon_\perp\cdot \boldsymbol{k}_\perp\, x f_g(x,\xi,k_\perp,\Delta_\perp)}$$

$$A_L^3 = \frac{ig_s^2 e_{em} e_q}{N_c}\frac{16\xi^2(\bar{z}-z)z\bar{z}QW}{(q_\perp^2+\mu^2)^3}\bar{u}(q_1)\gamma^- v(q_2)\int dx\frac{x}{(x^2-\xi^2+i\xi\varepsilon)^2}\left(1+\frac{8\xi^2(1-\beta)}{(x^2-\xi^2+i\xi\varepsilon)}\right)\boxed{\int d^2k_\perp\, q_\perp\cdot \boldsymbol{k}_\perp\, x f_g(x,\xi,k_\perp,\Delta_\perp)}$$

Probing gluon OAM through exclusive dijet production

Scattering amplitude

Twist expansion:

- **Twist-3 amplitude:** Proportional to gluon OAM

$$A_T^3 = -\frac{ig_s^2 e_{em} e_q}{N_c}\frac{2(\bar z - z)}{(q_\perp^2+\mu^2)^2}\bar u(q_1)\epsilon_\perp\cdot\gamma_\perp v(q_2)\int dx \frac{x}{(x^2-\xi^2+i\xi\varepsilon)^2}\left(2\xi + \frac{(2\xi)^3(1-2\beta)}{(x^2-\xi^2+i\xi\varepsilon)}\right)\int d^2k_\perp q_\perp\cdot \boldsymbol{k}_\perp\, x f_g(x,\xi,k_\perp,\Delta_\perp)$$

$$-\frac{ig_s^2 e_{em} e_q}{N_c}\frac{2(2\xi)^2 z\bar z W}{(q_\perp^2+\mu^2)^2}\bar u(q_1)\gamma^- v(q_2)\int dx \frac{x}{\cdots}\int d^2k_\perp \epsilon_\perp\cdot \boldsymbol{k}_\perp\, x f_g(x,\xi,k_\perp,\Delta_\perp)$$

Factorization-breaking third poles at $x=\pm\xi$

$$A_L^3 = \frac{ig_s^2 e_{em} e_q}{N_c}\frac{16\xi^2(\bar z - z) z\bar z Q W}{(q_\perp^2+\mu^2)^3}\bar u(q_1)\gamma^- v(q_2)\int dx \frac{x}{(x^2-\xi^2+i\xi\varepsilon)^2}\left(1 + \frac{8\xi^2(1-\beta)}{(x^2-\xi^2+i\xi\varepsilon)}\right)\int d^2k_\perp\, q_\perp\cdot \boldsymbol{k}_\perp\, x f_g(x,\xi,k_\perp,\Delta_\perp)$$

Probing gluon OAM through exclusive dijet production

Note: Gluon GPDs may contain $\sim \theta(\xi - |x|)(x^2 - \xi^2)^2$

(See Radyushkin, 9805342)

Hence, integrals containing third poles are divergent

Twist expansion:

- **Twist-3 amplitude:** Proportion

$$A_T^3 = -\frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(\bar{z} - z)}{(q_\perp^2 + \mu^2)^2} \bar{u}(q_1)\epsilon_\perp \cdot \gamma_\perp v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left(2\xi + \frac{(2\xi)^3(1 - 2\beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2k_\perp q_\perp \cdot \boldsymbol{k}_\perp \, x f_g(x, \xi, k_\perp, \Delta_\perp)$$

$$- \frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(2\xi)^2 z\bar{z}W}{(q_\perp^2 + \mu^2)^2} \bar{u}(q_1)\gamma^- v(q_2) \int dx \frac{x}{\dots}$$

Factorization-breaking third poles at $x = \pm\xi$

$$A_L^3 = \frac{ig_s^2 e_{em} e_q}{N_c} \frac{16\xi^2(\bar{z} - z)z\bar{z}QW}{(q_\perp^2 + \mu^2)^3} \bar{u}(q_1)\gamma^- v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left(1 + \frac{8\xi^2(1 - \beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2k_\perp \, q_\perp \cdot \boldsymbol{k}_\perp \, x f_g(x, \xi, k_\perp, \Delta_\perp)$$

Probing gluon OAM through exclusive dijet production

Scattering amplitude

Twist expansion:

Switch off the factorization-breaking third poles by setting $z = \bar{z} = \frac{1}{2}$

$$A_T^3 = -\frac{ig_s^2 e_{em} e_q}{N_c} \boxed{\frac{2(\bar{z}-z)}{(q_\perp^2+\mu^2)^2}} \bar{u}(q_1)\epsilon_\perp\cdot\gamma_\perp v(q_2) \int dx \frac{x}{(x^2-\xi^2+i\xi\varepsilon)^2}\left(2\xi + \frac{(2\xi)^3(1-2\beta)}{(x^2-\xi^2+i\xi\varepsilon)}\right)\int d^2k_\perp q_\perp\cdot \boldsymbol{k}_\perp\, x f_g(x,\xi,k_\perp,\Delta_\perp)$$

Recall: Not possible in SSA

Factorization-breaking third poles at $x = \pm\xi$

$$A_L^3 = \frac{ig_s^2 e_{em} e_q}{N_c} \frac{16\xi^2 \boxed{(\bar{z}-z)} z\bar{z} Q W}{(q_\perp^2+\mu^2)^3} \bar{u}(q_1)\gamma^- v(q_2) \int dx \frac{x}{(x^2-\xi^2+i\xi\varepsilon)^2}\left(1 + \frac{8\xi^2(1-\beta)}{(x^2-\xi^2+i\xi\varepsilon)}\right)\int d^2k_\perp\, q_\perp\cdot \boldsymbol{k}_\perp\, x f_g(x,\xi,k_\perp,\Delta_\perp)$$

Probing gluon OAM through exclusive dijet production

Scattering amplitude

Main result $(z = 1/2)$**:**

DSA is sensitive to OAM through an interference between L & T amplitudes:

$$\int d\phi_{q_\perp} L^{\mu\nu} A^*_\mu A_\nu = -\frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_\perp^2+\mu^2)^2} |l_\perp||\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$
$$\times \mathfrak{Re}\left[\left\{\mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2}\mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2+\mu^2}\left(\mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2}\mathcal{E}_g^{(2)*}\right)\right\}\mathcal{L}_g + \left(\mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2+\mu^2}\mathcal{E}_g^{(2)*}\right)\frac{\mathcal{O}}{2}\right]$$

DSA does not vanish for symmetric jet configurations $z = \bar{z} = \dfrac{1}{2}$

Probing gluon OAM through exclusive dijet production

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Signature of gluon OAM is cosine angular modulation

Probing gluon OAM through exclusive dijet production

Scattering amplitude

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DSA is sensitive to OAM through an interference between L & T amplitudes:

$$\int d\phi_{q_\perp} L^{\mu\nu} A^*_\mu A_\nu = -\frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_\perp^2+\mu^2)^2} |l_\perp||\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$
$$\times \mathfrak{Re}\left[\left\{\mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2}\mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2+\mu^2}\left(\mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2}\mathcal{E}_g^{(2)*}\right)\right\}\mathcal{L}_g + \left(\mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2+\mu^2}\mathcal{E}_g^{(2)*}\right)\frac{\mathcal{O}}{2}\right]$$

"Compton Form Factors":

$$\mathcal{L}_g(\xi) = \int_{-1}^{1} dx \frac{x^2 L_g(x,\xi)}{(x-\xi+i\epsilon)^2(x+\xi-i\epsilon)^2}$$

$$\mathcal{H}_g^{(1)}(\xi) = \int_{-1}^{1} dx \frac{H_g(x,\xi)}{(x-\xi+i\epsilon)(x+\xi-i\epsilon)} \qquad \mathcal{H}_g^{(2)}(\xi) = \int_{-1}^{1} dx \frac{\xi^2 H_g(x,\xi)}{(x-\xi+i\epsilon)^2(x+\xi-i\epsilon)^2}$$

Probing gluon OAM through exclusive dijet production

Scattering amplitude

Main result $(z = 1/2)$**:**

DSA is sensitive to OAM through an interference between L & T amplitudes:

$$\boxed{\int d\phi_{q_\perp} L^{\mu\nu} A^*_\mu A_\nu = -\frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_\perp^2+\mu^2)^2} |l_\perp||\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp}) \times \Re\mathfrak{e}\left[\left\{\mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2}\mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2+\mu^2}\left(\mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2}\mathcal{E}_g^{(2)*}\right)\right\}\mathcal{L}_g + \left(\mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2+\mu^2}\mathcal{E}_g^{(2)*}\right)\frac{\mathcal{O}}{2}\right]}$$

"Compton Form Factors":

$$O(x,\xi) \equiv \int d^2\widetilde{k}_\perp \frac{\widetilde{k}_\perp^2}{M^2} F_{1,2}(x,\xi,\widetilde{\Delta}_\perp = 0)$$

$$\mathcal{O}(\xi) = \int_{-1}^{1} dx \frac{xO(x,\xi)}{(x-\xi+i\epsilon)^2(x+\xi-i\epsilon)^2}$$

Probing gluon OAM through exclusive dijet production

Scattering amplitude

Not the end of the story:

Probing gluon OAM through exclusive dijet production

Scattering amplitude

Not the end of the story:

- **Interference between unpolarized & helicity GPD $(z = 1/2)$:**

Helicity GPD

$$\int d\phi_{q_\perp} L^{\mu\nu} A_\mu A_\nu = \frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1-\xi^2)\xi Q^2}{(q_\perp^2+\mu^2)^2} |l_\perp||\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp}) \mathfrak{Re}\left[\left(\mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2}\mathcal{E}_g^{(1)*}\right)\left(\tilde{\mathcal{H}}_g^{(2)} - \frac{\xi^2}{1-\xi^2}\tilde{\mathcal{E}}_g^{(2)}\right)\right]$$

- **Analogous contribution should enter SSA**

Probing gluon OAM through exclusive dijet production

Scattering amplitude

Not the end of the story:

- **Interference between unpolarized & helicity GPD** $(z = 1/2)$**:**

Helicity GPD

$$\int d\phi_{q_\perp} L^{\mu\nu} A_\mu A_\nu = \frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1-\xi^2)\xi Q^2}{(q_\perp^2+\mu^2)^2} |l_\perp||\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp}) \Re e \left[\left(\mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} \right) \left(\tilde{\mathcal{H}}_g^{(2)} - \frac{\xi^2}{1-\xi^2} \tilde{\mathcal{E}}_g^{(2)} \right) \right]$$

Helicity contributes to the same angular modulation as that of OAM

Probing gluon OAM through exclusive dijet production

Scattering amplitude

Not the end of the story:

- **Interference between unpolarized & helicity GPD** $(z = 1/2)$**:**

$$\int d\phi_{q_\perp} L^{\mu\nu} A_\mu A_\nu = \frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1-\xi^2)\xi Q^2}{(q_\perp^2+\mu^2)^2} |l_\perp||\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp}) \Re\mathfrak{e}\left[\left(\mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2}\mathcal{E}_g^{(1)*}\right)\left(\tilde{\mathcal{H}}_g^{(2)} - \frac{\xi^2}{1-\xi^2}\tilde{\mathcal{E}}_g^{(2)}\right)\right]$$

DSA does not vanish for symmetric jet configurations $z = \bar{z} = \frac{1}{2}$

Switch off the factorization-breaking third poles by setting $z = \bar{z} = \frac{1}{2}$

$$\int dx \frac{H_g(x,\xi)}{(x^2-\xi^2+i\xi\epsilon)^3} \qquad \int dx \frac{x\tilde{H}_g(x,\xi)}{(x^2-\xi^2+i\xi\epsilon)^3}$$

Probing gluon OAM through exclusive dijet production

Numerical estimate of cross section

Ingredients for non-perturbative functions

OAM

$$\int d\phi_{q_\perp} L^{\mu\nu} A^*_\mu A_\nu = -\frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_\perp^2+\mu^2)^2} |l_\perp||\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$
$$\times \mathfrak{Re}\left[\left\{\mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2}\mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2+\mu^2}\left(\mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2}\mathcal{E}_g^{(2)*}\right)\right\}\mathcal{L}_g + \left(\mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2+\mu^2}\mathcal{E}_g^{(2)*}\right)\frac{\mathcal{O}}{2}\right]$$

Helicity

$$\int d\phi_{q_\perp} L^{\mu\nu} A_\mu A_\nu = \frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1-\xi^2)\xi Q^2}{(q_\perp^2+\mu^2)^2} |l_\perp||\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$
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Probing gluon OAM through exclusive dijet production

Numerical estimate of cross section

Ingredients for non-perturbative functions

- **Neglect contributions from** $(E_g,\, \tilde{E}_g)\,,\; F_{1,2}$ $\longrightarrow$ **Very simple formula**

OAM

$$\int d\phi_{q_\perp} L^{\mu\nu} A^*_\mu A_\nu = -\frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_\perp^2+\mu^2)^2} |l_\perp||\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$
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Helicity

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Probing gluon OAM through exclusive dijet production

Numerical estimate of cross section

Ingredients for non-perturbative functions

- **Neglect contributions from** $(E_g,\, \tilde{E}_g)\,,\, F_{1,2}$ $\longrightarrow$ **Very simple formula**
- **Model** $(H_g,\, \tilde{H}_g)$ **according to the Double distribution approach (see for instance Radyushkin, 9805342)**

$$\begin{pmatrix} H_g(x,\xi) \\ \tilde{H}_g(x,\xi) \end{pmatrix} = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(\beta+\xi\alpha-x) \times \frac{15}{16} \frac{[(1-|\beta|)^2-\alpha^2]^2}{(1-|\beta|)^5} \times \begin{cases} \beta\, G(\beta) \\ \beta\, \Delta G(\beta) \end{cases}$$

Probing gluon OAM through exclusive dijet production

Numerical estimate of cross section

Ingredients for non-perturbative functions

- **Neglect contributions from** $(E_g, \tilde{E}_g)\,,\ F_{1,2}$ $\longrightarrow$ **Very simple formula**

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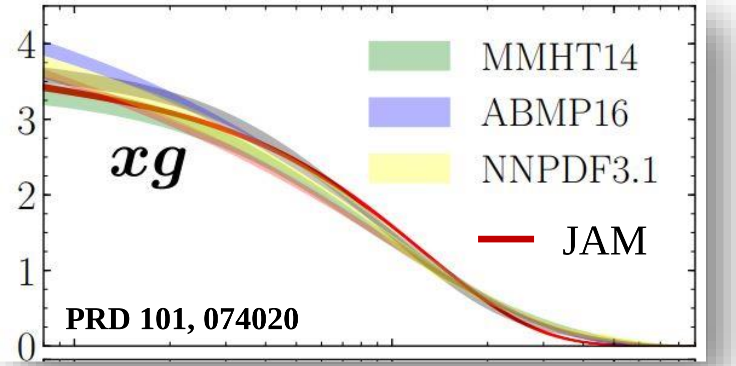
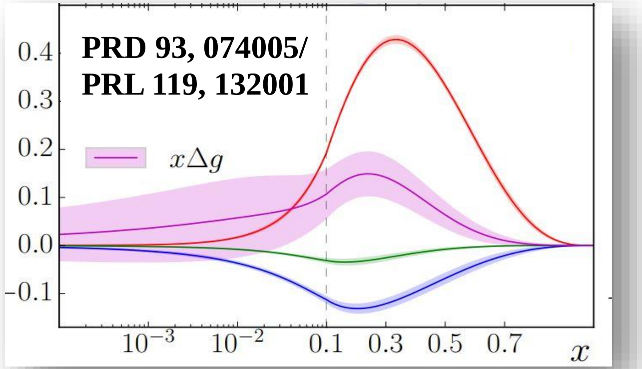
$$\begin{pmatrix} H_g(x,\xi) \\ \tilde{H}_g(x,\xi) \end{pmatrix} = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(\beta + \xi\alpha - x) \times \frac{15}{16}\frac{[(1-|\beta|)^2 - \alpha^2]^2}{(1-|\beta|)^5} \times \begin{cases} \beta\, G(\beta) \\ \beta\, \Delta G(\beta) \end{cases}$$

JAM PDFs





Probing gluon OAM through exclusive dijet production

Numerical estimate of cross section

Ingredients for non-perturbative functions

- **Neglect contributions from** $(E_g,\, \tilde{E}_g)\,,\, F_{1,2}$ $\longrightarrow$ **Very simple formula**
- **Model** $(H_g,\, \tilde{H}_g)$ **according to the Double distribution approach (see for instance Radyushkin, 9805342)**
- **Model for OAM:**

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1. “OAM density”: (Hatta, Yoshida, 1207.5332)

$$L^g_{can}(\boldsymbol{x}) \;=\; x\int_x^1 \frac{dx'}{x'^2}(H_g(x')+E_g(x')) \;-\; 2x\int_x^1 \frac{dx'}{x'^2}\Delta G(x') + \text{ genuine twist-three}$$

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$H_g(x') = x'G(x')$ Neglect E_g

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2. Use the Double distribution approach to construct $xL_g(x,\boldsymbol{\xi})$ from $xL_g(x)$ (GPD-like approach)

Probing gluon OAM through exclusive dijet production

Numerical estimate of cross section

Realistic EIC kinematics

$\sqrt{s}$ [GeV]	Q^2 [GeV^2]	y	ξ
120	2.7	0.7	$\lesssim 10^{-3}$
	4.8		
	10.0		

Focus on:

$$z = \bar{z} = \frac{1}{2}$$

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$$\frac{d\sigma}{dy dQ^2 d\phi_{l_\perp} dz dq_\perp^2 d^2\Delta_\perp} = \frac{\alpha_{em} y}{2^{11}\pi^7 Q^4} \frac{\int d\phi_{q_\perp} L^{\mu\nu} A_\mu^* A_\nu}{(W^2+Q^2)(W^2-M_J^2)z\bar{z}}$$

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Study cross section as differential in the skewness variable

$$\frac{d\sigma}{dy\,dQ^2\,dz\,d\xi\,d\delta\phi}$$

Relation between skewness & jet momenta:

$$\boxed{\xi = \frac{q_\perp^2 + z\bar{z}Q^2}{-q_\perp^2 + z\bar{z}(Q^2 + 2W^2)}}$$

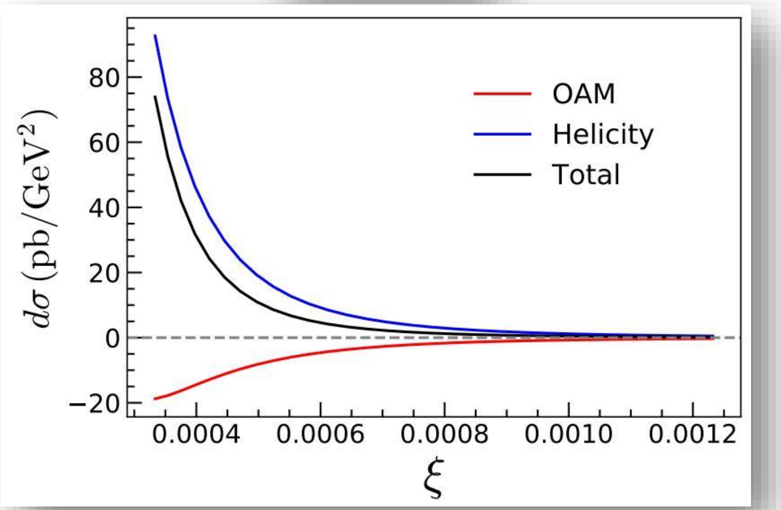
Cross section:

$$\frac{d\sigma}{dy\,dQ^2\,d\phi_{l_\perp}\,dz\,dq_\perp^2\,d^2\Delta_\perp} = \frac{\alpha_{em}y}{2^{11}\pi^7 Q^4}($$

Probing gluon OAM through exclusive dijet production

Numerical estimate of cross section

$Q^2 = 2.7$



Probing gluon OAM through exclusive dijet production

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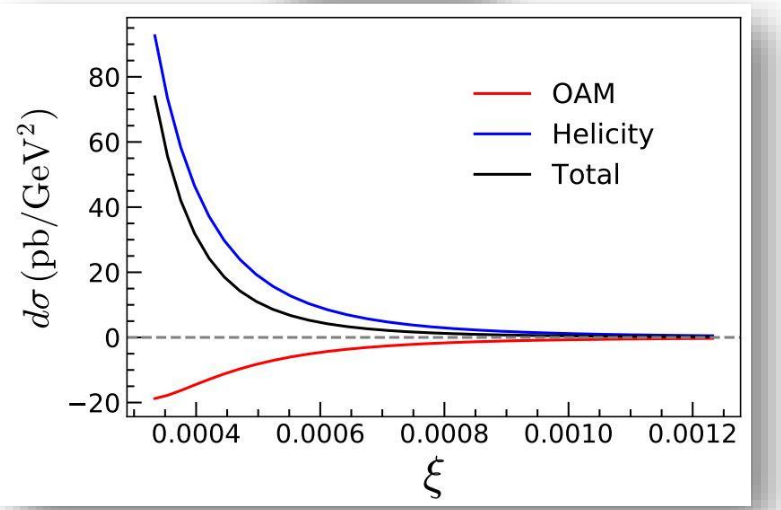

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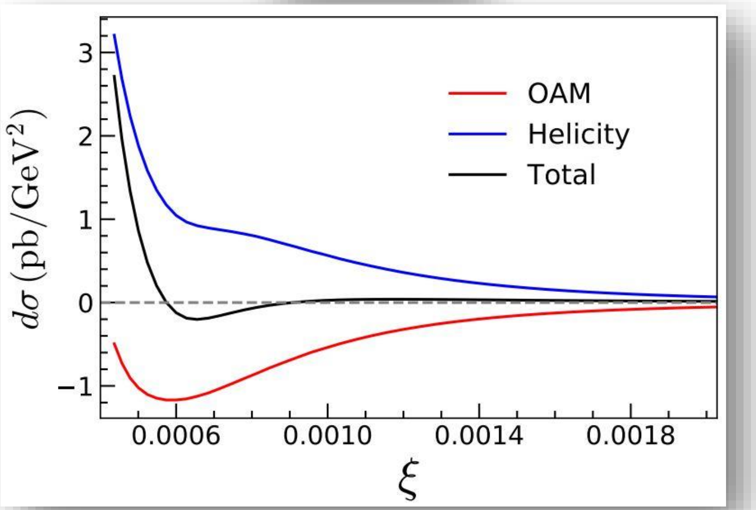

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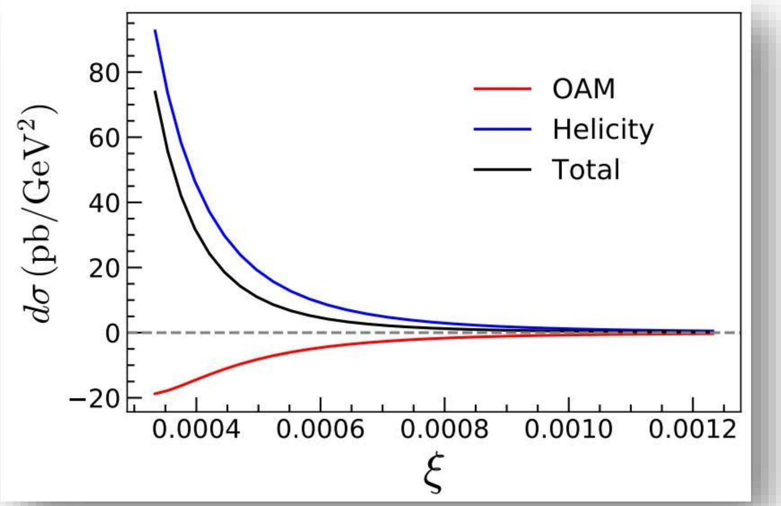

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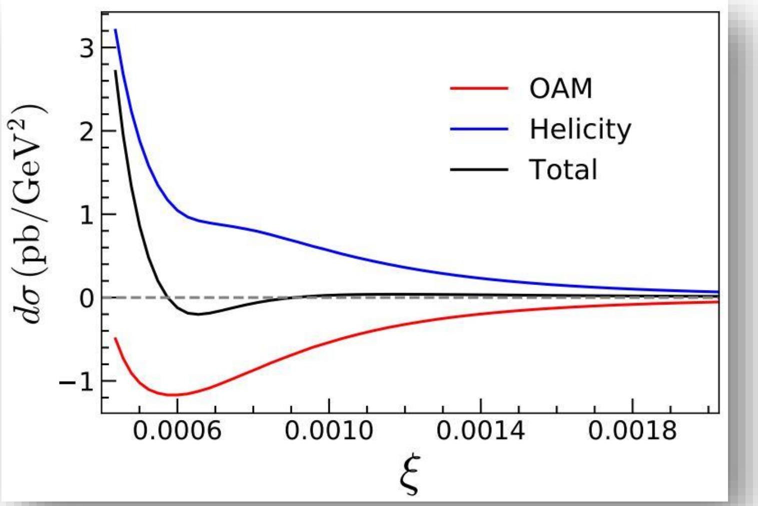
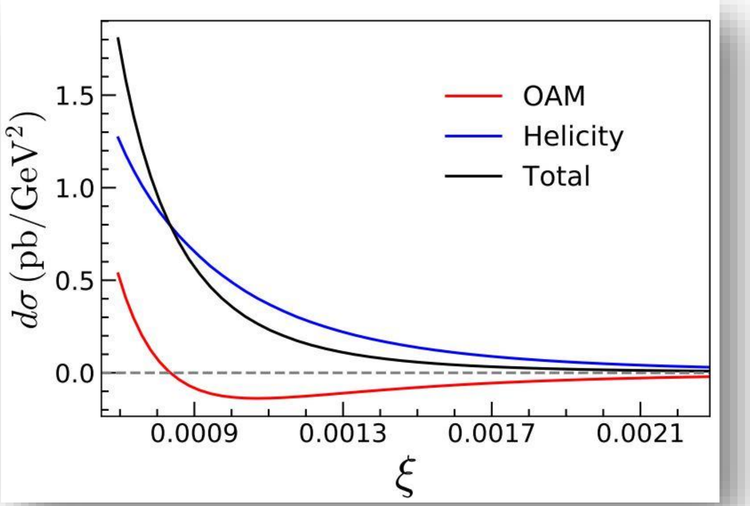


$Q^2 = 10$



Probing gluon OAM through exclusive dijet production

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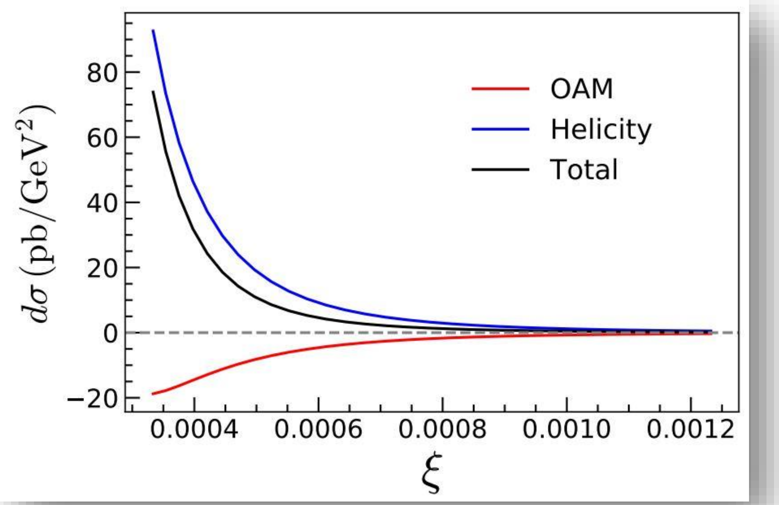

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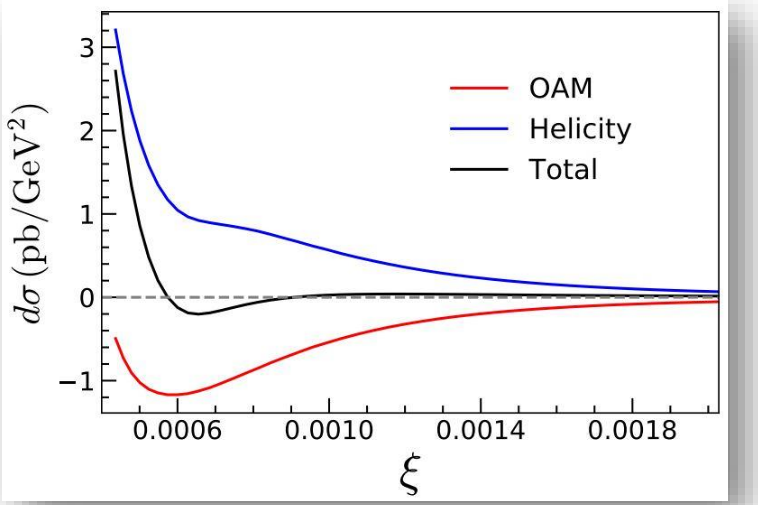

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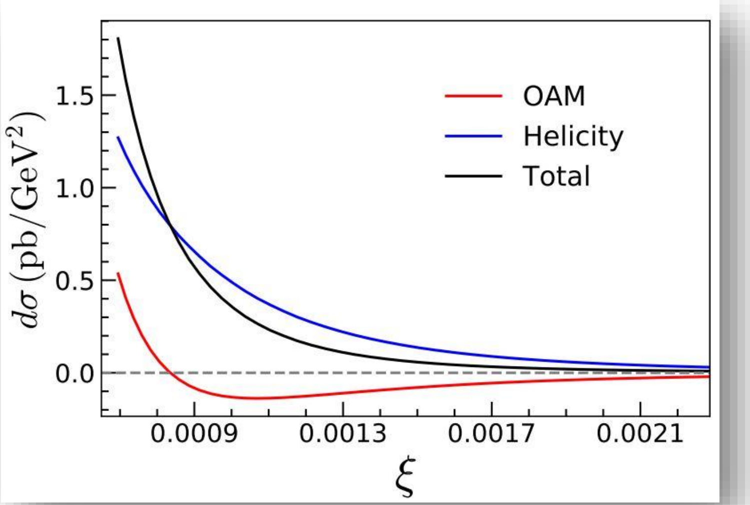
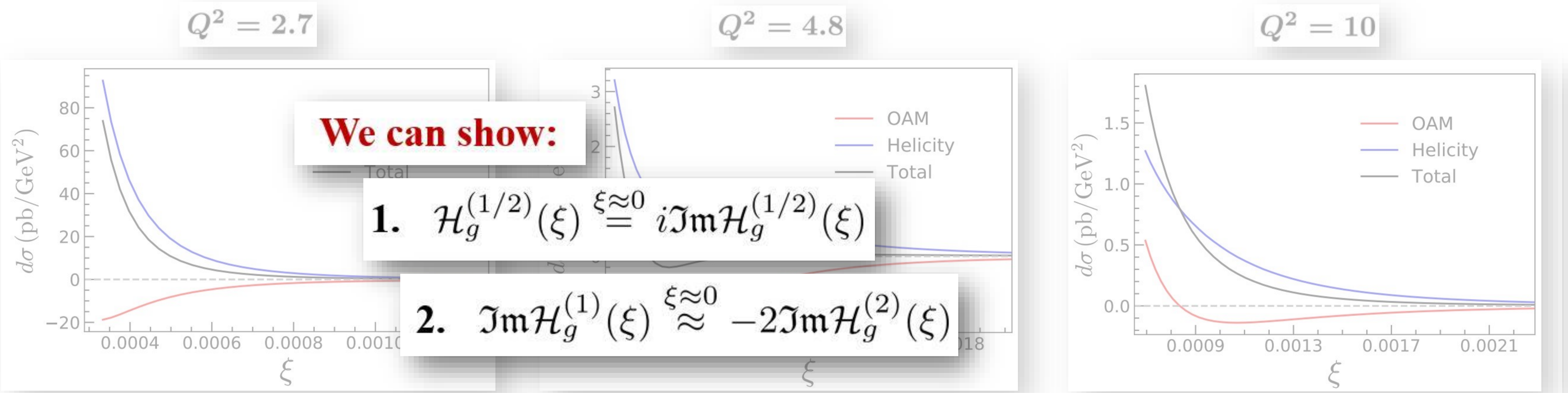


DSA: $$\int d\phi_{q_\perp} L^{\mu\nu} A^*_\mu A_\nu \Big|_{\delta\phi=0} \sim \mathfrak{Re}\left[\mathcal{H}_g^{(1)*}(\xi)\,\tilde{\mathcal{H}}_g^{(2)}(\xi)\right] - \mathfrak{Re}\left[\left\{\mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2 + Q^2/4}\mathcal{H}_g^{(2)*}(\xi)\right\}\mathcal{L}_g(\xi)\right]$$

Probing gluon OAM through exclusive dijet production

Numerical estimate of cross section



We can show:

1. $\mathcal{H}_g^{(1/2)}(\xi) \overset{\xi\approx 0}{=} i\mathfrak{Im}\mathcal{H}_g^{(1/2)}(\xi)$

2. $\mathfrak{Im}\mathcal{H}_g^{(1)}(\xi) \overset{\xi\approx 0}{\approx} -2\mathfrak{Im}\mathcal{H}_g^{(2)}(\xi)$

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Probing gluon OAM through exclusive dijet production

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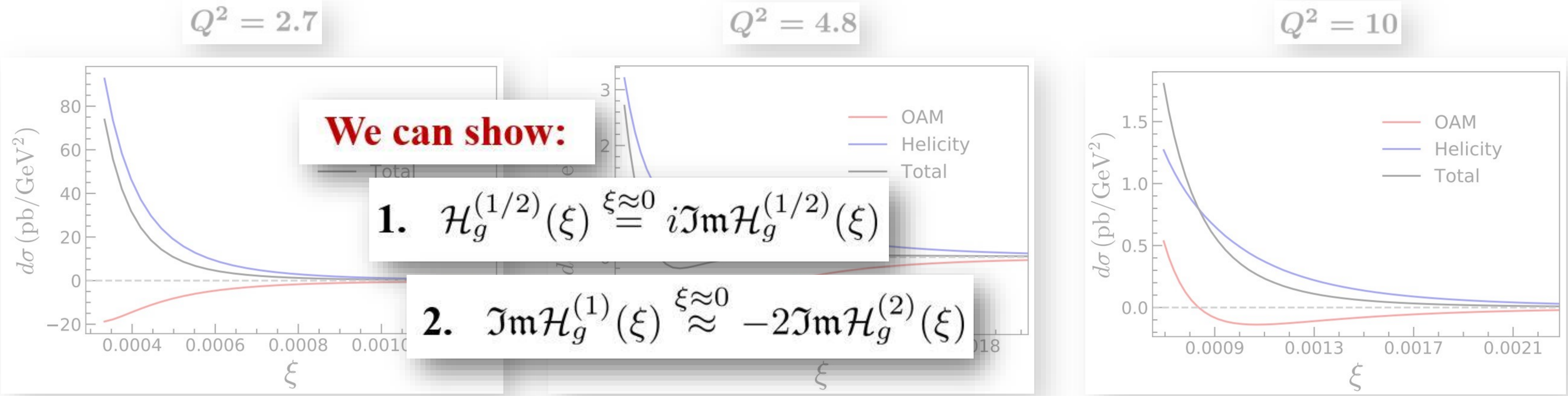

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Probing gluon OAM through exclusive dijet production

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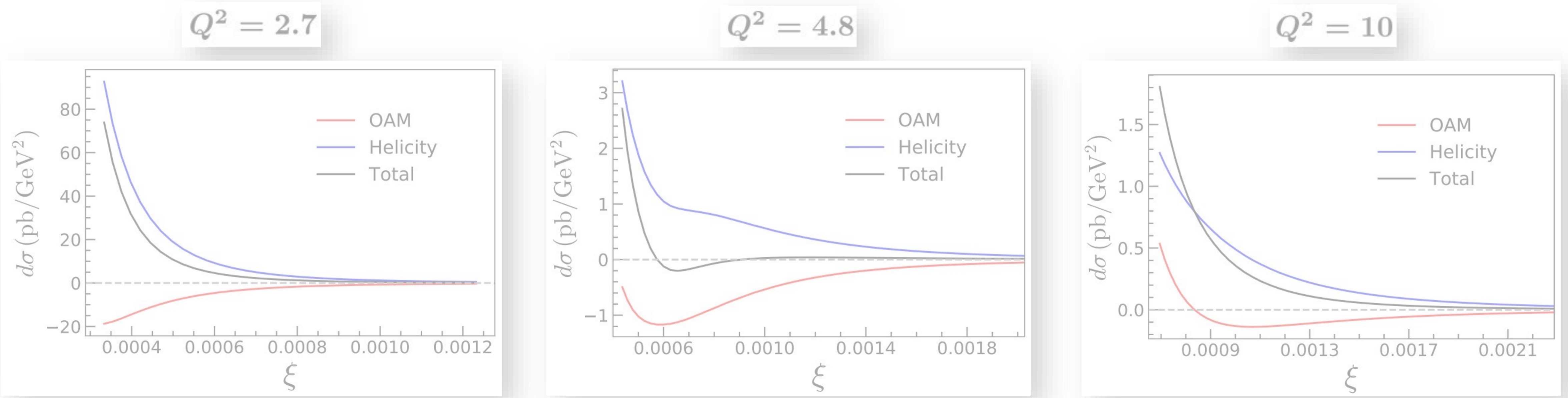

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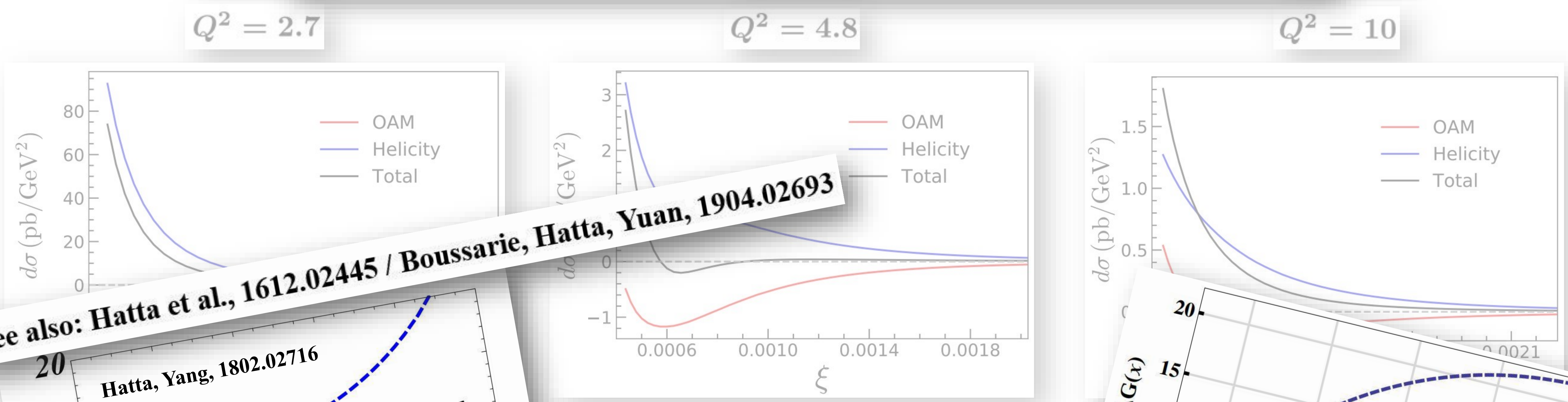
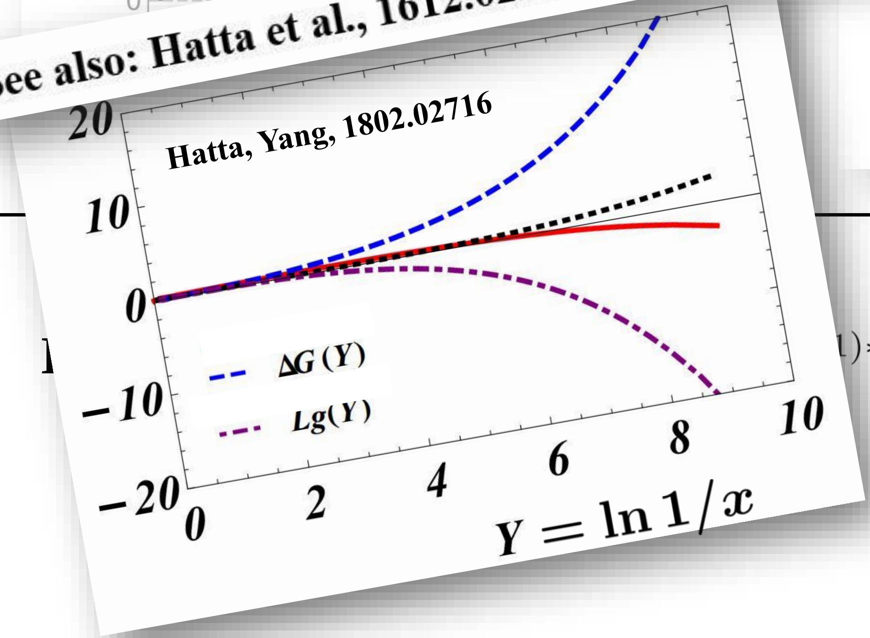
$\tilde{\mathcal{H}}_g^{(2)}$ **&** $\mathcal{L}_g$ **interfere positively/negatively depending upon sign of** $q_\perp^2 - \dfrac{Q^2}{4}$

Cancellation expected between Helicity & OAM at small x

$$\Delta G(x) \approx -L_g(x)$$



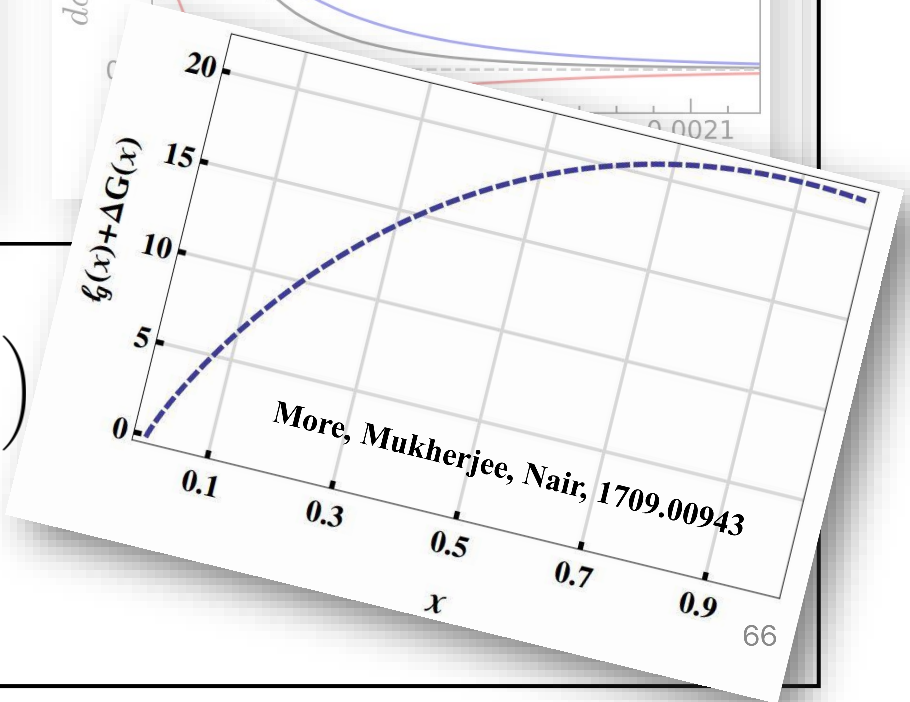
See also: Hatta et al., 1612.02445 / Boussarie, Hatta, Yuan, 1904.02693



$$)^{*}(\xi)\left(\tilde{\mathcal{H}}_g^{(2)}(\xi) + \frac{q_\perp^2 - Q^2/4}{q_\perp^2 + Q^2/4}\,\mathcal{L}_g(\xi)\right)$$

$\tilde{\mathcal{H}}_g^{(2)}(\xi) \rightarrow \Delta G(x)$

$\mathcal{L}_g(\xi) \rightarrow L_g(x)$



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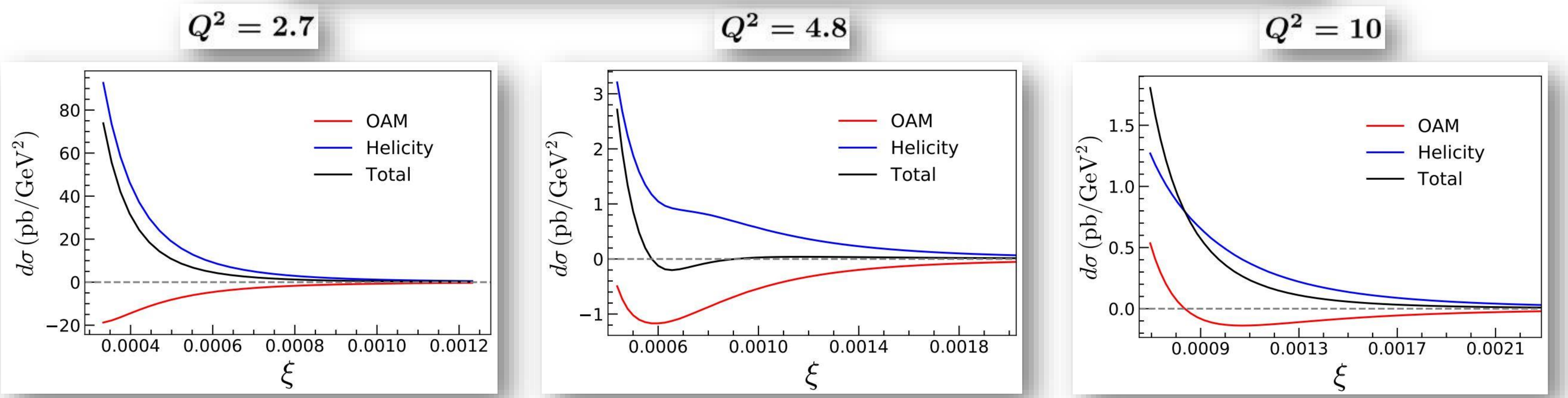


Unique opportunity to study interplay between

$\Delta G(x)$ & $L_g(x)$

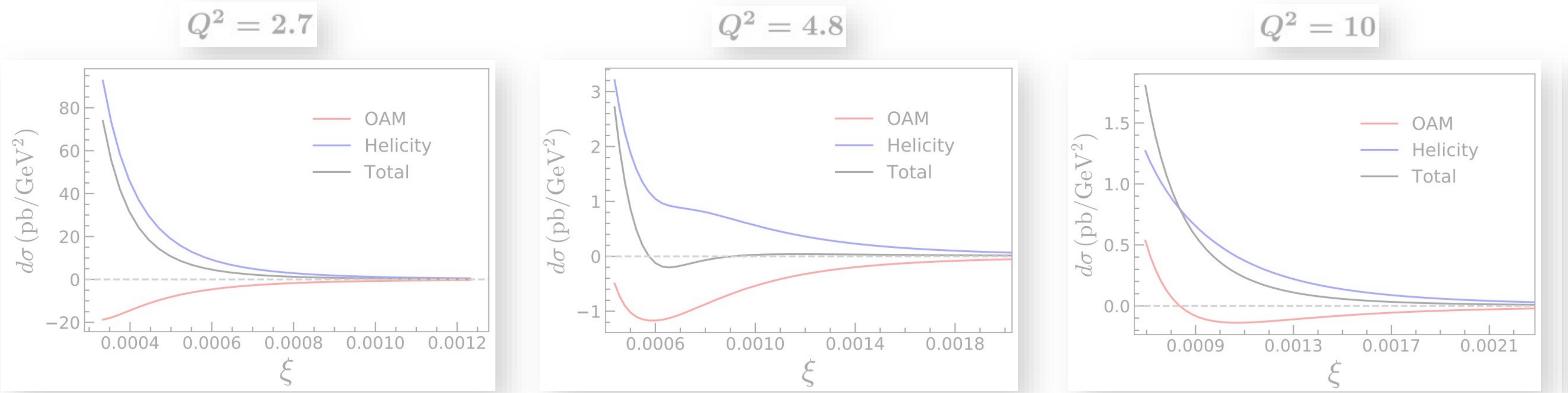
which has been so far only studied theoretically!

$$\left(\tilde{\mathcal{H}}_g^{(2)}(\xi) + \frac{q_\perp^2 - Q^2/4}{q_\perp^2 + Q^2/4}\, \mathcal{L}_g(\xi) \right)$$

$$\Delta G(x) \qquad L_g(x)$$

Probing gluon OAM through exclusive dijet production

Numerical estimate of cross section



Caveat:

- **In practice, measurements are done in a window in z around $z = 1/2$**

  Corrections of order $\sim (z - 1/2)^2$ should be calculable in k_t -factorization approach

Summary & Outlook

Summary

- **Gluon OAM related to the Wigner distribution**

Summary & Outlook

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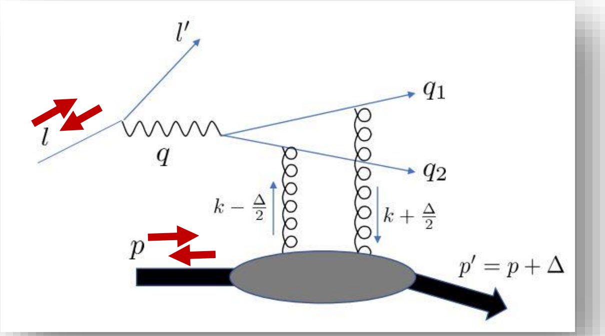
- **DSA in exclusive dijet production is a unique observable to access the gluon OAM @ EIC:**



$$\int d\phi_{q_\perp} L^{\mu\nu} A^*_\mu A_\nu \sim -\,\mathfrak{Re}\left[\left\{\mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2+\mu^2}\mathcal{H}_g^{(2)*}(\xi)\right\}\mathcal{L}_g(\xi)\right]\cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

$$+\,\mathfrak{Re}\left[\mathcal{H}_g^{(1)*}(\xi)\,\tilde{\mathcal{H}}_g^{(2)}(\xi)\right]\cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

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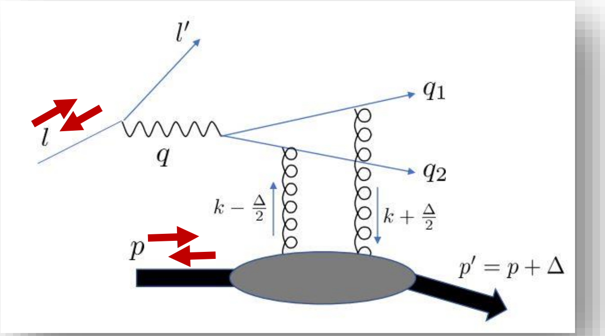

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Signature of gluon OAM is cosine angular modulation

Summary & Outlook

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DSA does not vanish for symmetric jet configurations $z = \bar{z} = \frac{1}{2}$

Gluon OAM related to the Wigner distribution

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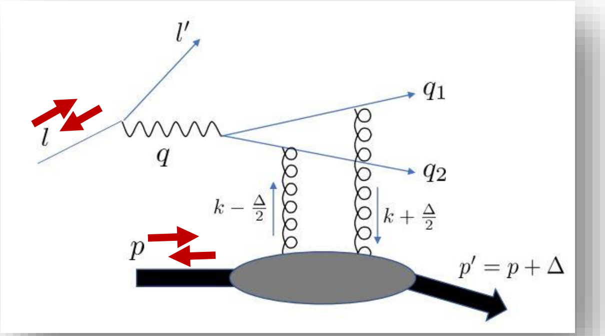

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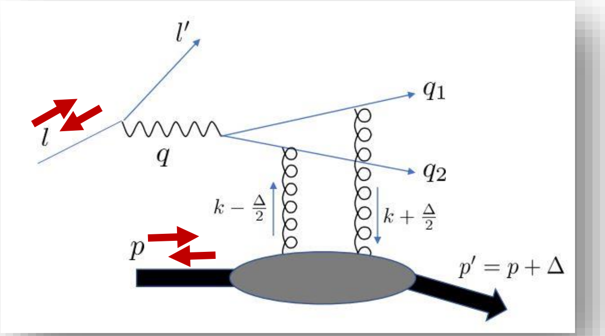
Consequence:

Elimination of factorization-breaking third poles at $x = \pm\xi$



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Signature of gluon OAM is cosine angular modulation

Summary & Outlook

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Consequence:

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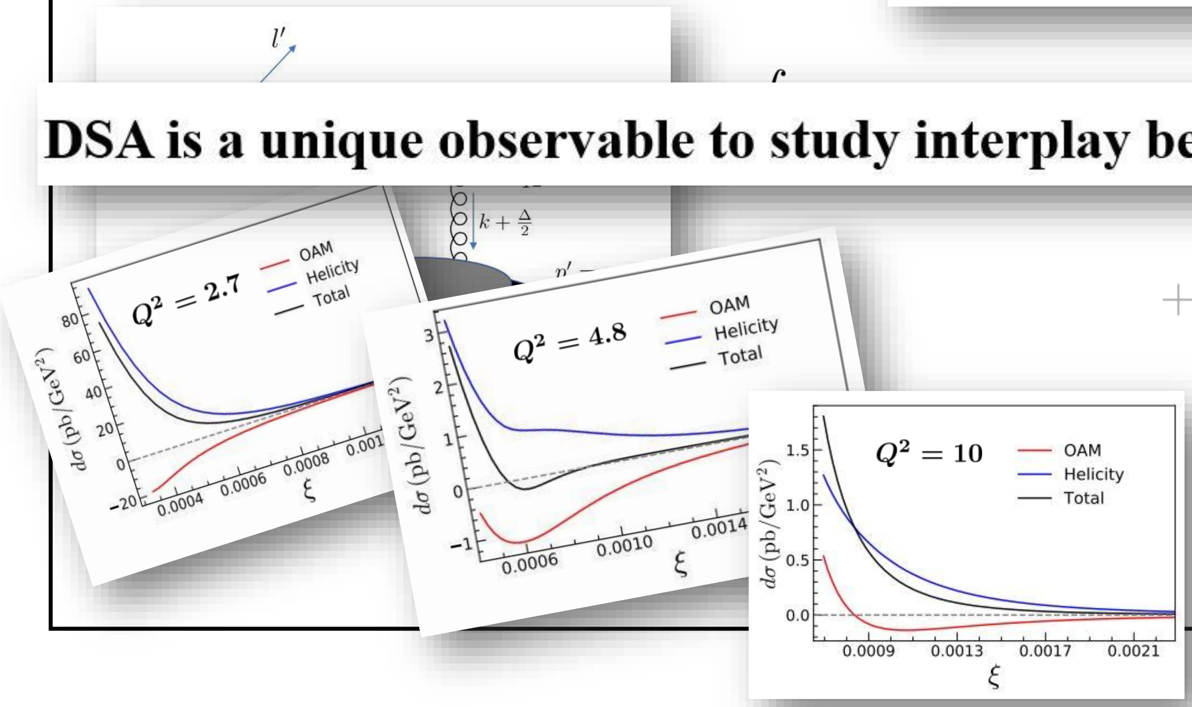
- DSA in exclusive dijet production is

DSA is a unique observable to study interplay between gluon OAM & helicity

$\mathcal{L}_g(\xi)\Big]\cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$

$+\,\Re\mathrm{e}\Big[\mathcal{H}_g^{(1)*}(\xi)\,\tilde{\mathcal{H}}_g^{(2)}(\xi)\Big]\cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$



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Summary & Outlook

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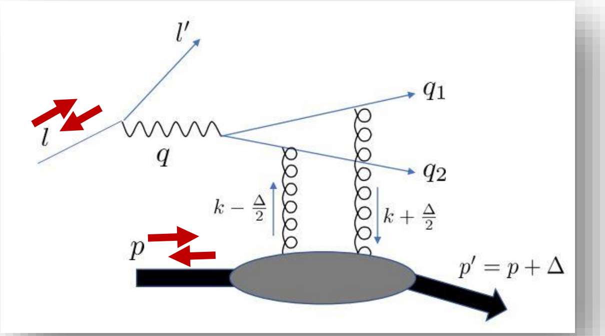

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$$+\,\mathfrak{Re}\left[\mathcal{H}_g^{(1)*}(\xi)\,\tilde{\mathcal{H}}_g^{(2)}(\xi)\right]\cos(\phi_{l_\perp}-\phi_{\Delta_\perp})$$

- **First realistic numerical calculation of observable sensitive to OAM @ EIC**

Summary & Outlook

Outlook

What about quark OAM?

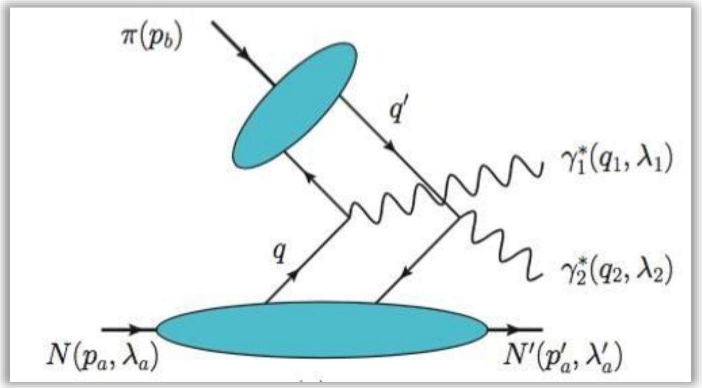
Summary & Outlook

Outlook

arXiv: 1702.04387 (2017)

Generalized TMDs and the exclusive double Drell-Yan process

Shohini Bhattacharya,[1] Andreas Metz,[1] and Jian Zhou[2]



What about quark OAM?

- **Exclusive double Drell-Yan is the only known process sensitive to quark OAM**
- **Low count rate (Amplitude $\sim \alpha_{em}^2$)**
- **Alternatively, access quark OAM through dijet production in ep collisions**

  (SB, Boussarie, Hatta, Work in progress)

Backup slides

Probing gluon OAM through exclusive dijet production

Cross section

Jet azimuthal angle ($\phi_{q_\perp}$) integrated out

$$\frac{d\sigma}{dydQ^2d\phi_{l_\perp}dzdq_\perp^2d^2\Delta_\perp} = \frac{\alpha_{em}y}{2^{11}\pi^7Q^4}\frac{\int d\phi_{q_\perp}L^{\mu\nu}A_\mu^*A_\nu}{(W^2+Q^2)(W^2-M_J^2)z\bar{z}}$$

Integrate assuming a Gaussian form factor

$$\sim e^{-b\Delta_\perp^2}$$

Slope = 5

(See Braun, Ivanov, 0505263)