

Illuminating the nucleon spin

Miguel G. Echevarría



Universidad
del País Vasco

Euskal Herriko
Unibertsitatea

eman ta zabal zazu

EHU QC

EHU Quantum Center

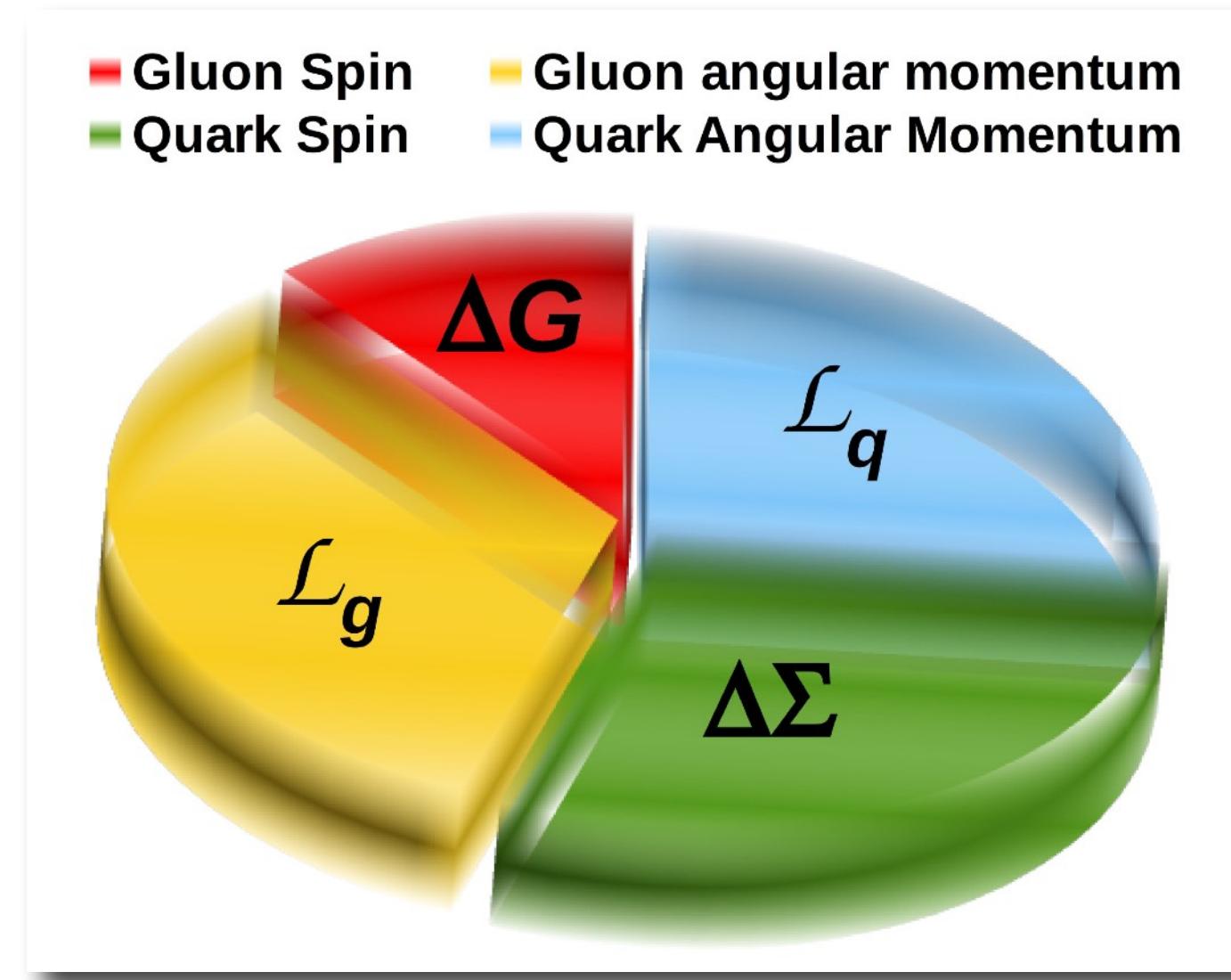
DIS2022
Santiago de Compostela, May 2-6, 2022



[M.G. Echevarria 2202.12249 (PRD)]

Nucleon spin sum rule in QCD

- Spin of the proton given in terms of quark and gluon angular momentum (AM) distributions
- Different ways to split the AM terms: I will take the Jaffe-Manohar sum rule

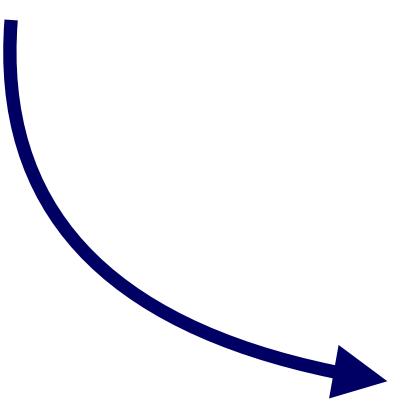


$$\frac{1}{2} \Delta\Sigma(Q^2) + \Delta G(Q^2) + L_q(Q^2) + L_g(Q^2) = \frac{1}{2}$$

$$\begin{aligned}\Delta\Sigma(Q^2) &= \sum_f \int_0^1 dx (\Delta q_f(x, Q^2) + \Delta \bar{q}_f(x, Q^2)) \\ \Delta G(Q^2) &= \int_0^1 dx \Delta G(x, Q^2) \\ L_q(Q^2) &= \sum_f \int_0^1 dx (L_f(x, Q^2) + \bar{L}_f(x, Q^2)) \\ L_g(Q^2) &= \int_0^1 dx L_g(x, Q^2)\end{aligned}$$

What if we include QED corrections? (i.e. How?)

Evolution equations in QCD

$$\frac{d}{d\ln Q^2} \begin{pmatrix} \Delta\Sigma(Q^2) \\ \Delta G(Q^2) \\ L_q(Q^2) \\ L_g(Q^2) \end{pmatrix} = \int_0^1 dx \int_x^1 \frac{dz}{z} \begin{pmatrix} \Delta P(z, Q^2) & \Delta \hat{P}(z, Q^2) \\ \Omega \hat{P}(z, Q^2) & \Omega P(z, Q^2) \end{pmatrix} \begin{pmatrix} \Delta\Sigma(\frac{x}{z}, Q^2) \\ \Delta G(\frac{x}{z}, Q^2) \\ L_q(\frac{x}{z}, Q^2) \\ L_g(\frac{x}{z}, Q^2) \end{pmatrix}$$

$$\Delta P(z, Q^2) = \begin{pmatrix} \Delta P_{qq}(z, Q^2) & \Delta P_{qg}(z, Q^2) \\ \Delta P_{gq}(z, Q^2) & \Delta P_{gg}(z, Q^2) \end{pmatrix}$$

- Coupled evolution equations
- Evolution of helicity distributions driven only by themselves: $\Delta \hat{P}(z, Q^2) = 0$

Evolution equations in QCD

- Take the evolution of quark helicity as an example:

$$\frac{d}{d\ln Q^2} \left(\frac{1}{2} \Delta \Sigma \right) = \int_0^1 dx \int_x^1 \frac{dz}{z} \left[\Delta P_{qq}(z, Q^2) \Delta \Sigma + \Delta P_{qg}(z, Q^2) \Delta G + \Delta \hat{P}_{qq}(z, Q^2) L_q + \Delta \hat{P}_{qg}(z, Q^2) L_g \right]$$

- Need perturbative expansions:

$$\Delta \Sigma(z, Q^2) = 2n_f \delta(1-z) + \sum_{n=1} \left(\frac{\alpha_s(Q)}{2\pi} \right)^n \Delta \Sigma^{(n)}(z, Q^2)$$

$$\Delta G(z, Q^2) = \delta(1-z) + \sum_{n=1} \left(\frac{\alpha_s(Q)}{2\pi} \right)^n \Delta G^{(n)}(z, Q^2)$$

$$\Delta P_{qq}(z, Q^2) = \sum_{n=1} \left(\frac{\alpha_s(Q)}{2\pi} \right)^n \Delta P_{qq}^{(n)}(z, Q^2) \quad \longrightarrow$$

- q/g helicity kernels known up to NNLO
- q/g OAM kernels known at LO

- At LO we get:

$$\frac{d}{d\ln Q^2} \left(\frac{1}{2} \Delta \Sigma \right) = \frac{\alpha_s}{2\pi} \left(\frac{1}{2} \Delta P_{qj}^{(1)} \right) (2n_f \delta_{jq} + \delta_{jg}) + O(\alpha_s^2)$$

Integrated kernels!

Spin sum rule at LO in QCD

$$\Delta P_{qq}^{(1)} = C_F \int_0^1 dx \left(\frac{1+x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right) = 0$$

$$\Delta P_{qg}^{(1)} = 2 n_f T_R \int_0^1 dx (2x-1) = 0$$

$$\Delta P_{gq}^{(1)} = C_F \int_0^1 dx (2-x) = \frac{3}{2} C_F$$

$$\Delta P_{gg}^{(1)} = 2 C_A \int_0^1 dx \left(\frac{1}{(1-x)_+} - 2x + 1 \right) + \frac{\beta^{(1)}}{2} \delta(x-1) = \frac{\beta^{(1)}}{2}$$

$$\Omega \hat{P}_{qq}^{(1)} = C_F \int_0^1 dx (x^2 - 1) = -\frac{2}{3} C_F$$

$$\Omega \hat{P}_{qg}^{(1)} = 2 n_f T_R \int_0^1 dx (1-x)(1-2x+2x^2) = \frac{2}{3} n_f T_R$$

$$\Omega \hat{P}_{gq}^{(1)} = C_F \int_0^1 dx (x-1)(-x+2) = -\frac{5}{6} C_F$$

$$\Omega \hat{P}_{gg}^{(1)} = 2 C_A \int_0^1 dx (x-1)(x^2 - x + 2) = -\frac{11}{6} C_A$$

$$\Delta \hat{P}^{(1)}(z, Q^2) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Omega P_{qq}^{(1)} = C_F \int_0^1 dx \left(\frac{x(1+x^2)}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right) = -\frac{4}{3} C_F$$

$$\Omega P_{qg}^{(1)} = 2 n_f T_R \int_0^1 dx x(x^2 + (1-x)^2) = \frac{2}{3} n_f T_R$$

$$\Omega P_{gq}^{(1)} = C_F \int_0^1 dx (1 + (1-x)^2) = \frac{4}{3} C_F$$

$$\Omega P_{gg}^{(1)} = 2 C_A \int_0^1 dx \frac{(x^2 - x + 1)^2}{(1-x)_+} + \frac{\beta^{(1)}}{2} \delta(x-1) = \frac{\beta^{(1)}}{2} - \frac{11}{6} C_A$$

As expected!

$$\frac{d}{d \ln Q^2} \left(\frac{1}{2} \Delta \Sigma + \Delta G + L_q + L_g \right) = \frac{\alpha_s}{2\pi} \sum_{i,j=q,g} \left(\Delta P_{ij}^{(1)} \left(\frac{1}{2} \delta_{iq} + \delta_{ig} \right) + \Omega \hat{P}_{ij}^{(1)} + \Omega \hat{P}_{ij}^{(1)} \right) (2n_f \delta_{jq} + \delta_{jg}) + O(\alpha_s^2) = 0$$

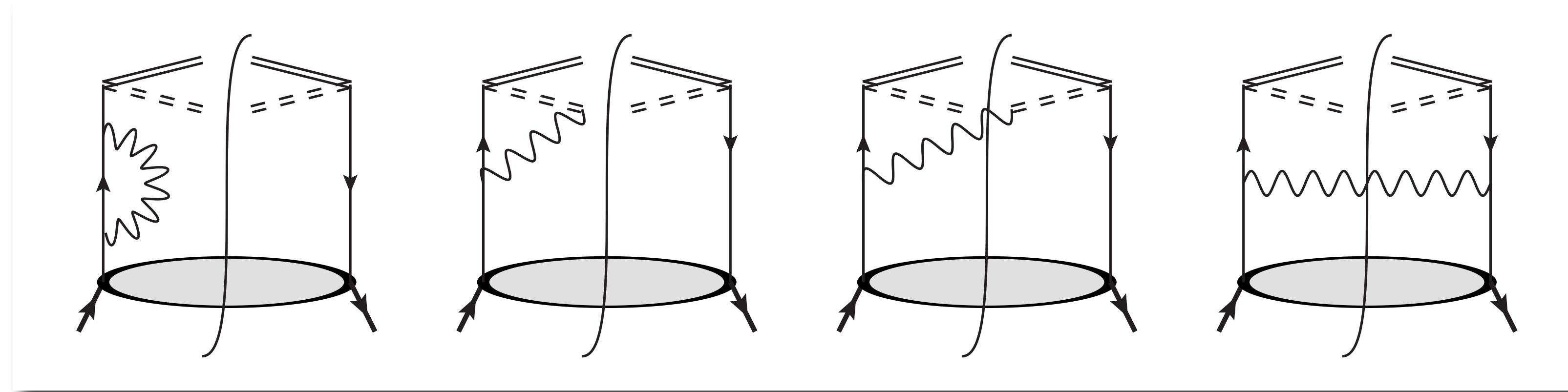
Inclusion of QED corrections

- Need to consider QED corrections in interactions and also modify (some of the) operators
- Take quark helicity as an example:

$$\Delta q_f(x, Q^2) = \int \frac{dy^-}{2\pi} e^{-iy^-xP^+} \langle PS| [\bar{\psi}_f W \widehat{W}_f] (y^-) \frac{\gamma^+ \gamma_5}{2} [\widehat{W}_f^\dagger W^\dagger \psi_f] (0) |PS\rangle$$

$$W(x) = \bar{P} \exp \left[ig_s \int_{-\infty}^0 ds A^+(x + s\bar{n}) \right] \longrightarrow \widehat{W}_f(x) = \exp \left[ieQ_f \int_{-\infty}^0 ds B^+(x + s\bar{n}) \right]$$

photon field



- Quark OAM distribution also acquires photon Wilson lines
- Gluon operators don't

Nucleon spin sum rule in QCDxQED

- Need to include also **lepton and photon distributions!!**

$$\frac{1}{2}\Delta\Sigma + \Delta G + \frac{1}{2}\Delta l + \Delta\gamma + L_q + L_g + L_l + L_\gamma = \frac{1}{2}$$

Defined analogously to
their QCD counterparts

$$\Delta l(Q^2) = \sum_f \int_0^1 dx (\Delta l_f(x, Q^2) + \Delta \bar{l}_f(x, Q^2))$$

$$\Delta\gamma(Q^2) = \int_0^1 dx \Delta\gamma(x, Q^2)$$

$$L_l(Q^2) = \sum_f \int_0^1 dx (L_f(x, Q^2) + \bar{L}_f(x, Q^2))$$

$$L_\gamma(Q^2) = \int_0^1 dx L_\gamma(x, Q^2)$$

$$\frac{d}{d\ln Q^2} \left(\frac{1}{2}\Delta\Sigma + \Delta G + \frac{1}{2}\Delta l + \Delta\gamma + L_q + L_g + L_l + L_\gamma \right) = 0 \quad ???$$

Evolution equations in QCDxQED

- QED is sensitive to **electric charge**: we need to **distinguish U-type** and **D-type** quark distributions:

$$\frac{d}{d\ln Q^2} \begin{pmatrix} \Delta\Sigma_U(Q^2) \\ \Delta\Sigma_D(Q^2) \\ \Delta G(Q^2) \\ \Delta l(Q^2) \\ \Delta\gamma(Q^2) \\ L_U(Q^2) \\ L_D(Q^2) \\ L_g(Q^2) \\ L_l(Q^2) \\ L_\gamma(Q^2) \end{pmatrix} = \int_0^1 dx \int_x^1 \frac{dz}{z} \begin{pmatrix} \Delta P(z, Q^2) & \Delta \hat{P}(z, Q^2) \\ \Omega \hat{P}(z, Q^2) & \Omega P(z, Q^2) \end{pmatrix} \begin{pmatrix} \Delta\Sigma_U(\frac{x}{z}, Q^2) \\ \Delta\Sigma_D(\frac{x}{z}, Q^2) \\ \Delta G(\frac{x}{z}, Q^2) \\ \Delta l(\frac{x}{z}, Q^2) \\ \Delta\gamma(\frac{x}{z}, Q^2) \\ L_U(\frac{x}{z}, Q^2) \\ L_D(\frac{x}{z}, Q^2) \\ L_g(\frac{x}{z}, Q^2) \\ L_l(\frac{x}{z}, Q^2) \\ L_\gamma(\frac{x}{z}, Q^2) \end{pmatrix}$$

5x5 matrices
100 kernels!

- Perturbative expansions generalized as:

$$\Delta\Sigma(x, Q^2) = \Delta\Sigma_U(x, Q^2) + \Delta\Sigma_D(x, Q^2)$$

$$\Delta\Sigma_{U(D)}(x, Q^2) = n_f \delta(1-x) + \sum_{n,m=0} \left(\frac{\alpha_s(Q^2)}{2\pi} \right)^n \left(\frac{\alpha(Q^2)}{2\pi} \right)^m \Delta\Sigma_{U(D)}^{(n,m)}(x, Q^2)$$

Evolution kernels at LO in QCD

$$\begin{array}{ccccccccc}
 & U & D & g & l & \gamma & U & D & g & l & \gamma \\
 \text{U} & \left(\begin{array}{ccccc}
 \Delta P_{qq}^{(1,0)} & 0 & \frac{1}{2}\Delta P_{qg}^{(1,0)} & 0 & 0 \\
 0 & \Delta P_{qq}^{(1,0)} & \frac{1}{2}\Delta P_{qg}^{(1,0)} & 0 & 0 \\
 \Delta P_{gq}^{(1,0)} & \Delta P_{gq}^{(1,0)} & \Delta P_{gg}^{(1,0)} & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0
 \end{array} \right) & & & & \Delta \hat{P}(z, Q^2) = 0 & & & \\
 \text{D} & & & & & & & & & \\
 \text{g} & & & & & & & & & \\
 \text{l} & & & & & & & & & \\
 \gamma & & & & & & & & & \\
 \text{U} & \left(\begin{array}{ccccc}
 \Omega \hat{P}_{qq}^{(1,0)} & 0 & \frac{1}{2}\Omega \hat{P}_{qg}^{(1,0)} & 0 & 0 \\
 0 & \Omega \hat{P}_{qq}^{(1,0)} & \frac{1}{2}\Omega \hat{P}_{qg}^{(1,0)} & 0 & 0 \\
 \Omega \hat{P}_{gq}^{(1,0)} & \Omega \hat{P}_{gq}^{(1,0)} & \Omega \hat{P}_{gg}^{(1,0)} & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0
 \end{array} \right) & & & \left(\begin{array}{ccccc}
 \Omega P_{qq}^{(1,0)} & 0 & \frac{1}{2}\Omega P_{qg}^{(1,0)} & 0 & 0 \\
 0 & \Omega P_{qq}^{(1,0)} & \frac{1}{2}\Omega P_{qg}^{(1,0)} & 0 & 0 \\
 \Omega P_{gq}^{(1,0)} & \Omega P_{gq}^{(1,0)} & \Omega P_{gg}^{(1,0)} & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0
 \end{array} \right) & & & \\
 \text{D} & & & & & & & & & \\
 \text{g} & & & & & & & & & \\
 \text{l} & & & & & & & & & \\
 \gamma & & & & & & & & &
 \end{array}$$

Kernels for lepton & photon distributions
at LO in QCD are all zero!!

**These are all
functions of (z, Q^2)**

Evolution kernels at LO in QED: recipe

- We can obtain the QED kernels from their QCD analogues
- The ***abelianization*** recipe is simple at LO:

$$C_F \longrightarrow Q_i^2$$

$$C_A \longrightarrow 0$$

$$T_R \longrightarrow 1$$

$$2n_f \longrightarrow \sum_{i=q,\bar{q},l,\bar{l}} N_{C,i} Q_i^2 = 2N_C \frac{n_f}{2} (Q_U^2 + Q_D^2) + 2n_l$$

- For example:

$$\beta^{(1,0)} = \frac{11}{3} C_A - \frac{4}{3} T_R n_f \quad \longrightarrow \quad \hat{\beta}^{(0,1)} = -\frac{4}{3} \left(N_C \frac{n_f}{2} (Q_U^2 + Q_D^2) + n_l \right)$$

Evolution kernels at LO in QED: results (1/4)

- All these are new (zeros too!!)
- Can be computed **carefully** from the analogous ones in QCD
- Towards the precision test of saturation physics (CGC) at RHIC and LHC. Key!.

$$\left(\begin{array}{ccccc} \Delta P_{UU}^{(0,1)} & 0 & 0 & 0 & \Delta P_{U\gamma}^{(0,1)} \\ 0 & \Delta P_{DD}^{(0,1)} & 0 & 0 & \Delta P_{D\gamma}^{(0,1)} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta P_{ll}^{(0,1)} & \Delta P_{l\gamma}^{(0,1)} \\ \Delta P_{\gamma U}^{(0,1)} & \Delta P_{\gamma D}^{(0,1)} & 0 & \Delta P_{\gamma l}^{(0,1)} & \Delta P_{\gamma\gamma}^{(0,1)} \\ \Omega \hat{P}_{UU}^{(0,1)} & 0 & 0 & 0 & \Omega \hat{P}_{U\gamma}^{(0,1)} \\ 0 & \Omega \hat{P}_{DD}^{(0,1)} & 0 & 0 & \Omega \hat{P}_{D\gamma}^{(0,1)} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Omega \hat{P}_{ll}^{(0,1)} & \Omega \hat{P}_{l\gamma}^{(0,1)} \\ \Omega \hat{P}_{\gamma U}^{(0,1)} & \Omega \hat{P}_{\gamma D}^{(0,1)} & 0 & \Omega \hat{P}_{\gamma l}^{(0,1)} & \Omega \hat{P}_{\gamma\gamma}^{(0,1)} \end{array} \right) \quad \Delta \hat{P}(z, Q^2) = 0$$

$$\left(\begin{array}{ccccc} \Omega P_{UU}^{(0,1)} & 0 & 0 & 0 & \Omega P_{U\gamma}^{(0,1)} \\ 0 & \Omega P_{DD}^{(0,1)} & 0 & 0 & \Omega P_{D\gamma}^{(0,1)} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Omega P_{ll}^{(0,1)} & \Omega P_{l\gamma}^{(0,1)} \\ \Omega P_{\gamma U}^{(0,1)} & \Omega P_{\gamma D}^{(0,1)} & 0 & \Omega P_{\gamma l}^{(0,1)} & \Omega P_{\gamma\gamma}^{(0,1)} \end{array} \right)$$

Evolution kernels at LO in QED: results (2/4)

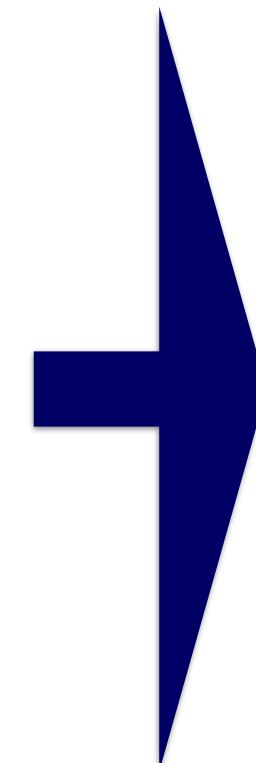
- Evolution of helicity distributions driven by helicity distributions

$$\Delta P_{qq}^{(1)} = C_F \int_0^1 dx \left(\frac{1+x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right) = 0$$

$$\Delta P_{qg}^{(1)} = 2 n_f T_R \int_0^1 dx (2x-1) = 0$$

$$\Delta P_{gq}^{(1)} = C_F \int_0^1 dx (2-x) = \frac{3}{2} C_F$$

$$\Delta P_{gg}^{(1)} = 2 C_A \int_0^1 dx \left(\frac{1}{(1-x)_+} - 2x + 1 \right) + \frac{\beta^{(1)}}{2} \delta(x-1) = \frac{\beta^{(1)}}{2}$$



$$\left. \begin{aligned} \Delta P_{ii}^{(0,1)} &= Q_i^2 \int_0^1 dx \left(\frac{1+x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right) = 0 \\ \Delta P_{i\gamma}^{(0,1)} &= n_f N_C Q_i^2 \int_0^1 dx (2x-1) = 0 \\ \Delta P_{\gamma i}^{(0,1)} &= Q_i^2 \int_0^1 dx (2-x) = \frac{3}{2} Q_i^2 \\ \Delta P_{\gamma\gamma}^{(0,1)} &= \int_0^1 dx \frac{\hat{\beta}^{(0,1)}}{2} \delta(x-1) = \frac{\hat{\beta}^{(0,1)}}{2} \\ \Delta P_{ll}^{(0,1)} &= Q_l^2 \int_0^1 dx \left(\frac{1+x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right) = 0 \\ \Delta P_{l\gamma}^{(0,1)} &= 2n_l Q_l^2 \int_0^1 dx (2x-1) = 0 \\ \Delta P_{\gamma l}^{(0,1)} &= Q_l^2 \int_0^1 dx (2-x) = \frac{3}{2} Q_l^2 \end{aligned} \right\}, \quad i = U, D$$

QCD@LO

QED@LO

Evolution kernels at LO in QED: results (3/4)

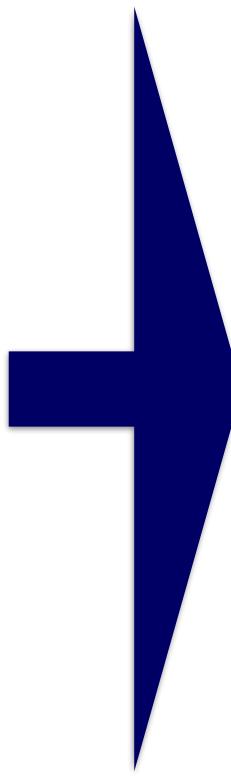
- Evolution of OAM distributions driven by helicity distributions

$$\Omega \hat{P}_{qq}^{(1)} = C_F \int_0^1 dx (x^2 - 1) = -\frac{2}{3}C_F$$

$$\Omega \hat{P}_{qg}^{(1)} = 2 n_f T_R \int_0^1 dx (1-x)(1-2x+2x^2) = \frac{2}{3}n_f T_R$$

$$\Omega \hat{P}_{gq}^{(1)} = C_F \int_0^1 dx (x-1)(-x+2) = -\frac{5}{6}C_F$$

$$\Omega \hat{P}_{gg}^{(1)} = 2 C_A \int_0^1 dx (x-1)(x^2-x+2) = -\frac{11}{6}C_A$$



$$\left. \begin{aligned} \Omega \hat{P}_{ii}^{(0,1)} &= Q_i^2 \int_0^1 dx (x^2 - 1) = -\frac{2}{3}Q_i^2 \\ \Omega \hat{P}_{i\gamma}^{(0,1)} &= n_f N_C Q_i^2 \int_0^1 dx (1-x)(1-2x+2x^2) = \frac{1}{3}n_f N_C Q_i^2 \\ \Omega \hat{P}_{\gamma i}^{(0,1)} &= Q_i^2 \int_0^1 dx (x-1)(-x+2) = -\frac{5}{6}Q_i^2 \\ \Omega \hat{P}_{\gamma\gamma}^{(0,1)} &= 0 \\ \Omega \hat{P}_{ll}^{(0,1)} &= Q_l^2 \int_0^1 dx (x^2 - 1) = -\frac{2}{3}Q_l^2 \\ \Omega \hat{P}_{l\gamma}^{(0,1)} &= 2n_l Q_l^2 \int_0^1 dx (1-x)(1-2x+2x^2) = \frac{2}{3}n_l \\ \Omega \hat{P}_{\gamma l}^{(0,1)} &= Q_l^2 \int_0^1 dx (x-1)(-x+2) = -\frac{5}{6}Q_l^2 \end{aligned} \right\}, \quad i = U, D$$

QCD@LO

QED@LO

Evolution kernels at LO in QED: results (4/4)

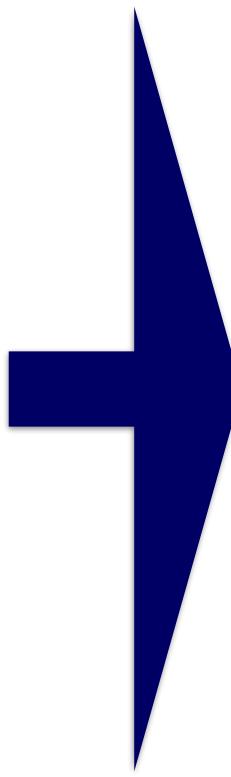
- Evolution of OAM distributions driven by OAM distributions

$$\Omega P_{qq}^{(1)} = C_F \int_0^1 dx \left(\frac{x(1+x^2)}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right) = -\frac{4}{3}C_F$$

$$\Omega P_{qg}^{(1)} = 2 n_f T_R \int_0^1 dx \ x(x^2 + (1-x)^2) = \frac{2}{3}n_f T_R$$

$$\Omega P_{gq}^{(1)} = C_F \int_0^1 dx \ (1 + (1-x)^2) = \frac{4}{3}C_F$$

$$\Omega P_{gg}^{(1)} = 2 C_A \int_0^1 dx \ \frac{(x^2 - x + 1)^2}{(1-x)_+} + \frac{\beta^{(1)}}{2} \delta(x-1) = \frac{\beta^{(1)}}{2} - \frac{11}{6}C_A$$



$$\left. \begin{aligned} \Omega P_{ii}^{(0,1)} &= Q_i^2 \int_0^1 dx \left(\frac{x(1+x^2)}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right) = -\frac{4}{3}Q_i^2 \\ \Omega P_{i\gamma}^{(0,1)} &= n_f N_C Q_i^2 \int_0^1 dx \ x(x^2 + (1-x)^2) = \frac{1}{3}n_f N_C Q_i^2 \\ \Omega P_{\gamma i}^{(0,1)} &= Q_i^2 \int_0^1 dx \ (1 + (1-x)^2) = \frac{4}{3}Q_i^2 \\ \Omega P_{\gamma\gamma}^{(0,1)} &= \int_0^1 dx \ \frac{\hat{\beta}^{(0,1)}}{2} \delta(x-1) = \frac{\hat{\beta}^{(0,1)}}{2} \\ \Omega P_{ll}^{(0,1)} &= Q_l^2 \int_0^1 dx \left(\frac{x(1+x^2)}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right) = -\frac{4}{3}Q_l^2 \\ \Omega P_{\gamma l}^{(0,1)} &= Q_l^2 \int_0^1 dx \ (1 + (1-x)^2) = \frac{4}{3}Q_l^2 \\ \Omega P_{l\gamma}^{(0,1)} &= 2n_l Q_l^2 \int_0^1 dx \ x(x^2 + (1-x)^2) = \frac{2}{3}n_l \end{aligned} \right\}, \quad i = U, D$$

QCD@LO

QED@LO

Scale independence of the nucleon spin in QCDxQED

$$\frac{1}{2}\Delta\Sigma(Q^2) + \Delta G(Q^2) + \frac{1}{2}\Delta l(Q^2) + \Delta\gamma(Q^2) + L_q(Q^2) + L_g(Q^2) + L_l(Q^2) + L_\gamma(Q^2) = \frac{1}{2}$$

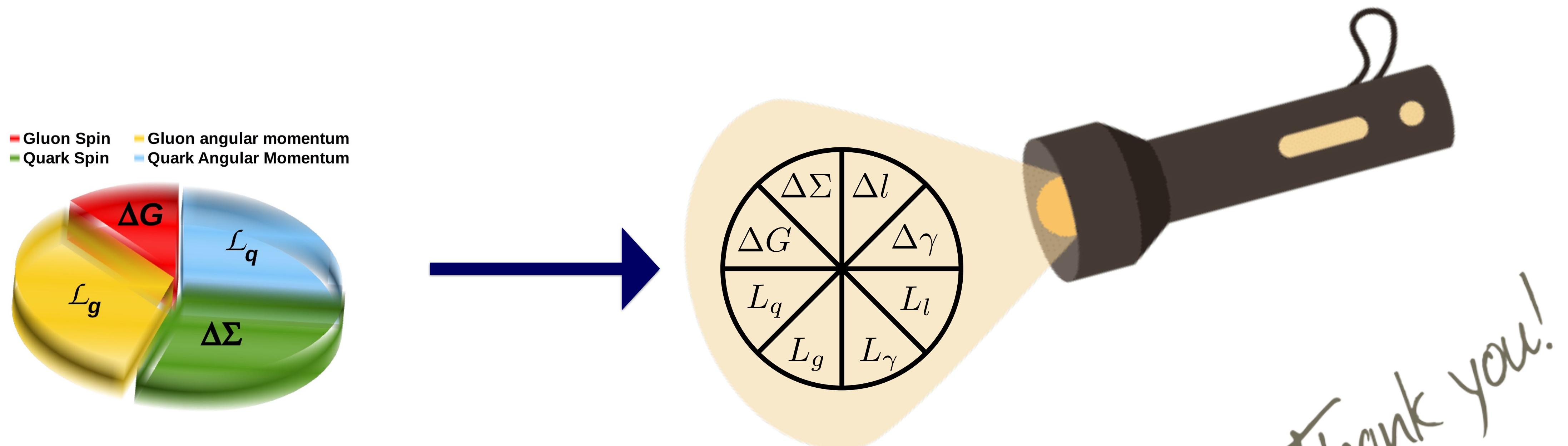
$$\begin{aligned}
& \frac{d}{d \ln Q^2} \left(\frac{1}{2} \Delta \Sigma(Q^2) + \Delta G(Q^2) + \frac{1}{2} \Delta l(Q^2) + \Delta \gamma(Q^2) + L_q(Q^2) + L_g(Q^2) + L_l(Q^2) + L_\gamma(Q^2) \right) = \\
& \left[\frac{\alpha_s(Q^2)}{2\pi} \sum_{i,j=U,D,g,\gamma,l} \left(\Delta P_{ij}^{(1,0)} \left(\frac{1}{2} \delta_{iU} + \frac{1}{2} \delta_{iD} + \delta_{ig} + \frac{1}{2} \delta_{il} + \delta_{i\gamma} \right) + \Omega \hat{P}_{ij}^{(1,0)} + \Omega P_{ij}^{(1,0)} \right) \right. \\
& \quad \left. + \frac{\alpha(Q^2)}{2\pi} \sum_{i,j=U,D,g,\gamma,l} \left(\Delta P_{ij}^{(0,1)} \left(\frac{1}{2} \delta_{iU} + \frac{1}{2} \delta_{iD} + \delta_{ig} + \frac{1}{2} \delta_{il} + \delta_{i\gamma} \right) + \Omega \hat{P}_{ij}^{(0,1)} + \Omega P_{ij}^{(0,1)} \right) \right] \\
& \times \left(n_f \delta_{jU} + n_f \delta_{jD} + \delta_{jg} + 2n_l \delta_{jl} + \delta_{j\gamma} \right) + O(\alpha^2, \alpha_s^2, \alpha \alpha_s) = 0
\end{aligned}$$

**Scale independent!!
(at LO)**

- This is a (perturbative) check of the extended sum rule and the new computed kernels

Conclusions

- Extended the nucleon spin sum rule to **QCDxQED: QED corrections + new distributions**
- Computed **QED corrections at LO for all q/g/lepton/photon distributions**
- Computed **QCD corrections at LO for the newly introduced lepton/photon distributions**
- Checked explicitly the scale-invariance of the spin sum rule at LO in both **QCD and QED**
- These results will allow in the future for a **more precise pheno** studies of the spin sum rule



Thank you!