



UNIVERSITÀ DI PAVIA

# Global Extraction of unpolarised TMDs at $N^3LL$

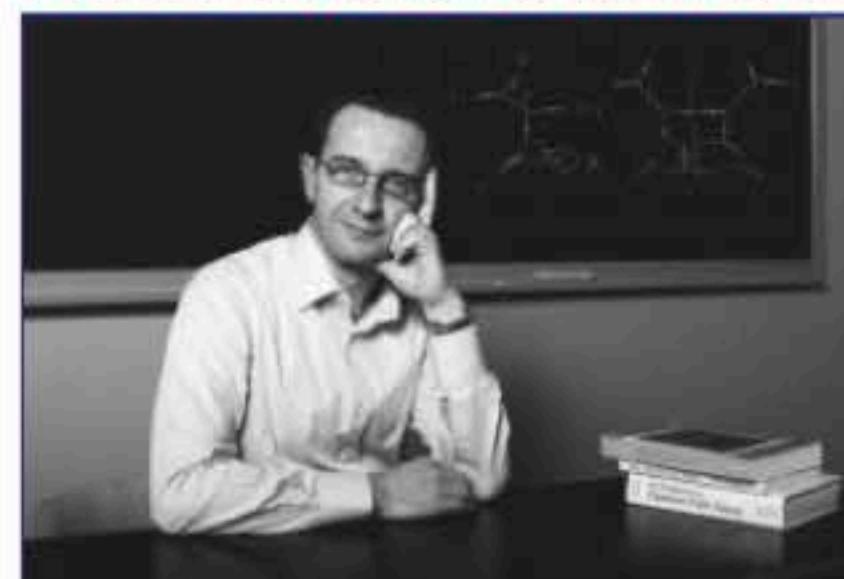
MAP Collaboration

Matteo Cerutti

DIS2022

# Results obtained with contribution from

**Alessandro Bacchetta**



**Marco Radici**



**Andrea Signori**



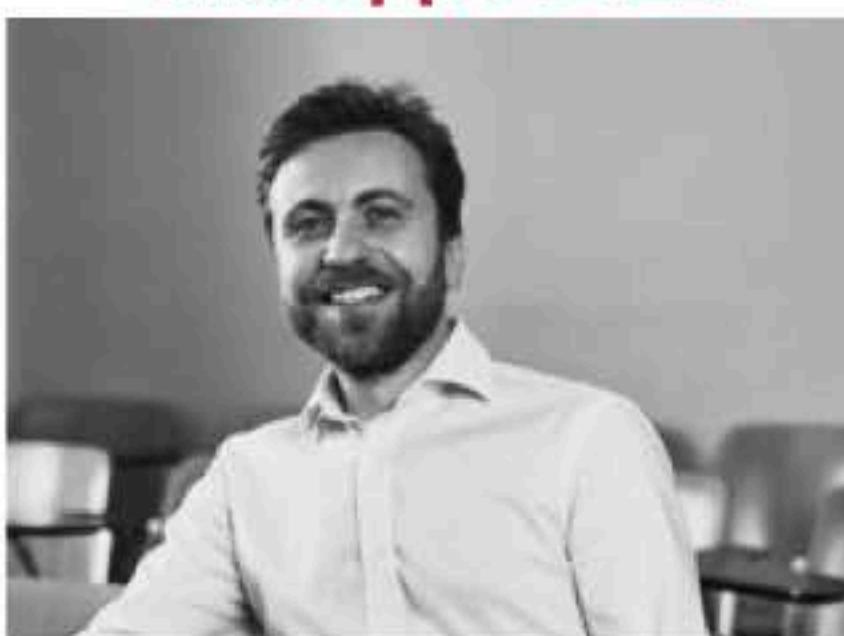
**Valerio Bertone**



**Chiara Bissolotti**



**Giuseppe Bozzi**



**Fulvio Piacenza**



# A new extraction of quark TMDs

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- Global analysis of Drell-Yan and Semi-Inclusive DIS data sets: **2031** data points

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- Number of fitted parameters: **21**

# A new extraction of quark TMDs

- Global analysis of Drell-Yan and Semi-Inclusive DIS data sets: **2031** data points
- Perturbative accuracy:  **$N^3LL^-$**
- **Normalisation** of SIDIS multiplicities beyond NLL
- Number of fitted parameters: **21**
- Extremely good description:  $\chi^2/N_{data} \simeq 1.00$

# Global analysis of DY and SIDIS data sets

$F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2)$

$$= \sum_a \mathcal{H}_{UU}^{1a}(Q^2, \mu^2) \int d^2 \mathbf{k}_{\perp A} d^2 \mathbf{k}_{\perp B} f_1^a(x_A, \mathbf{k}_{\perp A}^2; \mu^2) f_1^{\bar{a}}(x_B, \mathbf{k}_{\perp B}^2; \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} - \mathbf{q}_T + \mathbf{k}_{\perp B})$$

$$+ Y_{UU}^1(Q^2, \mathbf{q}_T^2) + \mathcal{O}(M^2/Q^2)$$

Arnold, Metz and Schlegel, Phys.Rev.D 79 (2009)

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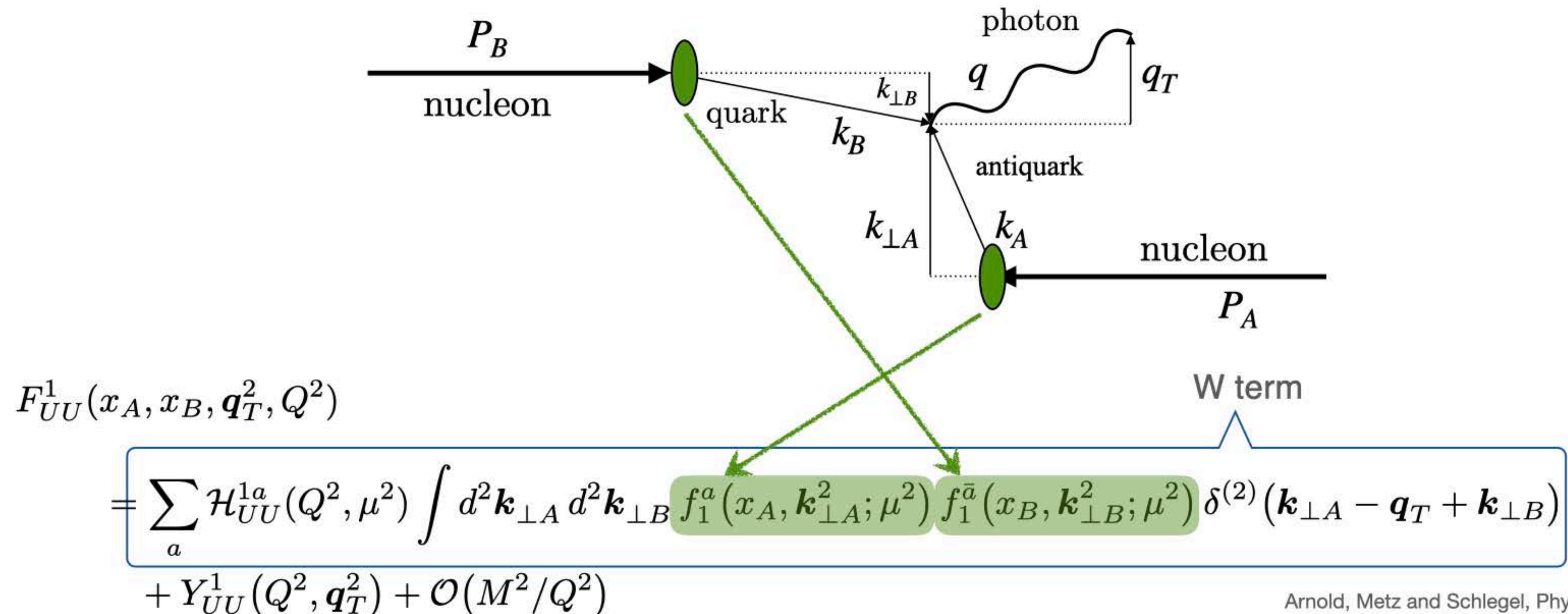
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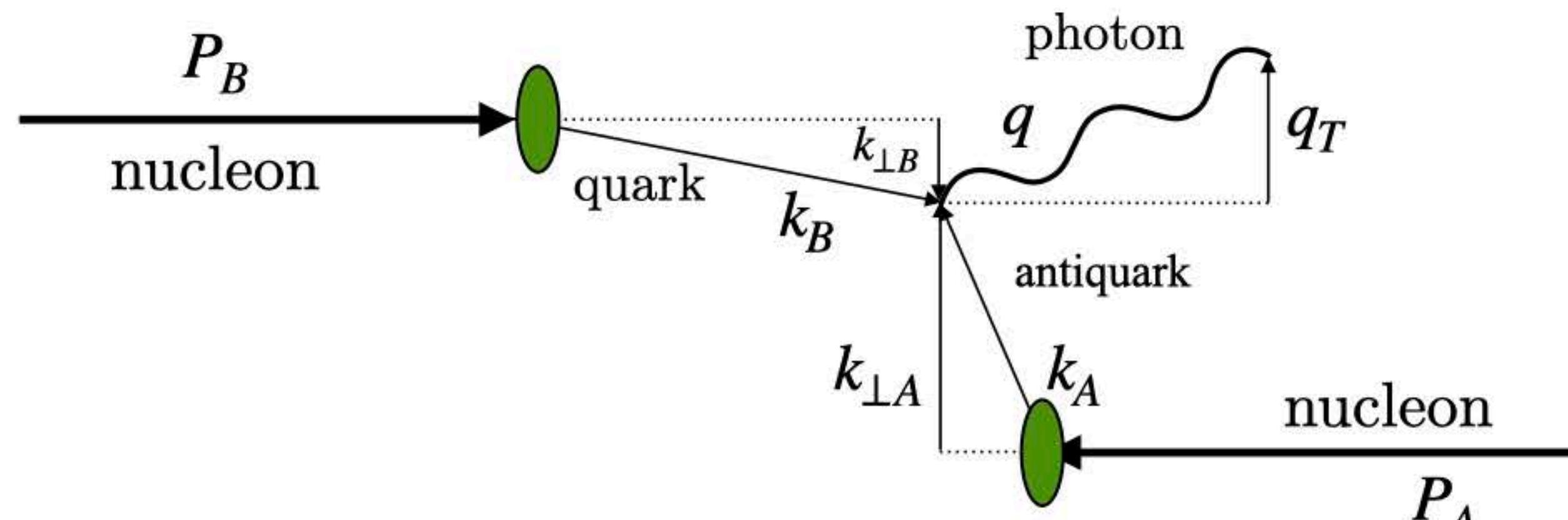
- The W term dominates in the region where  $q_T \ll Q$

# Global analysis of DY and SIDIS data sets



- The W term dominates in the region where  $q_T \ll Q$
- Y term has been excluded in the Pavia analyses

# Global analysis of DY and SIDIS data sets



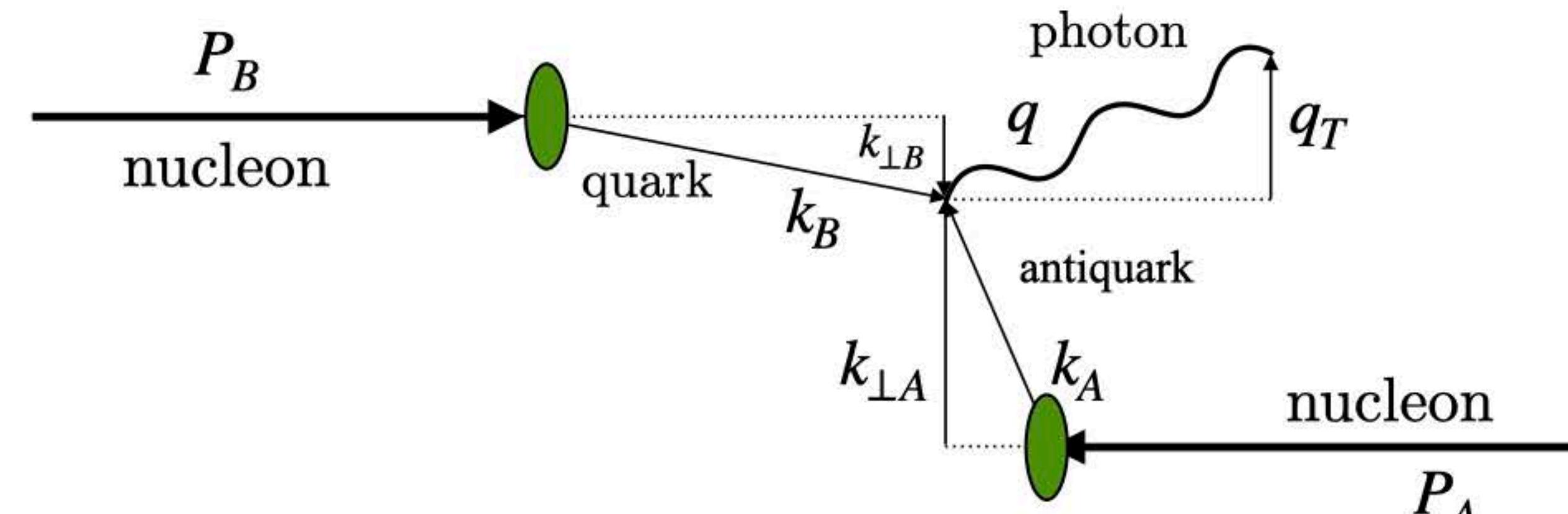
$$F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2)$$

$$\approx \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int d^2\mathbf{k}_{\perp A} d^2\mathbf{k}_{\perp B} f_1^q(x_A, \mathbf{k}_{\perp A}^2; \mu^2) f_1^{\bar{q}}(x_B, \mathbf{k}_{\perp B}^2; \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} - \mathbf{q}_T + \mathbf{k}_{\perp B})$$

$$= \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{q}_T|) \hat{f}_1^q(x_A, b_T^2; \mu^2) \hat{f}_1^{\bar{q}}(x_B, b_T^2; \mu^2)$$

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# Global analysis of DY and SIDIS data sets



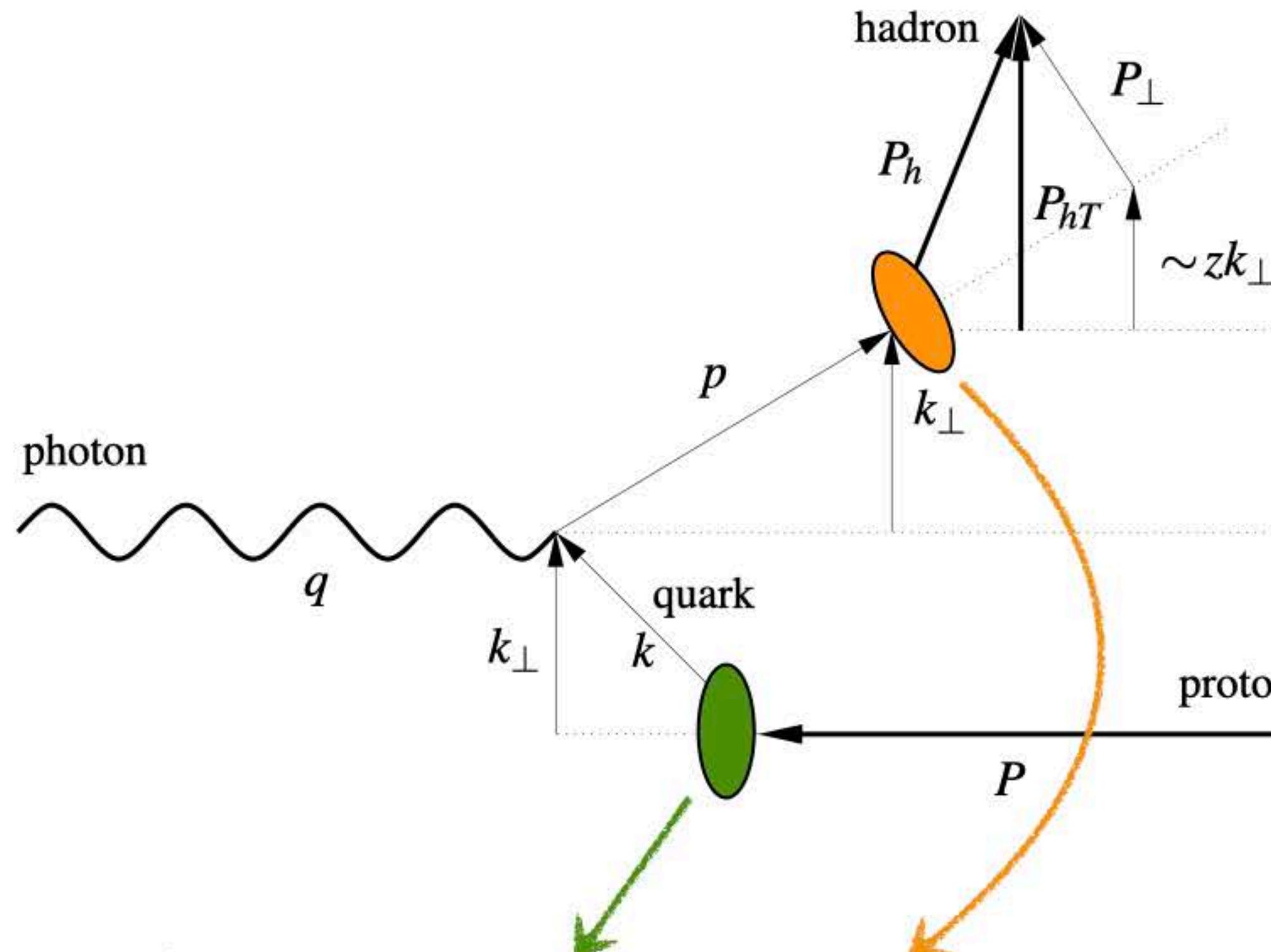
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Arnold, Metz and Schlegel, Phys.Rev.D 79 (2009)

- At small  $\mathbf{q}_T$  the dominant part is given by TMDs
- Fourier-transformed space to avoid convolutions
- TMDs formally depend on two scales, but we set them equal.

# Global analysis of DY and SIDIS data sets



$$F_{UU,T}(x, z, \mathbf{P}_{hT}^2, Q^2)$$

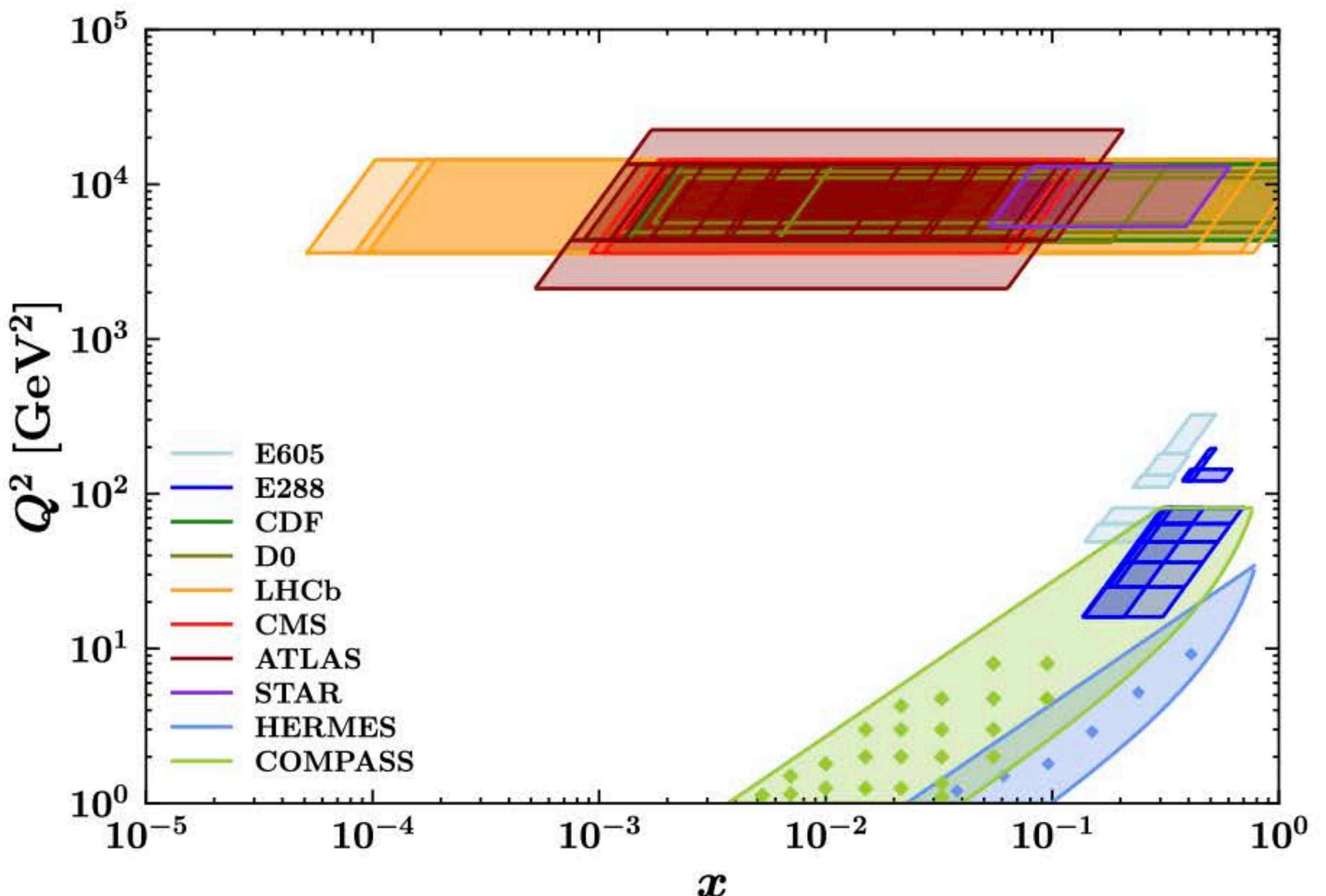
$$= x \sum_q \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int d^2\mathbf{k}_\perp d^2\mathbf{P}_\perp f_1^a(x, \mathbf{k}_\perp^2; \mu^2) D_1^{a \rightarrow h}(z, \mathbf{P}_\perp^2; \mu^2) \delta(z\mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp)$$

Bacchetta, Diehl, et al., JHEP 02 (2007)

$$= x \sum_a \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{P}_{h\perp}|) \hat{f}_1^q(x, z^2 b_\perp^2; \mu^2) \hat{D}_1^{a \rightarrow h}(z, b_\perp^2; \mu^2)$$

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# Global analysis of DY and SIDIS data sets

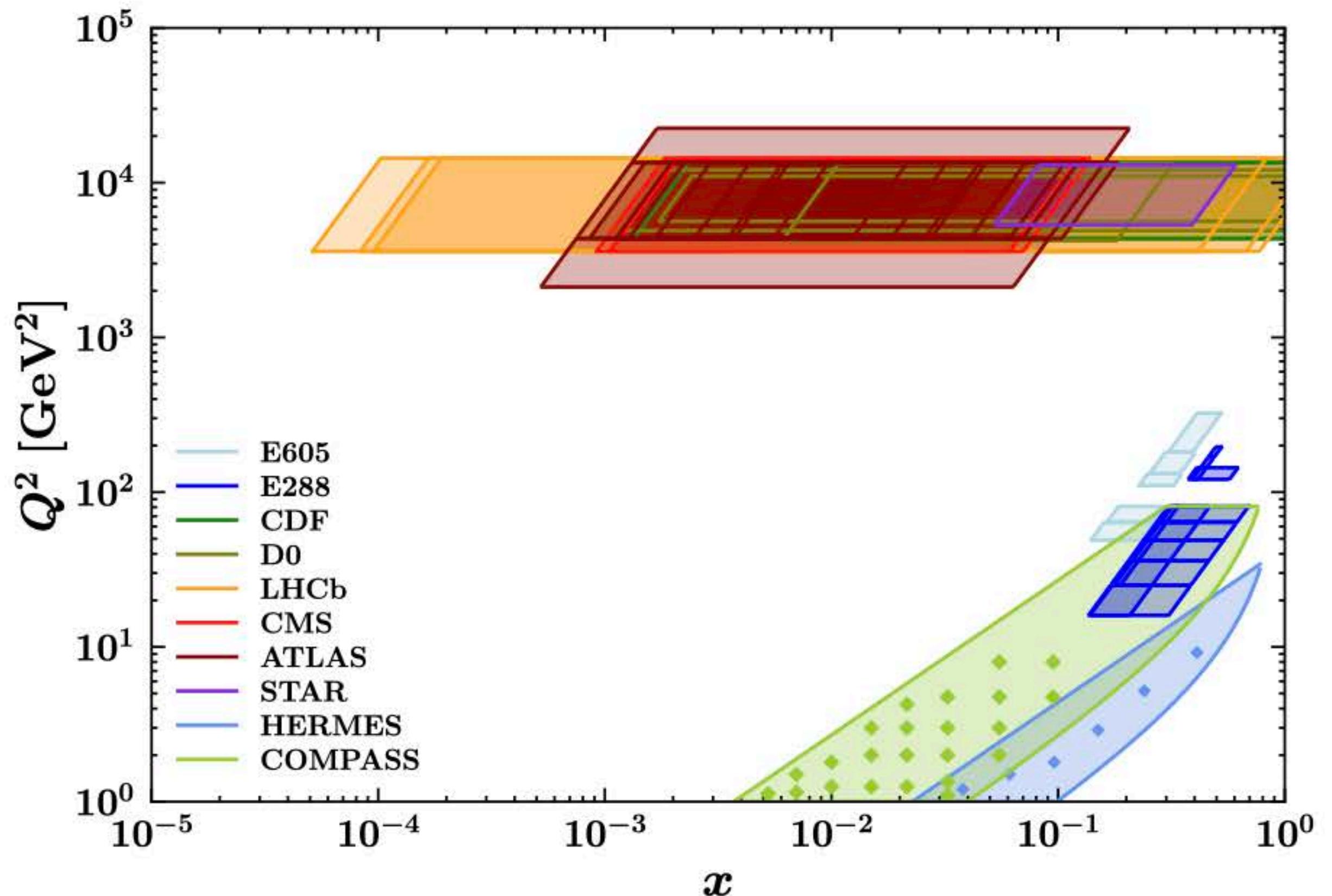


# Global analysis of DY and SIDIS data sets

Cuts on kinematics

$$\langle Q \rangle > 1.3 \text{ GeV}$$

$$0.2 < \langle z \rangle < 0.7$$



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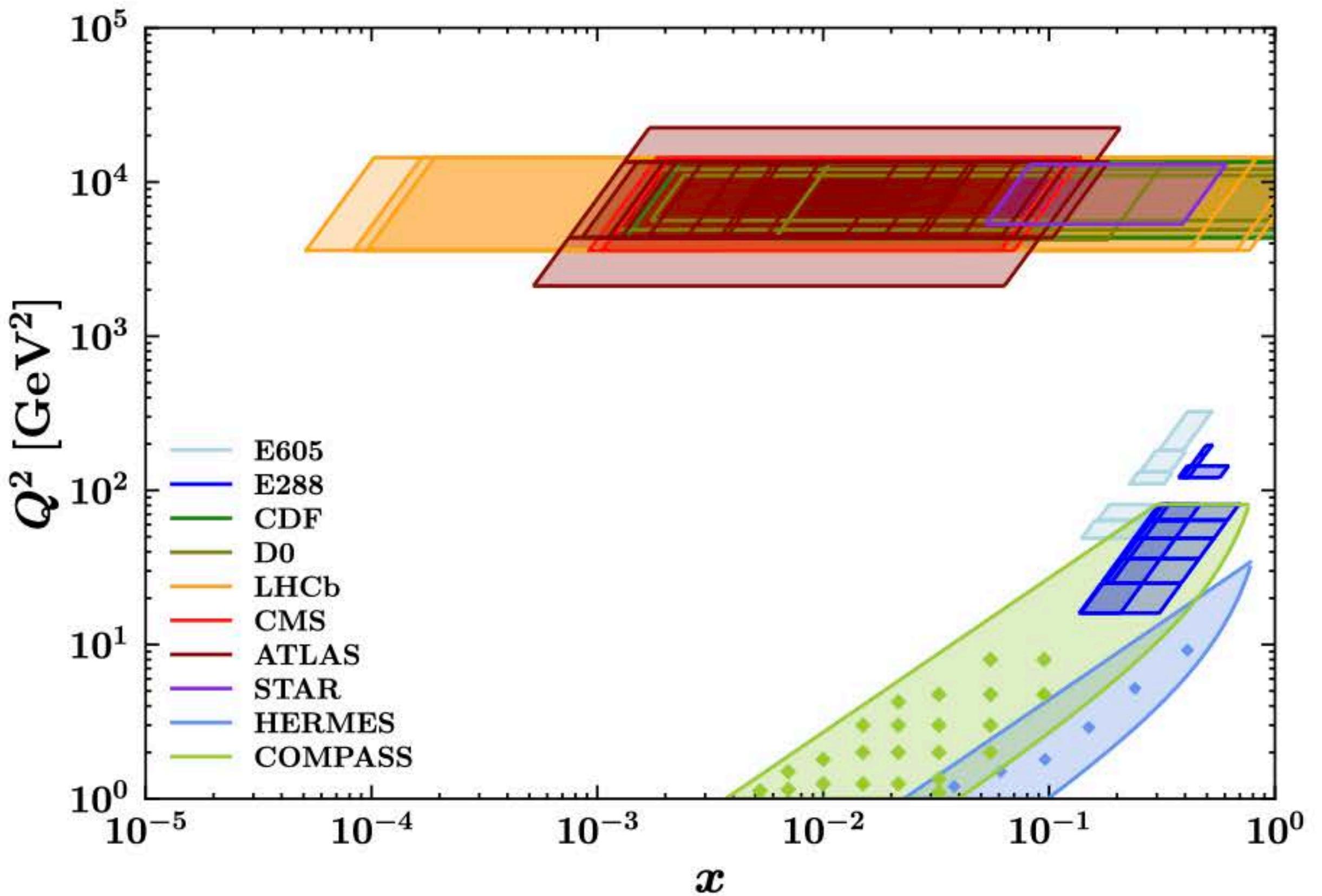
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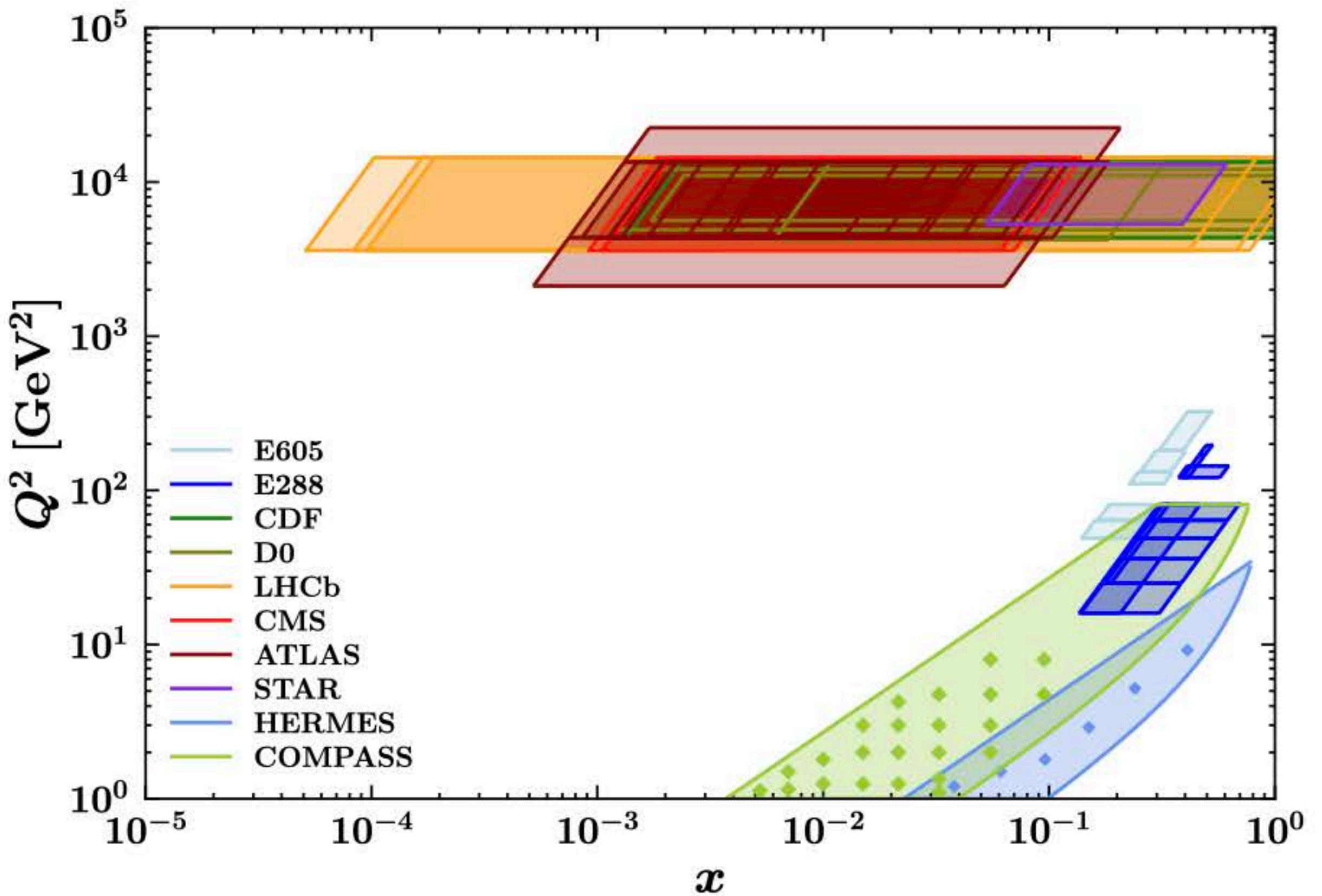
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$$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$

SIDIS



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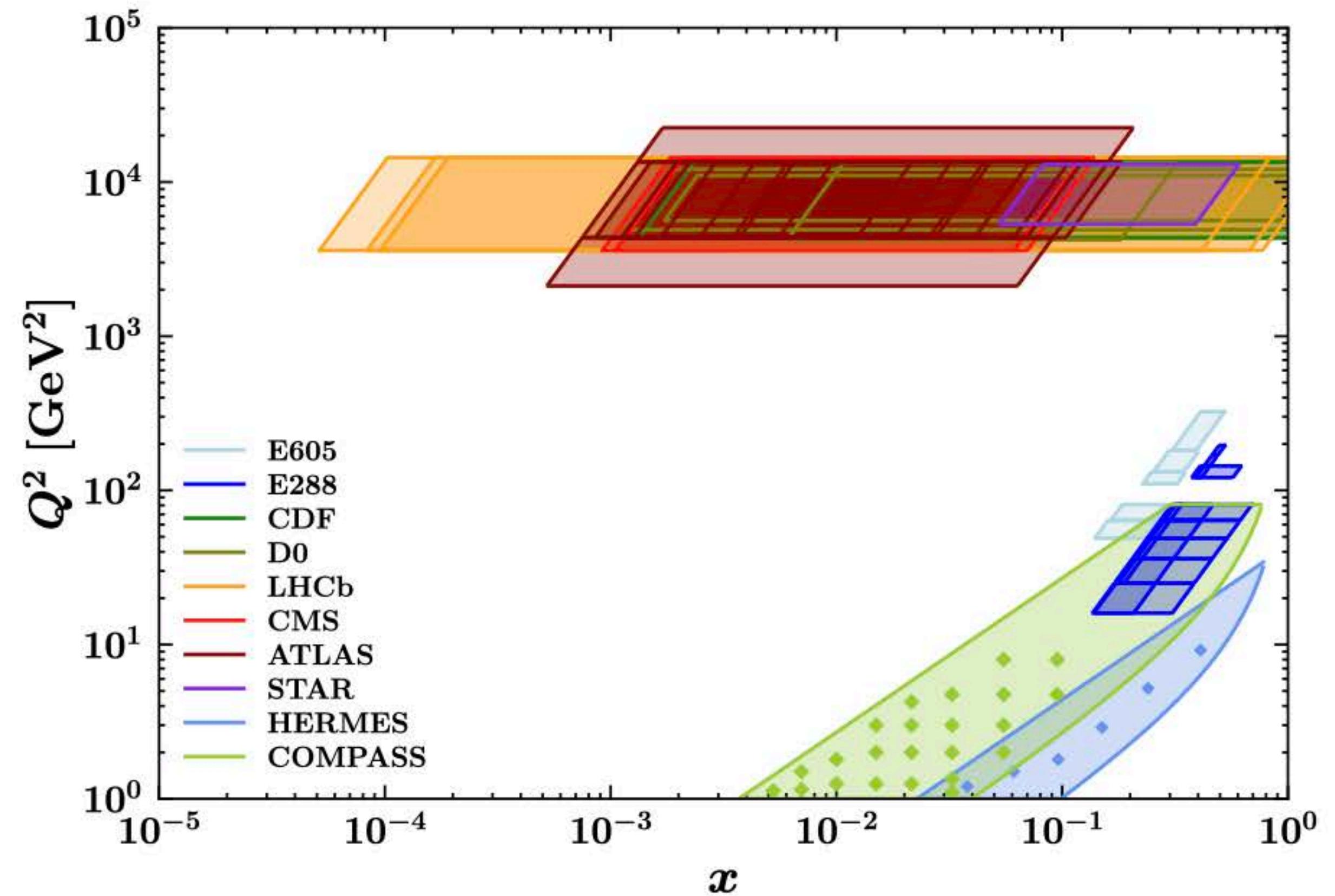
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SIDIS



**Number of experimental points = 2031**

# Perturbative accuracy: $N^3 LL^-$

TMD structure in b-space:

$$\hat{f}_1^q(x, b_T; \mu^2) = \int d^2 \mathbf{k}_\perp e^{i \mathbf{b}_T \cdot \mathbf{k}_\perp} f_1^q(x, \mathbf{k}_\perp^2; \mu^2)$$

*Collins, “Foundations of Perturbative QCD”*

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matching coefficients  
(perturbative)

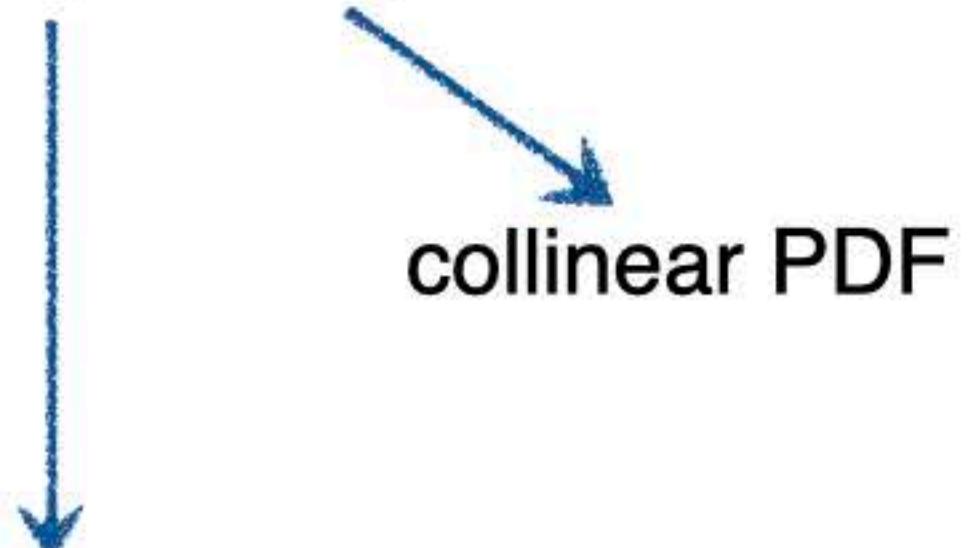
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collinear PDF

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perturbative Sudakov  
form factor

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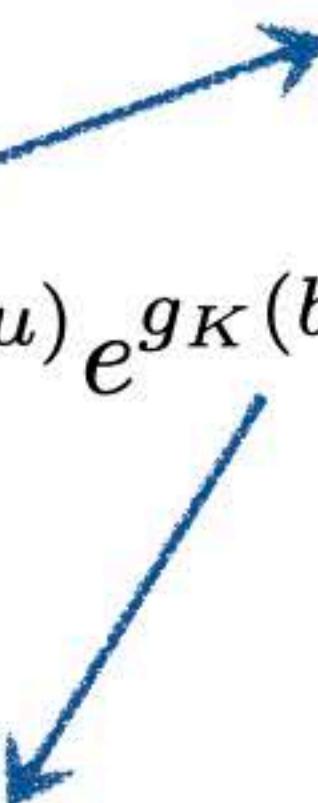
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matching coefficients  
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collinear PDF

nonperturbative part  
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# Perturbative accuracy: $N^3 LL^-$

Orders in powers of  $\alpha_S$

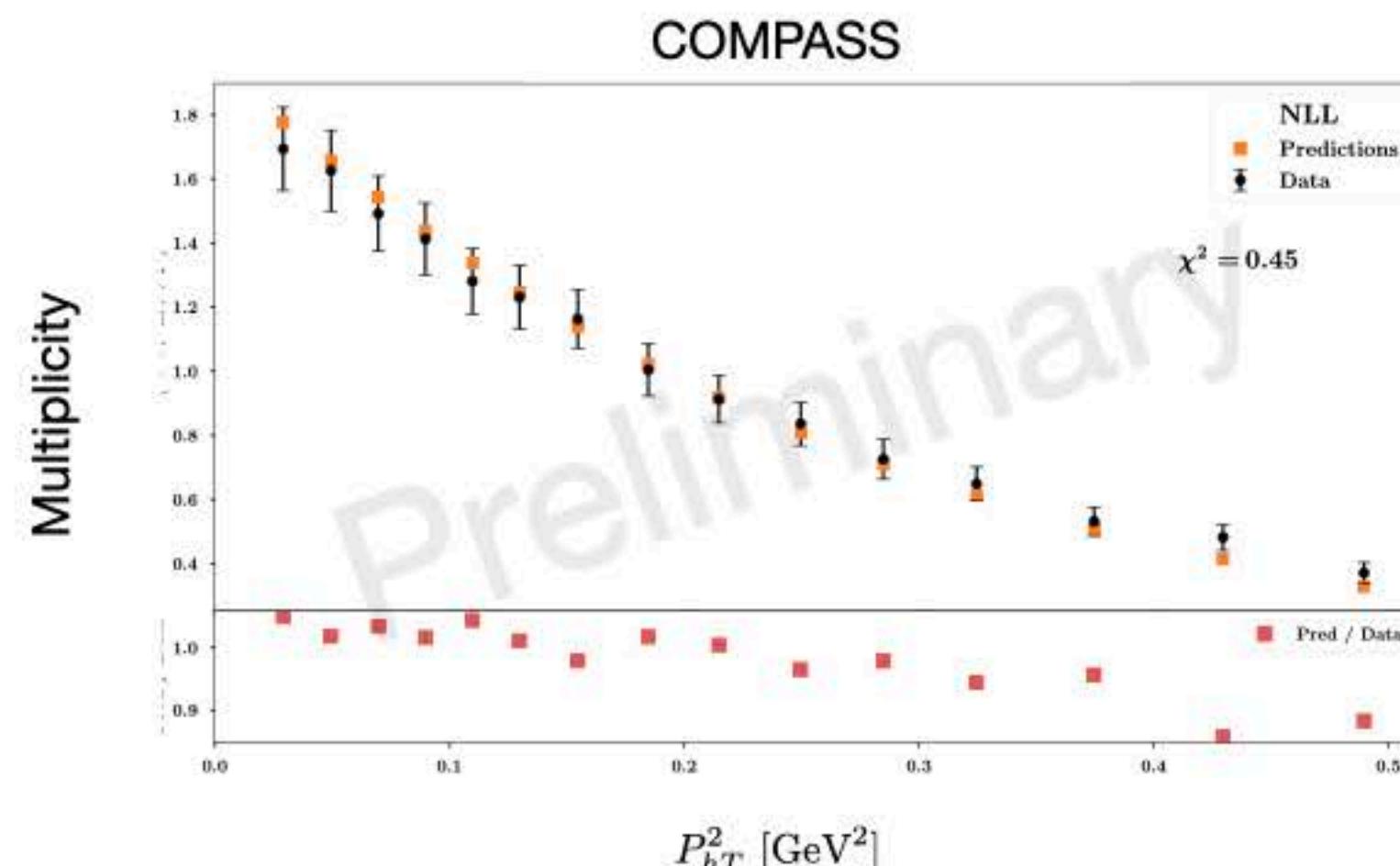
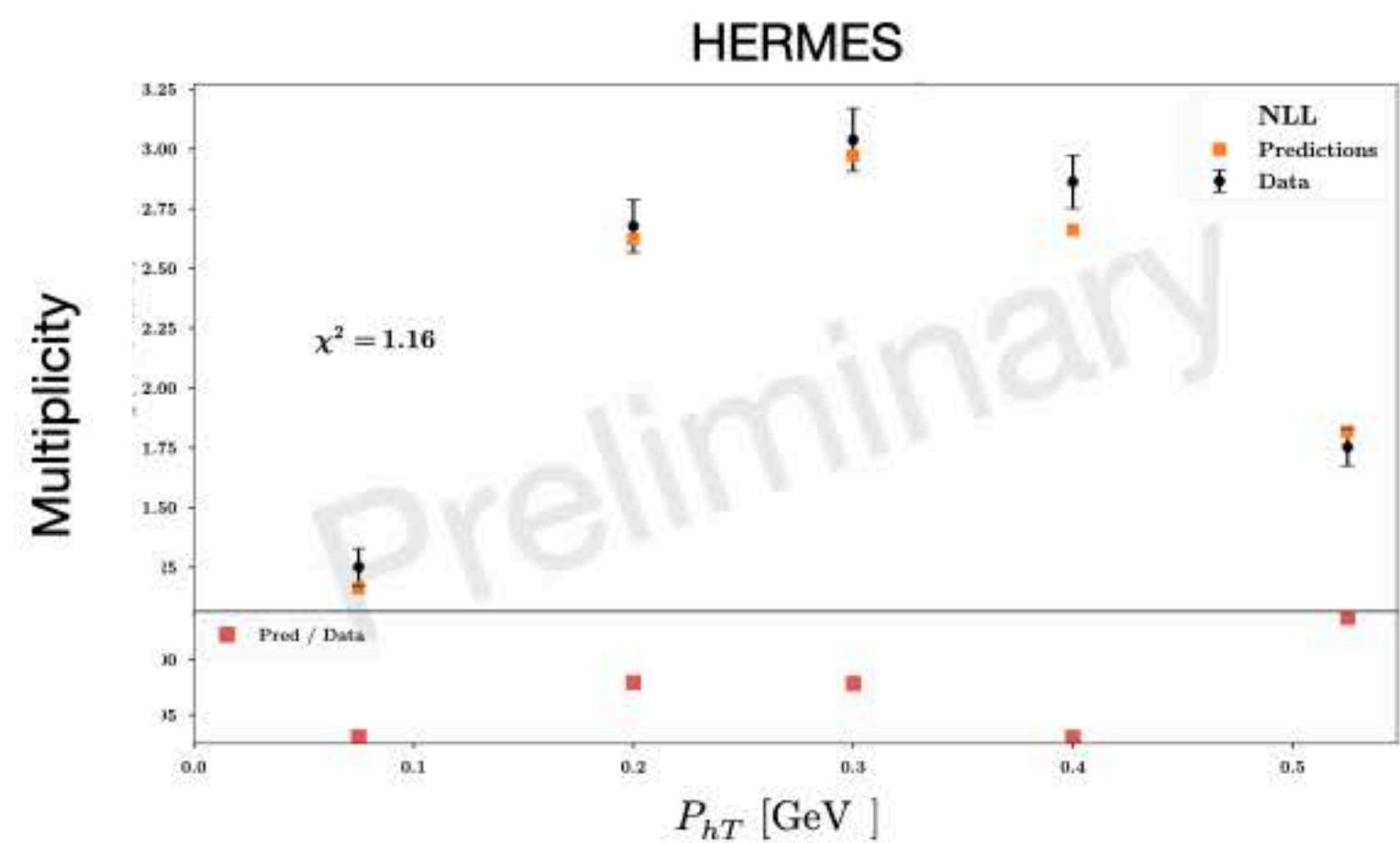
# Perturbative accuracy: $N^3 LL^-$

Orders in powers of  $\alpha_S$

Accuracy	Hard factor and matching coefficient	Ingredients in perturbative Sudakov form factor		PDF and $\alpha_S$ evol.
	$H$ and $C$	$K$ and $\gamma_F$	$\gamma_K$	
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
$N^3 LL^-$	2	3	4	NLO (FF only)
$N^3 LL$	2	3	4	NNLO
$N^3 LL'$	3	3	4	$N^3 LO$

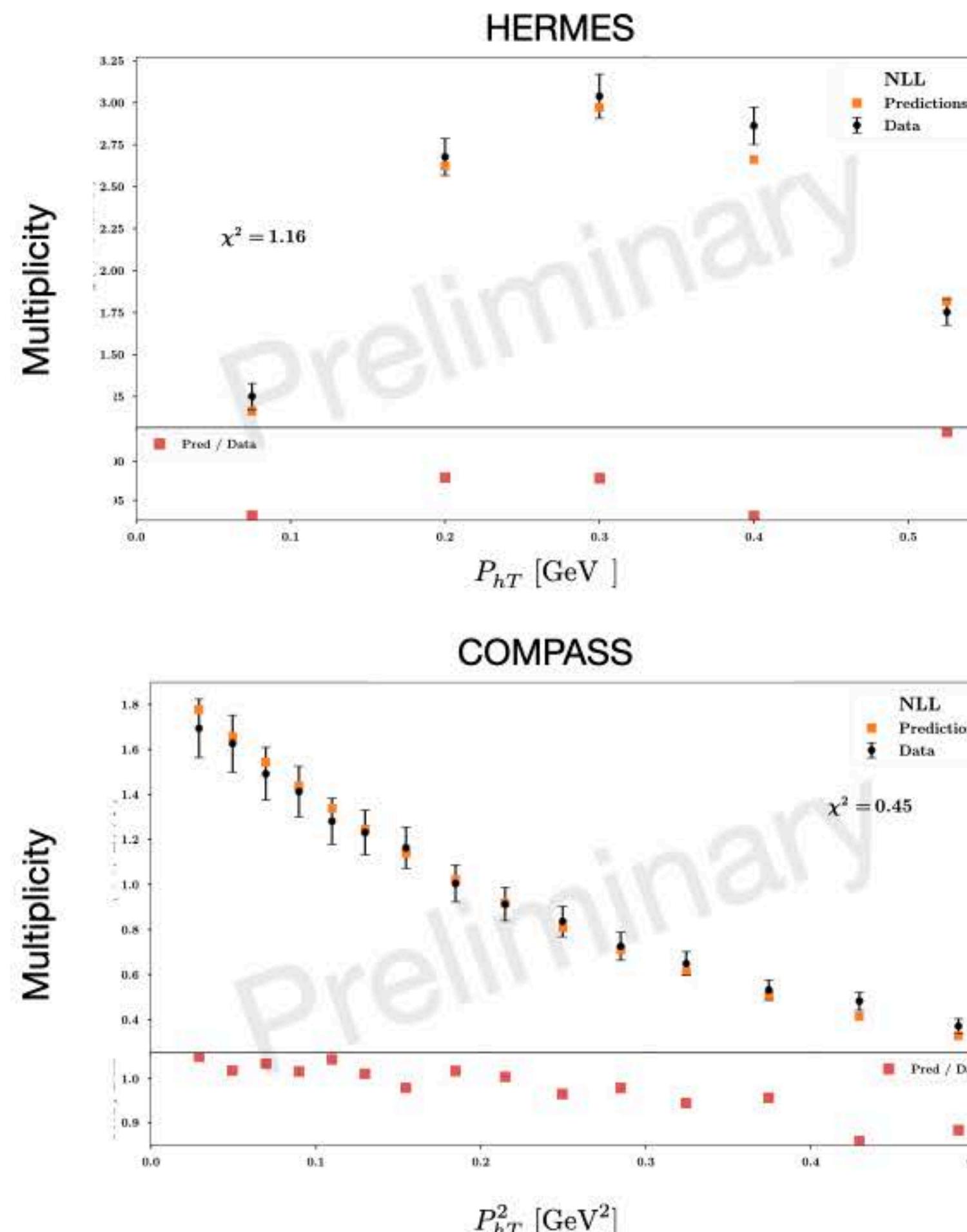
# Normalisation of SIDIS multiplicities

## SIDIS multiplicities at NLL

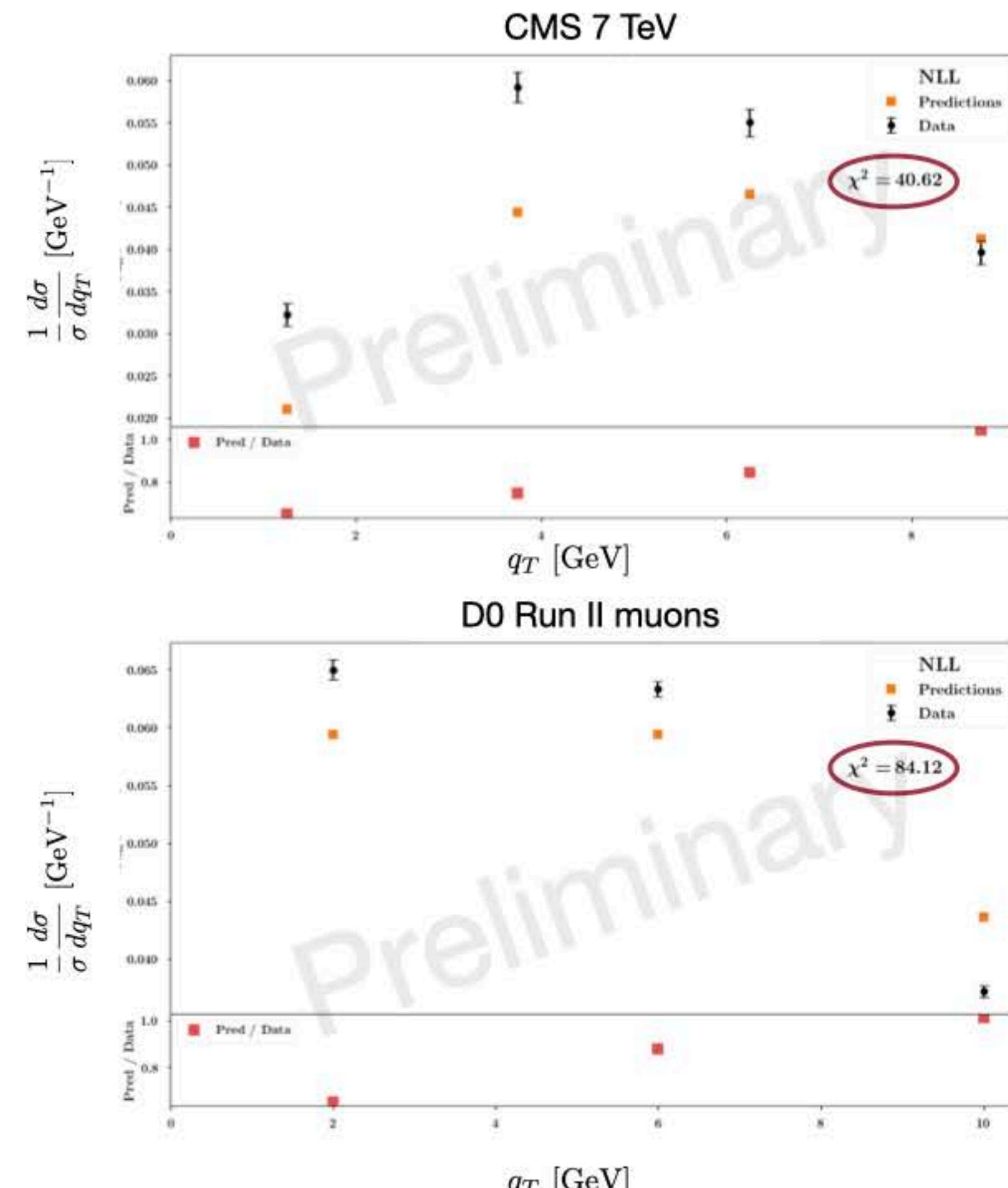


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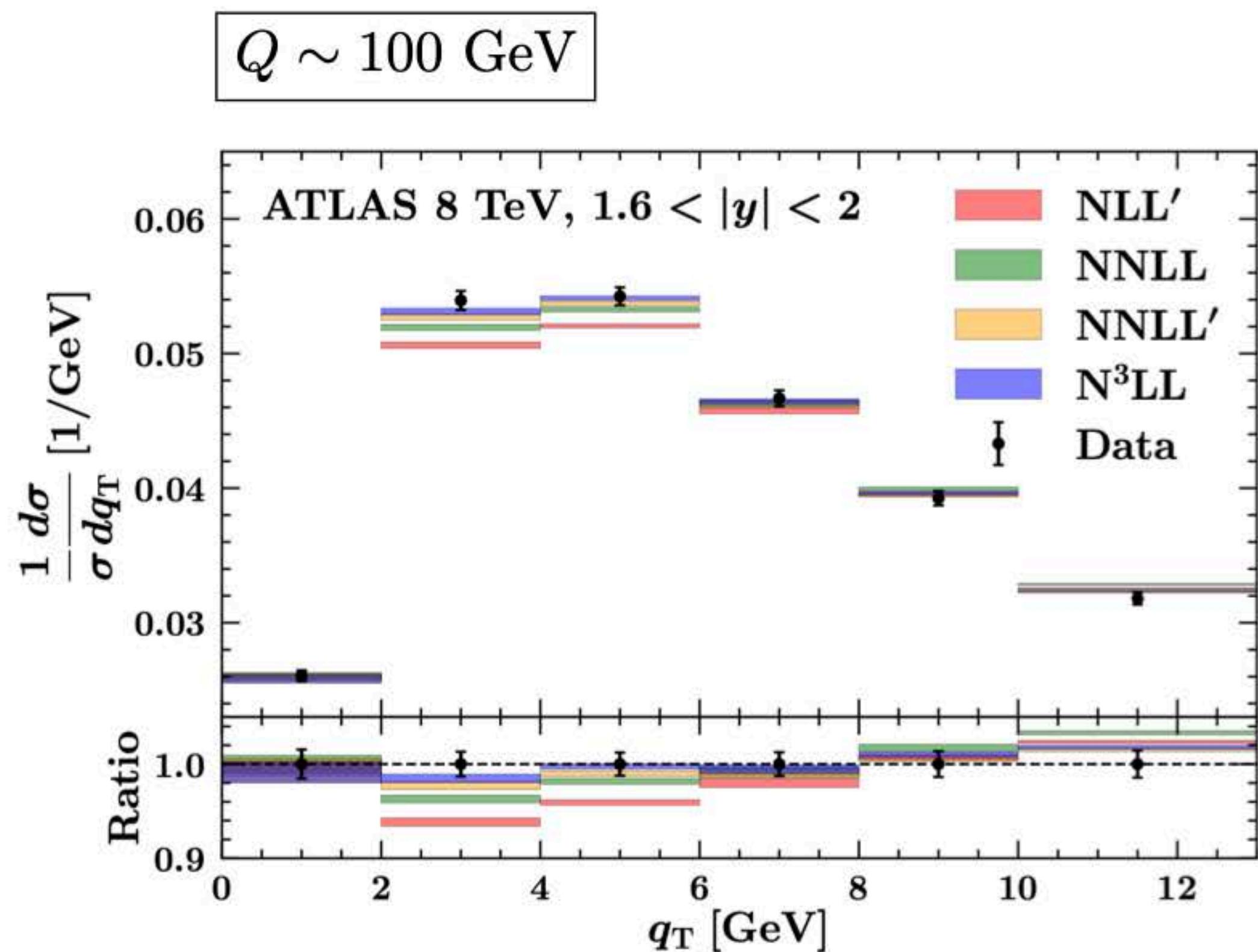
## High-Energy Drell-Yan at NLL



# Normalisation of SIDIS multiplicities

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High-Energy Drell-Yan beyond NLL

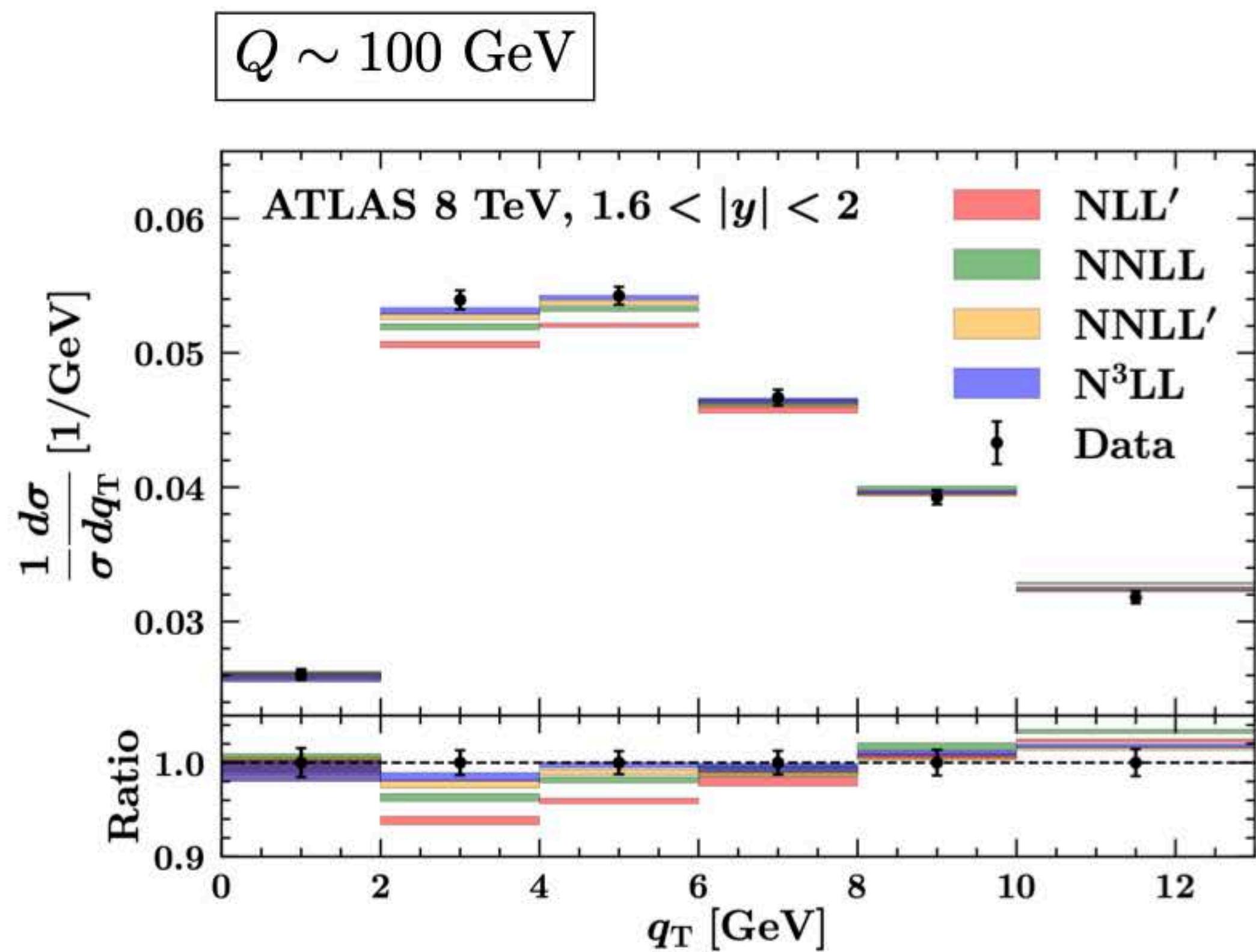


Bacchetta, Bertone, Bissoletti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550

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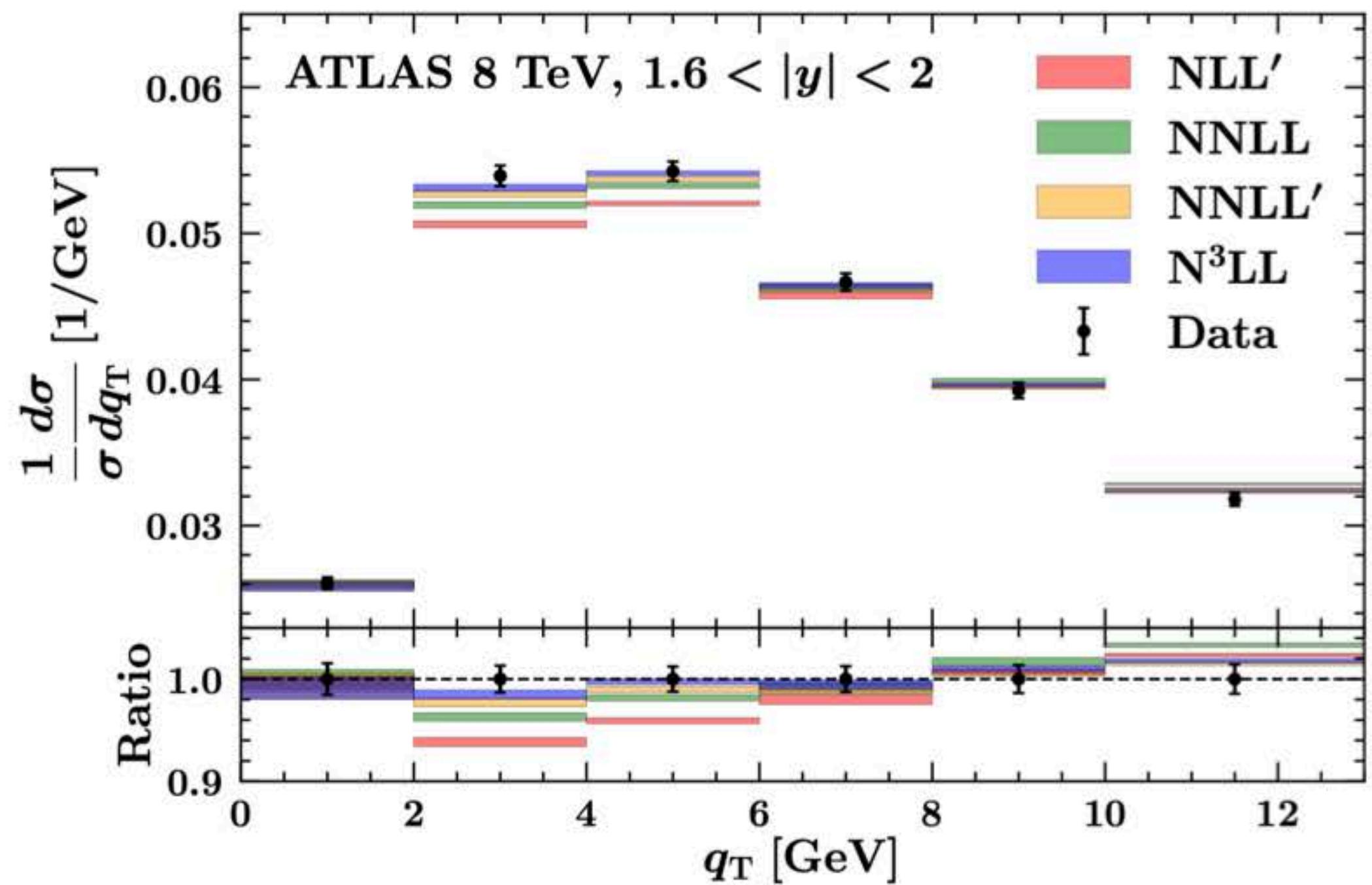
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SIDIS multiplicities beyond NLL

$$Q \sim 2 \text{ GeV}$$

High-Energy Drell-Yan beyond NLL

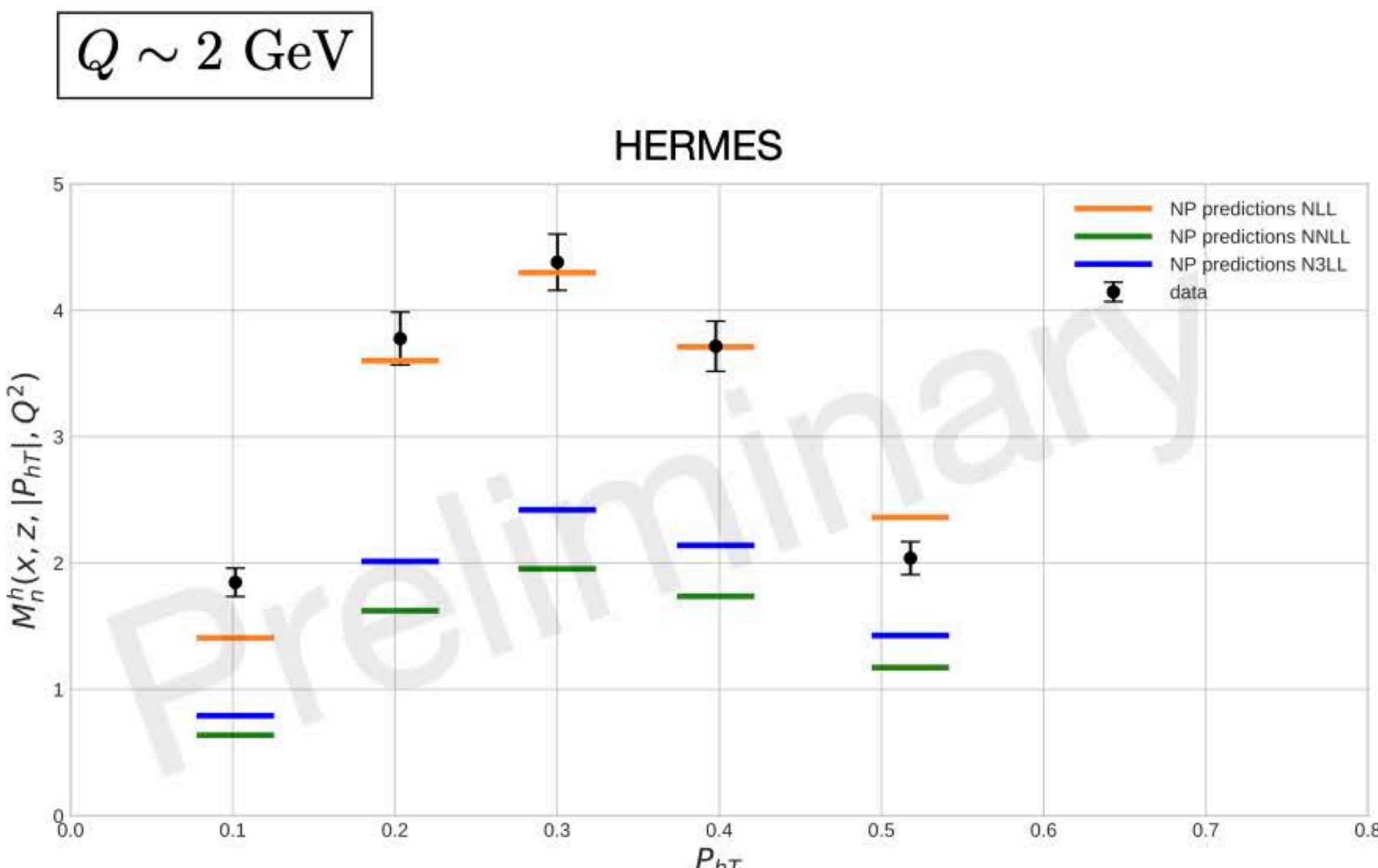
$$Q \sim 100 \text{ GeV}$$



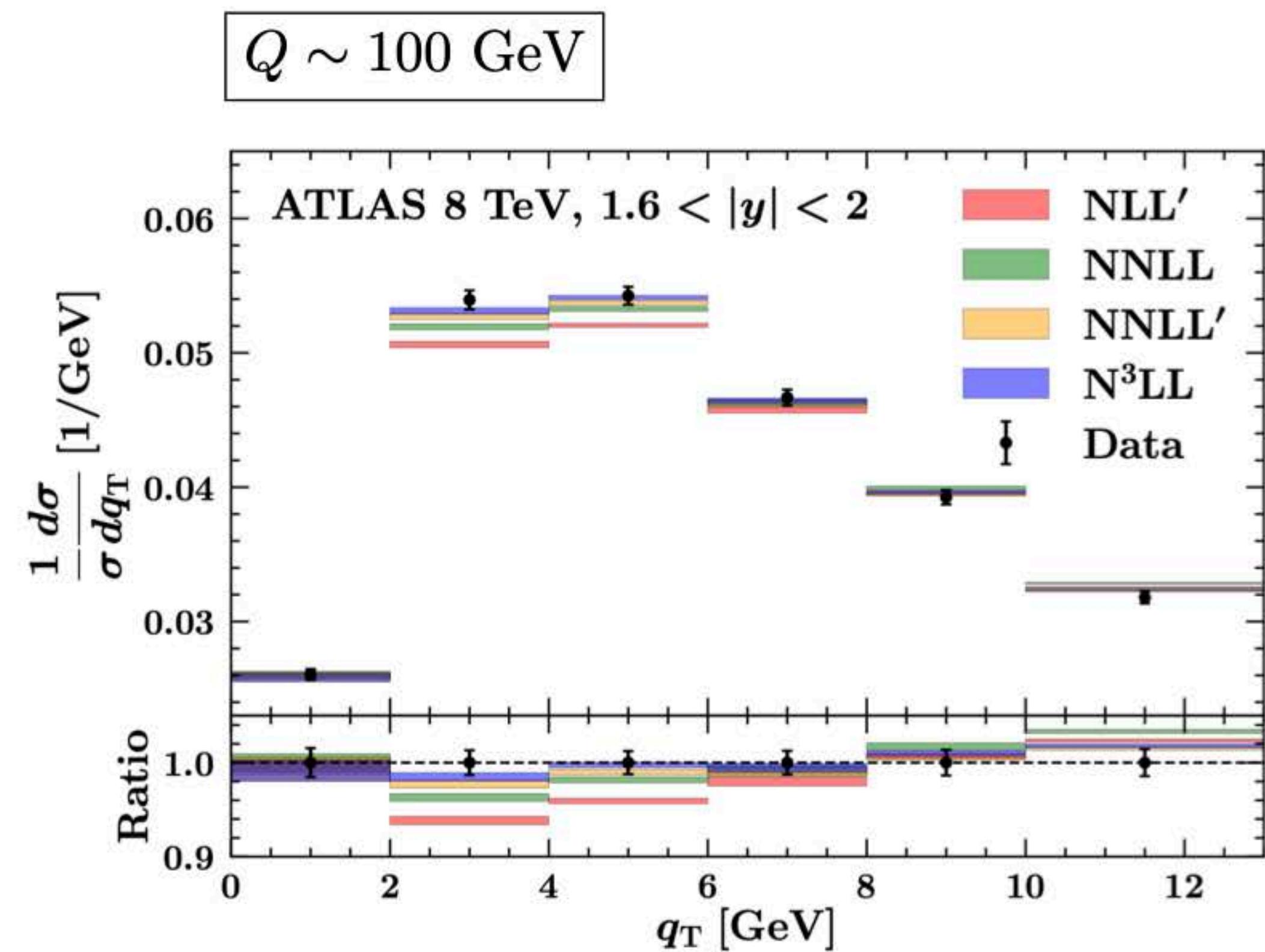
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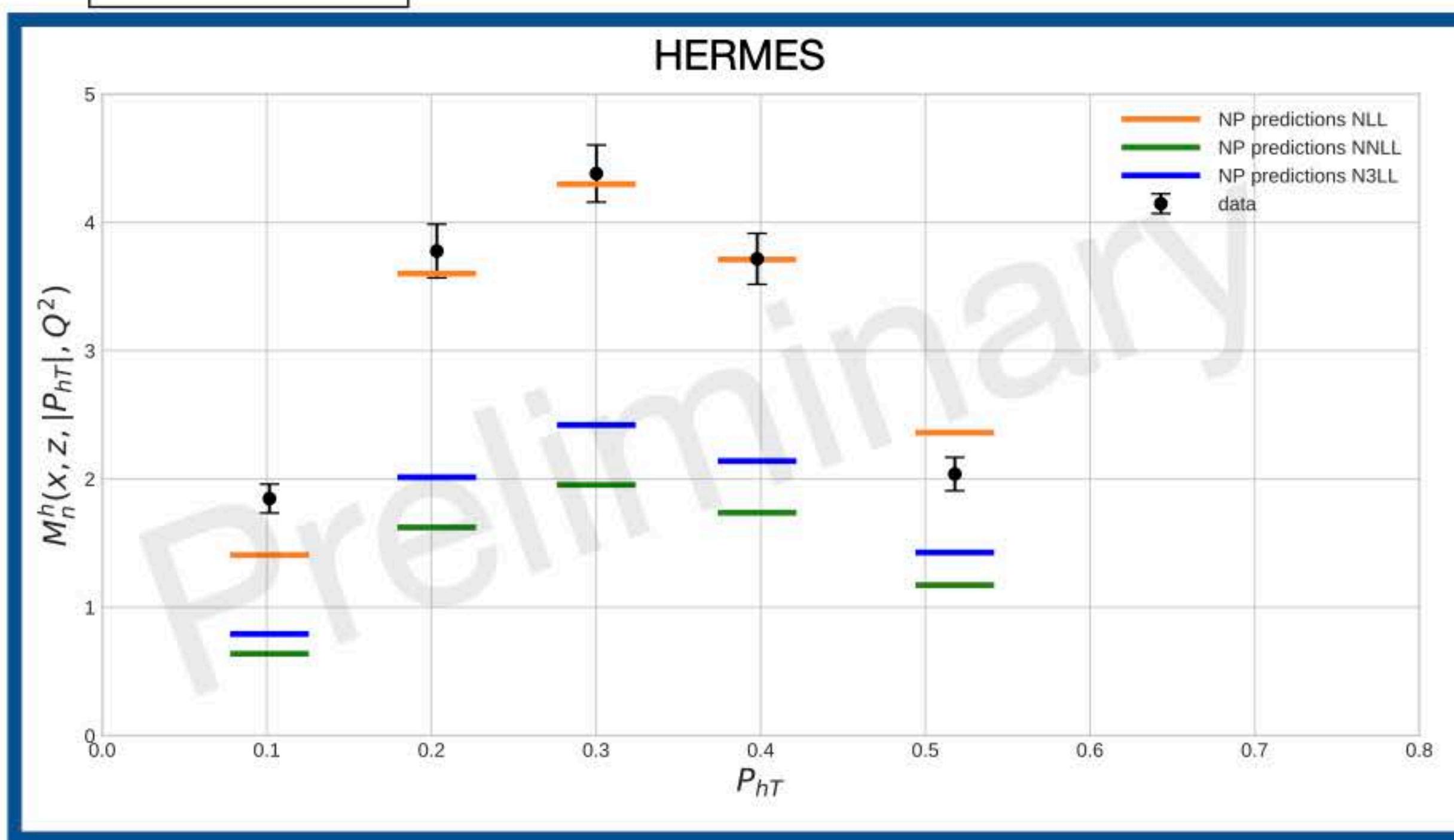
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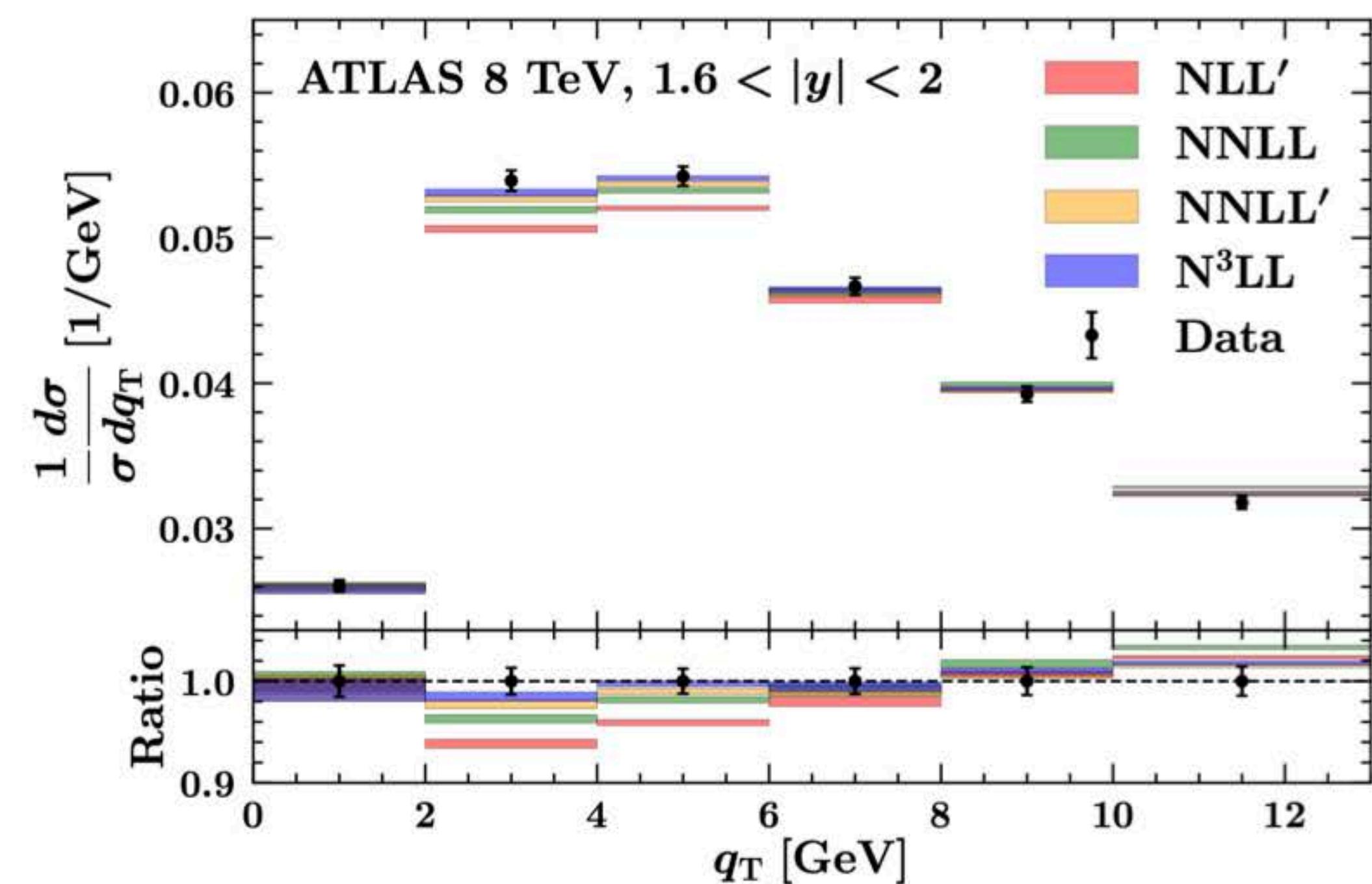
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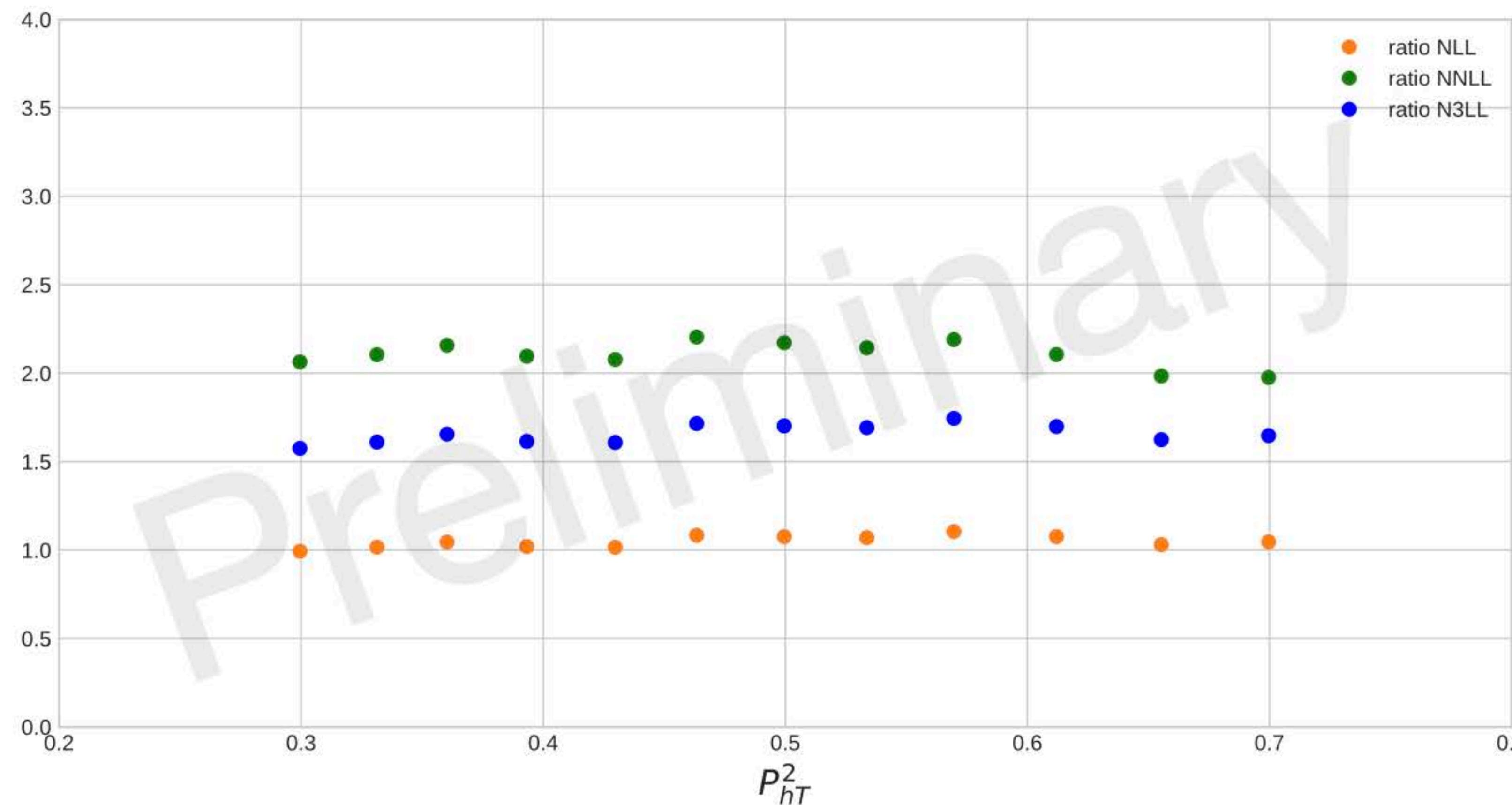
The description considerably worsens at higher orders!!

Bacchetta, Bertone, Bissoletti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550

# Normalisation of SIDIS multiplicities

COMPASS multiplicities (one of many bins)

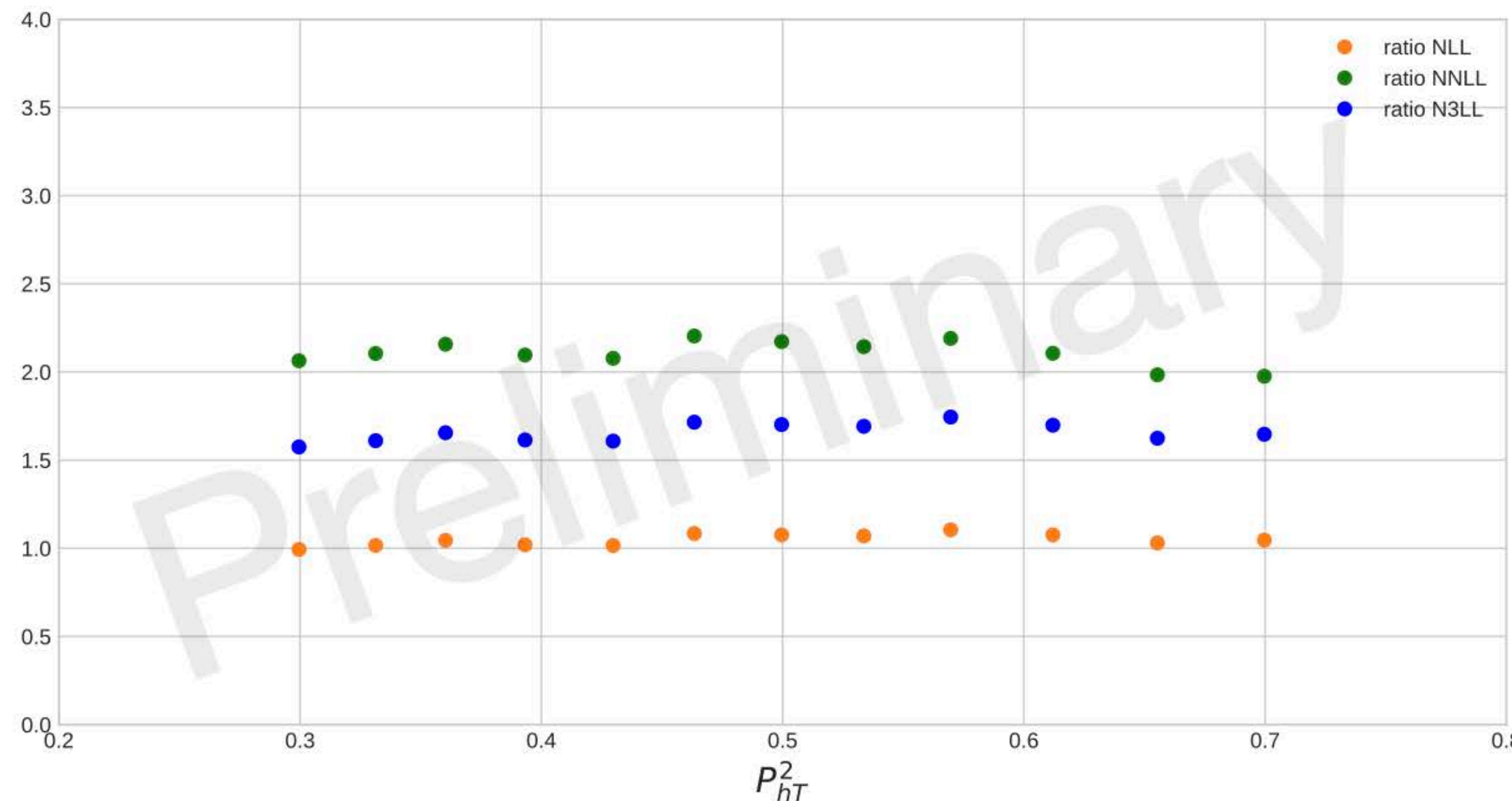
J.O. Gonzalez-Hernandez, PoS DIS2019 (2019) 176



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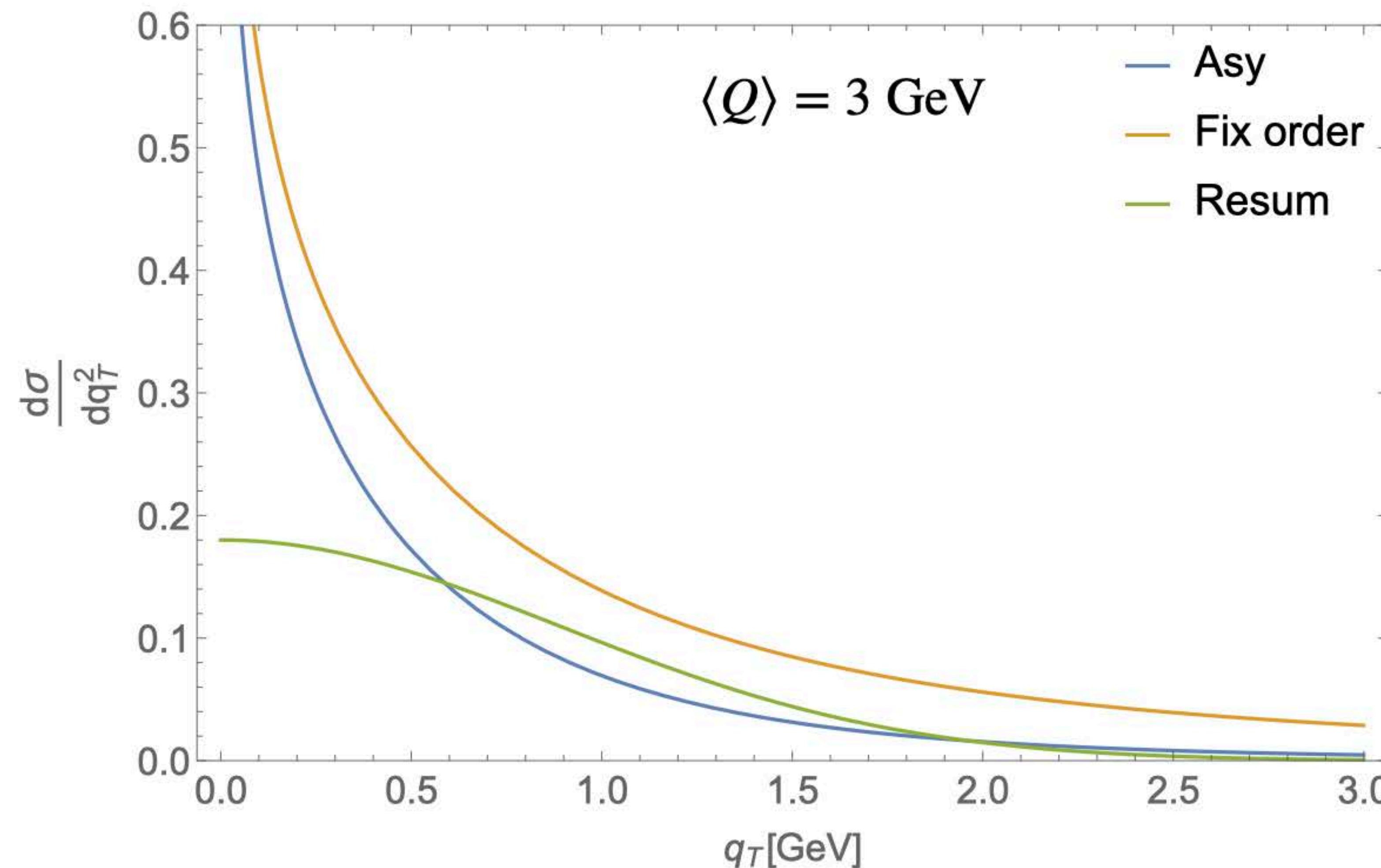
J.O. Gonzalez-Hernandez, PoS DIS2019 (2019) 176



***The discrepancy amounts to an almost constant factor!!***

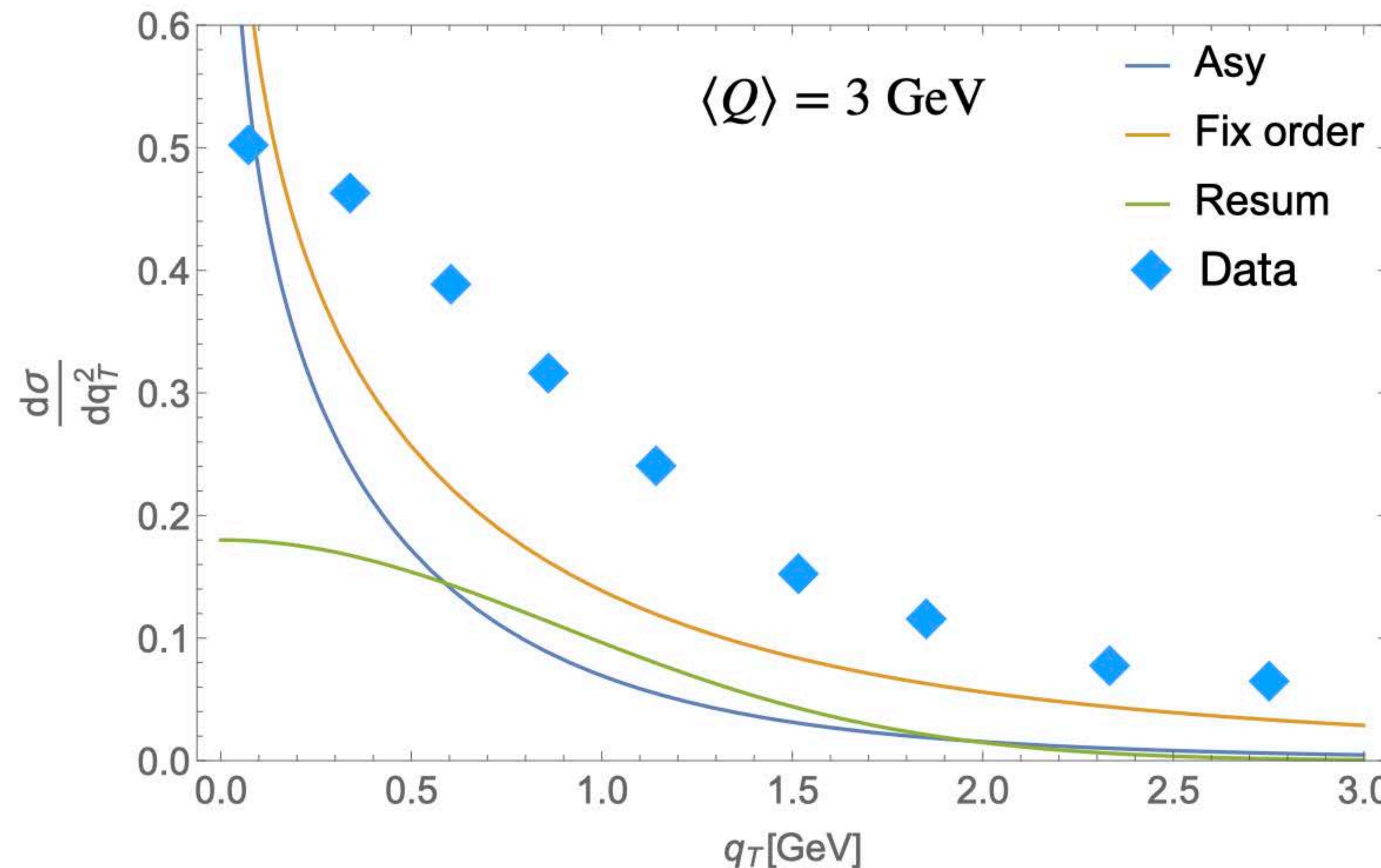
# Normalisation of SIDIS multiplicities

According to our formalism



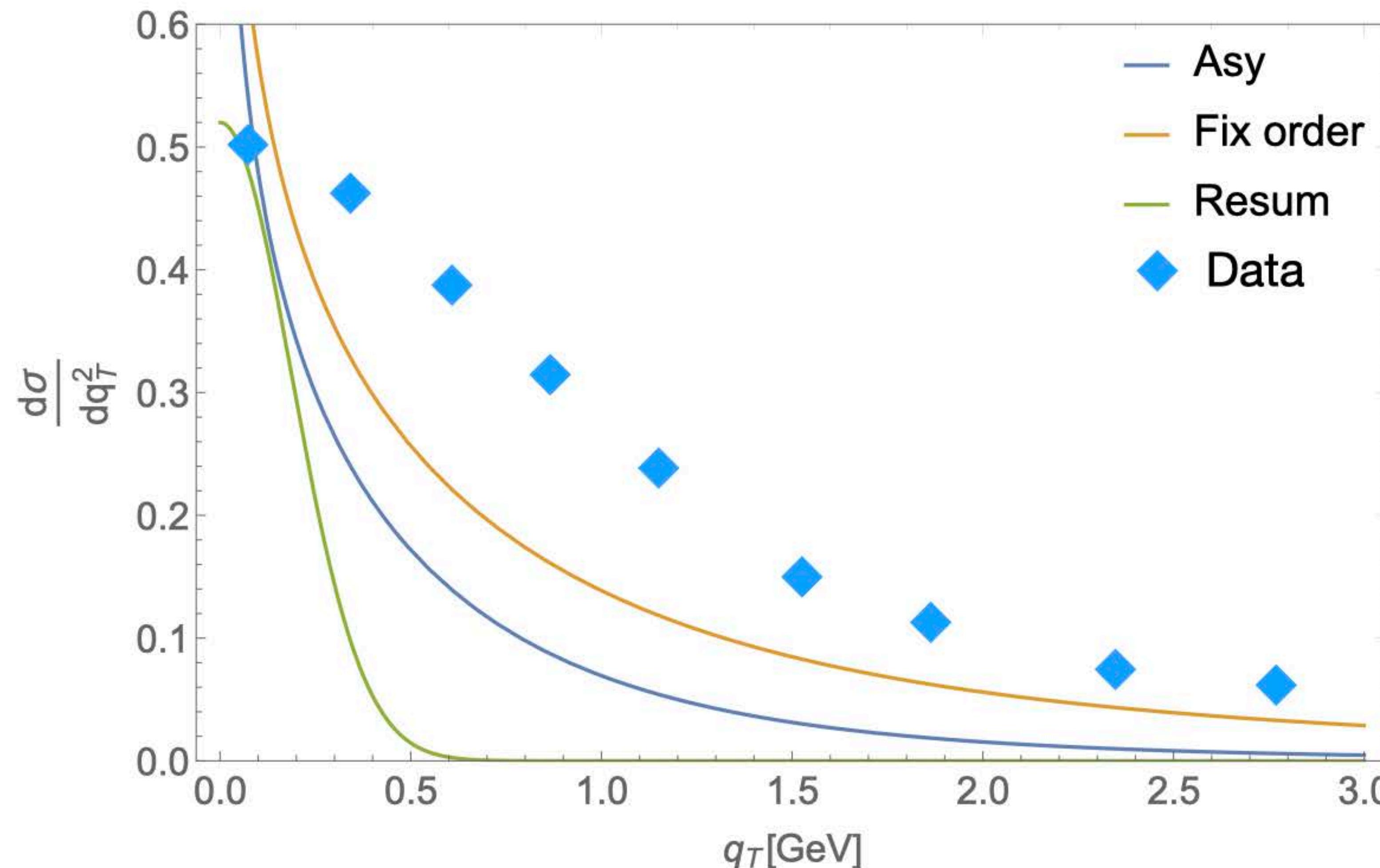
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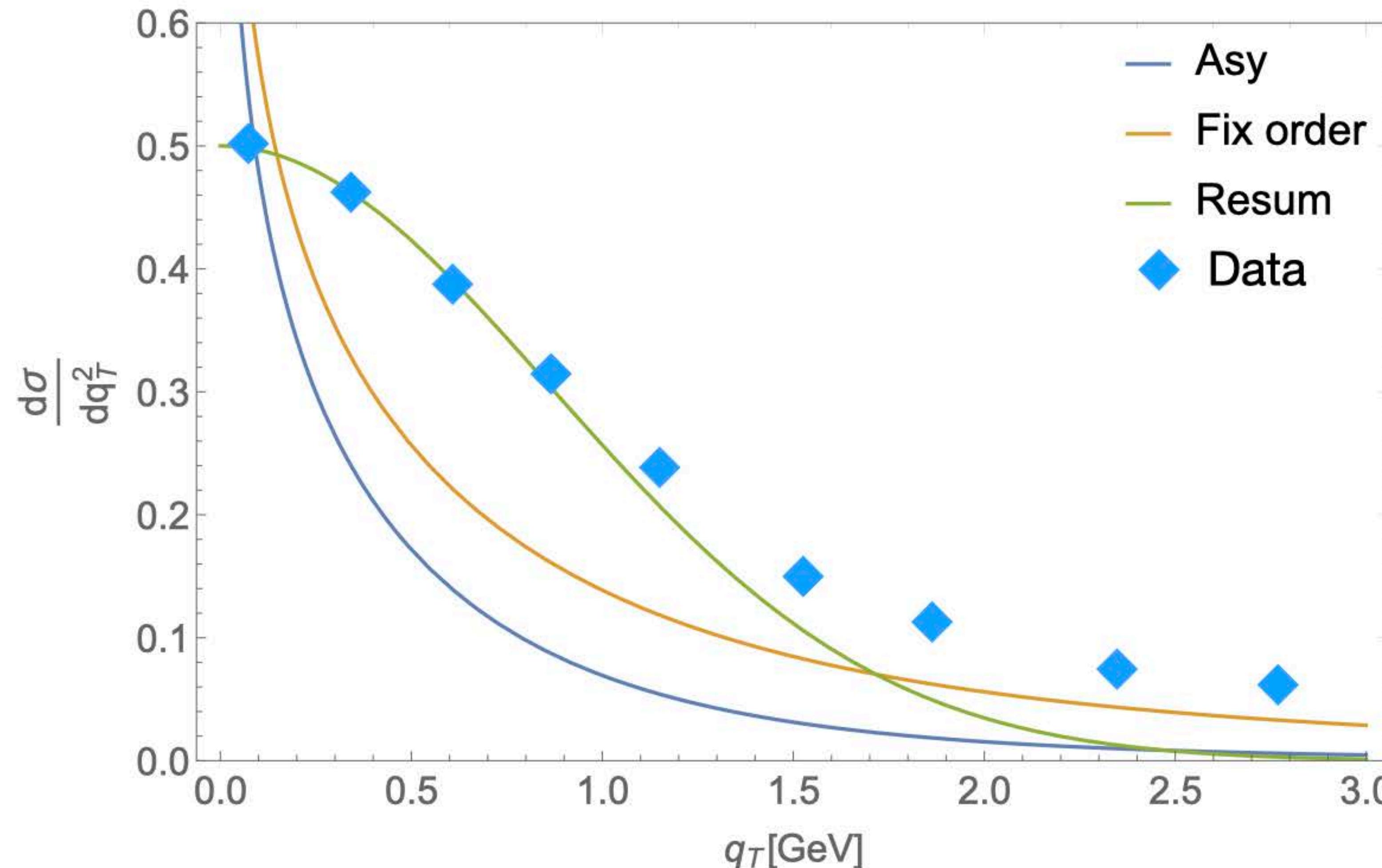
# Normalisation of SIDIS multiplicities

Solution1: restrict the TMD region



# Normalisation of SIDIS multiplicities

Solution2: enhance TMD contributions



# Normalisation of SIDIS multiplicities

Introduction of a normalisation prefactor

$$\text{PREFACTOR}(x, z, Q) = \frac{\frac{d\sigma^h}{dxdQ^2dz} \Big|_{\text{nonmix.}}}{\int W d^2q_T}$$

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$$\frac{d\sigma^h}{dxdQ^2dz} \Big|_{O(\alpha_S)} = \sigma_0 \sum_{f,f'} \frac{e_f^2}{z^2} (\delta_{f'f} + \delta_{f'g}) \frac{\alpha_S}{\pi} \left\{ [D_1^{h/f'} \otimes C_1^{f'f} \otimes f_1^{f/N}](x, z, Q) \right\} \Big|_{\text{nonmix.}}$$

$$\int W \Big|_{O(\alpha_S)} = \sigma_0 \sum_{f,f'} \frac{e_f^2}{z^2} (\delta_{f'f} + \delta_{f'g}) \frac{\alpha_S}{\pi} [D_1^{h/f'} \otimes C_{\text{TMD}}^{f'f} \otimes f_1^{f/N}](x, z, Q)$$

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Introduction of a normalisation prefactor

$$\text{PREFACTOR}(x, z, Q) = \frac{\frac{d\sigma^h}{dxdQ^2dz} \Big|_{\text{nonmix.}}}{\int W d^2q_T}$$

$$\frac{d\sigma^h}{dxdQ^2dz} \Big|_{O(\alpha_S)} = \sigma_0 \sum_{f,f'} \frac{e_f^2}{z^2} (\delta_{f'f} + \delta_{f'g}) \frac{\alpha_S}{\pi} \left\{ [D_1^{h/f'} \otimes C_1^{f'f} \otimes f_1^{f/N}](x, z, Q) \right\} \Big|_{\text{nonmix.}}$$

$$\int W \Big|_{O(\alpha_S)} = \sigma_0 \sum_{f,f'} \frac{e_f^2}{z^2} (\delta_{f'f} + \delta_{f'g}) \frac{\alpha_S}{\pi} [D_1^{h/f'} \otimes C_{\text{TMD}}^{f'f} \otimes f_1^{f/N}](x, z, Q)$$

***Independent of the fitting parameters!!***

# Non-perturbative part of TMDs

$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of} \left( e^{-\frac{k_\perp^2}{g^{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g^{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g^{1C}}} \right)$$

# Non-perturbative part of TMDs

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$$g_1(x) = N_1 \frac{(1-x)^\alpha \ x^\sigma}{(1-\hat{x})^\alpha \ \hat{x}^\sigma}$$

# Non-perturbative part of TMDs

$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of} \left( e^{-\frac{k_\perp^2}{g^{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g^{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g^{1C}}} \right)$$



$$D_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of} \left( e^{-\frac{P_\perp^2}{g_{3A}}} + \lambda_{FB} k_\perp^2 e^{-\frac{P_\perp^2}{g_{3B}}} \right)$$

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# Non-perturbative part of TMDs

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$$g_1(x) = N_1 \frac{(1-x)^\alpha}{(1-\hat{x})^\alpha} \frac{x^\sigma}{\hat{x}^\sigma}$$

$$g_3(z) = N_3 \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}$$



# Non-perturbative part of TMDs

$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of} \left( e^{-\frac{k_\perp^2}{g^{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g^{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g^{1C}}} \right)$$



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$$g_K(b_T^2) = -g_2^2 \frac{b_T^2}{4}$$

# Non-perturbative part of TMDs

$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of} \left( e^{-\frac{k_\perp^2}{g^{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g^{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g^{1C}}} \right)$$

$$g_1(x) = N_1 \frac{(1-x)^\alpha}{(1-\hat{x})^\alpha} \frac{x^\sigma}{\hat{x}^\sigma}$$

$$D_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of} \left( e^{-\frac{P_\perp^2}{g_{3A}}} + \lambda_{FB} k_\perp^2 e^{-\frac{P_\perp^2}{g_{3B}}} \right)$$

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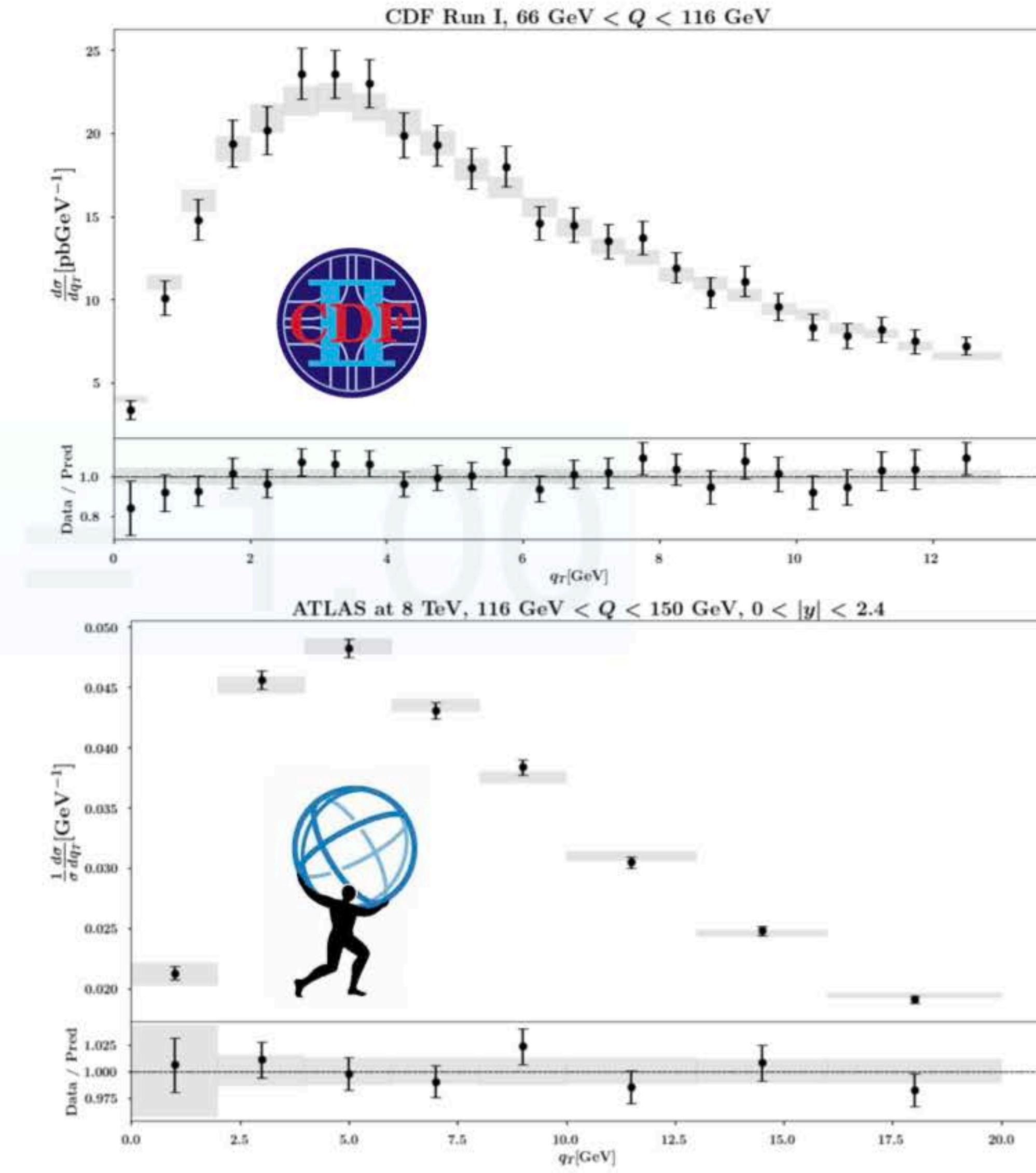
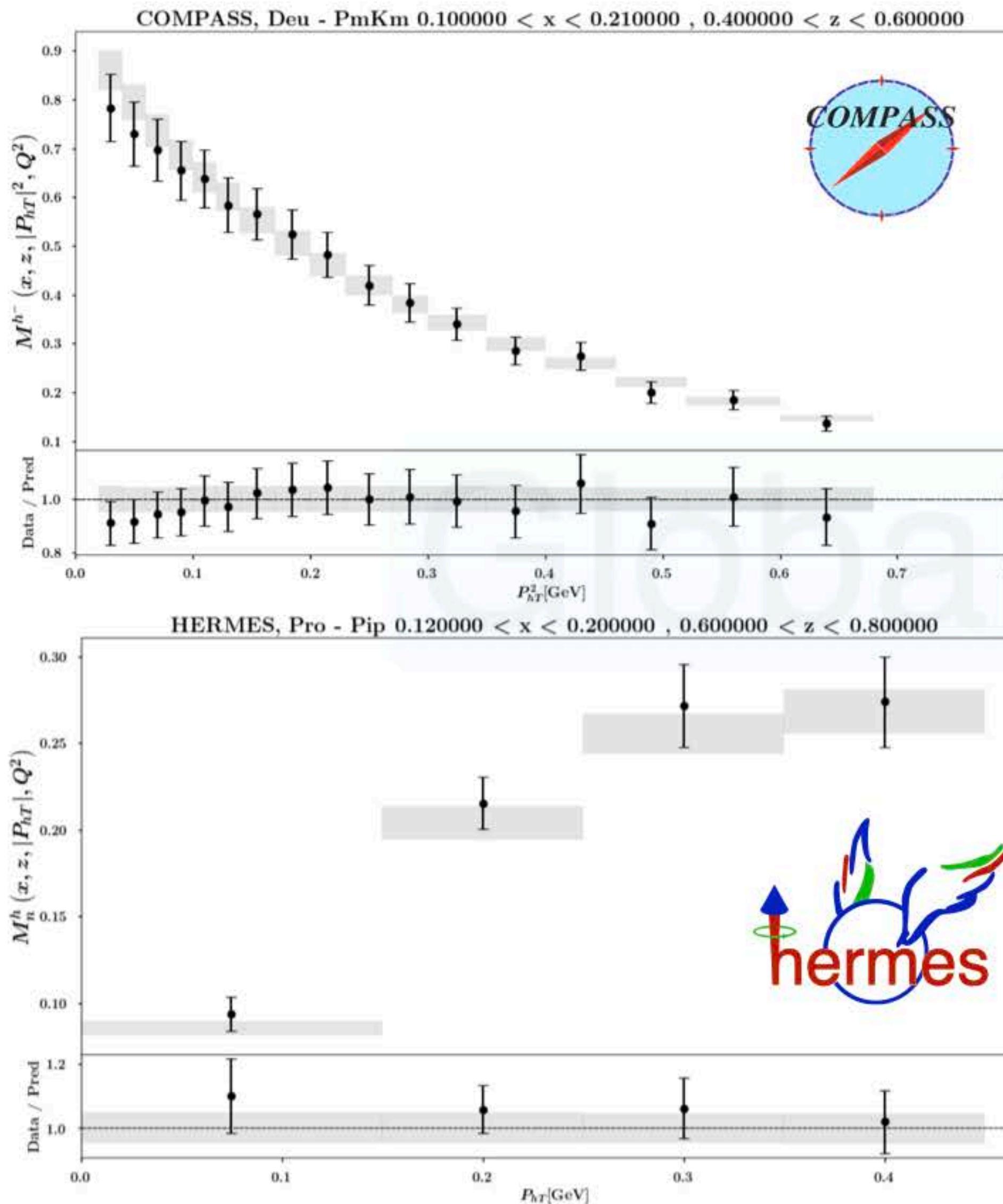
$$g_K(b_T^2) = -g_2^2 \frac{b_T^2}{4}$$

11 parameters for TMD PDF  
+ 1 for NP evolution + 9 for TMD FF  
= 21 free parameters

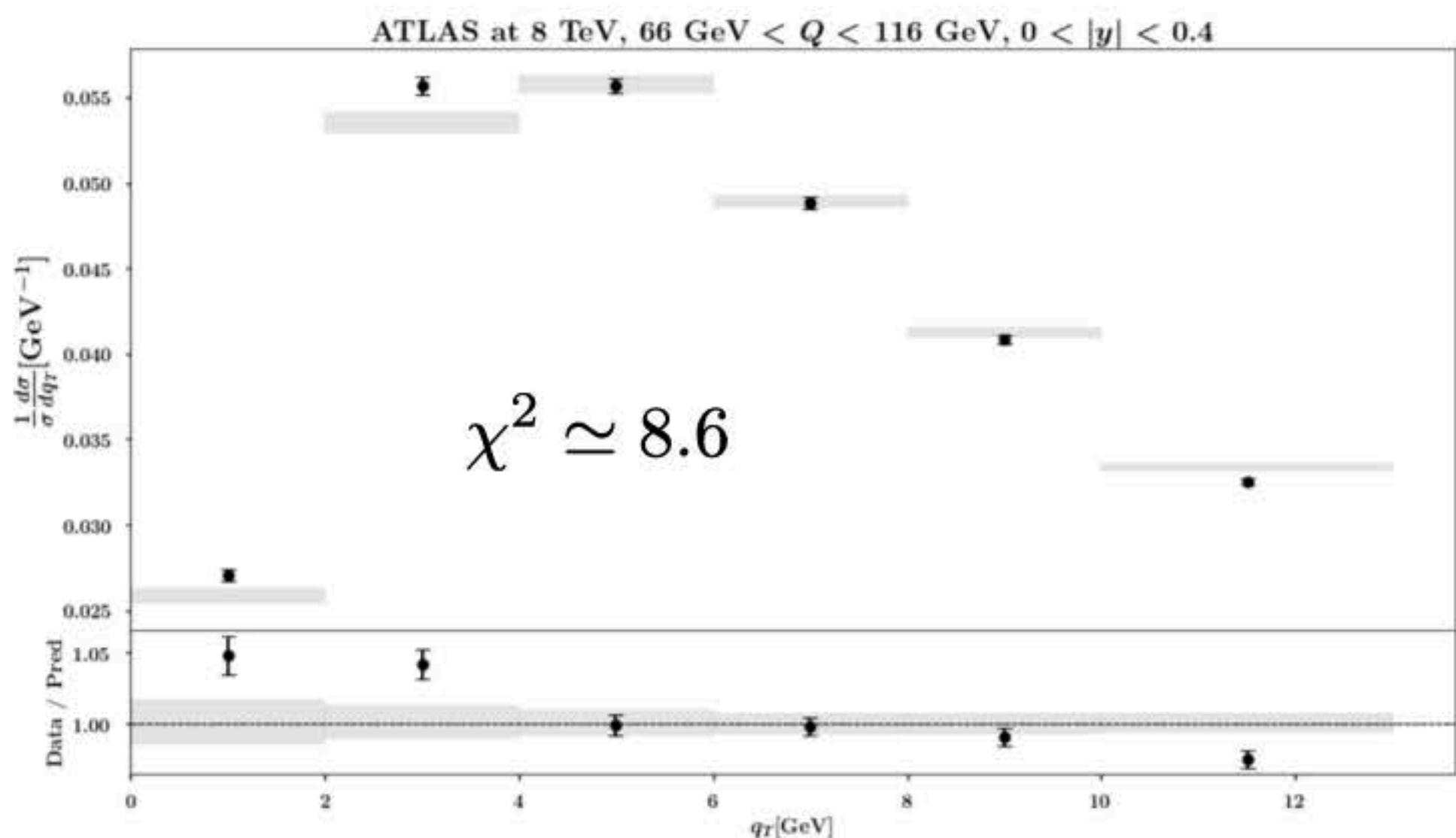
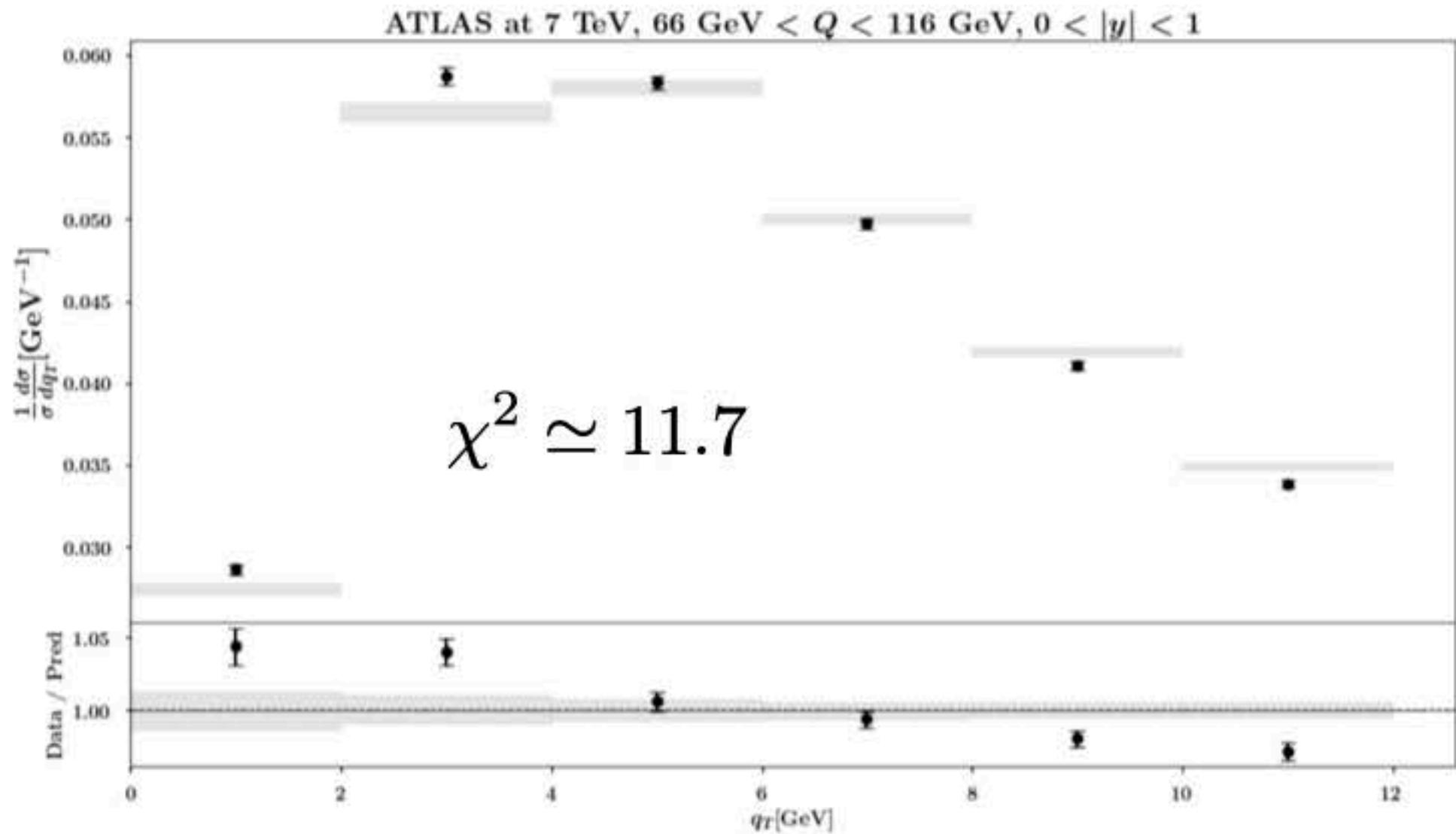
## Results of the baseline fit

Global  $\chi^2 = 1.00$

# Results of the baseline fit



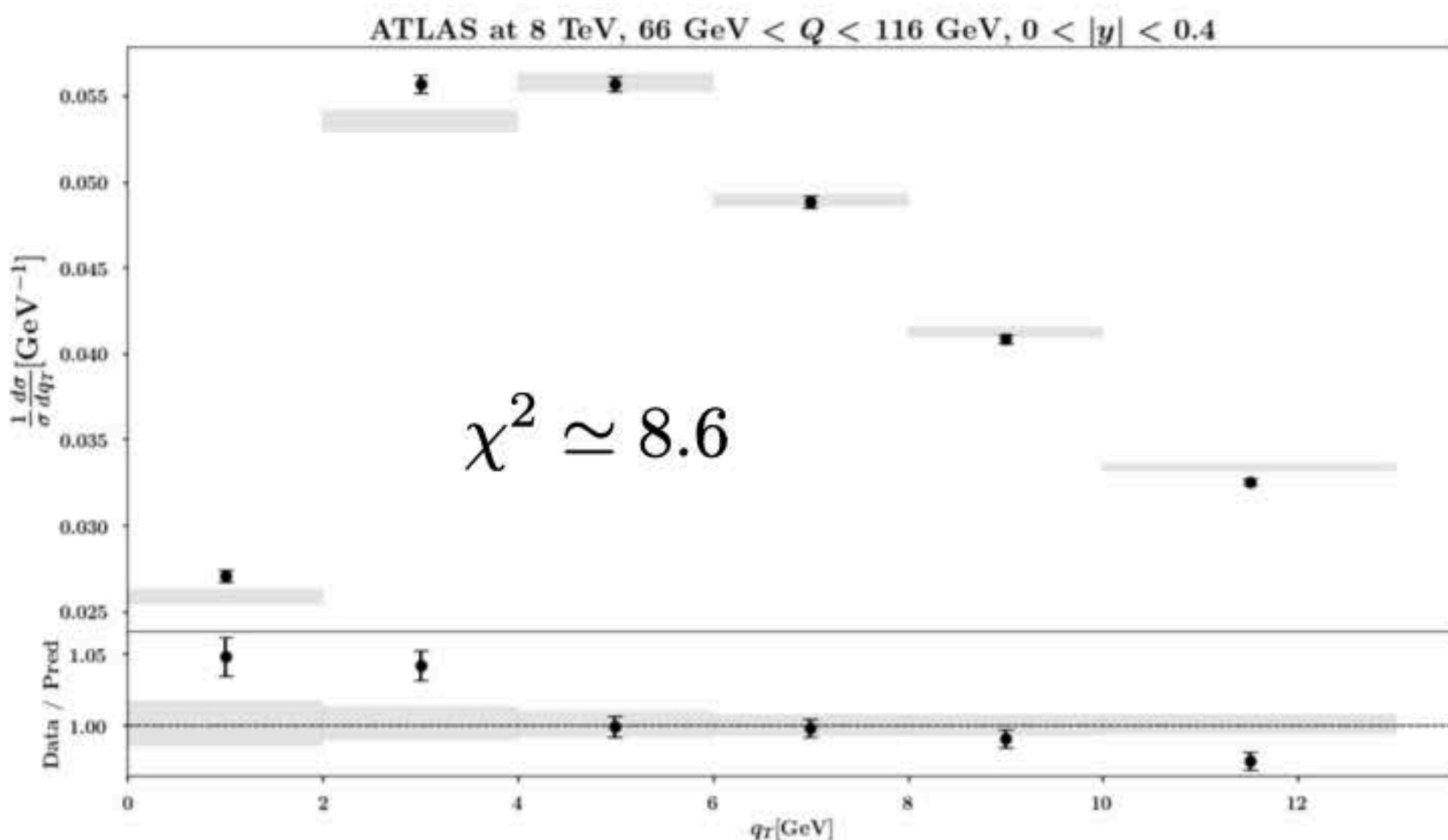
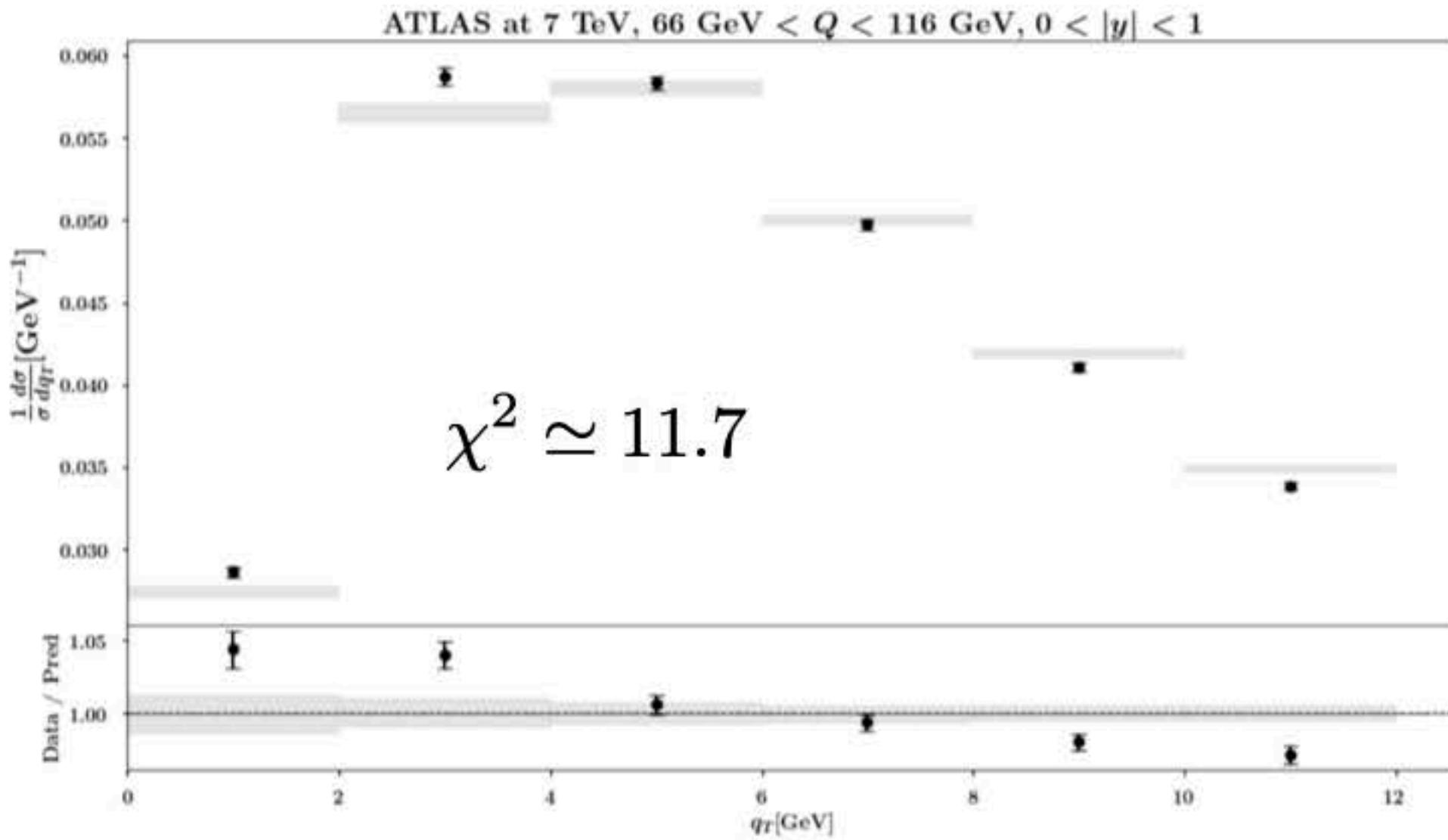
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# Results of the baseline fit

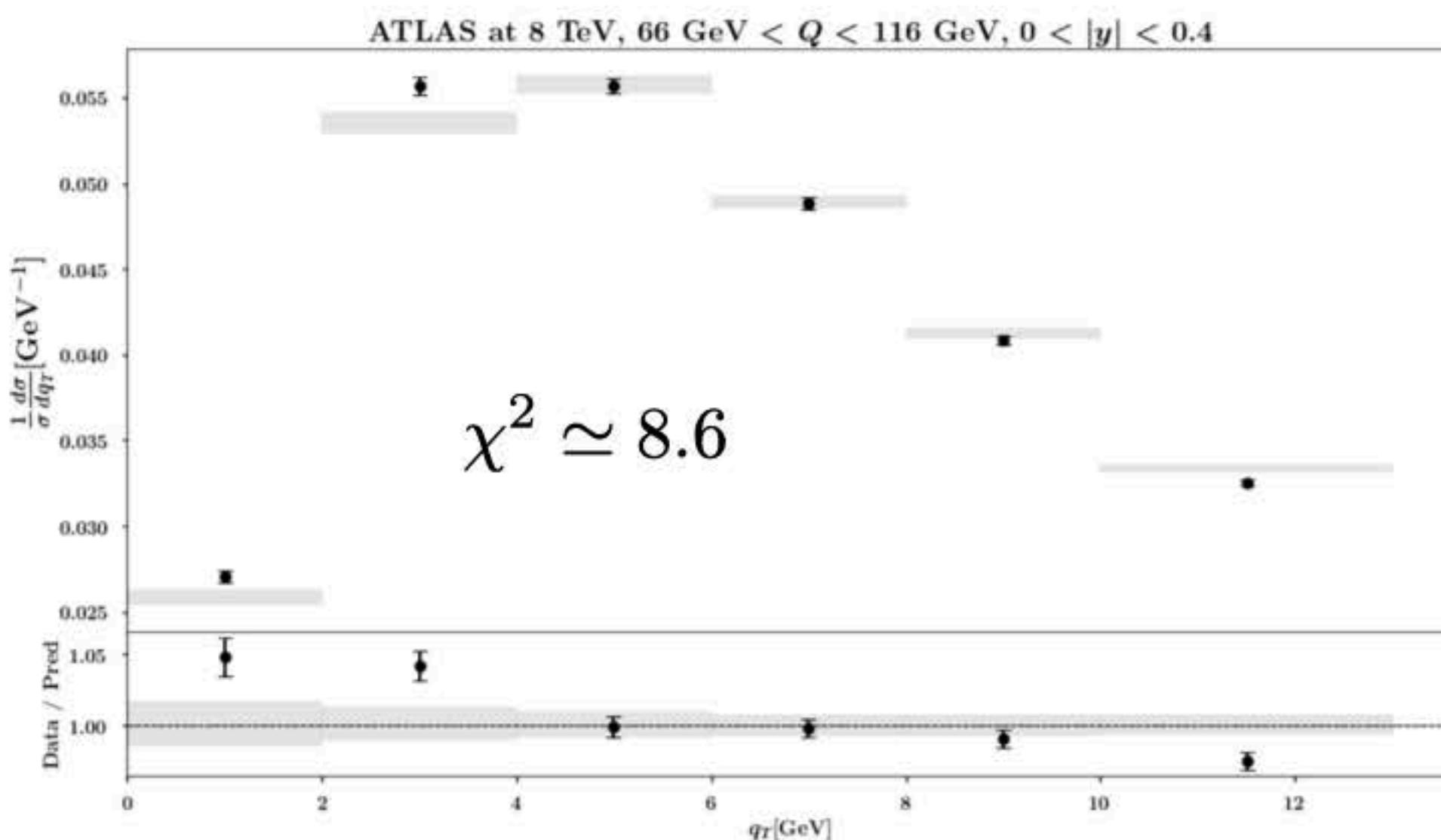
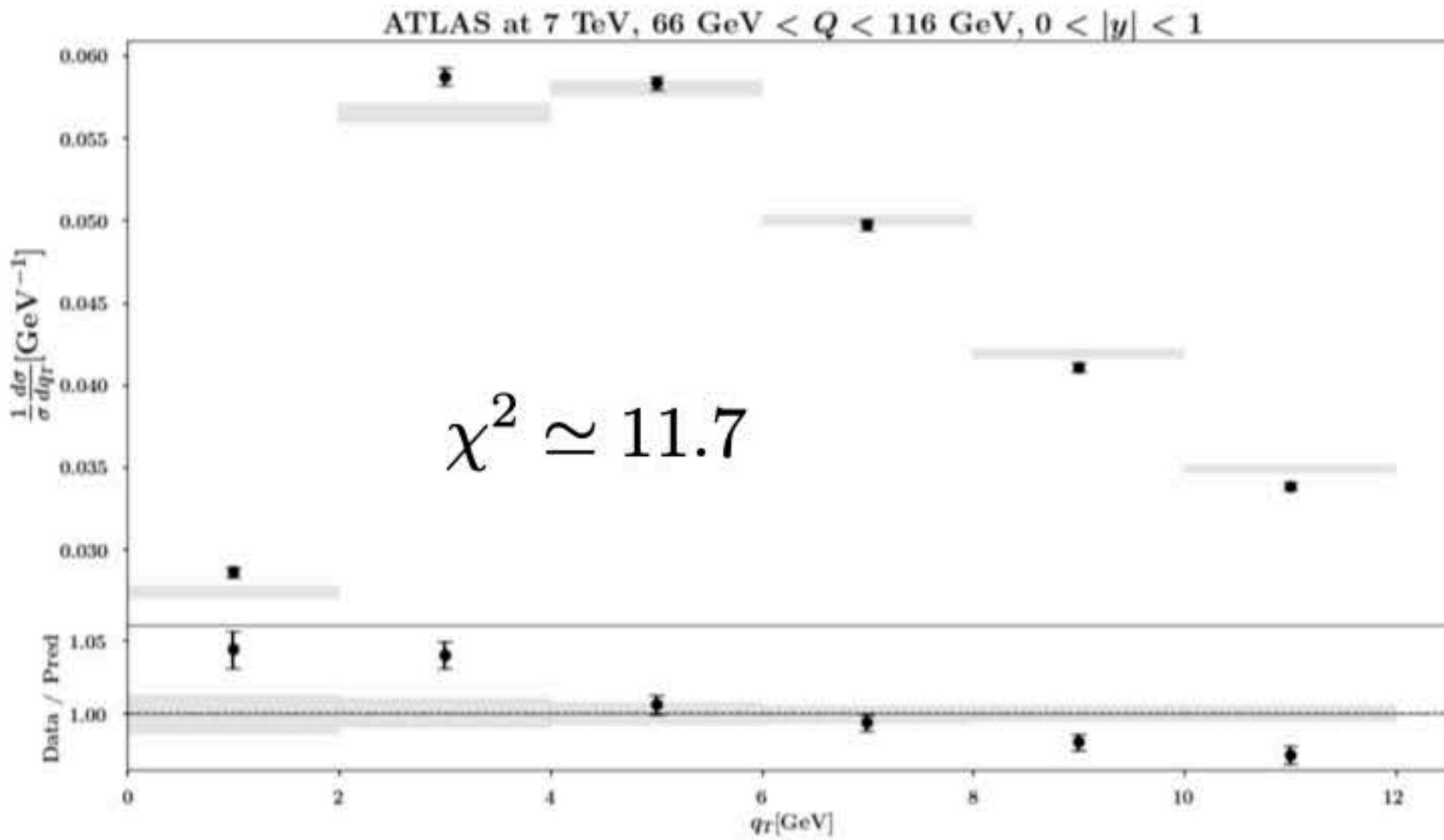


Global  $\chi^2 = 1.00$

Possible justifications:



# Results of the baseline fit



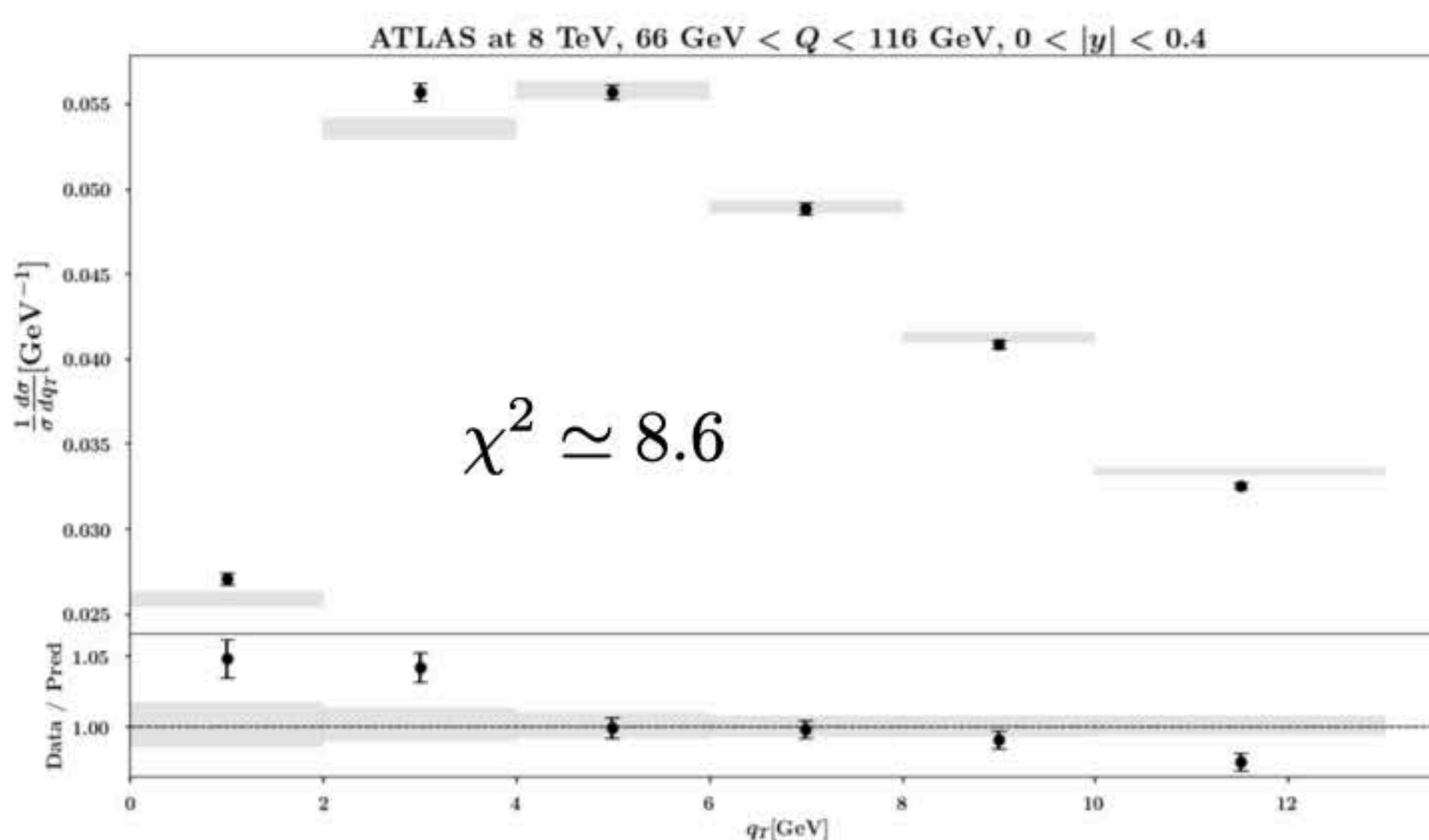
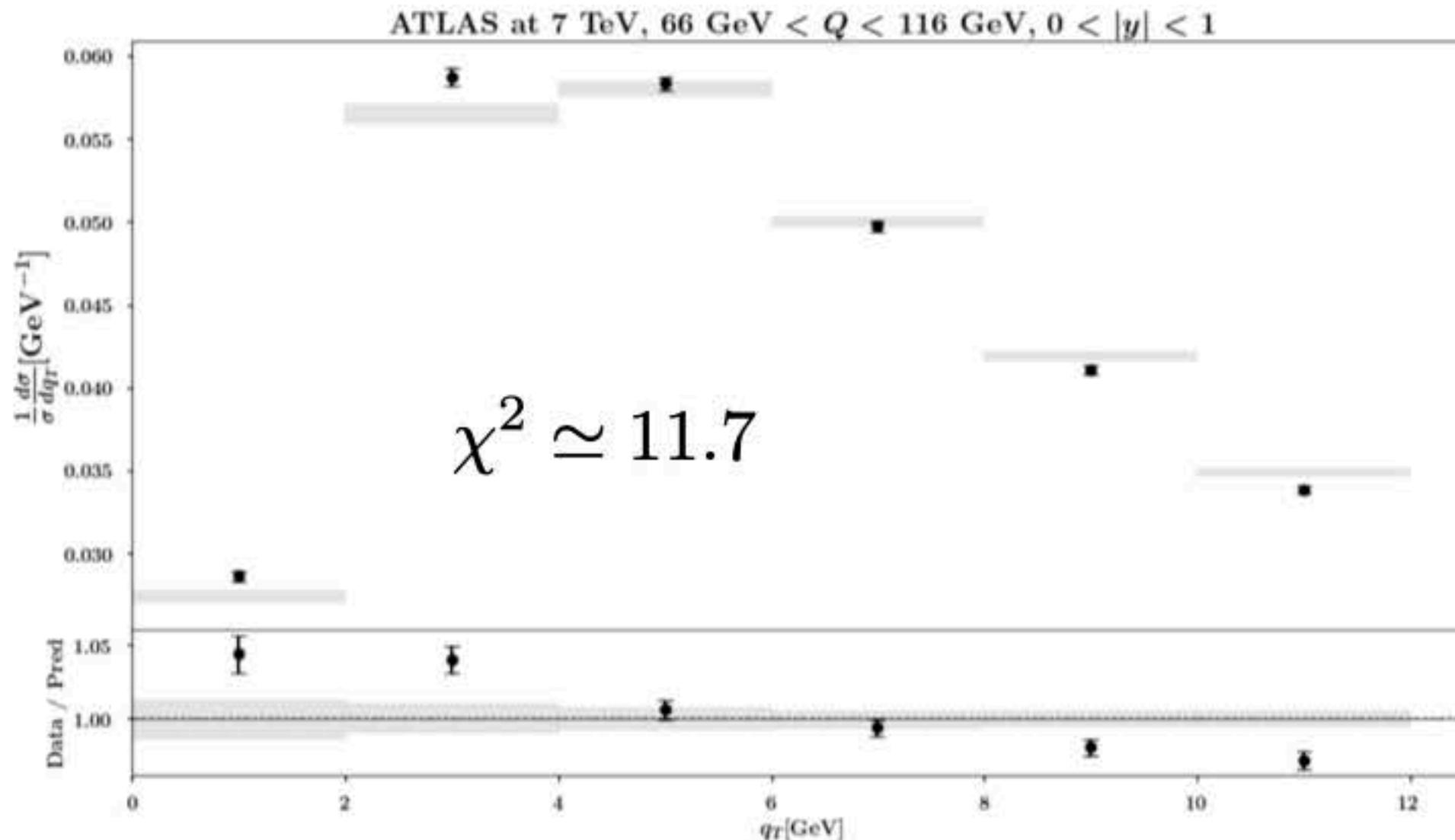
Global  $\chi^2 = 1.00$

Possible justifications:

- ⊖ Small experimental uncertainties



# Results of the baseline fit



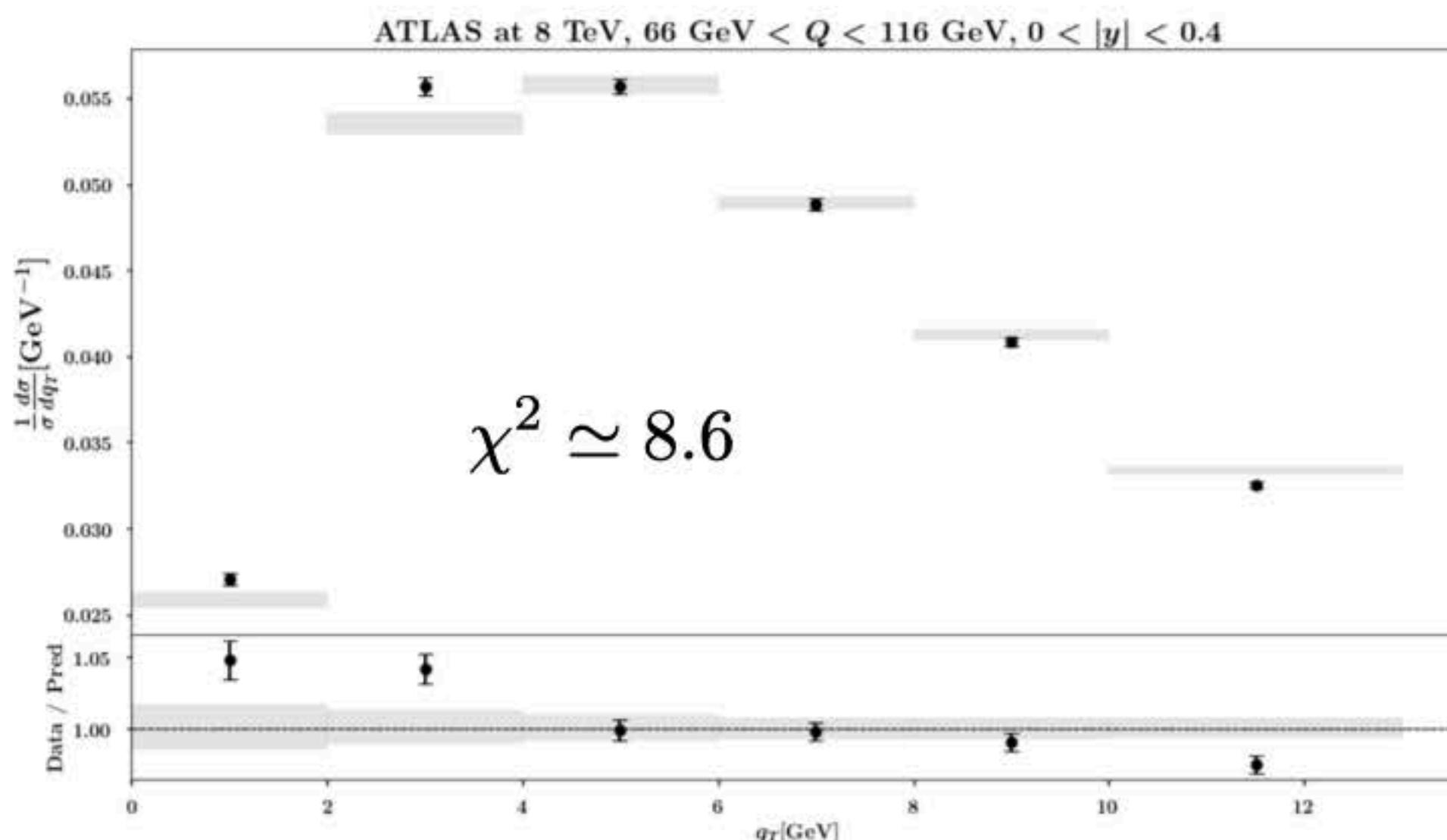
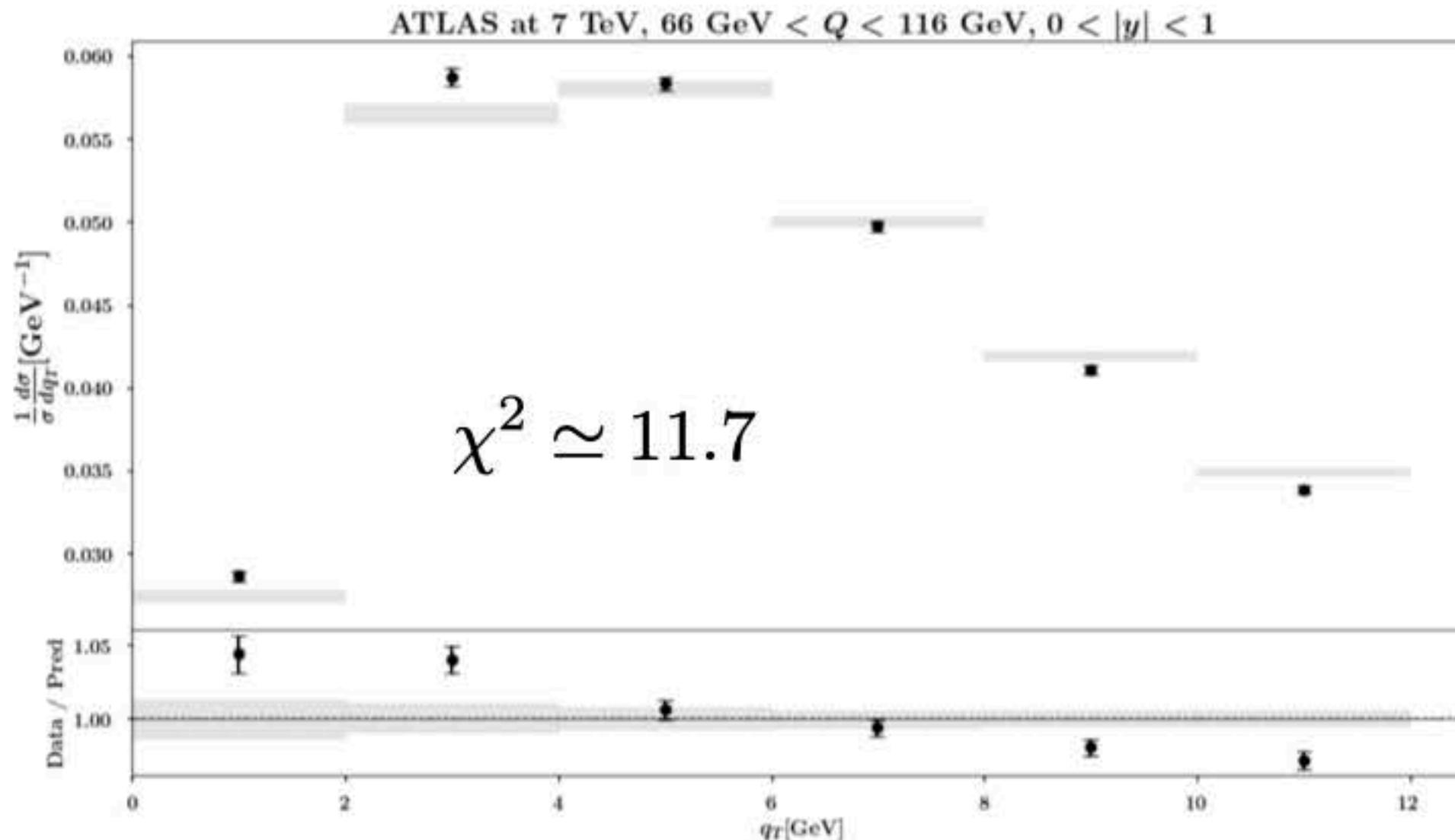
Global  $\chi^2 = 1.00$

Possible justifications:

- ➡ Small experimental uncertainties
- ➡ Approximation of lepton cuts



# Results of the baseline fit



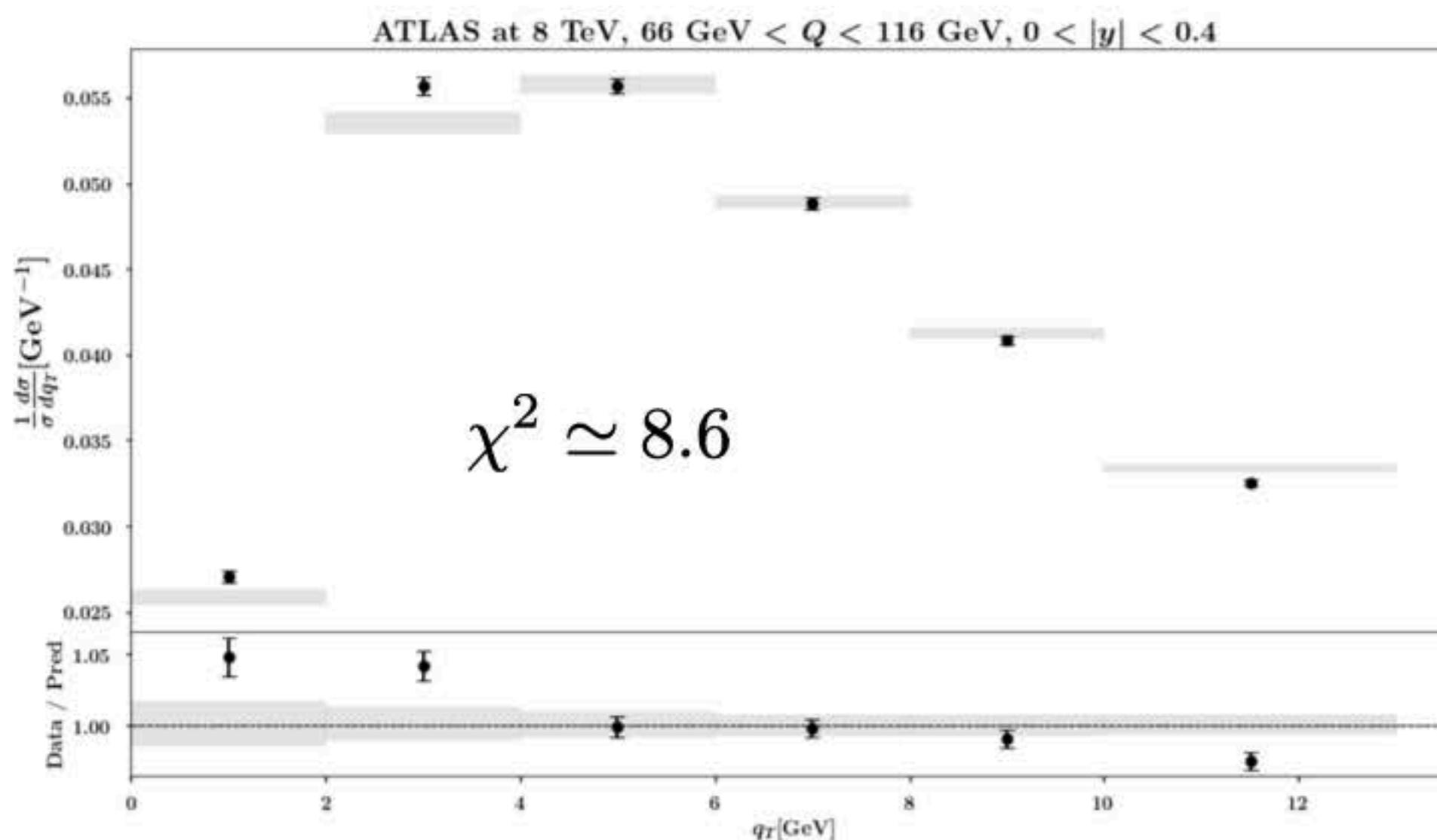
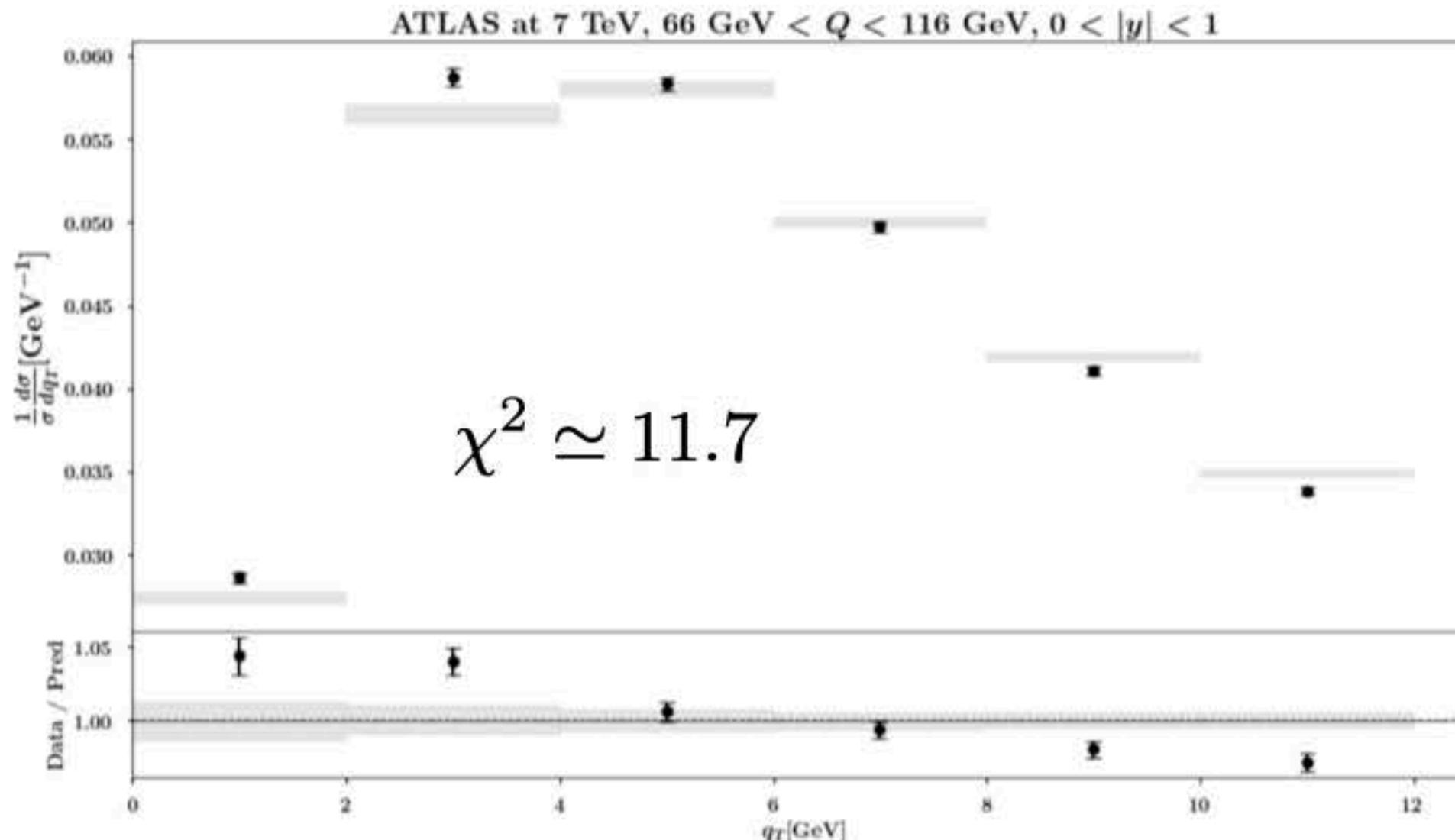
Global  $\chi^2 = 1.00$

## Possible justifications:

- ➡ Small experimental uncertainties
- ➡ Approximation of lepton cuts
- ➡ Effects of power corrections



# Results of the baseline fit



Global  $\chi^2 = 1.00$

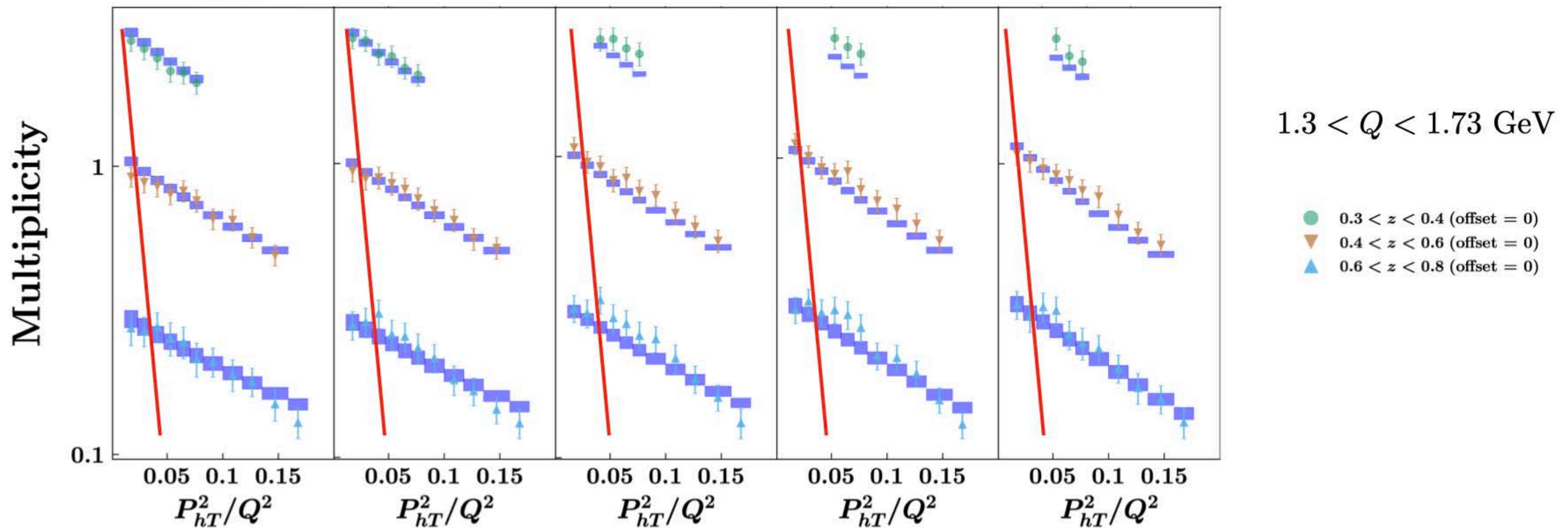
## Possible justifications:

- ➡ Small experimental uncertainties
- ➡ Approximation of lepton cuts
- ➡ Effects of power corrections
- ➡ Effects of the matching between perturbative and non-perturbative physics



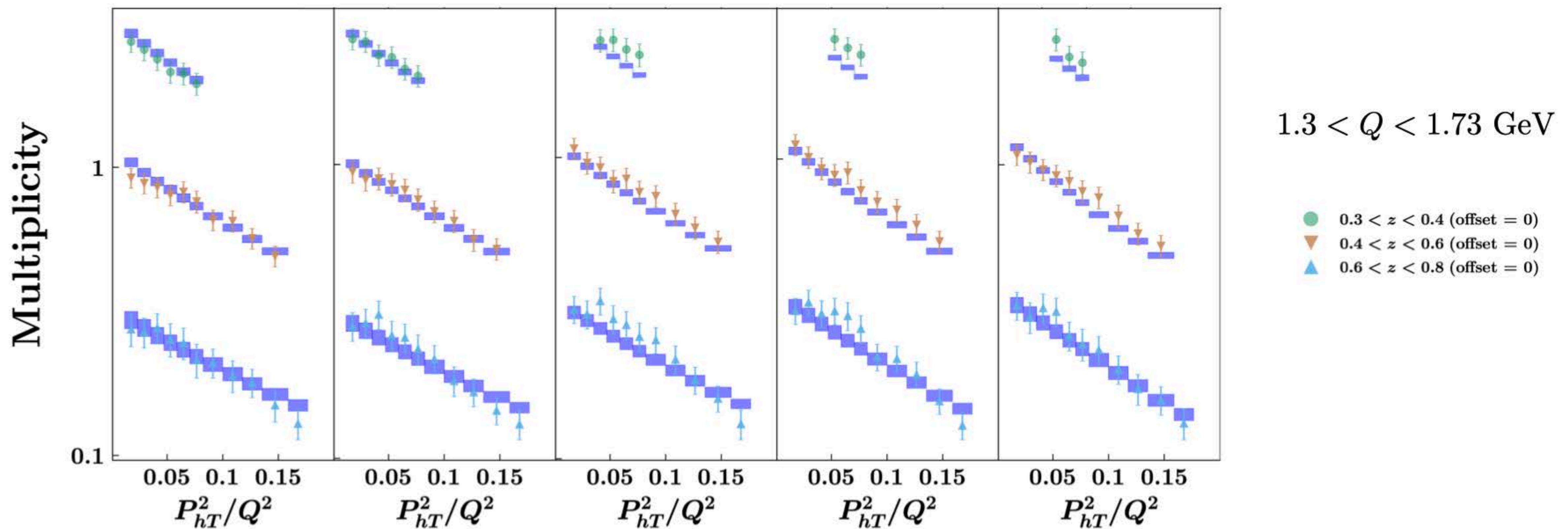
# Results of the baseline fit

$$\left. \frac{P_{hT}}{zQ} \right|_{\max} = 0.25$$



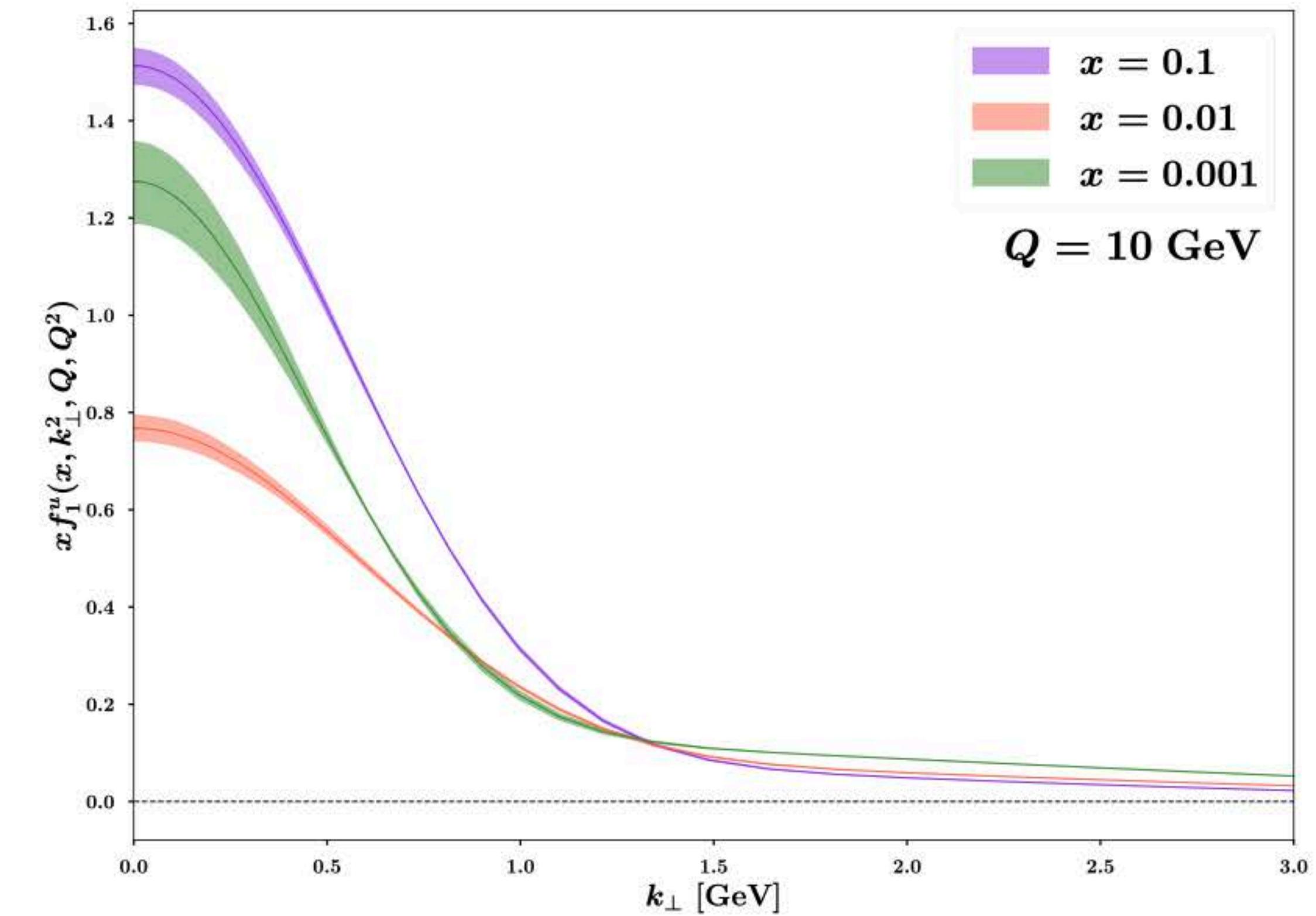
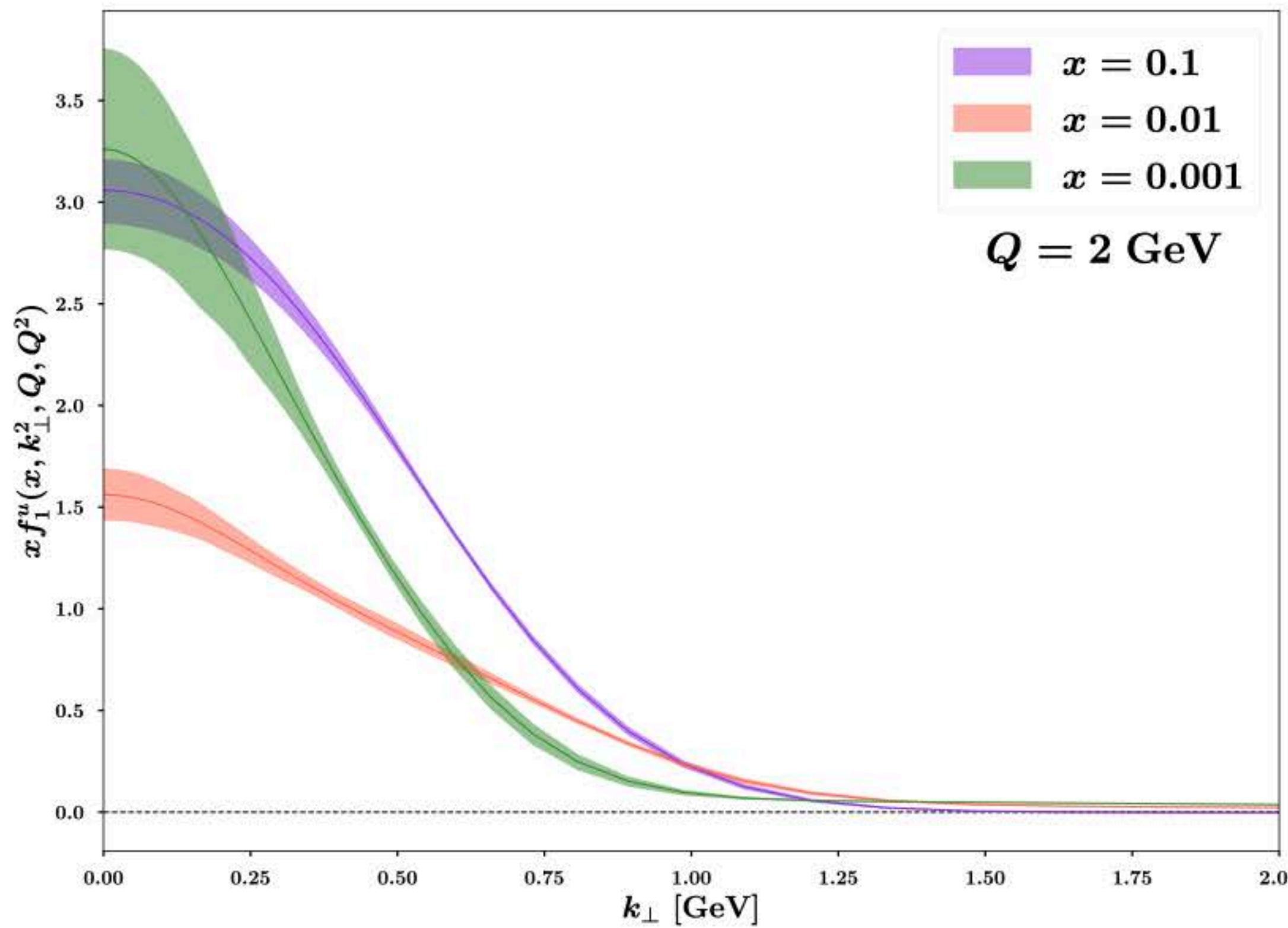
# Results of the baseline fit

$$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$



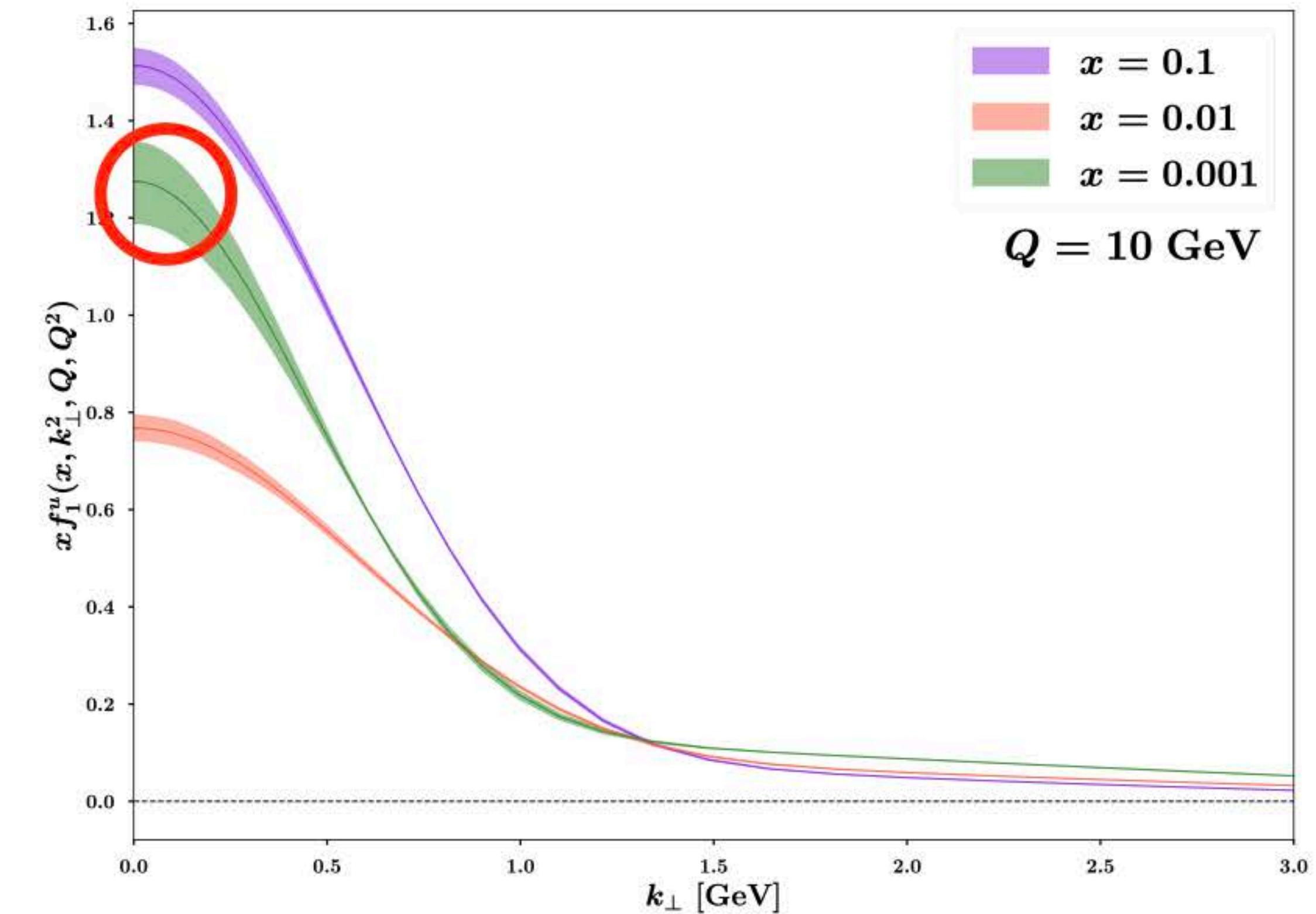
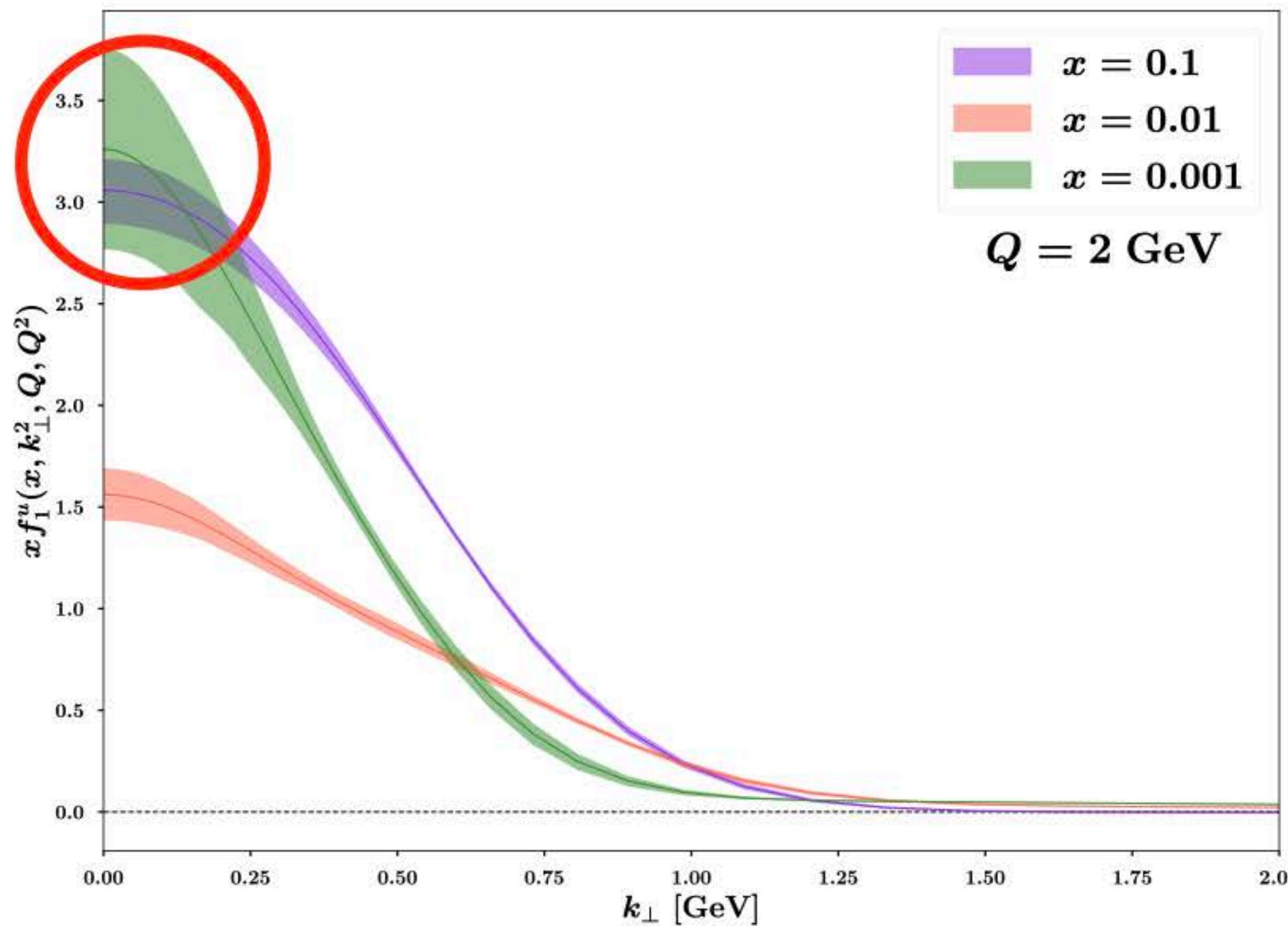
# Results of the baseline fit

## Visualisation of TMD PDFs



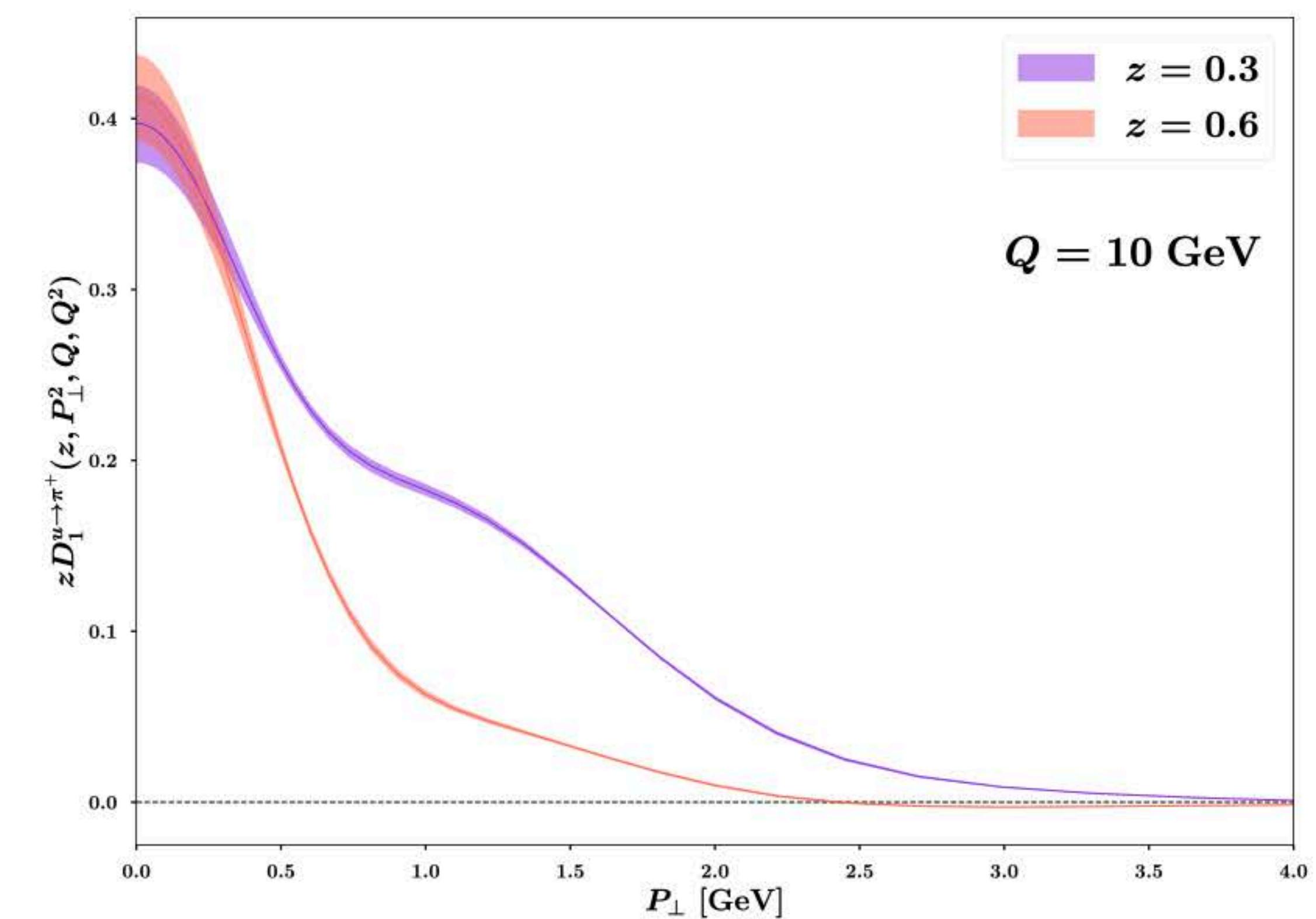
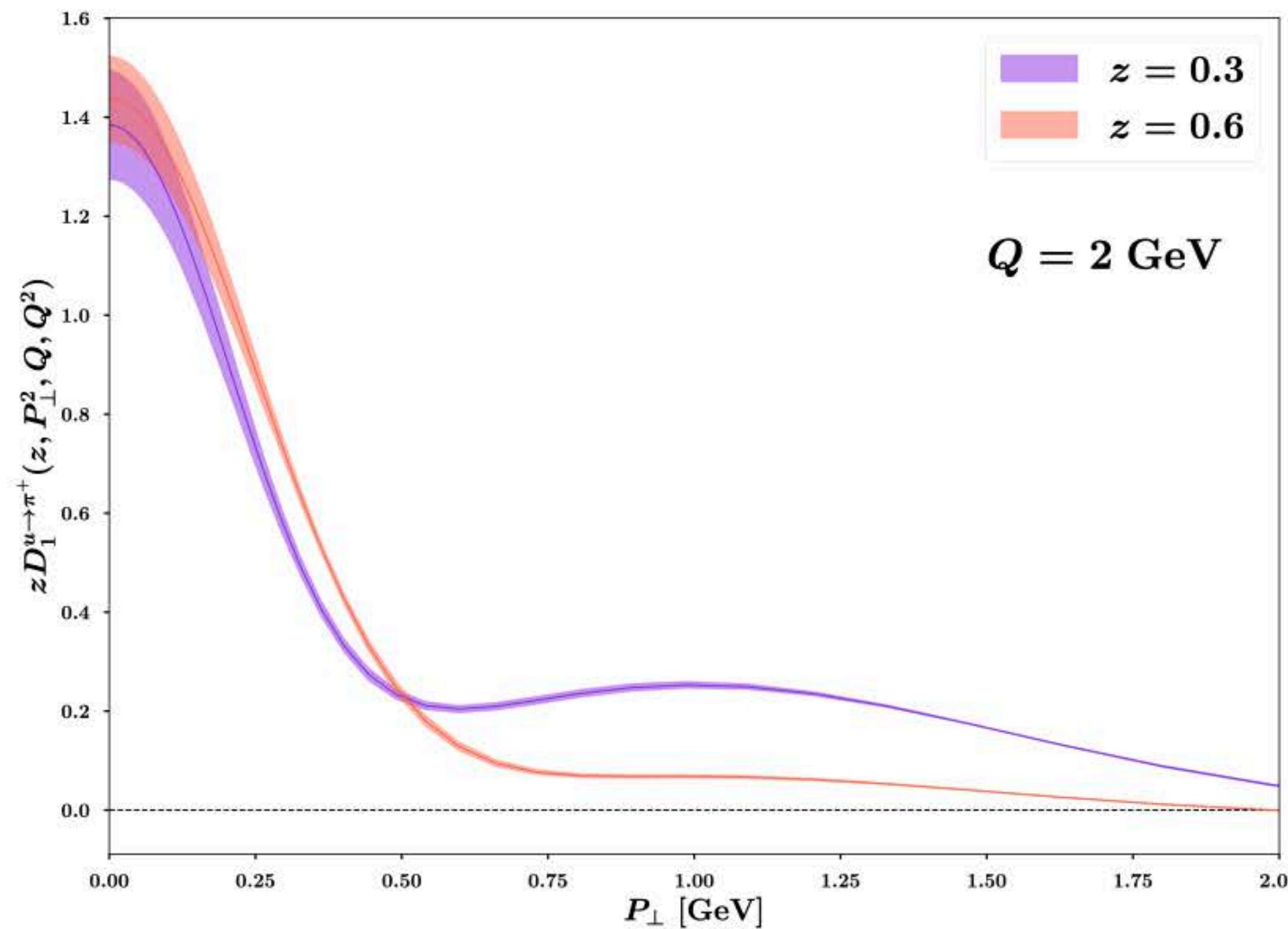
# Results of the baseline fit

## Visualisation of TMD PDFs



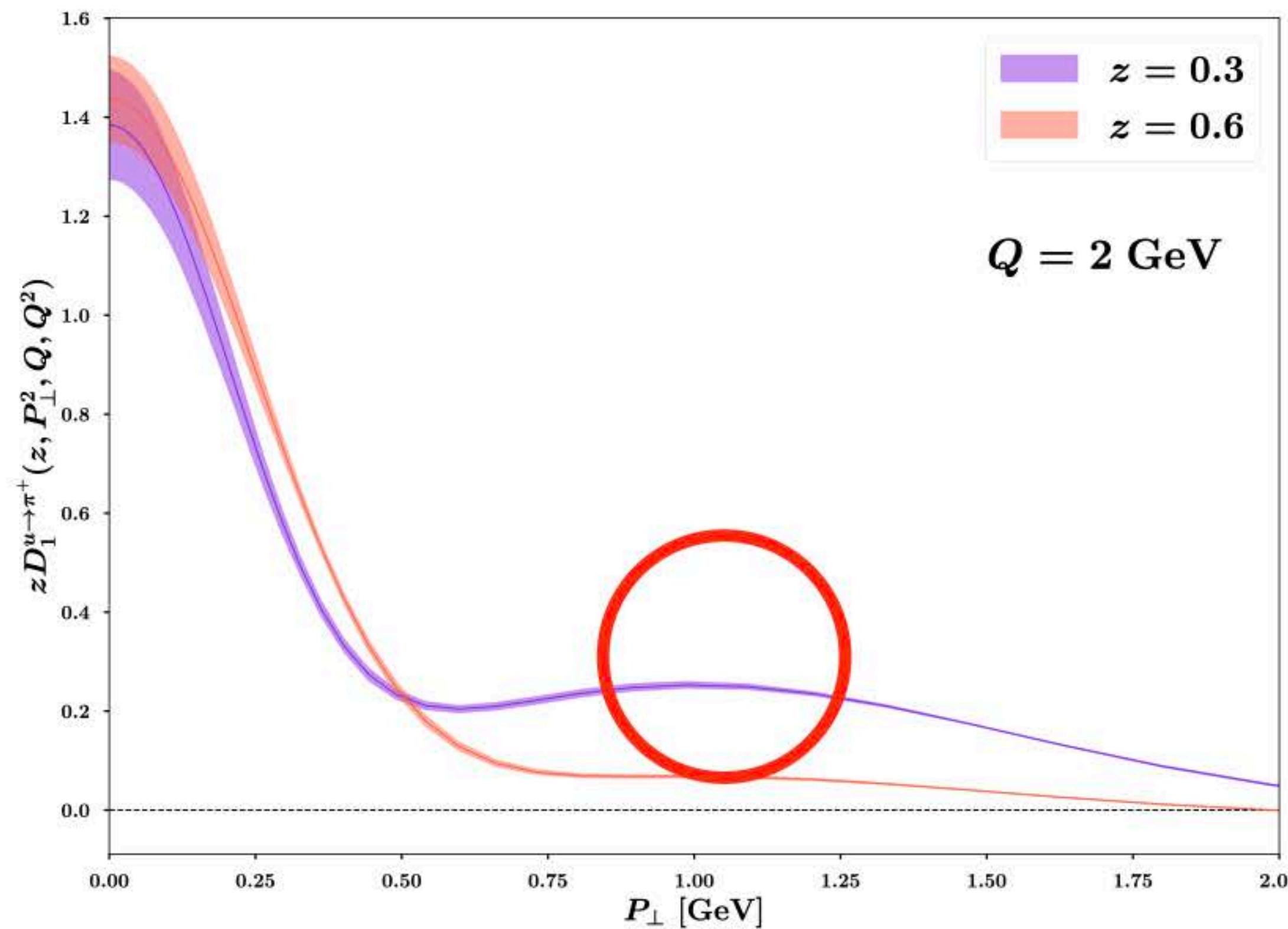
# Results of the baseline fit

## Visualisation of TMD FFs

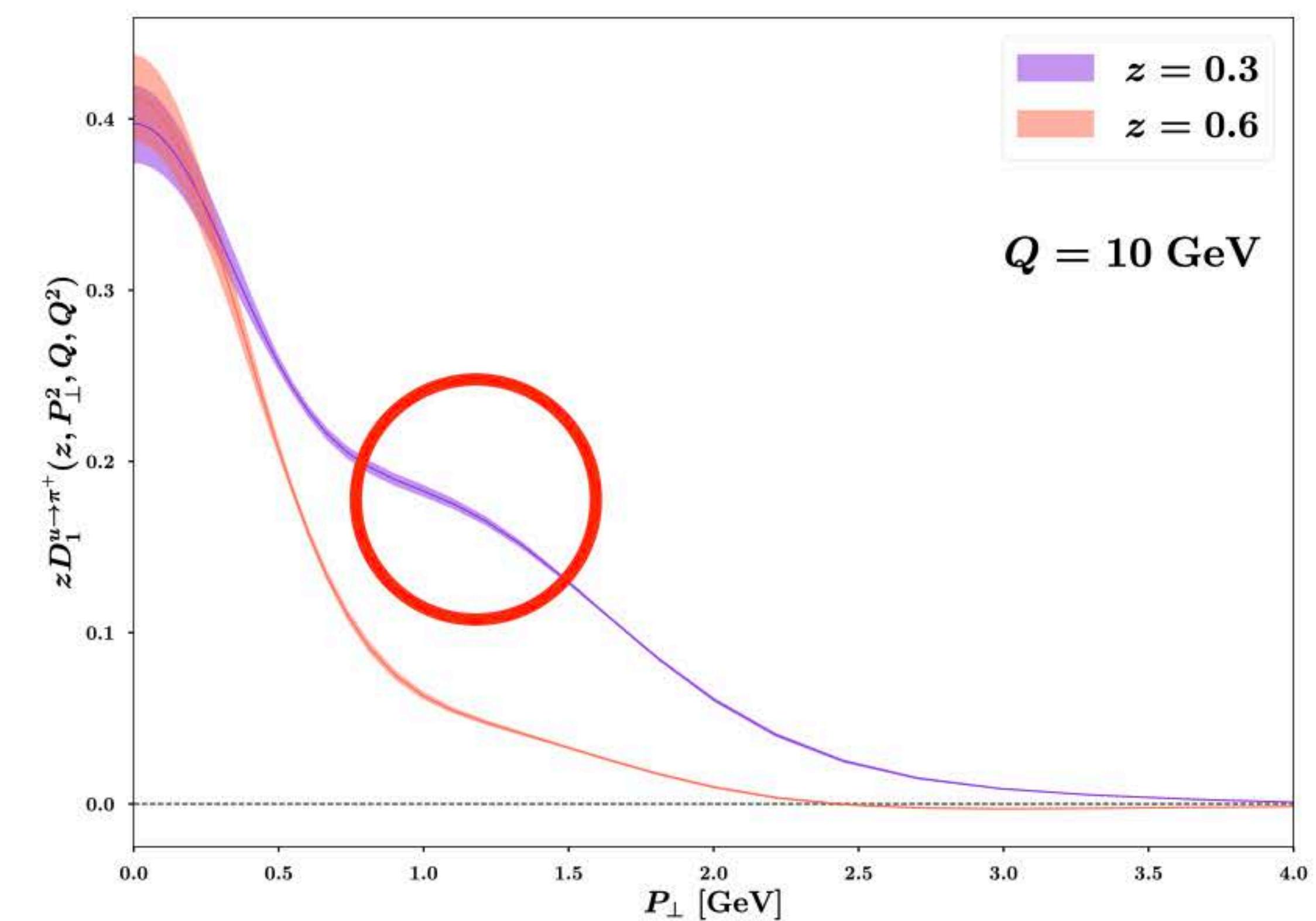


# Results of the baseline fit

## Visualisation of TMD FFs



$Q = 2 \text{ GeV}$

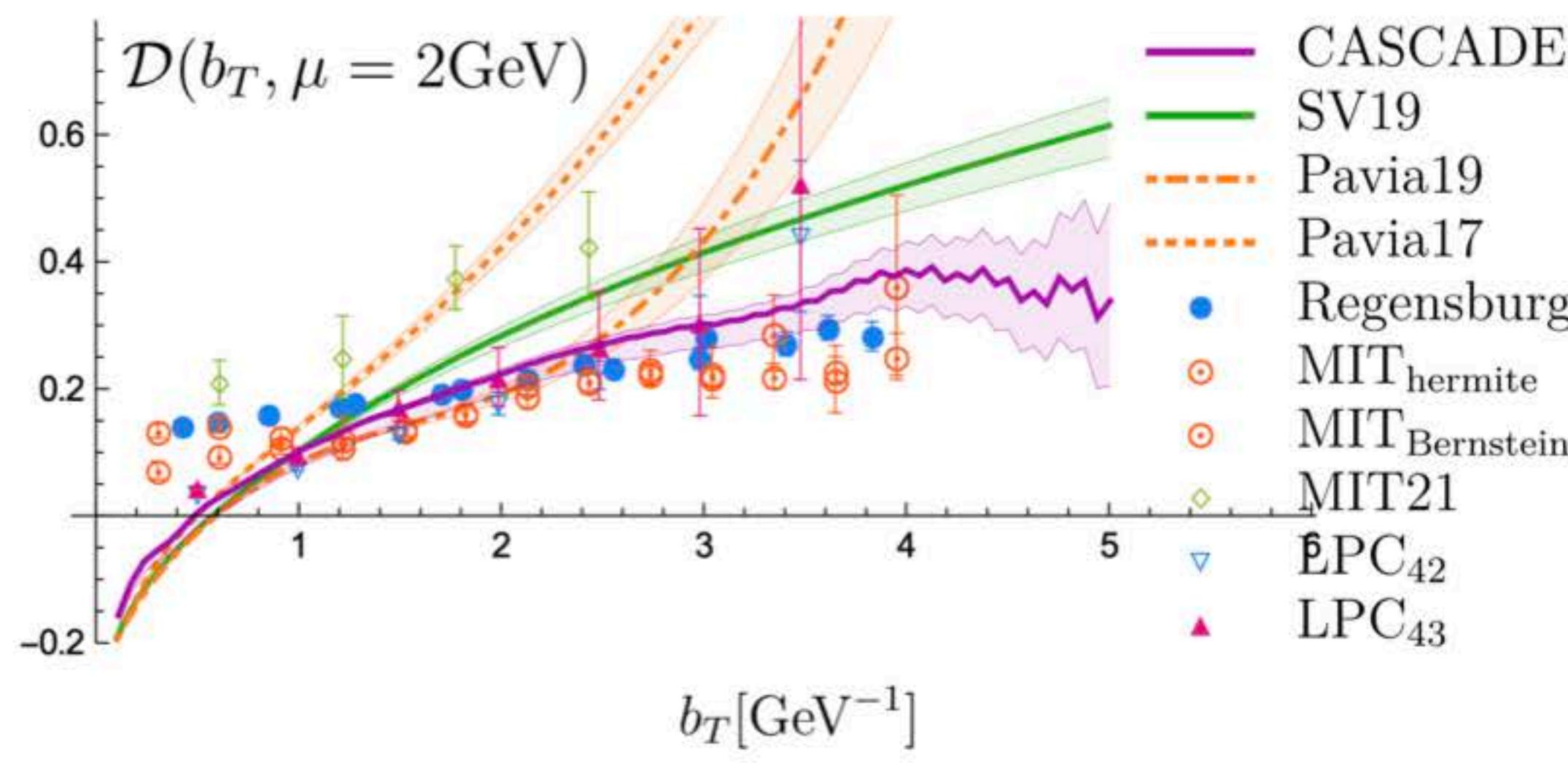


$Q = 10 \text{ GeV}$

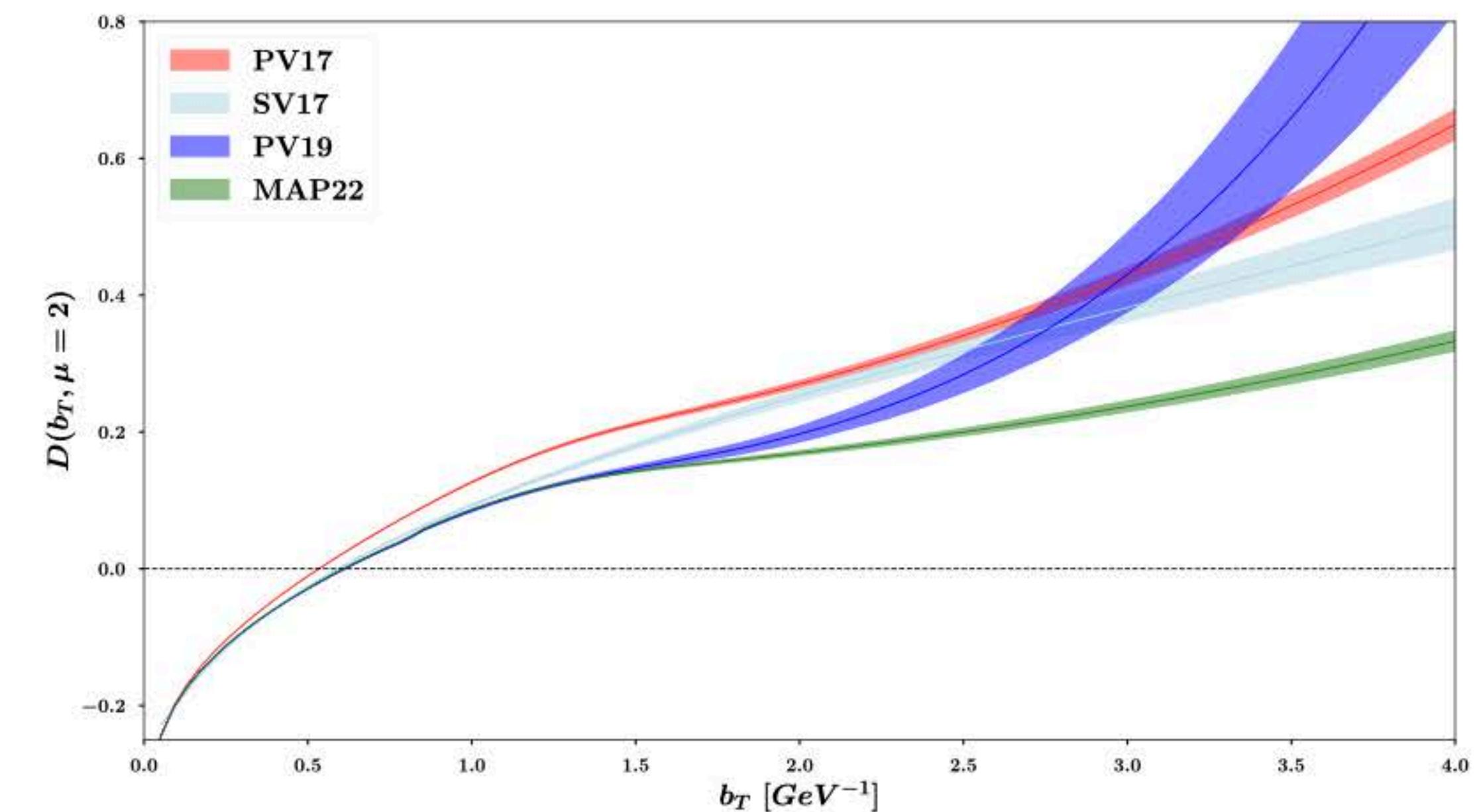
# Results of the baseline fit

## Collins-Soper Kernel

A. Vladimirov talk (CPHI 2022)



MAP Collaboration results



# Summary

**MAP22 GLOBAL FIT - A new extraction of quark TMDs**

*In preparation*

# Summary

**MAP22 GLOBAL FIT - A new extraction of quark TMDs**

*In preparation*

- Global analysis of Drell-Yan and Semi-Inclusive DIS data sets: **2031** data points
- Perturbative accuracy:  **$N^3LL^-$**
- **Normalisation** of SIDIS multiplicities beyond NLL
- Number of fitted parameters: **21**
- Extremely good description:  $\chi^2/N_{data} \simeq 1.00$

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# **BACKUP SLIDES**

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# Logarithmic Accuracy

	Sudakov form factor	Matching coefficient
LL	$\alpha_S^n \ln^{2n} \left( \frac{Q^2}{\mu_b^2} \right)$	$\tilde{C}^0$
NLL	$\alpha_S^n \ln^{2n} \left( \frac{Q^2}{\mu_b^2} \right), \quad \alpha_S^n \ln^{2n-1} \left( \frac{Q^2}{\mu_b^2} \right)$	$\tilde{C}^0$
NLL'	$\alpha_S^n \ln^{2n} \left( \frac{Q^2}{\mu_b^2} \right), \quad \alpha_S^n \ln^{2n-1} \left( \frac{Q^2}{\mu_b^2} \right)$	$(\tilde{C}^0 + \alpha_S \tilde{C}^1)$

the difference between the two is NNLL:  $\alpha_S^n \ln^{2n-2} \left( \frac{Q^2}{\mu_b^2} \right)$

# Non-mixed terms in collinear SIDIS cross section

$$\begin{aligned} \left. \frac{d\sigma^h}{dx dQ^2 dz} \right|_{O(\alpha_s^1)} = & \sigma_0 \sum_{ff'} \frac{e_f^2}{z^2} (\delta_{f'f} + \delta_{f'g}) \frac{\alpha_s}{\pi} \left\{ \left[ D_1^{h/f'} \otimes C_1^{f'f} \otimes f_1^{f/N} \right] (x, z, Q) \right. \\ & \left. + \frac{1-y}{1+(1-y)^2} \left[ D_1^{h/f'} \otimes C_L^{f'f} \otimes f_1^{f/N} \right] (x, z, Q) \right\}, \end{aligned}$$

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$$\begin{aligned} C_1^{qq} = & \frac{C_F}{2} \left\{ -8\delta(1-x)\delta(1-z) \right. \\ & + \delta(1-x) \left[ P_{qq}(z) \ln \frac{Q^2}{\mu_F^2} + L_1(z) + L_2(z) + (1-z) \right] \\ & + \delta(1-z) \left[ P_{qq}(x) \ln \frac{Q^2}{\mu^2} + L_1(x) - L_2(x) + (1-x) \right] \\ & \left. + 2 \frac{1}{(1-x)_+} \frac{1}{(1-z)_+} - \frac{1+z}{(1-x)_+} - \frac{1+x}{(1-z)_+} + 2(1+xz) \right\}, \end{aligned}$$

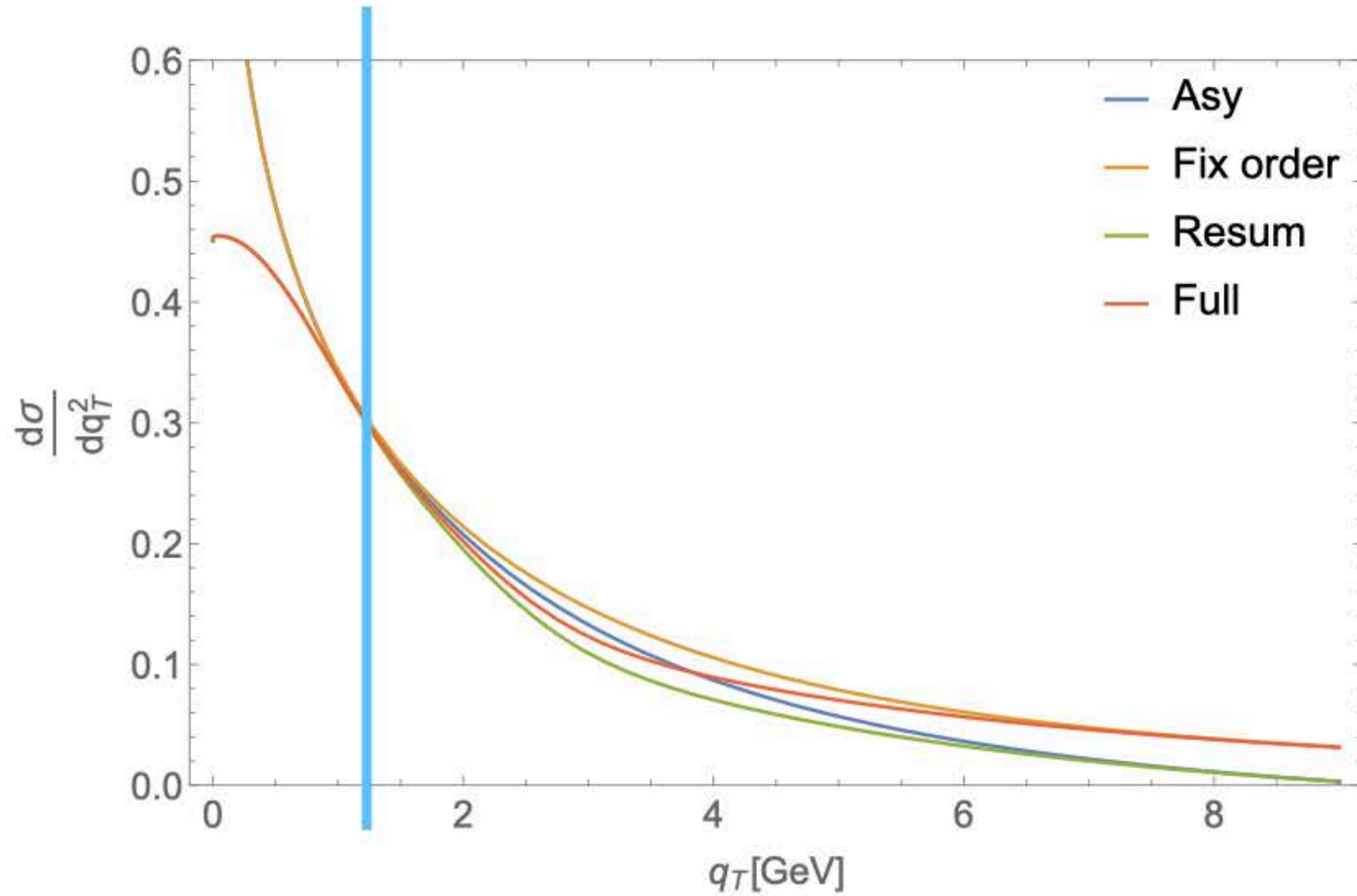
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$$C_1^{qq} = \frac{C_F}{2} \left\{ -8\delta(1-x)\delta(1-z) \right. \\ + \delta(1-x) \left[ P_{qq}(z) \ln \frac{Q^2}{\mu_F^2} + L_1(z) + L_2(z) + (1-z) \right] \\ + \delta(1-z) \left[ P_{qq}(x) \ln \frac{Q^2}{\mu_F^2} + L_1(x) - L_2(x) + (1-x) \right] \\ \left. + 2 \frac{1}{(1-x)_+} \frac{1}{(1-z)_+} - \frac{1+z}{(1-x)_+} - \frac{1+x}{(1-z)_+} + 2(1+xz) \right\},$$

# Source of W-term suppression

Ideal situation at high Q

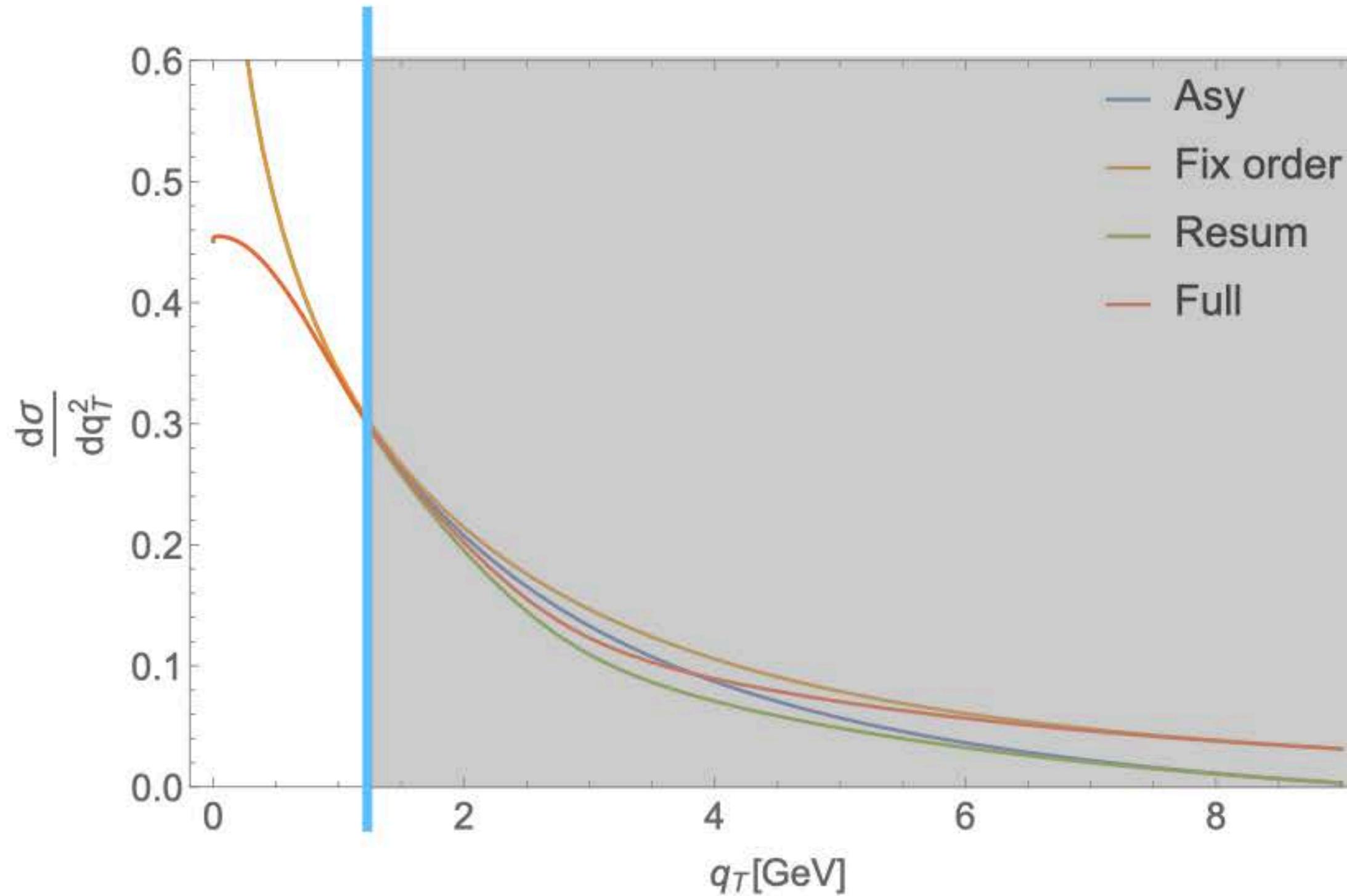


Standard approach

- Resummed contribution is dominant where the Asymptotic term is close to the Fixed Order

# Source of W-term suppression

Ideal situation at high Q

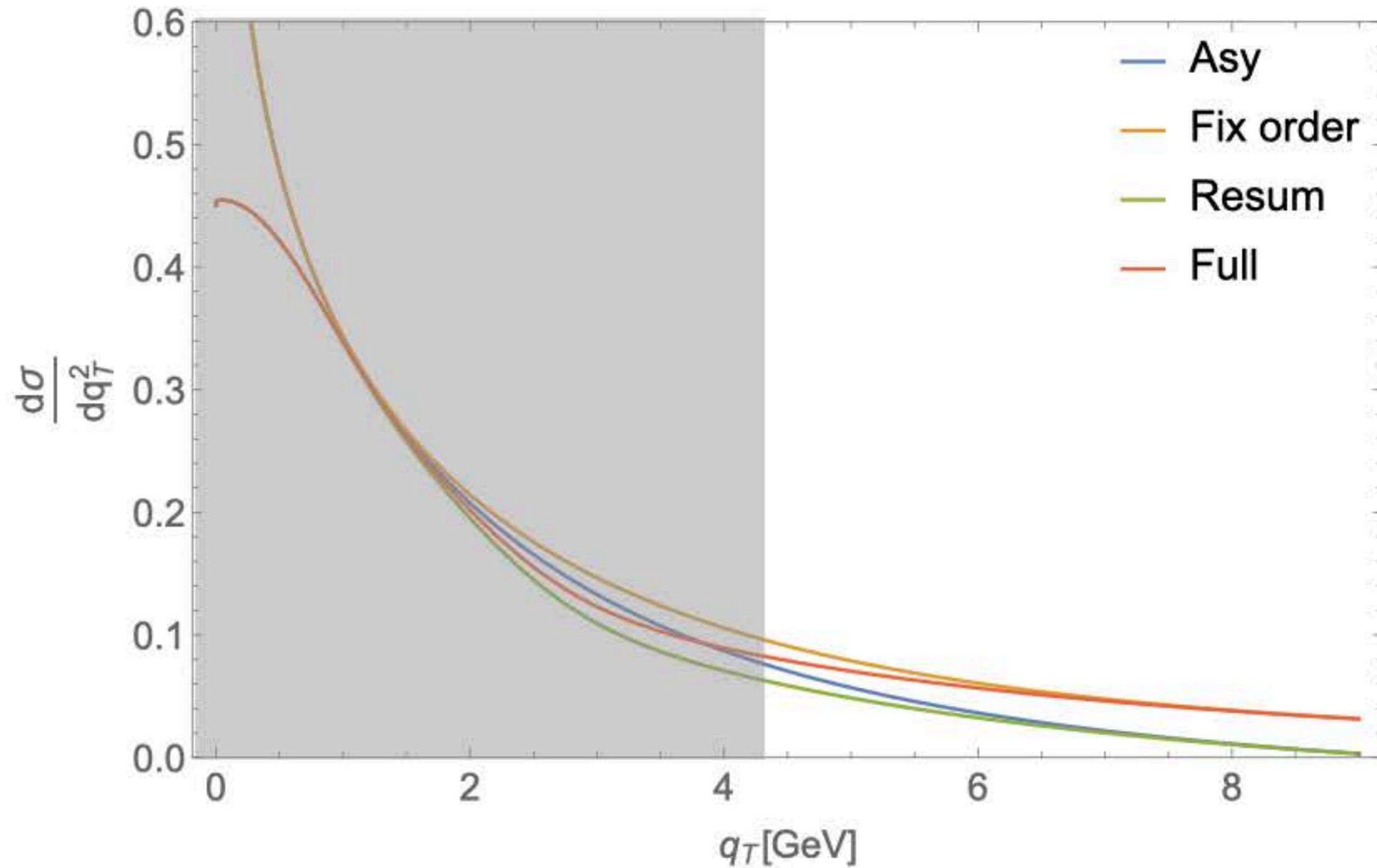


Standard approach

- Resummed contribution is dominant where the Asymptotic term is close to the Fixed Order → TMD Region

# Source of W-term suppression

Ideal situation at high Q

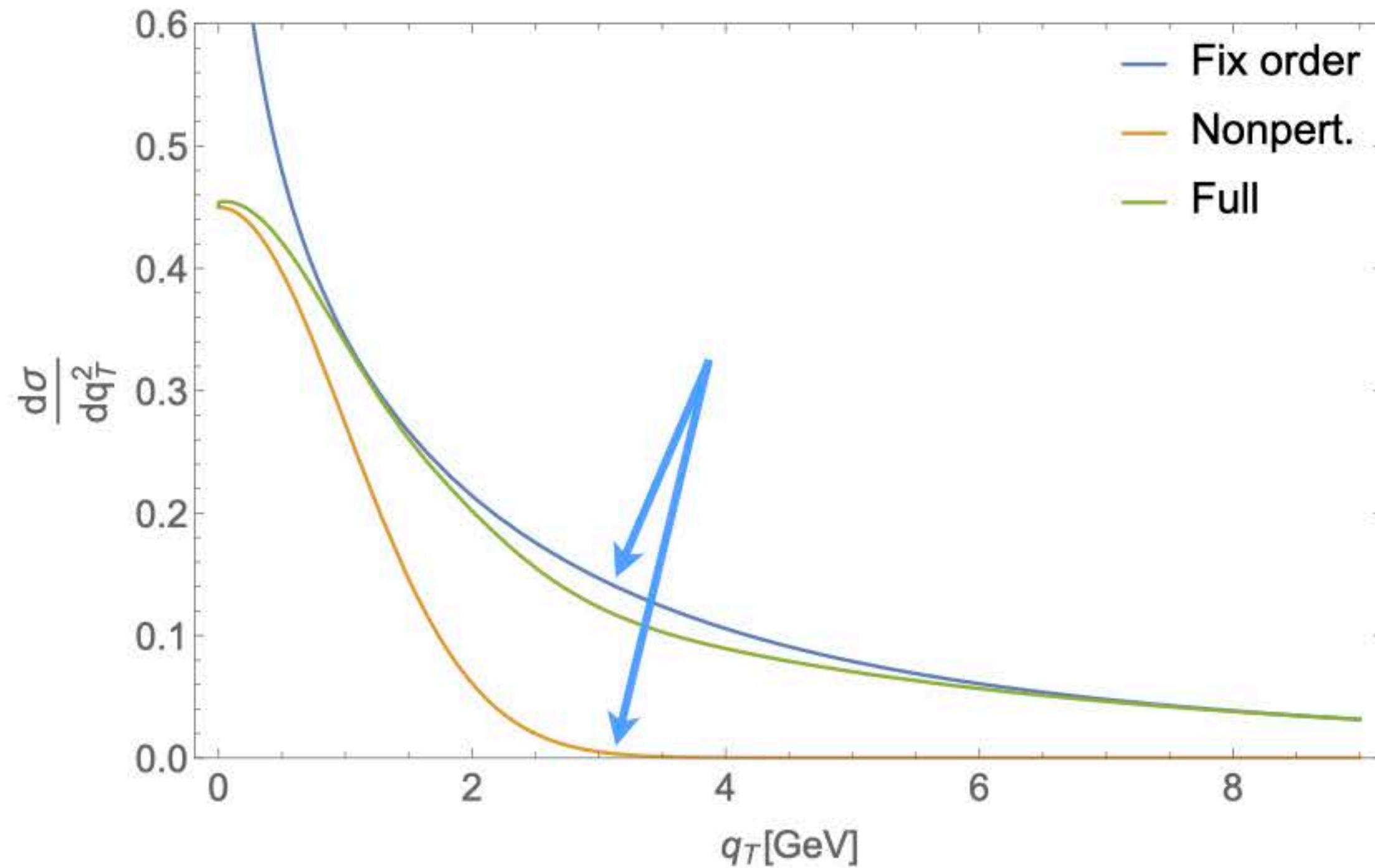


## Standard approach

- Resummed contribution is dominant where the Asymptotic term is close to the Fixed Order
- From a certain value of  $q_T$  the total cross section follows the Fixed Order term

# Source of W-term suppression

Ideal situation at high Q

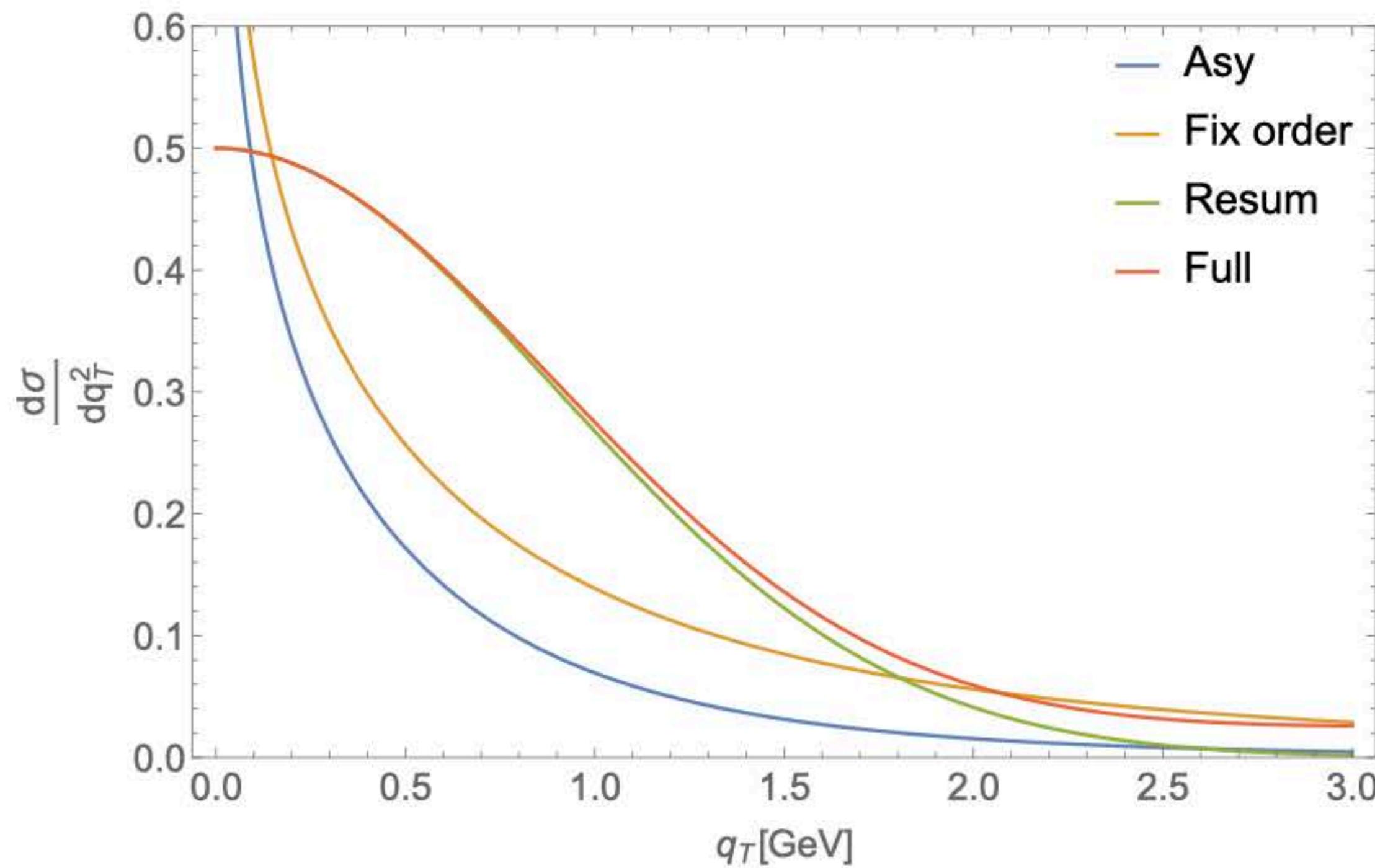


## Standard approach

- Collinear result is mostly given by the integral of the Fixed Order
- The Non-Perturbative term is only a small correction

# Source of W-term suppression

Ideal situation at low Q

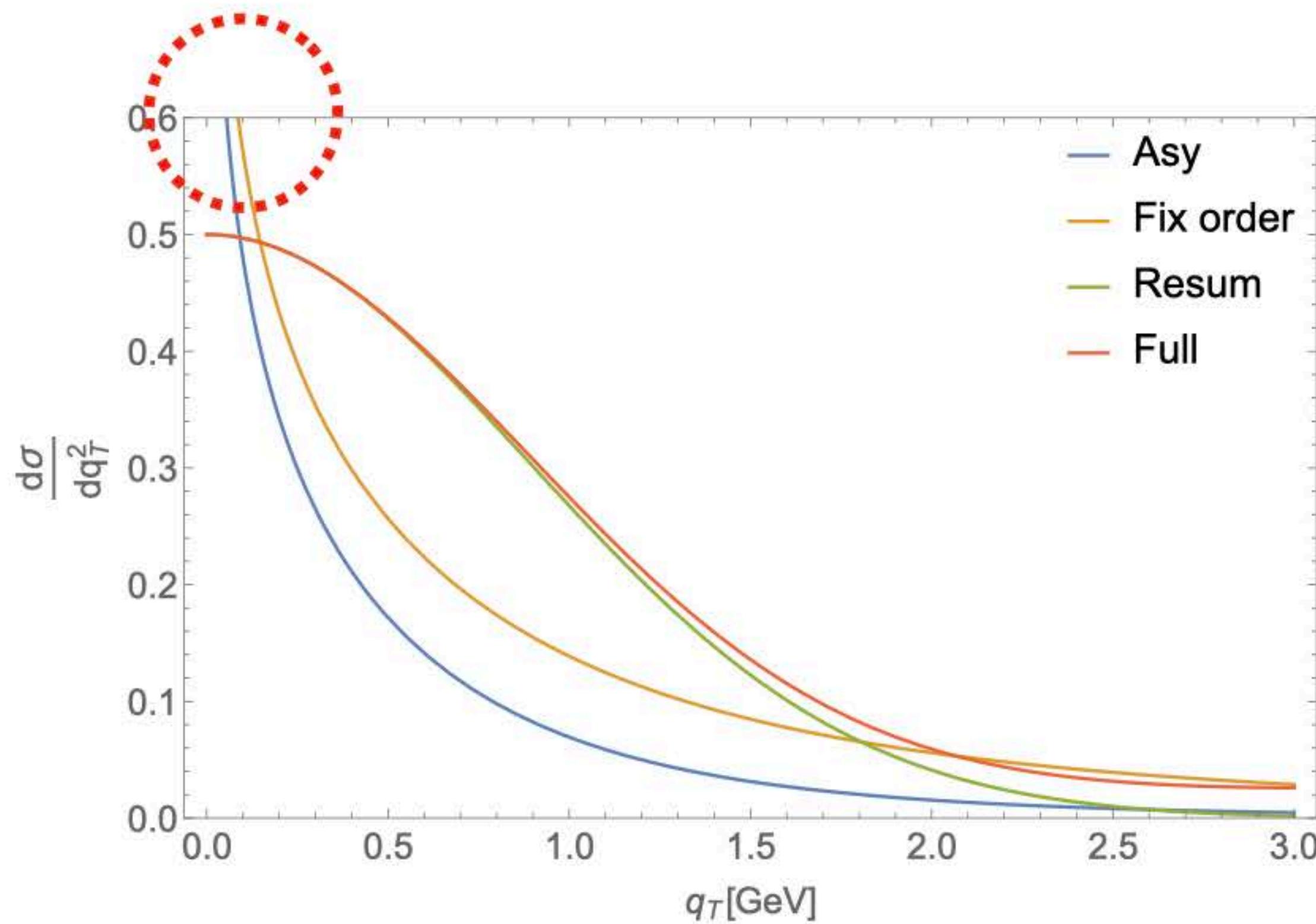


Standard approach

- Resummed contribution is dominant where the Asymptotic term is close to the Fixed Order

# Source of W-term suppression

Ideal situation at low Q

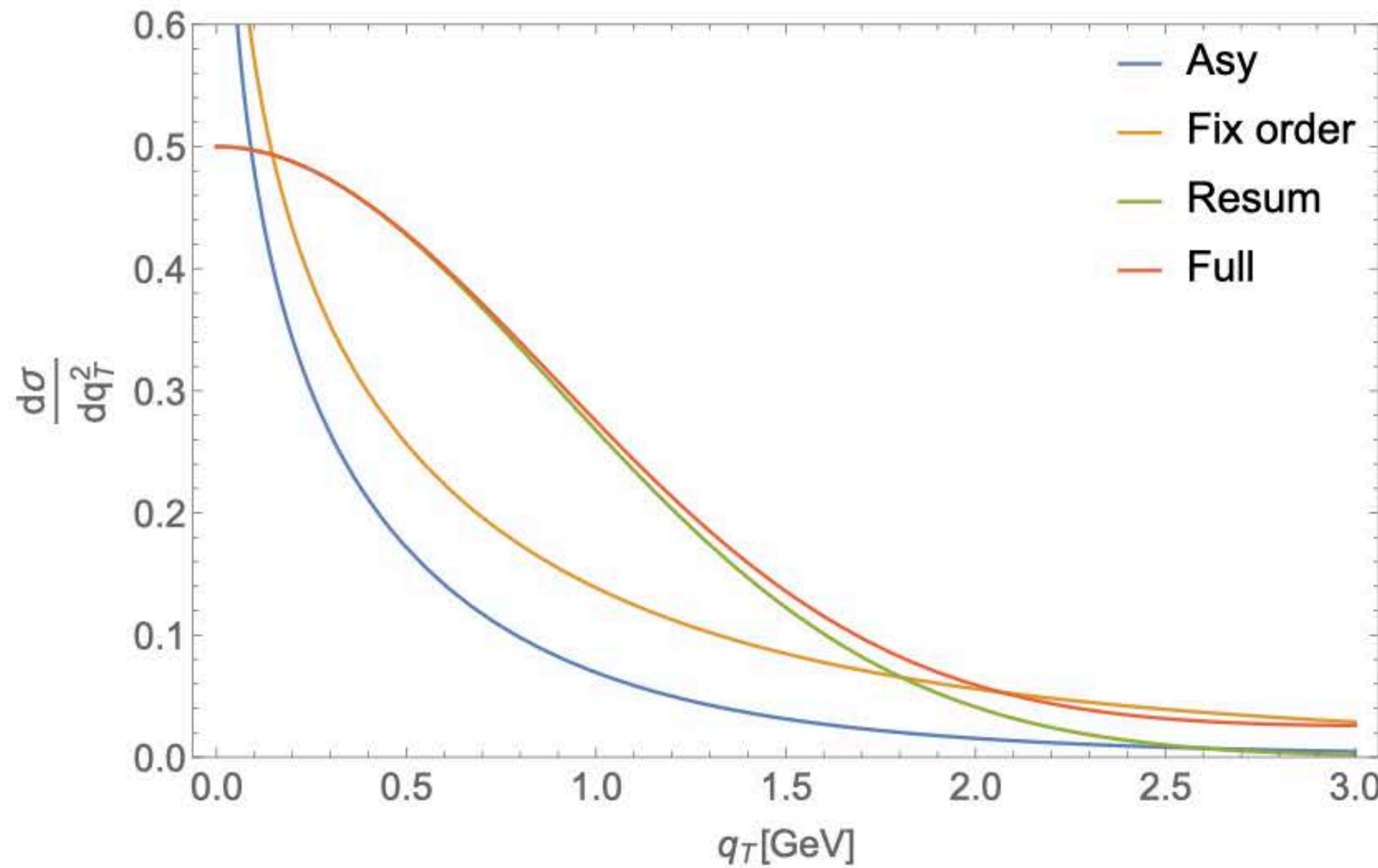


Standard approach

- Resummed contribution is dominant where the Asymptotic term is close to the Fixed Order → TMD Region?

# Source of W-term suppression

Ideal situation at low Q

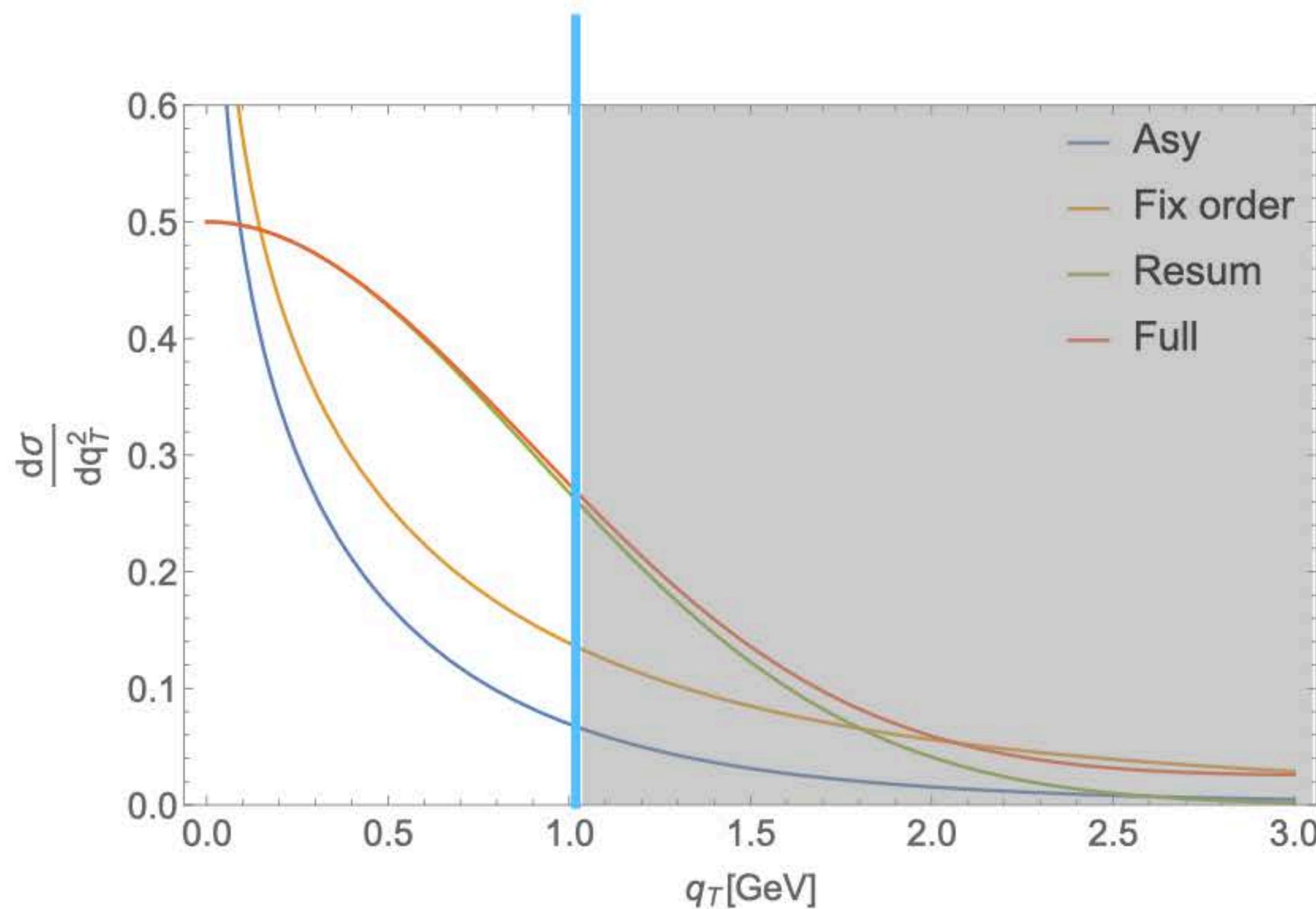


## Non-Perturbative approach

- Resummed contribution is dominant where the Asymptotic term is close to the Fixed Order OR the Non-Perturbative contributions dominates

# Source of W-term suppression

Ideal situation at low Q



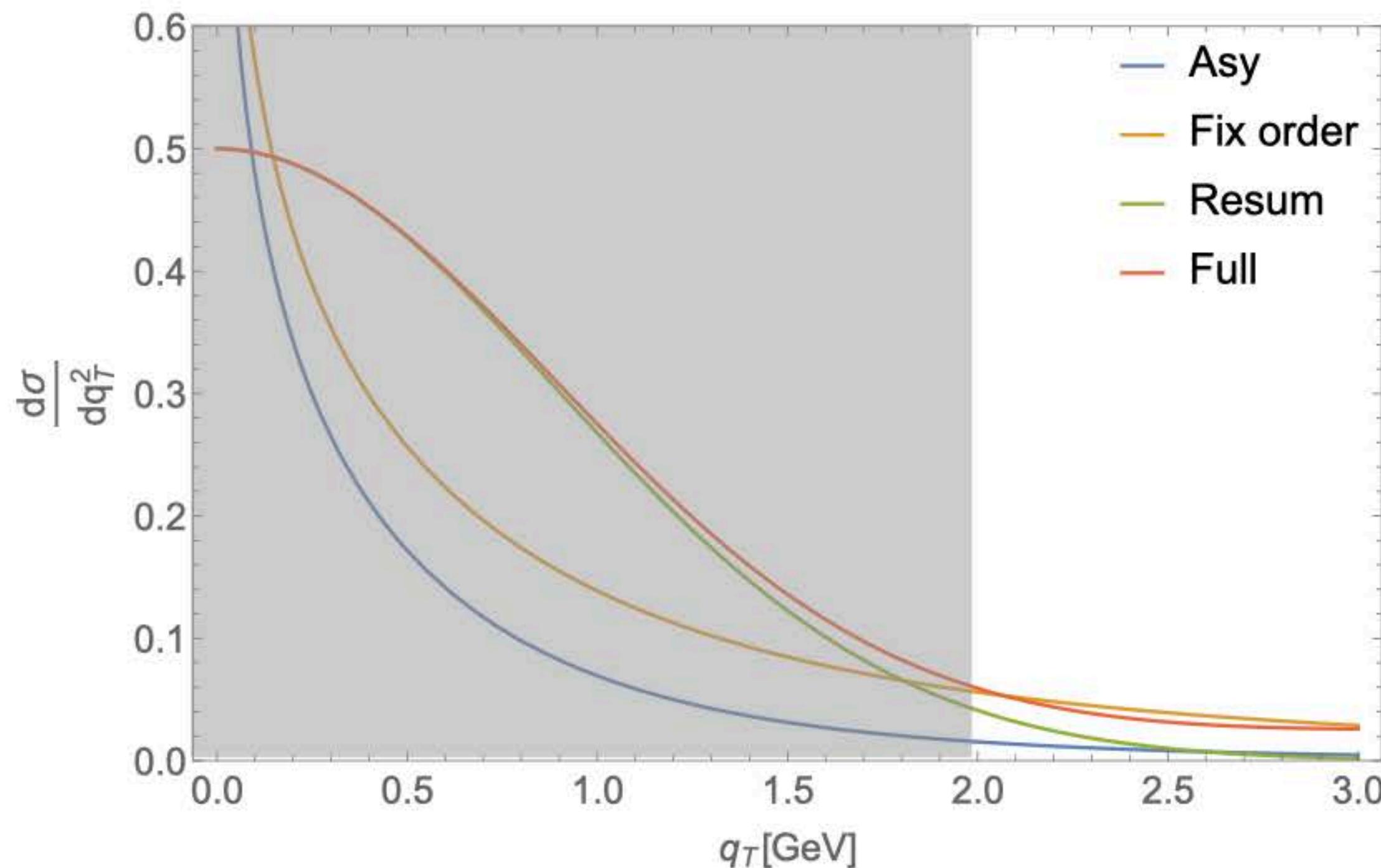
## Non-Perturbative approach

- Resummed contribution is dominant where the Asymptotic term is close to the Fixed Order OR the Non-Perturbative contributions dominates

→ TMD Region

# Source of W-term suppression

Ideal situation at low Q

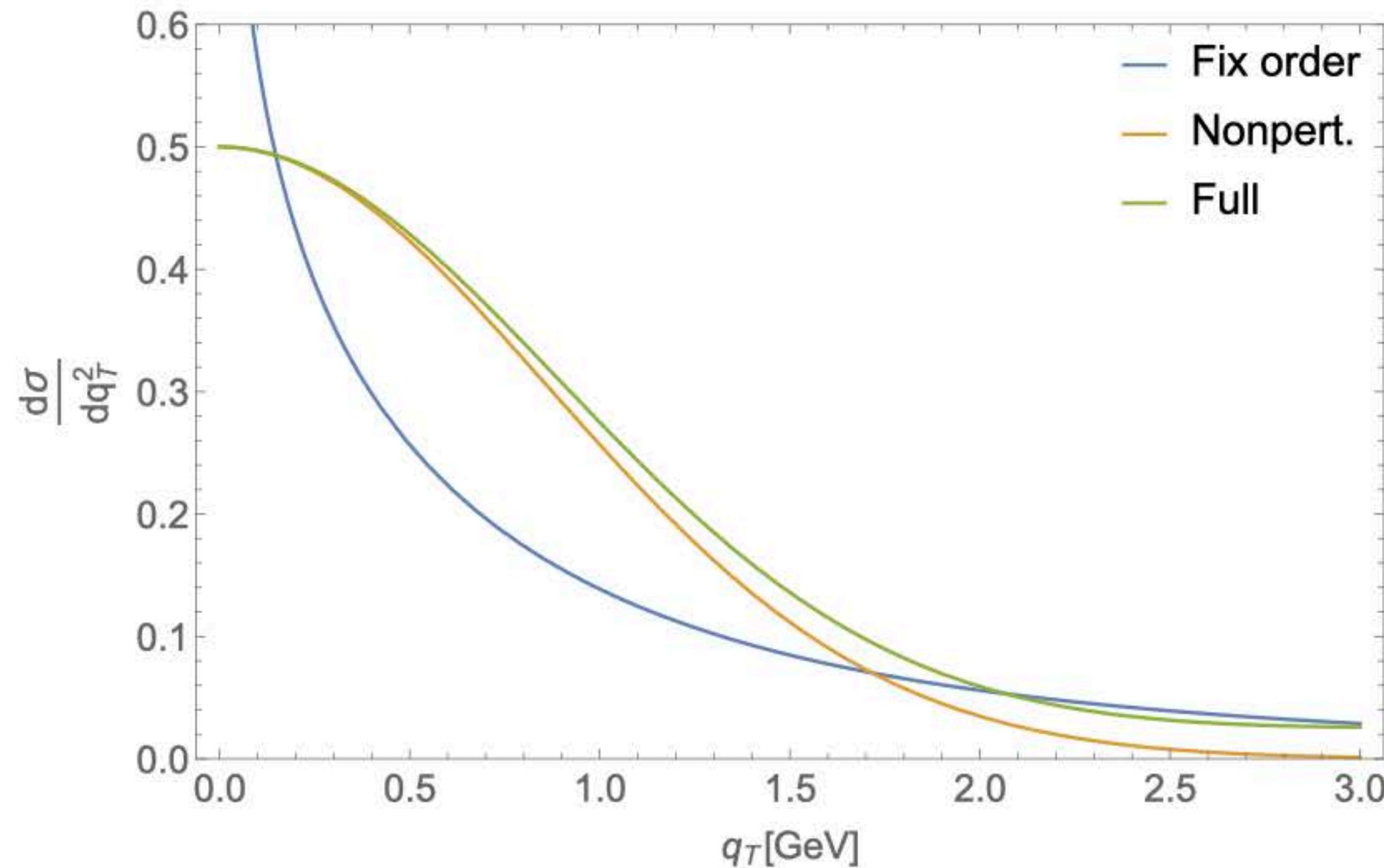


## Non-Perturbative approach

- Resummed contribution is dominant where the Asymptotic term is close to the Fixed Order OR the Non-Perturbative contributions dominates
- From a certain value of  $q_T$  the cross section follows the Fixed Order term

# Source of W-term suppression

Ideal situation at low Q



## Non-Perturbative approach

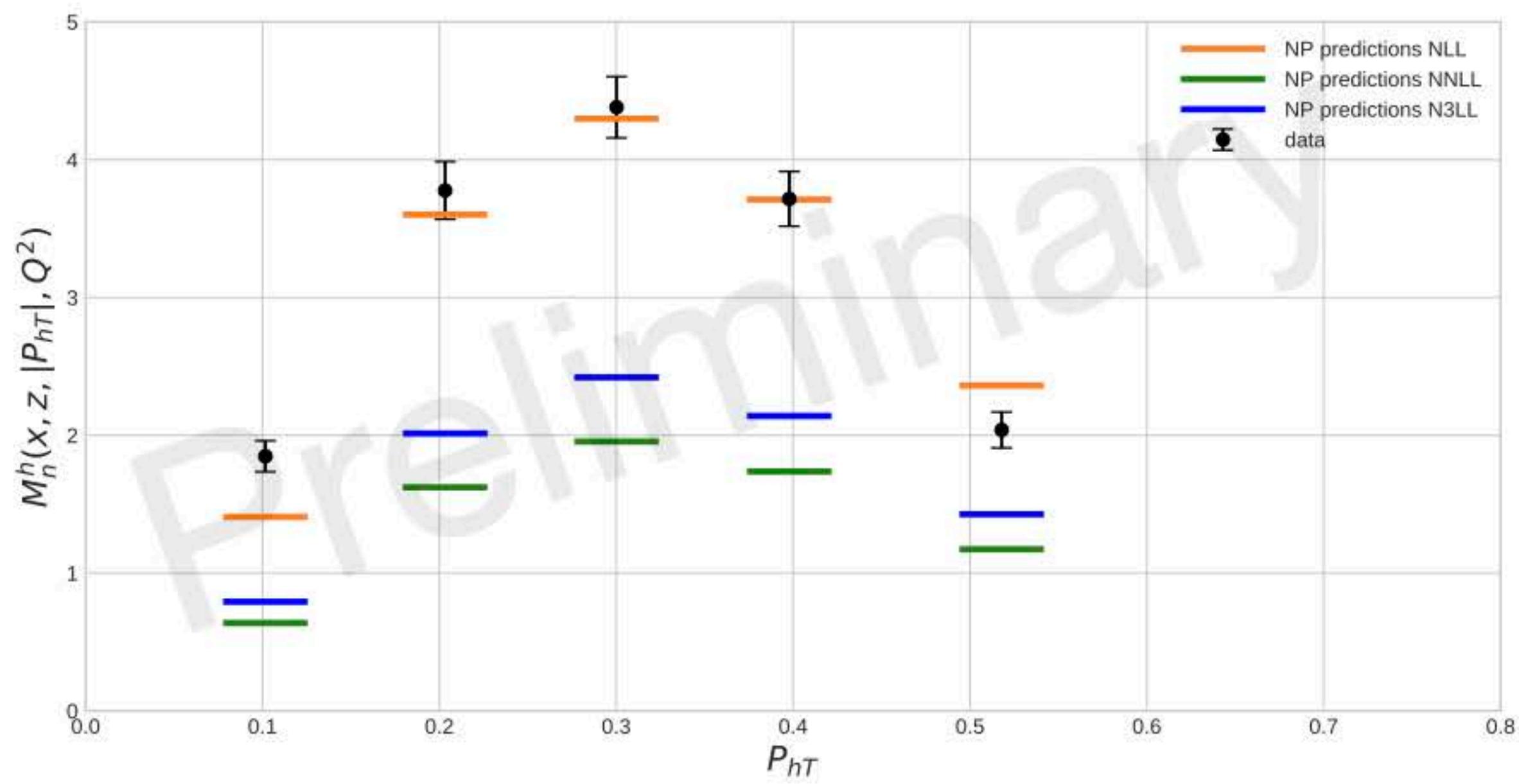
- Collinear result is no more mostly given by the integral of the Fixed Order
- The Non-Perturbative term is not only a small correction, but is even larger than the Fixed Order contribution

# Source of W-term suppression

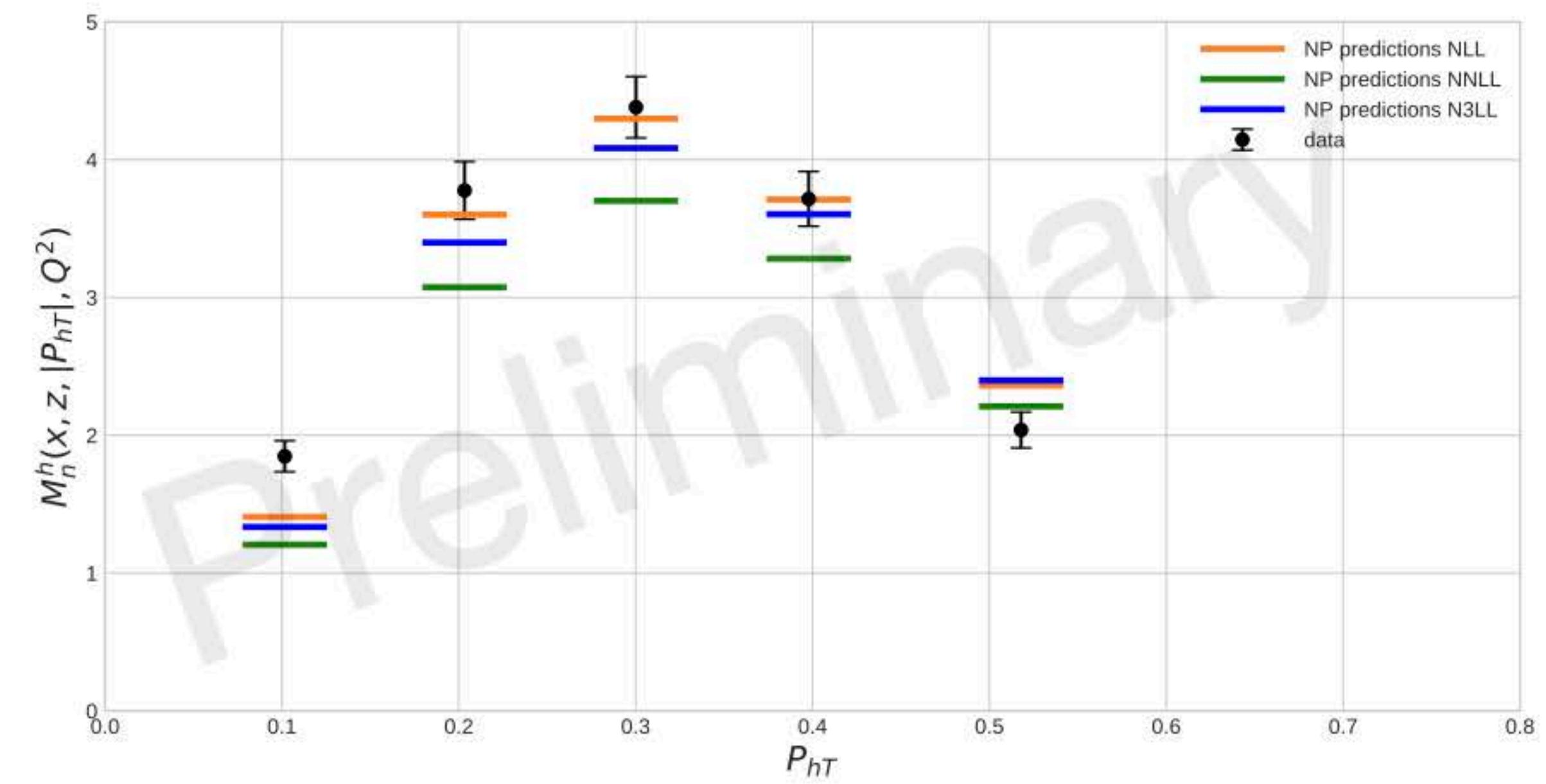
Present situation at low Q

HERMES multiplicity

Full Hard Factor



Hard Factor = 1

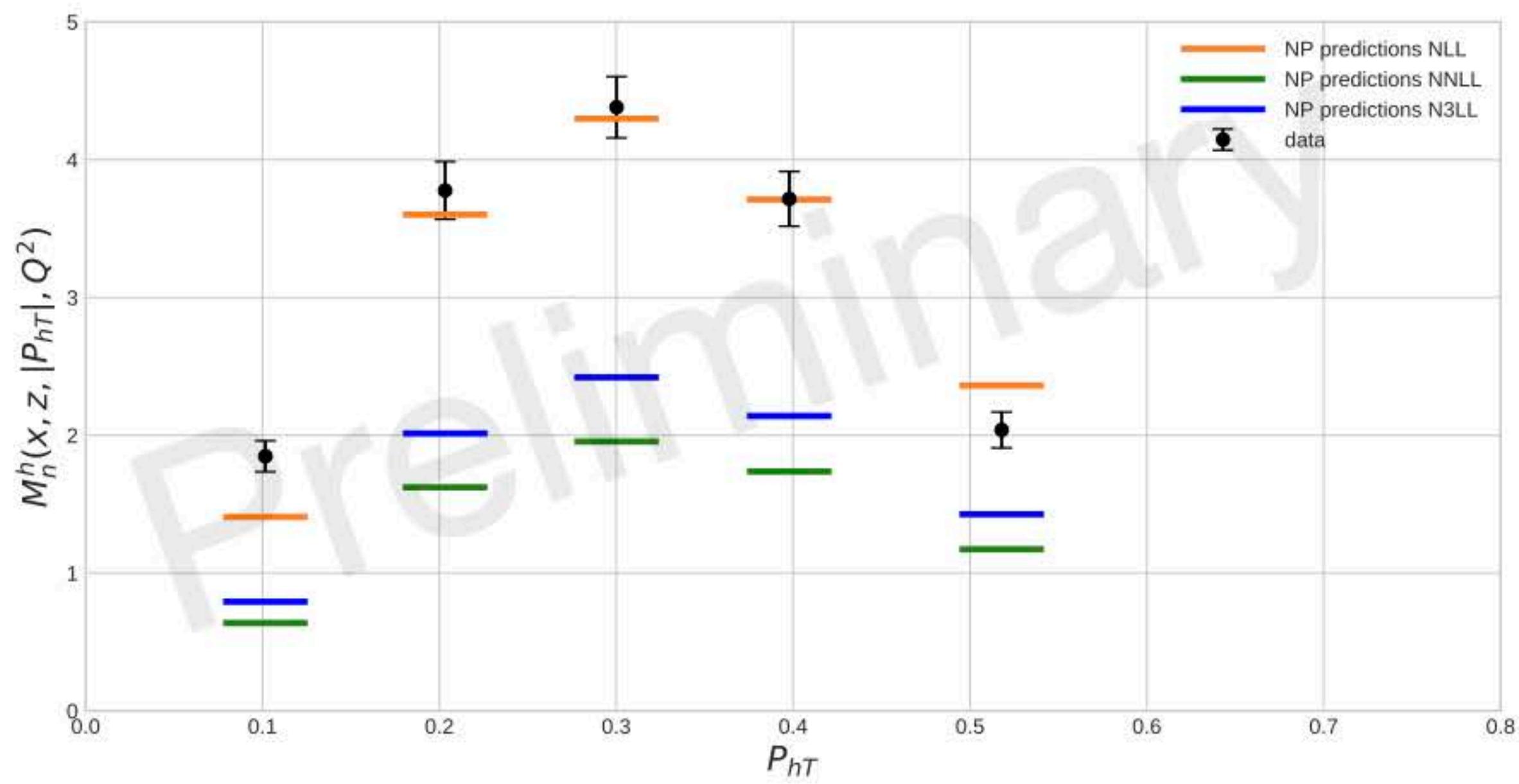


# Source of W-term suppression

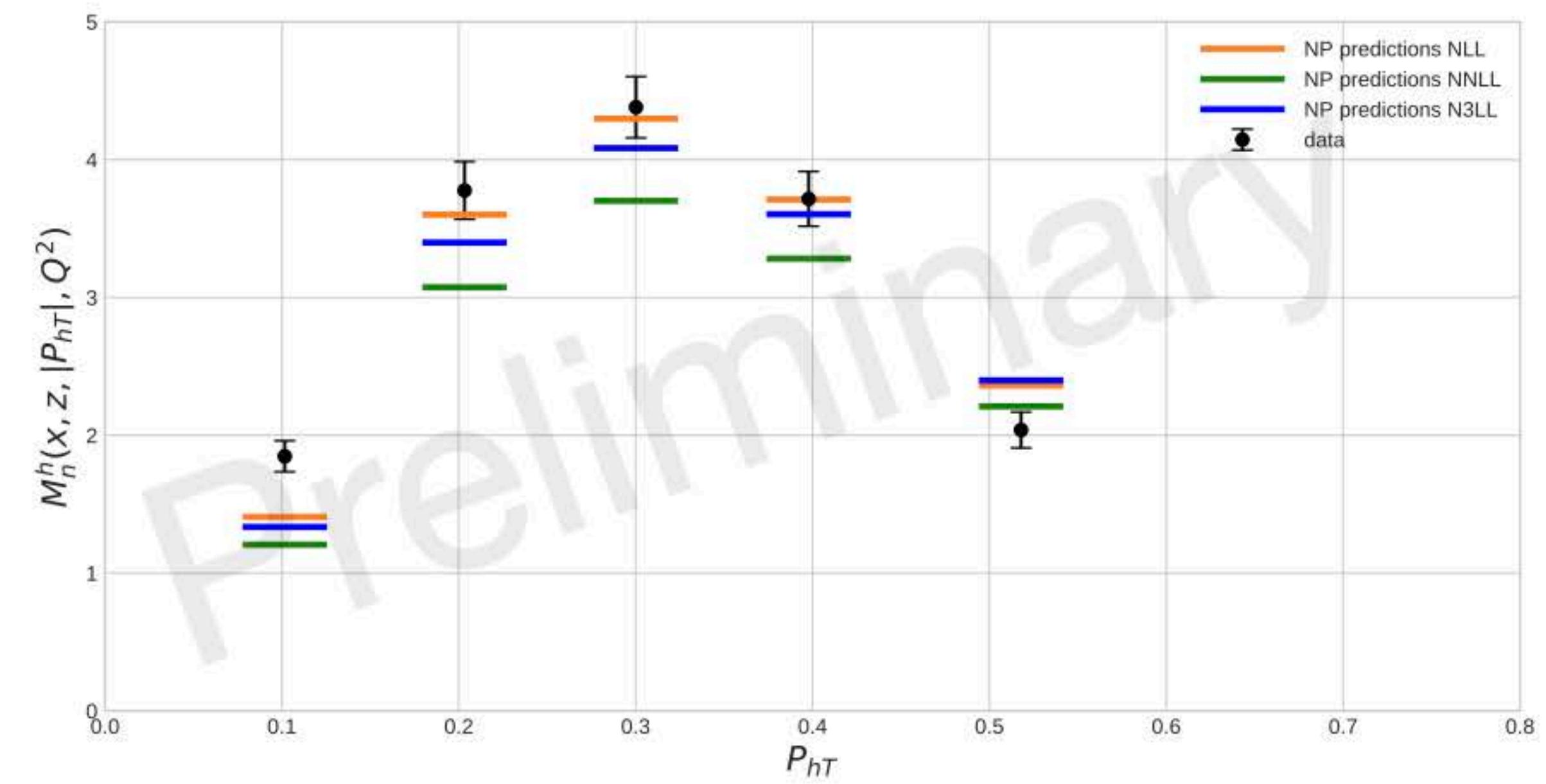
Present situation at low Q

HERMES multiplicity

Full Hard Factor



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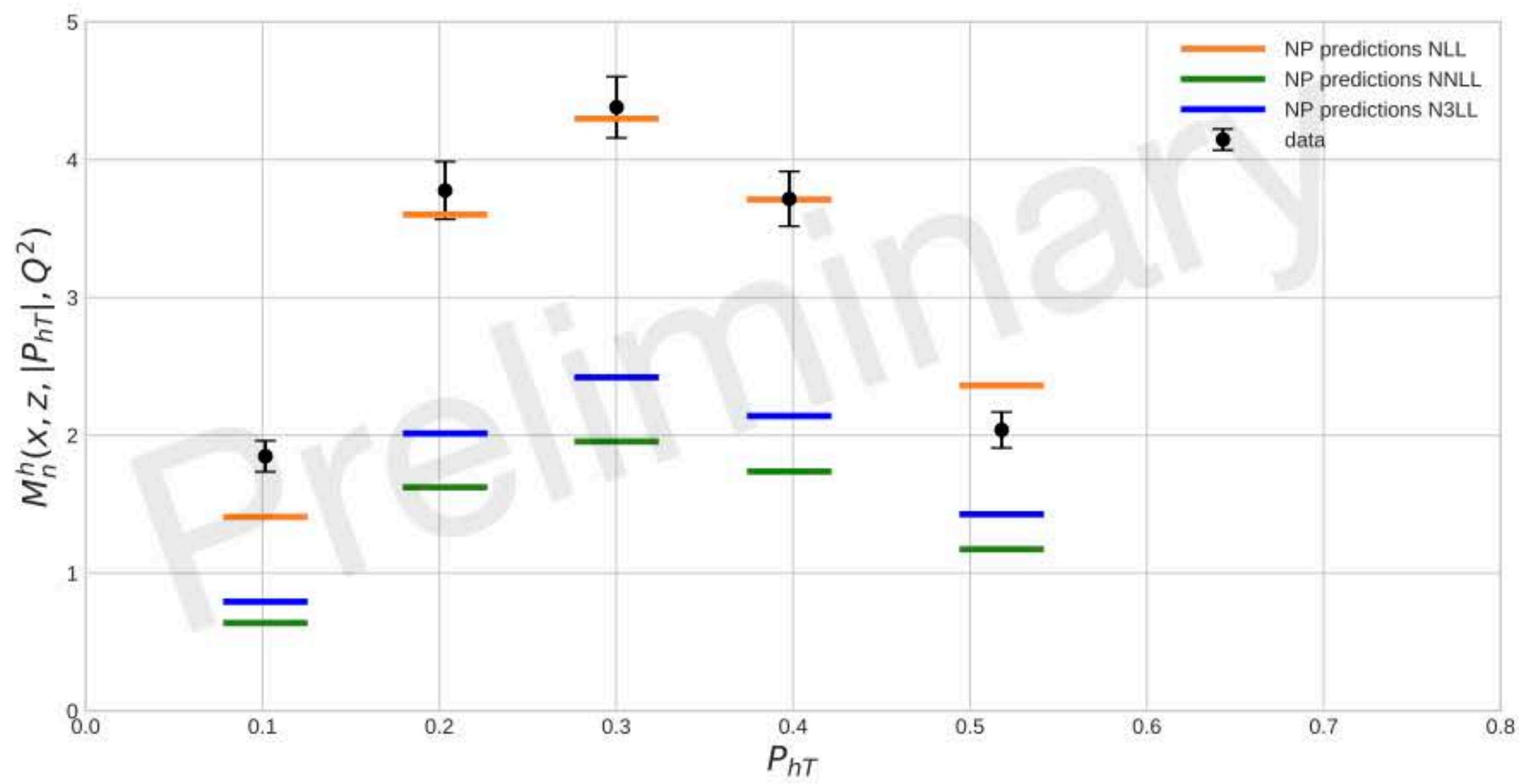


# Source of W-term suppression

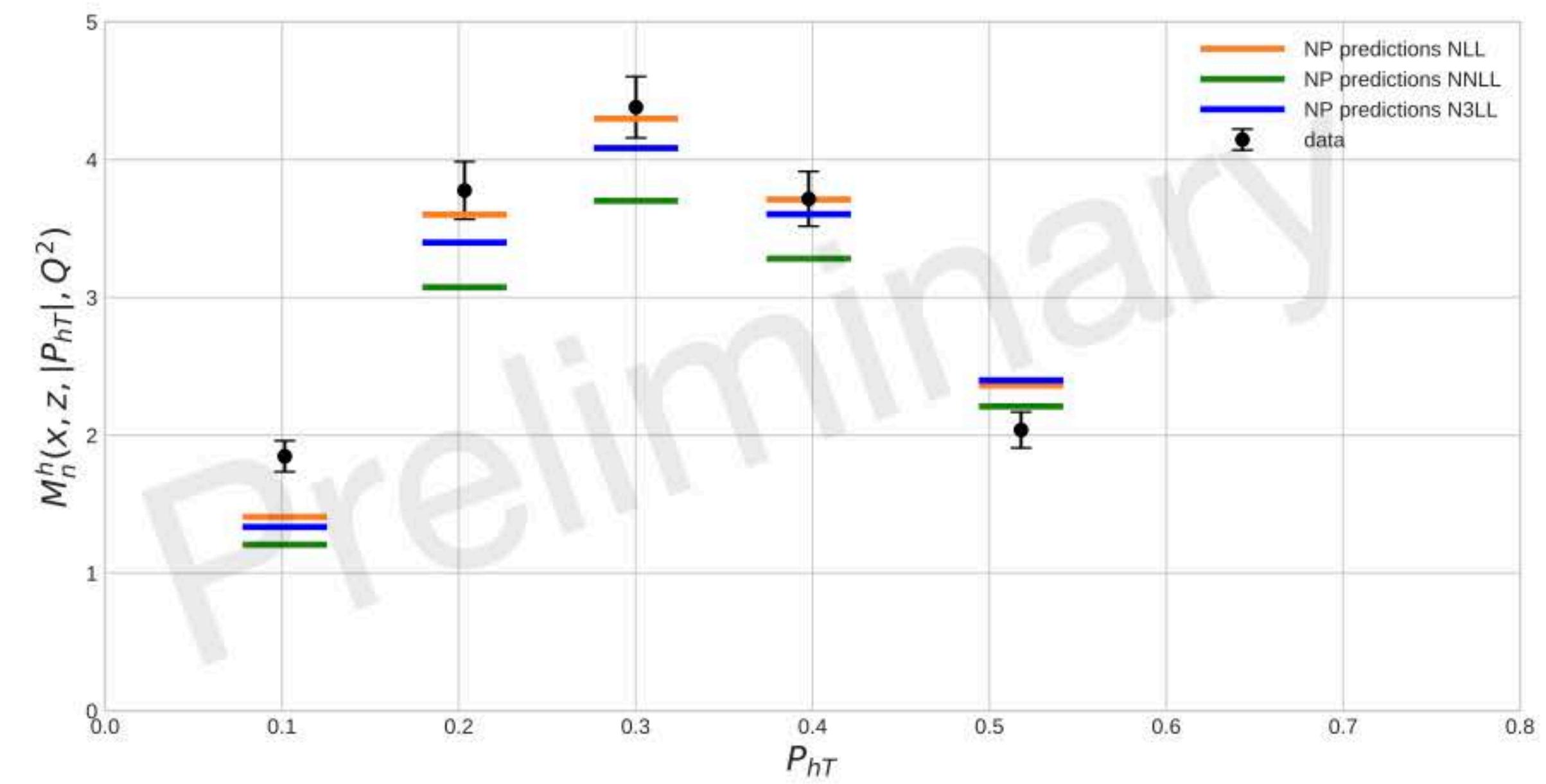
Present situation at low Q

HERMES multiplicity

Full Hard Factor



Hard Factor = 1

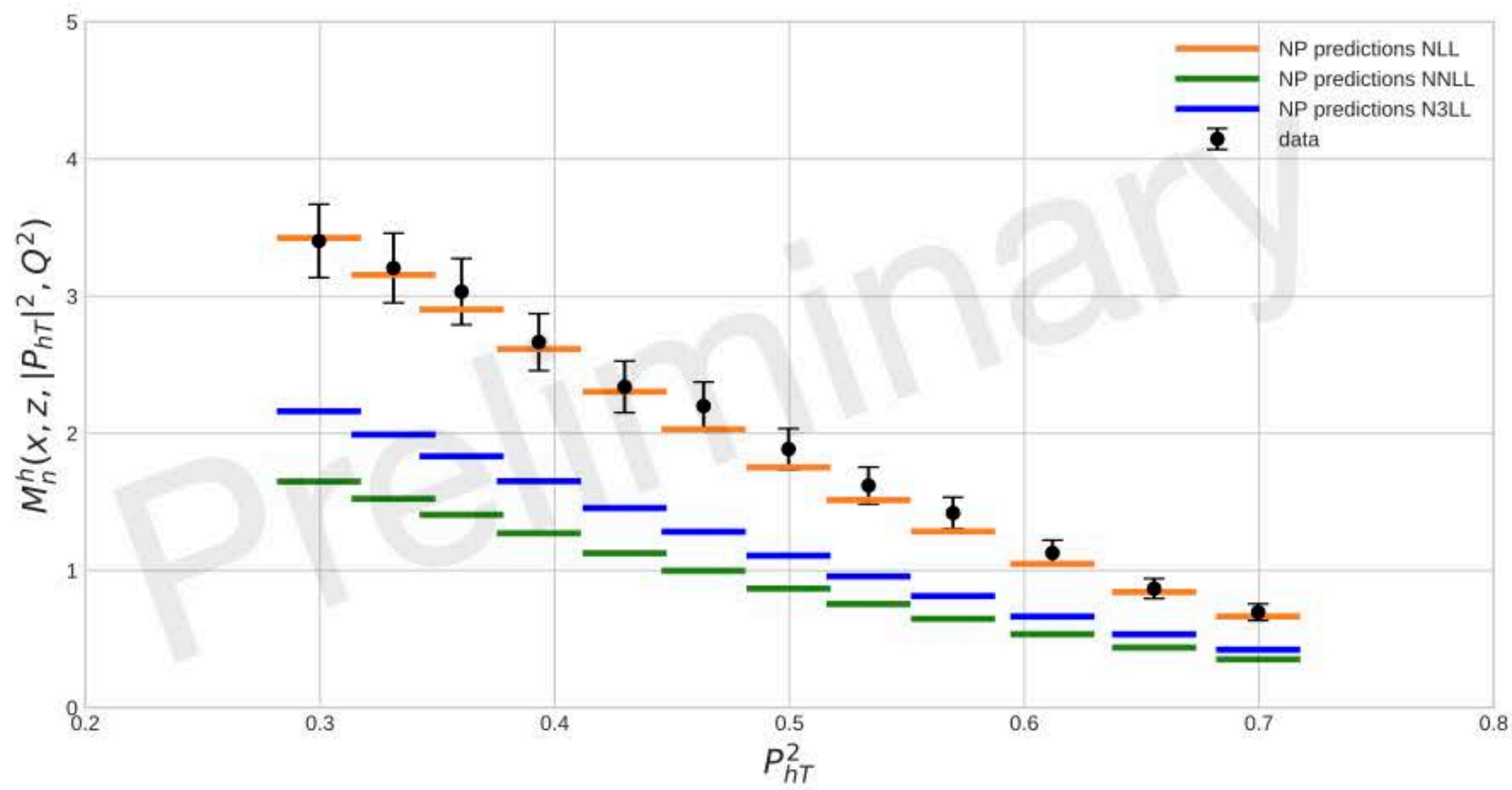


# Source of W-term suppression

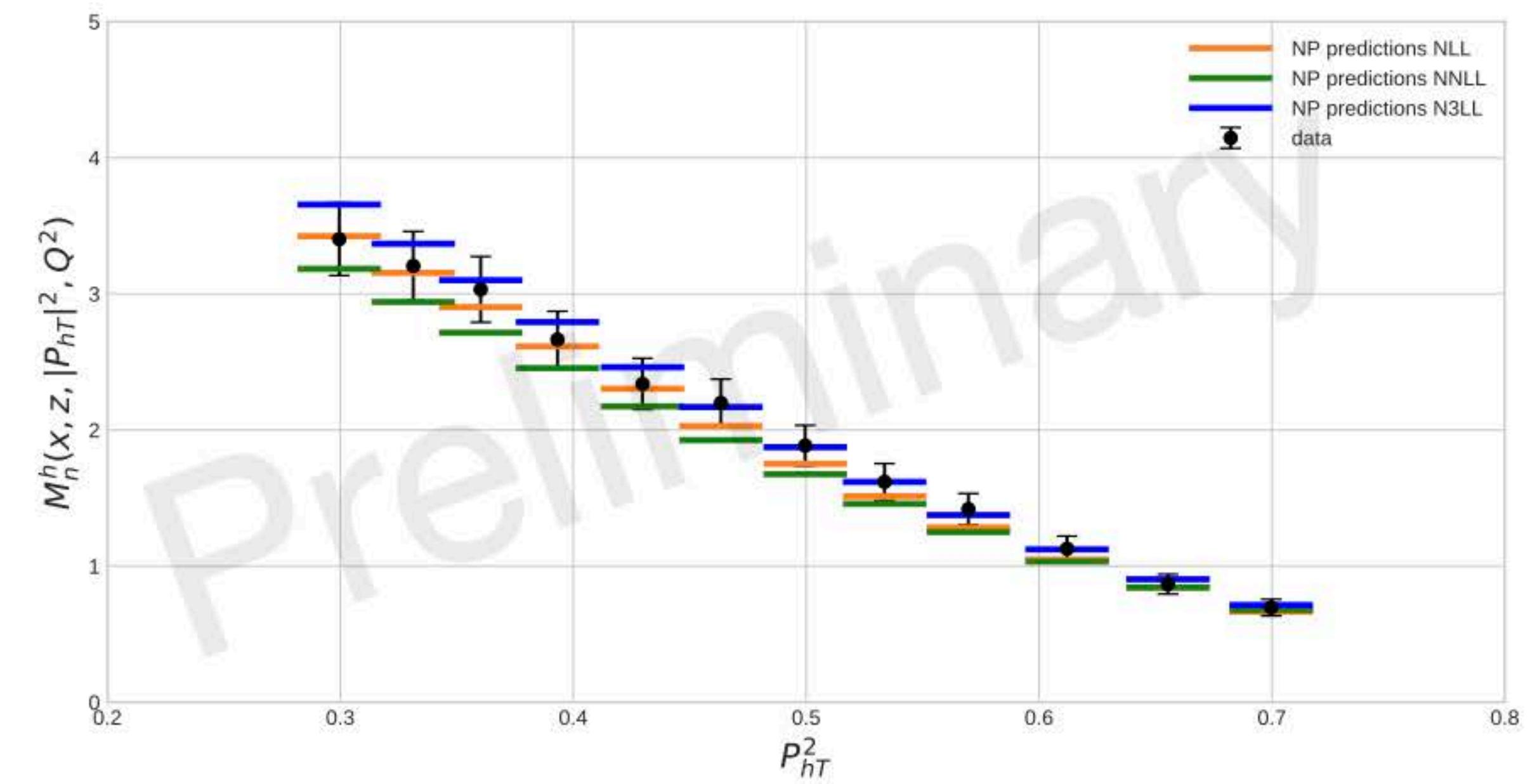
Present situation at low Q

COMPASS multiplicity

Full Hard Factor



Hard Factor = 1

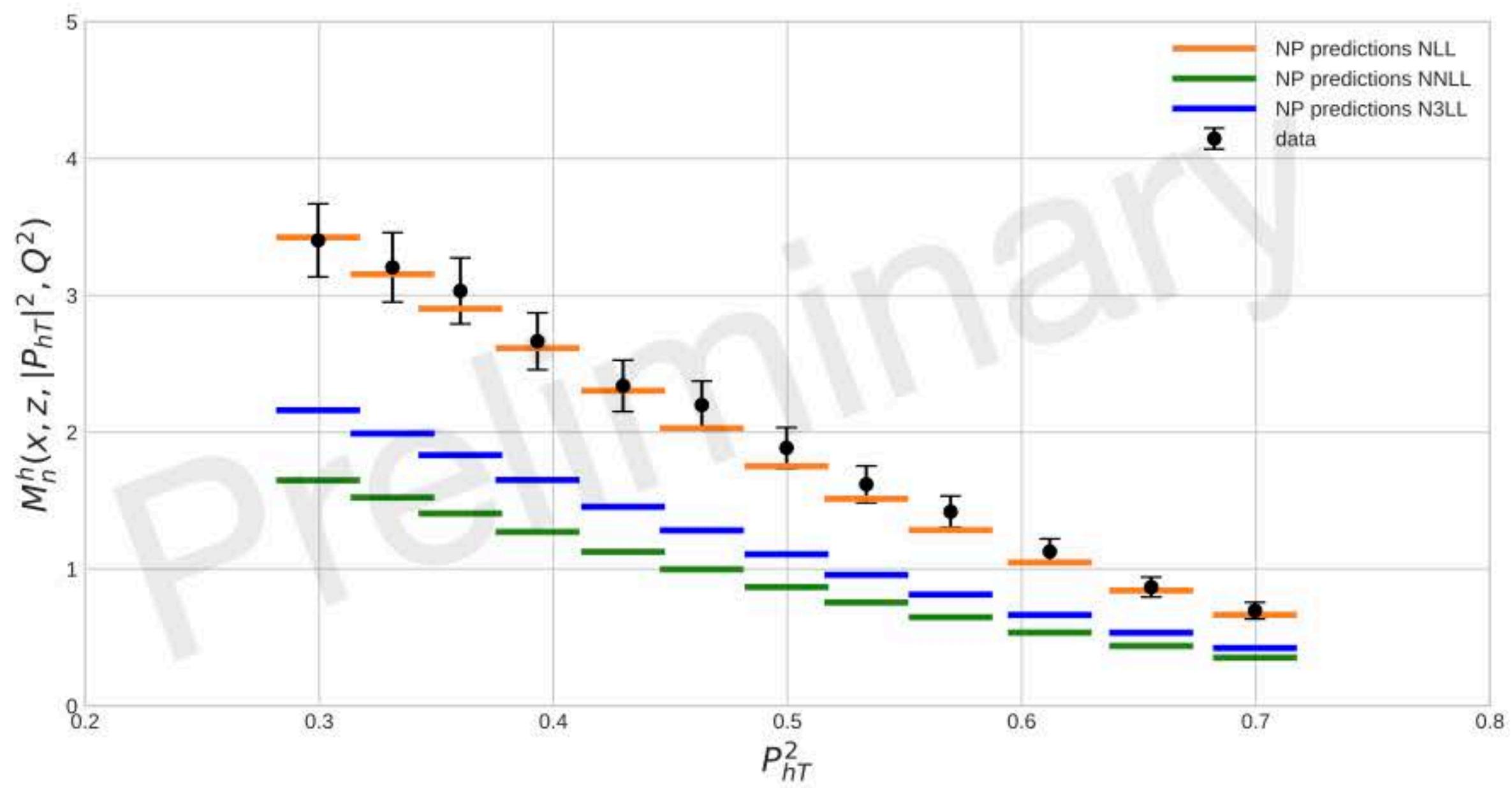


# Source of W-term suppression

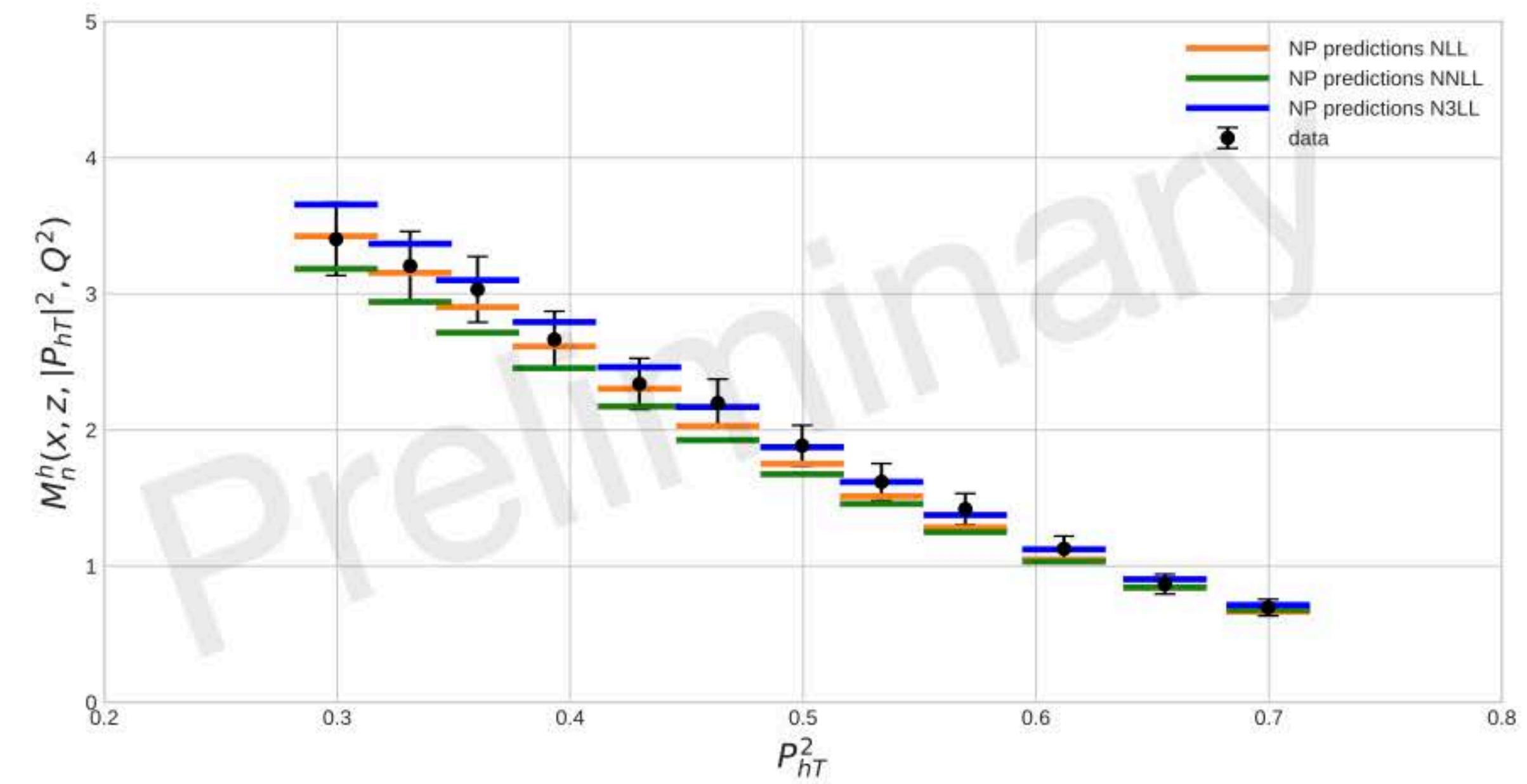
Present situation at low Q

COMPASS multiplicity

Full Hard Factor



Hard Factor = 1

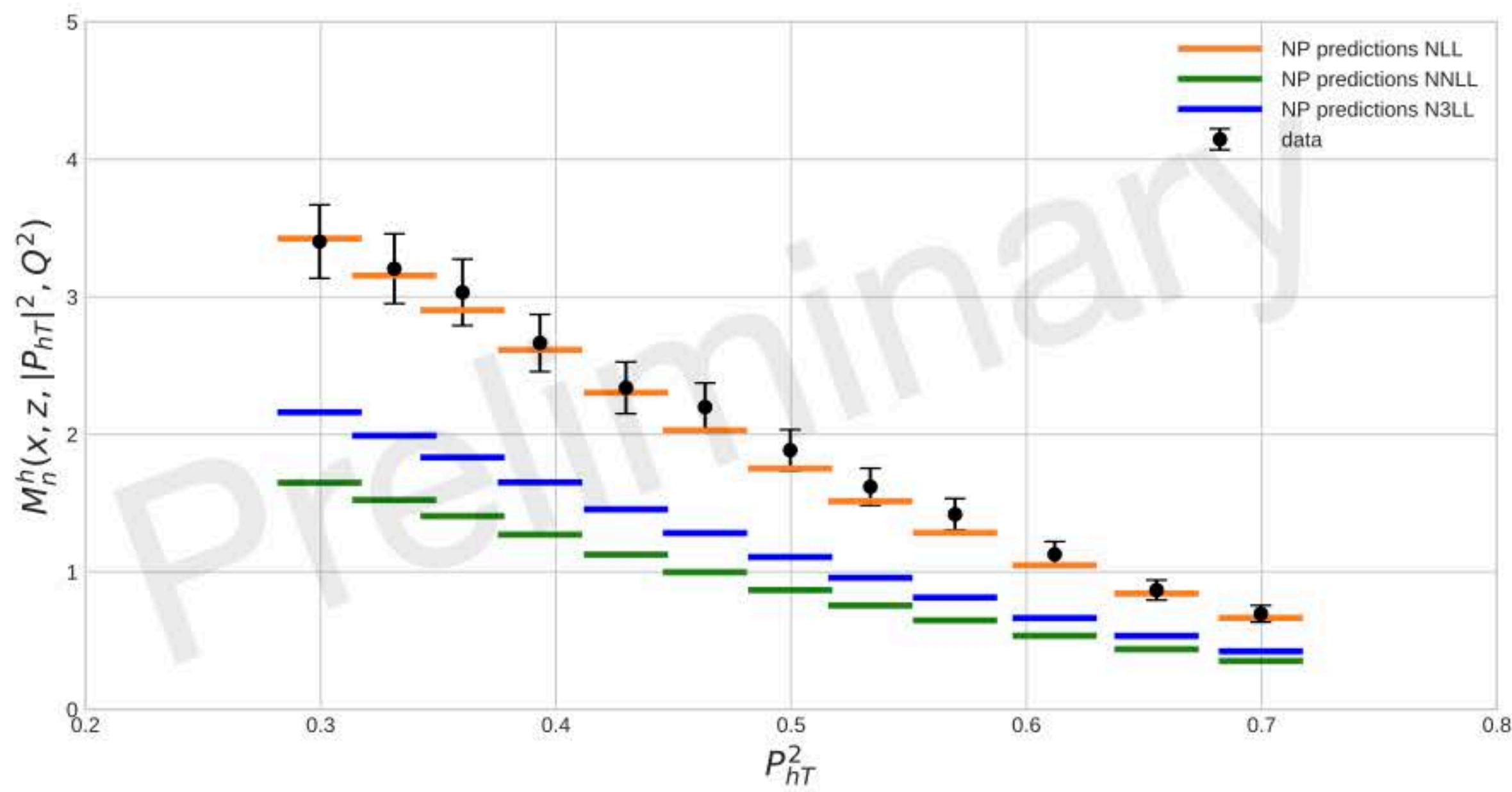


# Source of W-term suppression

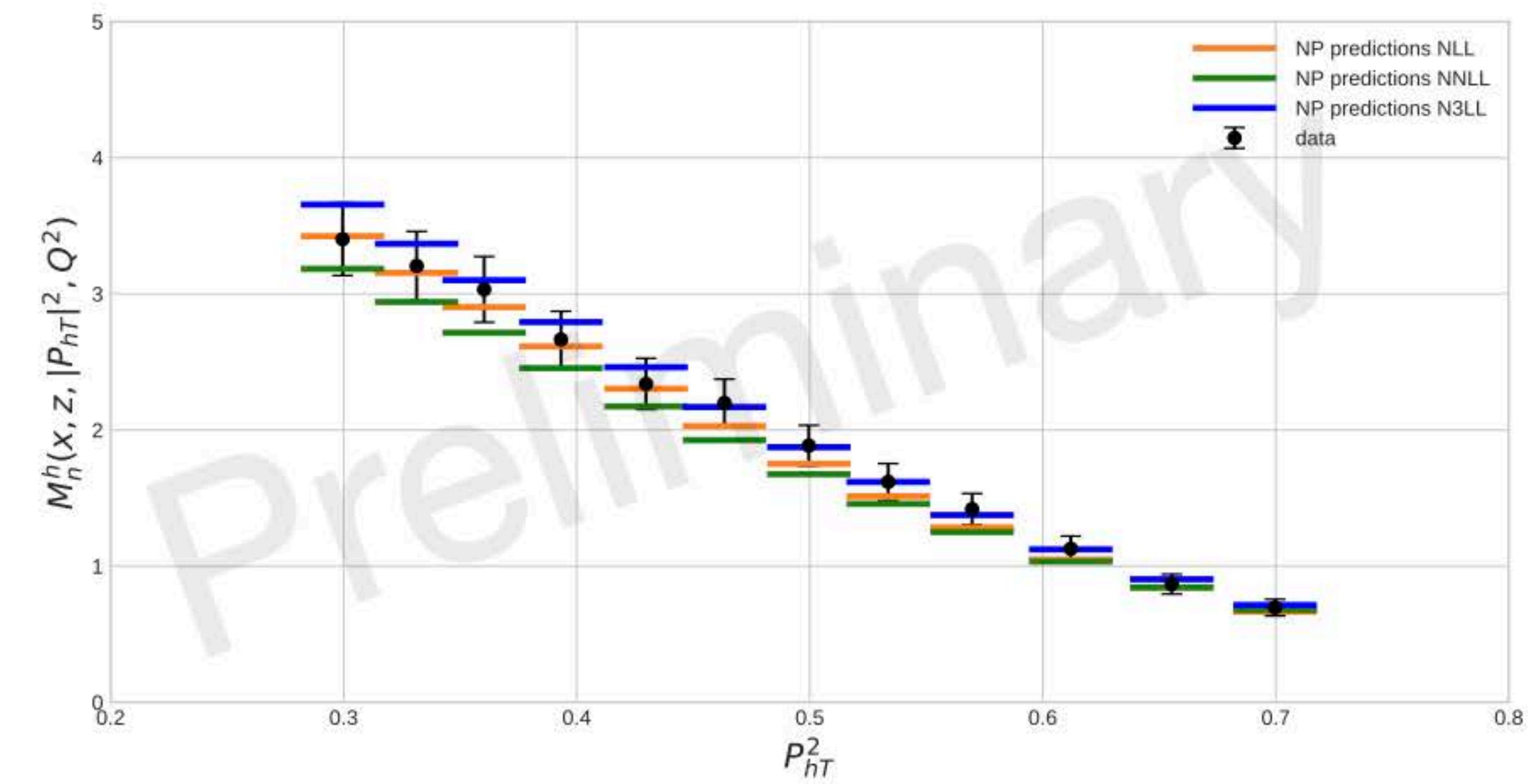
Present situation at low Q

COMPASS multiplicity

Full Hard Factor



Hard Factor = 1



# Cut qT/Q for SIDIS dataset

$$P_{hT}|_{\max} = \min[\min[c_1 Q, c_2 zQ] + c_3 \text{ GeV}, c_4 zQ]$$

$qT/Q = 0.4$

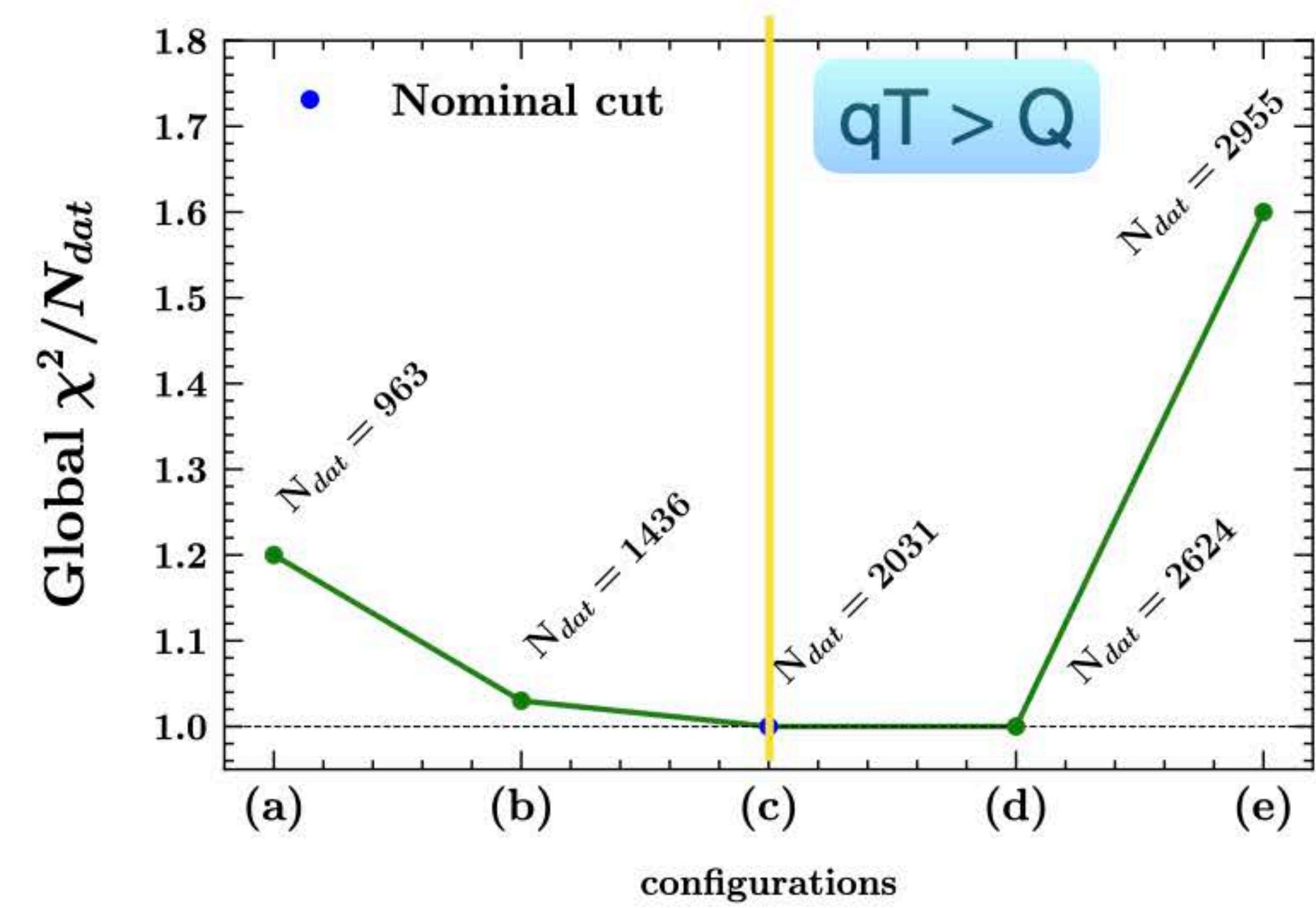
(a)  $\begin{cases} c_1 = 0.2 \\ c_2 = 0.5 \\ c_3 = 0.3 \\ c_4 = 0.4 \end{cases}$

> baseline  
(d)  $\begin{cases} c_1 = 0.2 \\ c_2 = 0.6 \\ c_3 = 0.4 \\ c_4 = \infty \end{cases}$

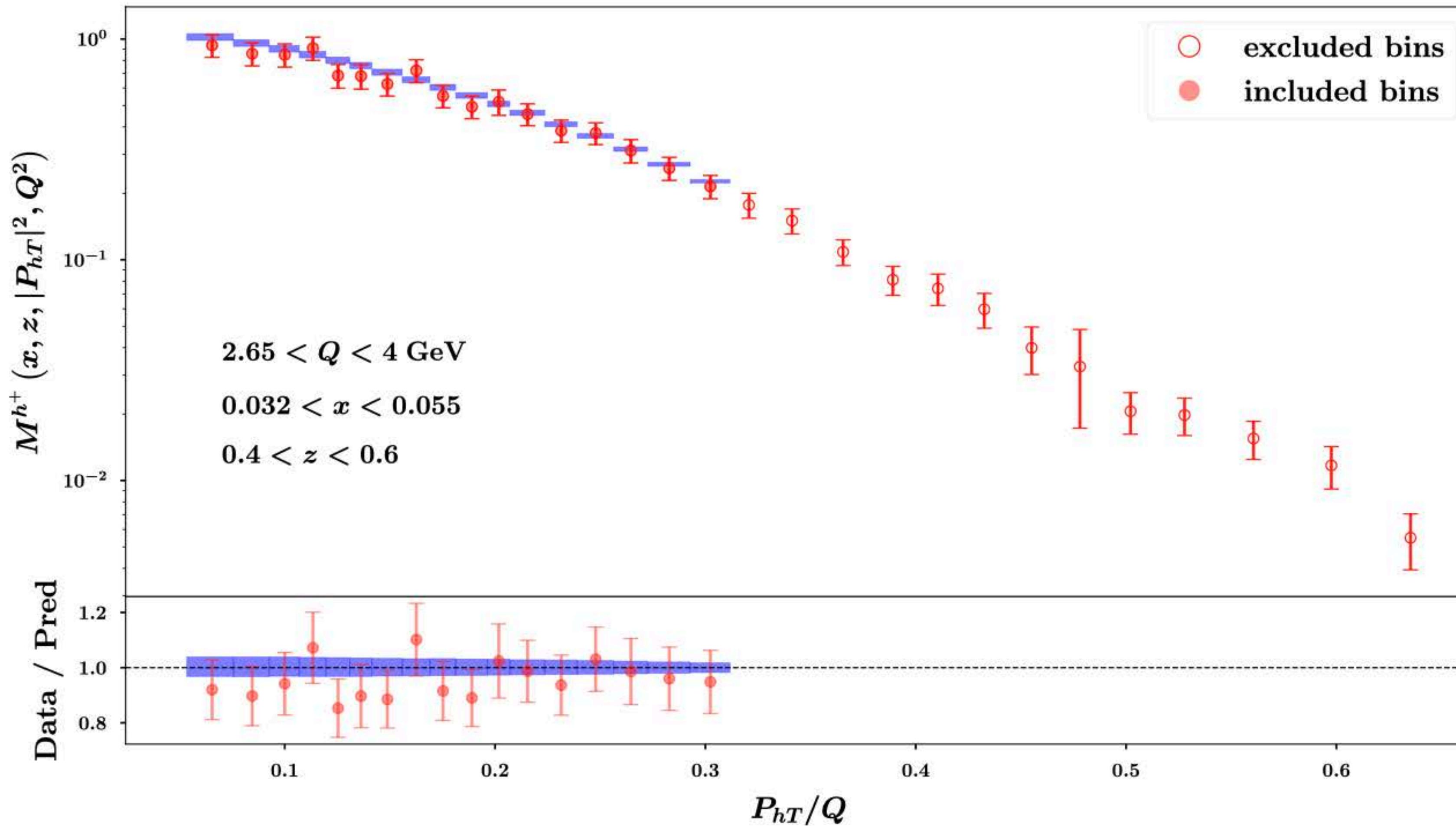
< baseline  
(b)  $\begin{cases} c_1 = 0.15 \\ c_2 = 0.4 \\ c_3 = 0.2 \\ c_4 = 1 \end{cases}$

PV17  
(e)  $\begin{cases} c_1 = 0.2 \\ c_2 = 0.7 \\ c_3 = 0.5 \\ c_4 = \infty \end{cases}$

baseline  
(c)  $\begin{cases} c_1 = 0.2 \\ c_2 = 0.5 \\ c_3 = 0.3 \\ c_4 = 1 \end{cases}$



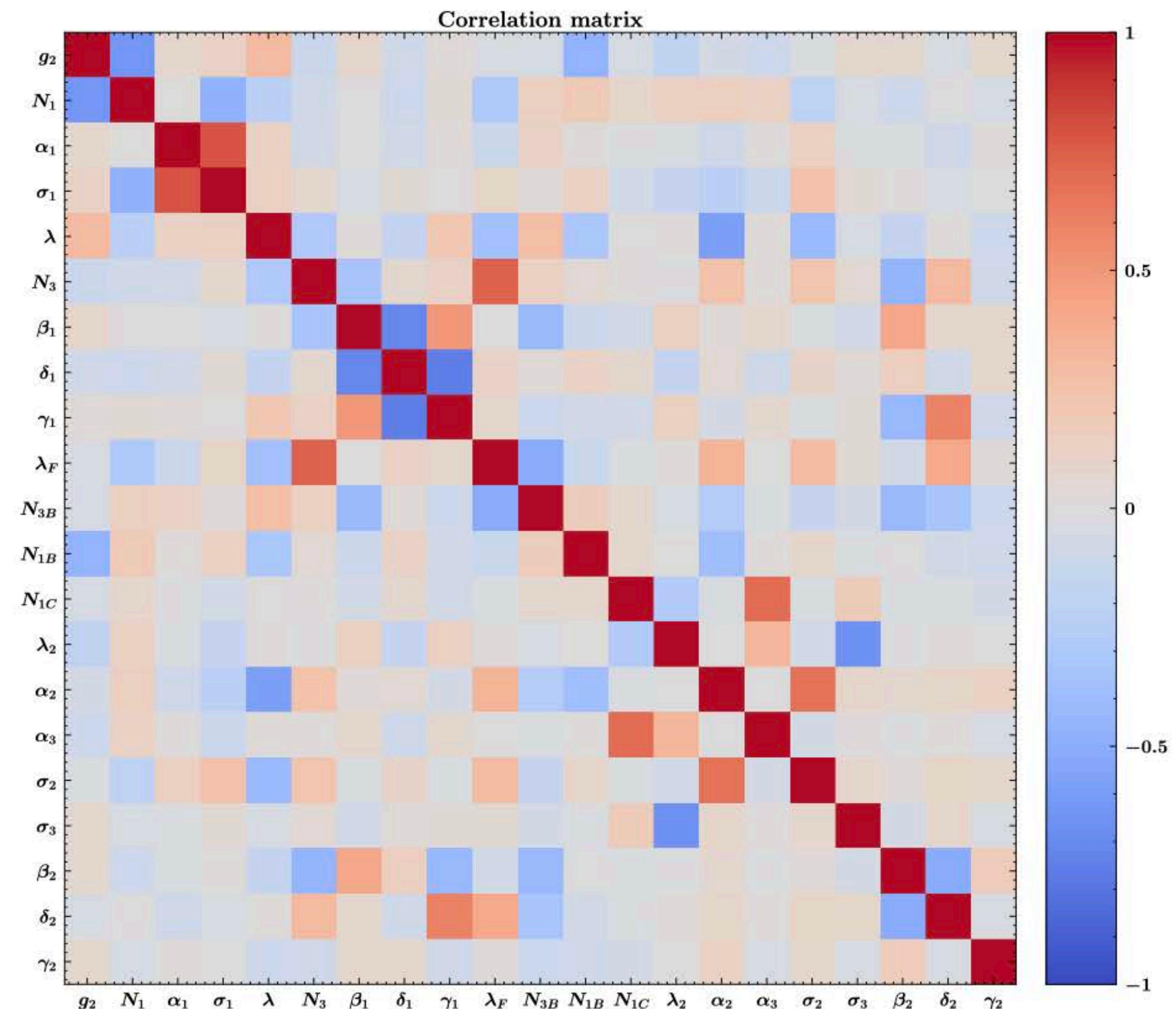
# Cut $qT/Q$ for SIDIS dataset



# Results of the baseline fit

Error propagation

↓  
250 Montecarlo replicas



# Results of the baseline fit

Error propagation



250 Montecarlo replicas

Correlation matrix



Hints of the appropriateness of the chosen functional form

