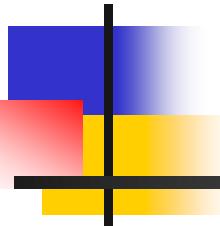


# **Longitudinal and Transverse polarizations of $\Lambda$ Hyperons in unpolarized SIDIS and $e^+e^-$ annihilation**



Yu-kun Song, University of Jinan, China

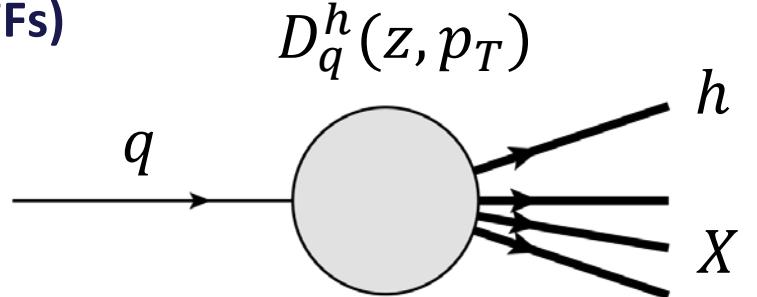
In collaboration with K.B.Chen, Z.T.Liang, Y.L.Pan and S.Y.Wei

2022/05/05

*K.B.Chen, Z.T.Liang, Y.L.Pan, YKS, S.Y.Wei, Phys. Lett. B 816 (2021) 136217*  
*K.B.Chen, Z.T.Liang, YKS, S.Y.Wei, Phys. Rev. D 105 (2022) 034027*

# Motivation-TMD FFs

- Transverse-Momentum-Dependent Fragmentation functions (TMD FFs)



The quark correlation matrix

$$\hat{\Xi}_q^\Lambda(z, p_T) = \sum_X \int \frac{p^+ d\xi^- d^2 \xi_\perp}{2\pi} e^{i(p^+ \xi^- - \mathbf{p}_T \cdot \boldsymbol{\xi}_\perp)/z} \langle 0 | \mathcal{L}^\dagger(0, \infty) \psi(0) | \Lambda X \rangle \langle \Lambda X | \bar{\psi}(\xi) \mathcal{L}(\xi, \infty) | 0 \rangle$$

Decomposition at leading twist

$$4\hat{\Xi}_q^\Lambda = \gamma^+ \left[ D_{1q}^\Lambda + \frac{(\hat{\mathbf{e}}_j \times \mathbf{p}_T) \cdot \mathbf{S}_\Lambda}{z_\Lambda M_\Lambda} D_{1Tq}^{\perp\Lambda} \right] + \gamma_5 \gamma^+ \left[ \lambda_\Lambda G_{1Lq}^\Lambda + \frac{\mathbf{p}_T \cdot \mathbf{S}_{\Lambda\perp}}{z_\Lambda M_\Lambda} G_{1Tq}^{\perp\Lambda} \right] \lambda_q$$

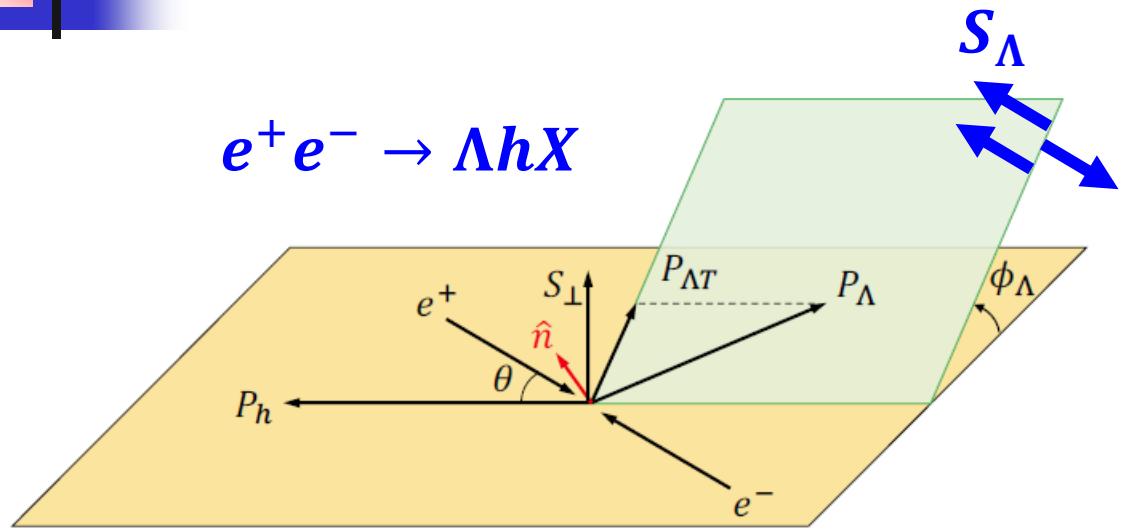
$$+ \frac{i p_T^i}{2 M_\Lambda} [\gamma_i, \gamma^+] H_{1q}^{\perp\Lambda} + \frac{S_{\Lambda\perp}^i}{2} [\gamma_i, \gamma^+] \gamma_5 H_{1Tq}^{\Lambda} + \frac{p_T^i}{2M} [\gamma_i, \gamma^+] \gamma_5 \left( \lambda_\Lambda H_{1Lq}^{\perp\Lambda} - \frac{\mathbf{p}_T \cdot \mathbf{S}_\Lambda}{M_\Lambda} H_{1Tq}^{\perp\Lambda} \right) s_{q\perp}$$

Where  $D_{iq}^\Lambda \equiv D_{iq}^\Lambda(z, p_T^2)$

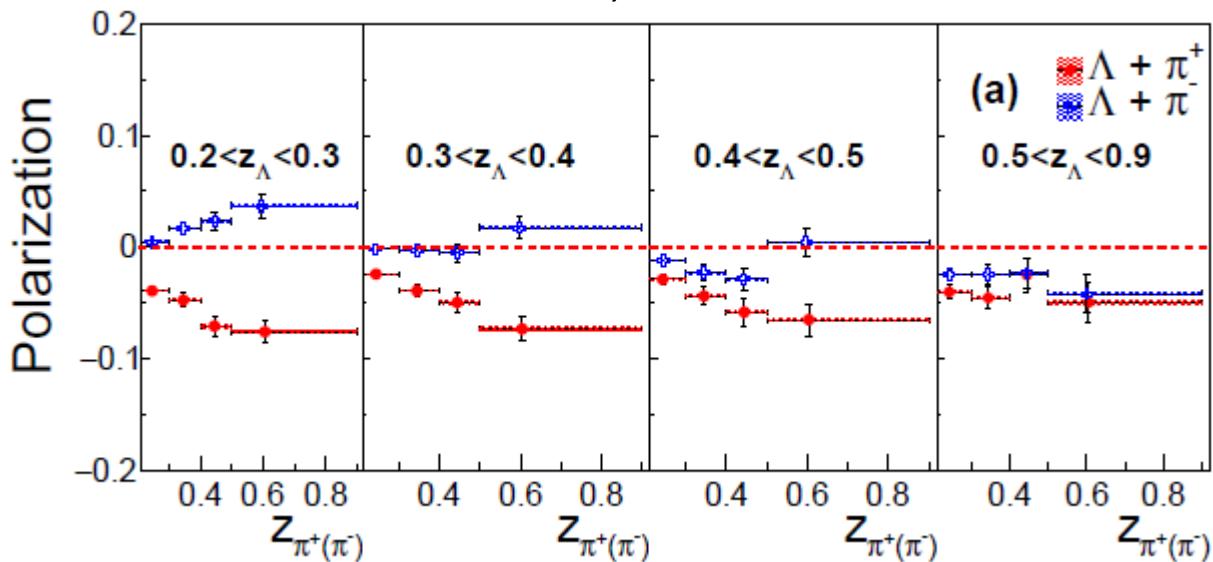
## Contents

- Isospin-symmetric polarized TMD FF  $D_{1T}^\perp$ 
  - Isospin asymmetry violation from EW decay
  - Fit to Belle Data with isospin-symmetry:  $D_{1Tu}^\Lambda = D_{1Td}^\Lambda$
- Chiral-odd TMD FFs  $H_{1Tq}^\Lambda$ ,  $H_{1Lq}^{\perp\Lambda}$  and  $H_{1Tq}^{\perp\Lambda}$ 
  - Longitudinal and transverse polarizations  $\vec{\mathcal{P}}_\Lambda = (\mathcal{P}_L, \mathcal{P}_T, \mathcal{P}_N)$  in unpolarized SIDIS and  $e^+e^-$
  - Nuclear effects in eA collision at EIC

# Hyperon polarization at Belle



Belle, PRL2019



➤ Polarization w.r.t. production plane

$$P_N = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow}$$

➤ In parton model

$$P_N \propto \sum_q c_q \frac{D_{1T,q}^{\perp,\Lambda}}{D_{1q}^\Lambda}$$

➤ Assuming favored FFs dominance

- $\Lambda(uds) + \pi^+(u\bar{d}) \sim d\bar{d}$  favored
- $\Lambda(uds) + \pi^-(d\bar{u}) \sim u\bar{u}$  favored

$$\Rightarrow \frac{D_{1T,u}^{\perp,\Lambda}}{D_{1u}^\Lambda} \neq \frac{D_{1T,d}^{\perp,\Lambda}}{D_{1d}^\Lambda} \Rightarrow D_{1T,u}^{\perp,\Lambda} \neq D_{1T,d}^{\perp,\Lambda}$$

# Unpolarized FFs

- Most of the global fits of unpolarized FFs respect isospin symmetry

*Metz, Vossen, Prog.Nucl.Part.Phys. 2016*

**Table 5**

Selection of global fits to  $D_1$ . The order of the fits from top to bottom is approximately chronologically.

Fit	Datasets	Uncertainties	Final States
EMC [396,453]	SIDIS	No	$\pi^\pm$
CGMG [454]	$e^+e^-$ , $pp(\bar{p})$	No	$\pi^0$
BKK [455–457]	$e^+e^-$	No	$h^\pm, \pi^\pm, K^\pm, K_S^0$
DSV [226]	$e^+e^-$	No	$\Lambda$
Kretzer [223]	$e^+e^-$ , SIDIS [458]	No	$h^\pm, \pi^\pm, K^\pm$
BFGW [459]	$e^+e^-$	Yes	$h^\pm$
KKP [460]	$e^+e^-$	No	$\pi^\pm, K^\pm, p(\bar{p})$
Bourrely, Soffer [461]	$e^+e^-$ , $pp(\bar{p})$	No	$\pi^0$
AKK08 [462]	$e^+e^-$ , $pp(\bar{p})$	No	$\pi^\pm, K^\pm, p/\bar{p}, K_S^0, \Lambda/\bar{\Lambda}$
HKNS [398]	$e^+e^-$	Yes	$\pi^\pm, K^\pm, p/\bar{p}$
AESSS [463]	$e^+e^-$ , $pp(\bar{p})$	Yes	$\eta$
DSS [131,464,132]	$e^+e^-$ , $pp(\bar{p})$ , SIDIS	Yes	$h^\pm, \pi^\pm, K^\pm, p/\bar{p}, \eta$
LSS [465]	SIDIS	No	$\pi^\pm$
ASR-NNLO [466]	$e^+e^-$	No	$\pi^\pm$
COMPASS-LO [397]	SIDIS	No	$\pi^\pm$

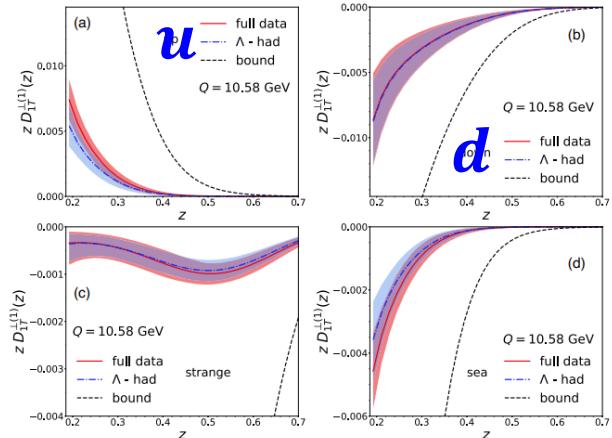
$$D_u^\Lambda(z) = D_d^\Lambda(z)$$

$$D_u^\Lambda(z) \neq D_d^\Lambda(z)$$

Some sets (such as **AKK08**) introduce large isospin symmetry violation.

# Parametrizations of polarized FF $D_{1T}^{\perp}$

➤ Fit to Belle data w/o isospin symmetry



$$D_{1T,u}^{\perp,\Lambda} \neq D_{1T,d}^{\perp,\Lambda}$$

*D'Alesio, Murgia, Zaccheddu, PRD 2020*

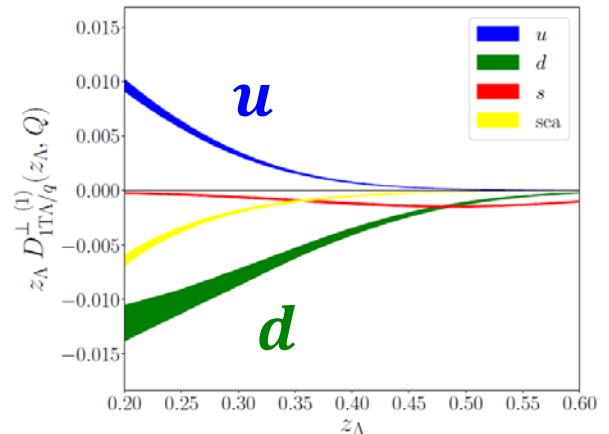
DMZ

➤ Spectator di-quark model results

*Li, Wang, Yang, Lu, EPJC 2021*

➤ QCD respect isospin symmetry, and QCD dominate hadronization process

How could there be any significant isospin symmetry violation?

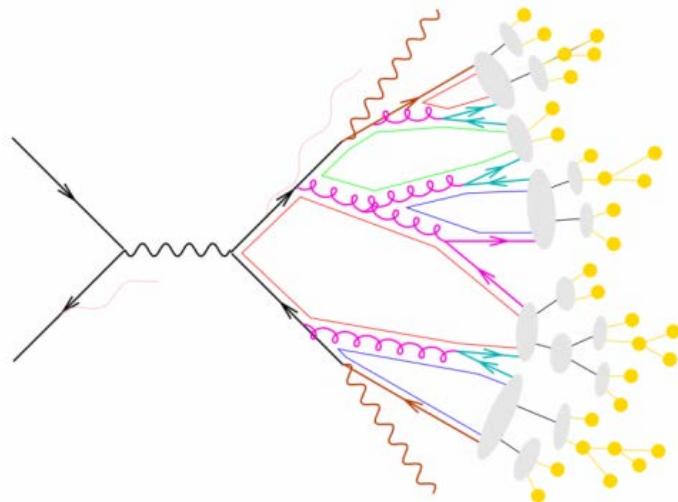


*Callos, Kang, Terry, PRD 2020*

CKT

# Isospin symmetry violation from EW resonance decay

- $e^-e^+$  annihilation



Directly produced hadrons

$$D_{1q}^h(z) = D_{1q}^{h,\text{dir}}(z) + D_{1q}^{h,\text{dec}}(z)$$

Hadrons from EW decays

$$D_{1q}^{h,h_j}(z) = Br(h, h_j) \int dz' K_{h,h_j}(z, z') D_{1q}^{h_j}(z')$$

- We assumed that  $D_{1q}^{h,\text{dir}}(z)$  respect isospin symmetry, while  $D_{1q}^{h,\text{dec}}(z)$  violate the symmetry.

unpolarized FF:

$$\frac{D_{1u}^{\Xi^0} - D_{1d}^{\Xi^-}}{D_{1u}^{\Xi^0} + D_{1d}^{\Xi^-}} \leq 5\%,$$

$$\frac{D_{1u}^\Lambda - D_{1d}^\Lambda}{D_{1u}^\Lambda + D_{1d}^\Lambda} \simeq 0$$

Chen, Liang, Pan, Song, Wei,

polarized FF:

$$\frac{G_{1Lu}^\Lambda - G_{1Ld}^\Lambda}{G_{1Lu}^\Lambda + G_{1Ld}^\Lambda} \sim \text{a tiny violation}$$

PLB 2021

- Based on these observations, we choose to use isospin symmetric  $D_1$  and  $D_{1T}^\perp$  to fit Belle data

# Fit to Belle data with isospin symmetric $D_{1T}^{\perp}$

- Factorized formulae for  $\Lambda$  transverse polarization

$$P_\Lambda(z_\Lambda, z_h) = \frac{\sqrt{\pi}}{2} \frac{z_h \Delta}{z_\Lambda M_\Lambda \sqrt{z_h^2 + z_\Lambda^2 \Delta_h^2 / \Delta^2}} \sum_q \left[ R_{1q}^{\Lambda h}(z_\Lambda, z_h) \frac{D_{1T,q}^{\perp,\Lambda}(z_\Lambda)}{D_{1q}^\Lambda(z_\Lambda)} + (q \rightarrow \bar{q}) \right]$$

- $R_{1q}^{\Lambda h}(z_\Lambda, z_h)$ : production weights of different quark flavors

$$R_{1q}^{\Lambda h}(z_\Lambda, z_h) = \frac{e_q^2 D_{1q}^\Lambda(z_\Lambda) D_{1\bar{q}}^h(z_h)}{\sum_q e_q^2 D_{1q}^\Lambda(z_\Lambda) D_{1\bar{q}}^h(z_h) + (q \rightarrow \bar{q})}$$

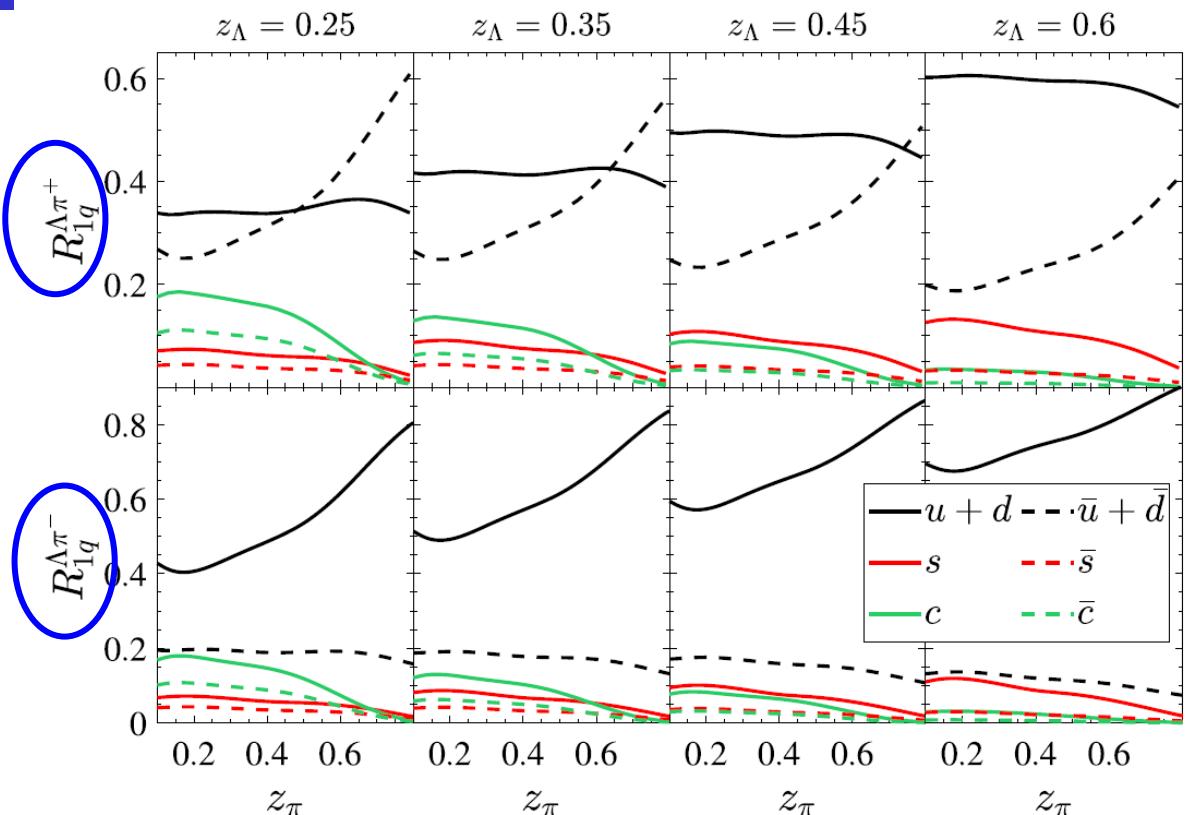
- For  $\Lambda$  fragmentation functions  $D_{1q}^\Lambda(z_\Lambda)$  we take **DSV** parametrizations

*de Florian, Stratmann, Vogelsang, PRD 1998*

- For  $\pi^\pm$  and  $K^\pm$  fragmentation functions  $D_{1q}^{\pi^\pm}(z_h)$  we take **DEHSS** parametrizations

*de Florian, Epele, Hernandez-Pinto, Sassot, Stratmann, PRD 2017*

# Fit to Belle data with isospin symmetric $D_{1T}^{\perp}$



$$R_{1q}^{\Lambda h}(z_\Lambda, z_h)$$

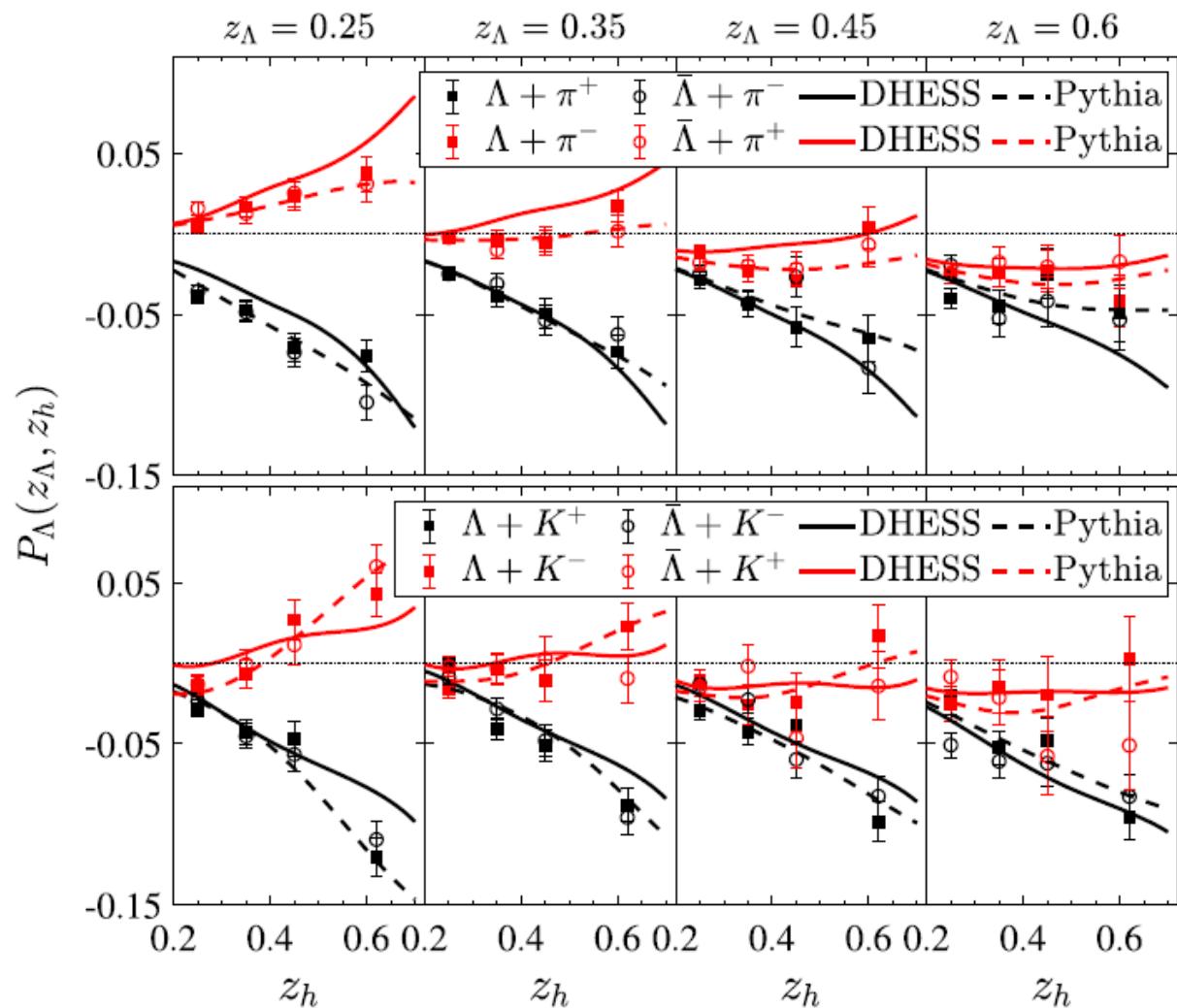
- Quark weights are **different** for  $\Lambda\pi^+$  and  $\Lambda\pi^-$
- Unfavored  $\bar{u} + \bar{d}$  FFs give large contributions
- $s, \bar{s}, c, \bar{c}$  contributions are non-negligible, and different with each other

- We take into account 6 different  $D_{1Tq}^{\perp\Lambda}$ 's

$$D_{1Tu}^{\perp\Lambda} = D_{1Td}^{\perp\Lambda}, \quad D_{1T\bar{u}}^{\perp\Lambda} = D_{1T\bar{d}}^{\perp\Lambda}, \quad D_{1Ts}^{\perp\Lambda}, \quad D_{1T\bar{s}}^{\perp\Lambda}, \quad D_{1Tc}^{\perp\Lambda}, \quad D_{1T\bar{c}}^{\perp\Lambda}$$

$$D_{1Tq}^{\perp\Lambda}(z) = N_{Tq} \frac{(\alpha_{Tq} + \beta_{Tq} - 1)^{\alpha_{Tq} + \beta_{Tq} - 1}}{(\alpha_{Tq} - 1)^{\alpha_{Tq} - 1} + \beta_{Tq}^{\beta_{Tq}}} z^{\alpha_{Tq}} (1-z)^{\beta_{Tq}} D_{1q}^{\Lambda}(z)$$

# Fit to Belle data



138 data points, 19 free parameters in all

$$\chi^2/d.o.f = \begin{cases} 2.215, & (\text{DSV}) \\ 0.978, & (\text{Pythia}) \end{cases}$$

Chen, Liang, Pan, Song, Wei,  
PLB 2021

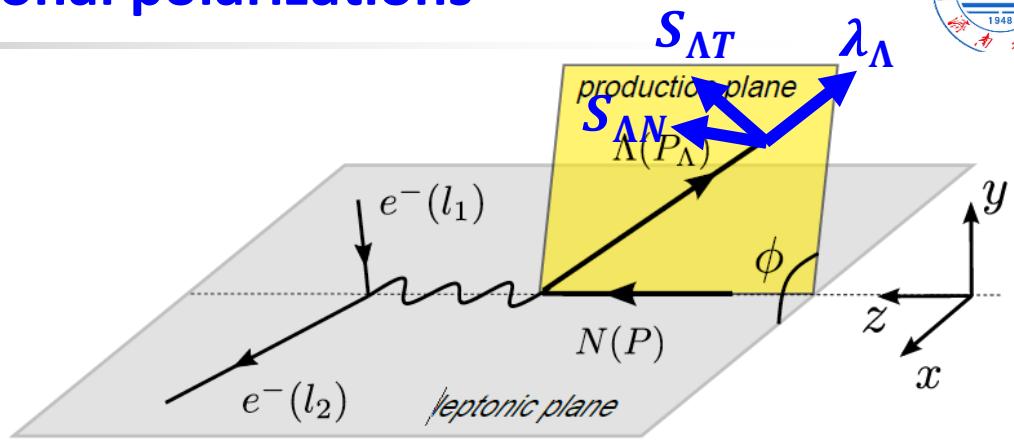
Isospin symmetric  $D_{1T,q}^\perp$ 's can fit Belle data well by considering all possible quarks flavors.

## Extension to SIDIS and 3-dimensional polarizations

$$\frac{d\sigma^{\text{SIDIS}}}{dxdydz_\Lambda d^2P_{\Lambda\perp}} = \frac{\pi\alpha_{\text{em}}^2}{2Q^4} \frac{y}{z_\Lambda} L_{\mu\nu} W^{\mu\nu}$$

$$\frac{d\sigma^{\text{SIDIS}}}{dxdydz_\Lambda d^2P_{\Lambda\perp}} = \frac{\pi\alpha_{\text{em}}^2}{2Q^4}$$

$$\begin{aligned} & \times \left\{ A(y) F_{UU}^T + B(y) F_{UU}^L + C(y) \cos \phi F_{UU}^{\cos \phi} + B(y) \cos 2\phi F_{UU}^{\cos 2\phi} \right. \\ & + \lambda_\Lambda \left[ C(y) \sin \phi F_{UL}^{\sin \phi} + B(y) \sin 2\phi F_{UL}^{\sin 2\phi} \right] \\ & + S_{\Lambda T} \left[ C(y) \sin \phi F_{UT}^{\sin \phi} + B(y) \sin 2\phi F_{UT}^{\sin 2\phi} \right] \\ & \left. + S_{\Lambda N} \left[ A(y) F_{UU}^T + B(y) F_{UU}^L + C(y) \cos \phi F_{UU}^{\cos \phi} + B(y) \cos 2\phi F_{UU}^{\cos 2\phi} \right] \right\} \end{aligned}$$



Non-zero polarizations  
along  $\hat{e}_L, \hat{e}_T, \hat{e}_N$   
 $\Rightarrow \mathcal{P}_L, \mathcal{P}_T, \mathcal{P}_N,$

➤ In the rest frame of  $\Lambda$ , the polarization vector  $\vec{S}_\Lambda = \lambda_\Lambda \hat{e}_L + S_{\Lambda T} \hat{e}_T + S_{\Lambda N} \hat{e}_N$

$\hat{e}_L$  :  $\Lambda$  3-momentum  $\vec{P}_\Lambda$  direction,

$\hat{e}_T, \hat{e}_N$  : transverse direction inside and normal to the production plane

# SIDIS and $e^+e^-$ annihilation

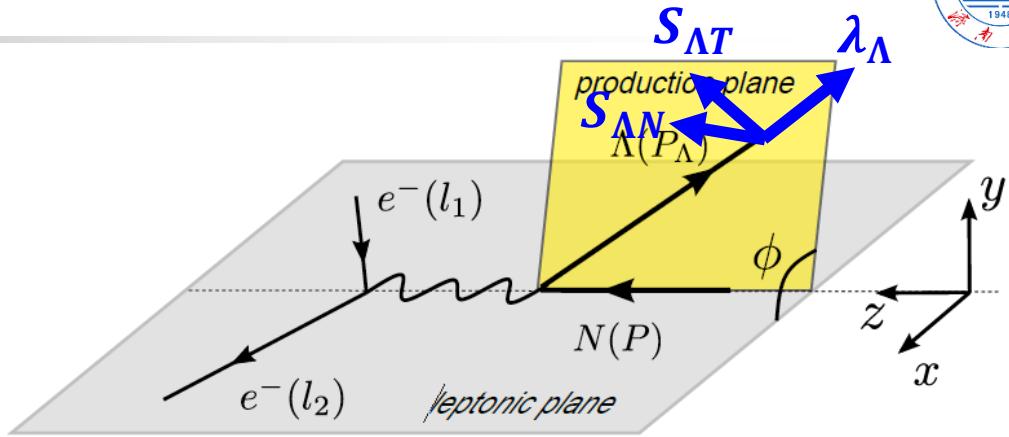
- In parton model at leading twist

$$\mathcal{P}_L = \frac{B(y) \mathcal{I}[w_2 h_1^\perp \mathbf{H}_{1L}^{\perp\Lambda}] \sin 2\phi}{A(y) \mathcal{I}[f_1 D_1^\Lambda] + B(y) \mathcal{I}[w_2 h_1^\perp \mathbf{H}_{1T}^{\perp\Lambda}] \cos 2\phi}$$

$$\mathcal{P}_T = \frac{B(y) \{-\mathcal{I}[w_1 h_1^\perp \mathbf{H}_{1T}^\Lambda] + \mathcal{I}[w_{3a} h_1^\perp \mathbf{H}_{1T}^{\perp\Lambda}]\} \sin 2\phi}{A(y) \mathcal{I}[f_1 D_1^\Lambda] + B(y) \mathcal{I}[w_2 h_1^\perp \mathbf{H}_{1T}^{\perp\Lambda}] \cos 2\phi}$$

$$\mathcal{P}_N = \frac{A(y) \mathcal{I}[\bar{w}_1 f_1 \mathbf{D}_{1T}^{\perp\Lambda}] / z_\Lambda + B(y) \{\mathcal{I}[w_1 h_1^\perp \mathbf{H}_{1T}^\Lambda] + \mathcal{I}[w_{3b} h_1^\perp \mathbf{H}_{1T}^{\perp\Lambda}]\} \cos 2\phi}{A(y) \mathcal{I}[f_1 D_1^\Lambda] + B(y) \mathcal{I}[w_2 h_1^\perp \mathbf{H}_{1T}^{\perp\Lambda}] \cos 2\phi}$$

- Measurements of these polarizations will help to study chiral-odd TMD FFs  $\mathbf{H}_1^\perp, \mathbf{H}_{1L}^\perp, \mathbf{H}_{1T}, \mathbf{H}_{1T}^{\perp\Lambda}$
- Quite similar kinematics and polarization observables at  $e^+e^-$  annihilations



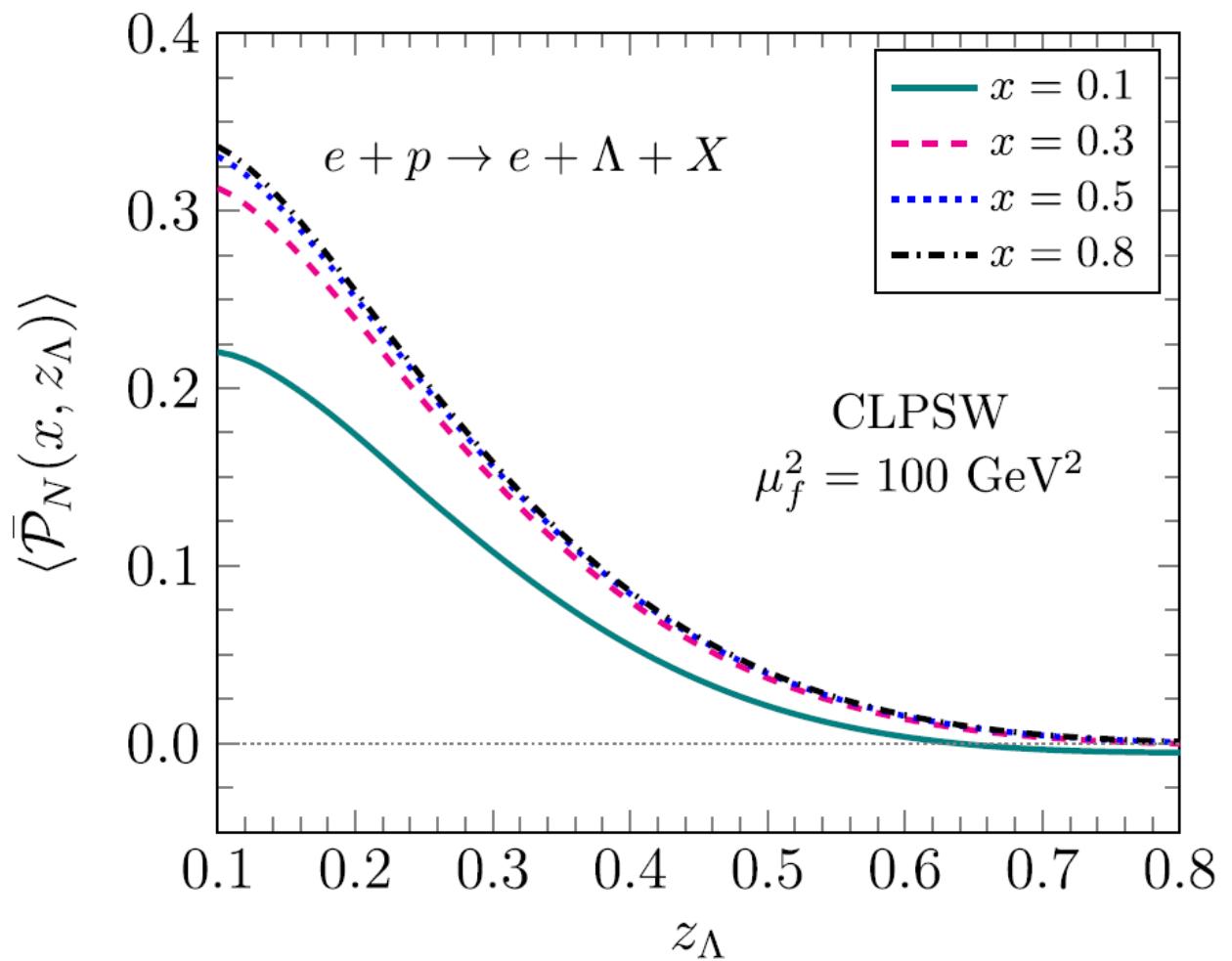
- Integrating over  $\vec{P}_{\Lambda\perp}$ , only  $\mathcal{P}_N$  survive.

Taking Gaussian ansatz for  $p_T$  distribution, we have

$$\langle \bar{\mathcal{P}}_N(x, z_\Lambda) \rangle = \frac{\sqrt{\pi} \kappa_3(z_\Lambda)}{2z_\Lambda} \frac{e_q^2 x f_{1q}(x) D_{1Tq}^{\perp\Lambda}(z_\Lambda)}{e_q^2 x f_{1q}(x) D_{1q}^\Lambda(z_\Lambda)}$$

- $f_{1q}$ : NLO CT14  
*Dulat, Hou, Gao, et al. PRD 2016*
- $D_{1q}$ : DSV  
*de Florian, Stratmann, Vogelsang, PRD 1998*
- $D_{1T}^{\perp\Lambda}$ : CLPSW  
*Chen, Liang, Pan, Song, Wei, PLB 2021*

*Chen, Liang, Song, Wei, PRD 2022*



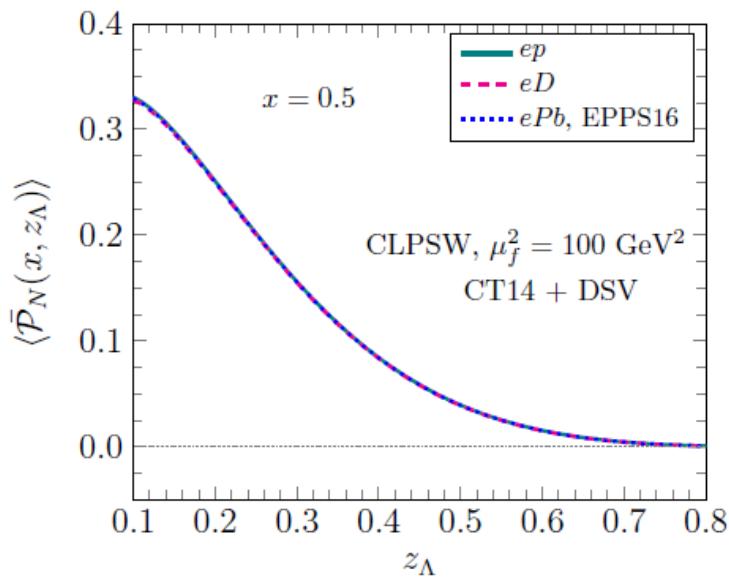
# Test of Isospin symmetry at the EIC with $\mathcal{P}_N$ for SIDIS

Different u/d ratio →  $\begin{cases} \text{same } \mathcal{P}_N, \\ \text{different } \mathcal{P}_N, \end{cases}$

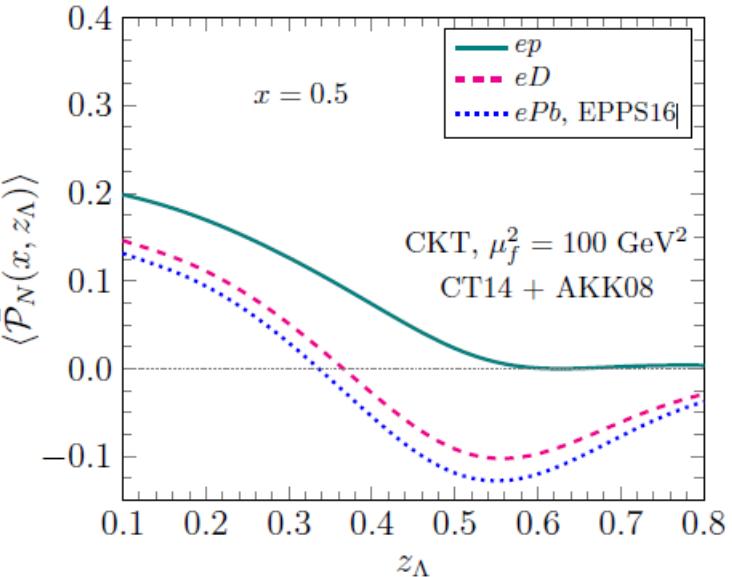
$$ep/eD/ePb \rightarrow e\Lambda X$$

$$\begin{aligned} & (\mathbf{D}_{1u}^\perp = \mathbf{D}_{1d}^\perp), \quad \text{CLPSW} \\ & (\mathbf{D}_{1u}^\perp \neq \mathbf{D}_{1d}^\perp), \quad \text{CKT, DMZ} \end{aligned}$$

- Chen, Liang, Pan, Song, Wei, PLB 2021
- Callos, Kang, Terry, PRD 2020
- D'Alesio, Murgia, Zaccheddu, PRD 2020

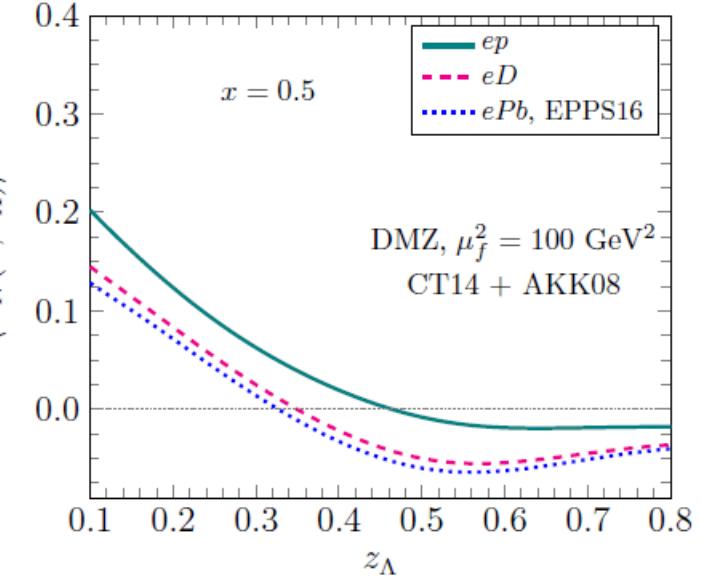


Isospin symmetric parametrization



Isospin symmetry violating parametrizations

Chen, Liang, Song, Wei, PRD 2022



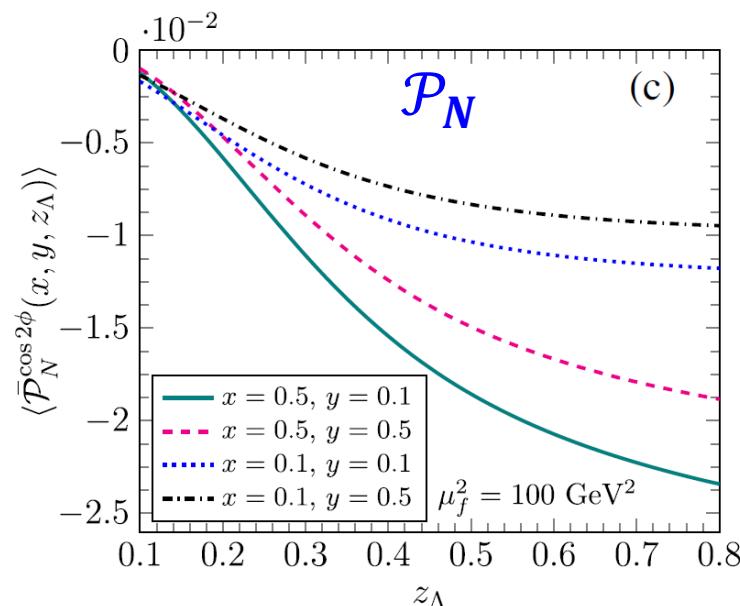
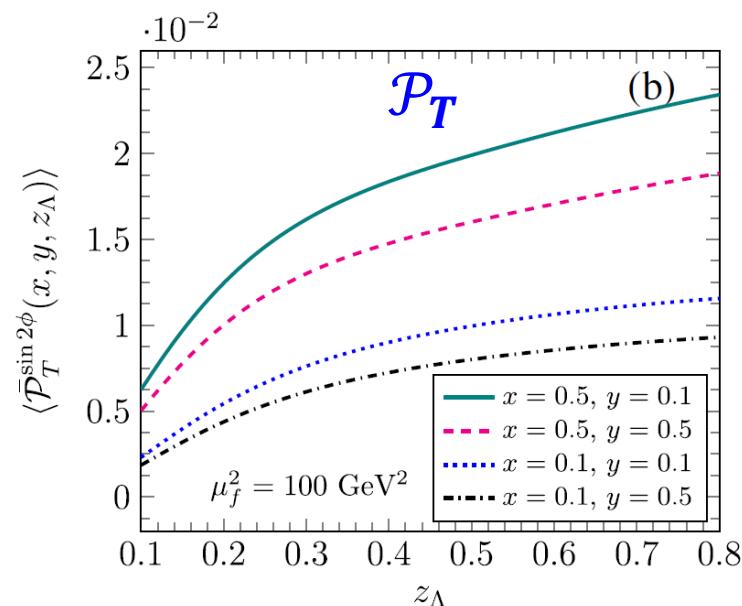
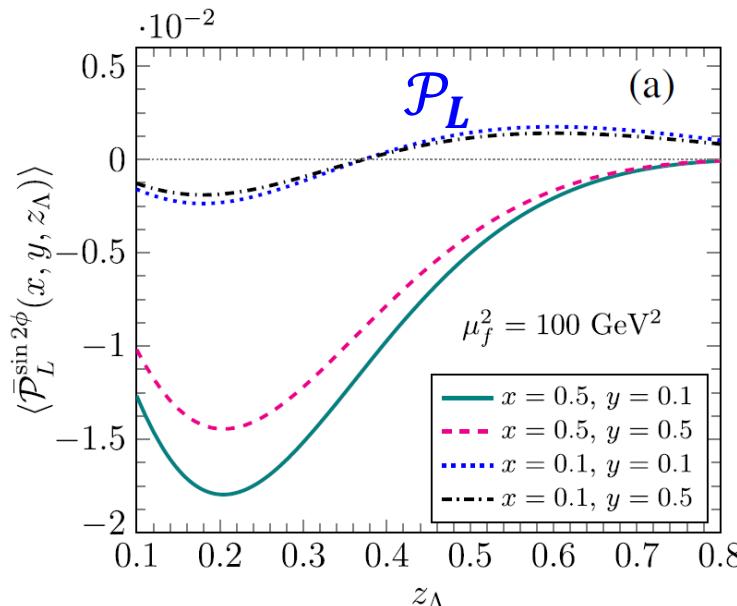
# Weighted polarization $\mathcal{P}_L, \mathcal{P}_T, \mathcal{P}_N$ in SIDIS

$$\langle \mathcal{P}_{L/T/N}^{\sin n\phi} \rangle \equiv \frac{\int d\phi \sigma \mathcal{P}_{L/T/N} \sin n\phi}{\int d\phi \sigma} \quad \Rightarrow \quad \langle \mathcal{P}_L^{\sin 2\phi} \rangle = \frac{B(y) \mathcal{I}[w_2 h_1^\perp H_{1L}^{\perp\Lambda}]}{2A(y) \mathcal{I}[f_1 D_1^\Lambda]}$$

$$\langle \mathcal{P}_T^{\sin 2\phi} \rangle = \frac{B(y) \{-\mathcal{I}[w_1 h_1^\perp H_{1T}^\Lambda] + \mathcal{I}[w_{3a} h_1^\perp H_{1T}^{\perp\Lambda}]\}}{2A(y) \mathcal{I}[f_1 D_1^\Lambda]}, \quad \langle \mathcal{P}_N^{\cos 2\phi} \rangle = \frac{B(y) \{\mathcal{I}[w_1 h_1^\perp H_{1T}^\Lambda] + \mathcal{I}[w_{3b} h_1^\perp H_{1T}^{\perp\Lambda}]\}}{2A(y) \mathcal{I}[f_1 D_1^\Lambda]}$$

To estimate their magnitudes, we take  $H_{1L}^{\perp\Lambda} \sim H_{1T}^{\perp\Lambda} \sim D_{1T}^{\perp\Lambda}/z_\Lambda$ ,  $H_{1T}^\Lambda \sim G_{1L}^\Lambda$

$D_{1T}^{\perp\Lambda}$ : CLPSW.  $f_1$ : NLO CT14.  $D_1^\Lambda, G_{1L}^\Lambda$ : DSV  $h_1^\perp$ : Lu, Schmidt, PRD 2010.



Chen, Liang, Song, Wei, PRD 2022

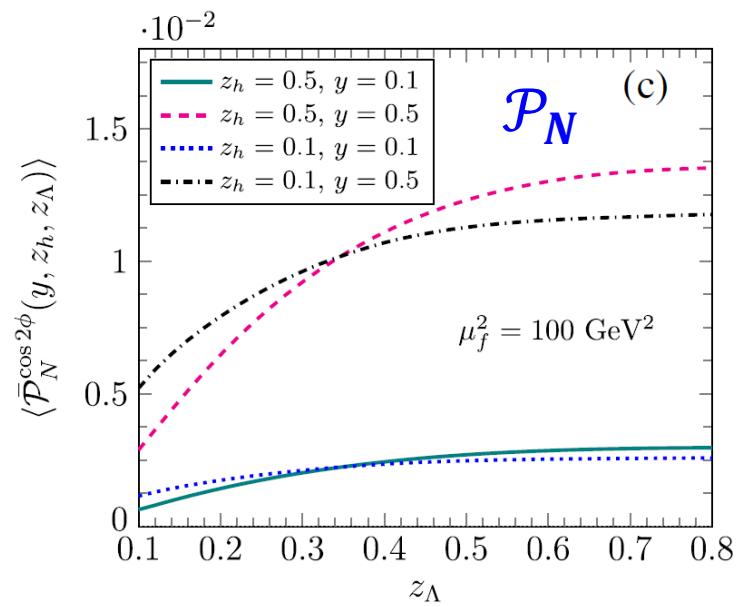
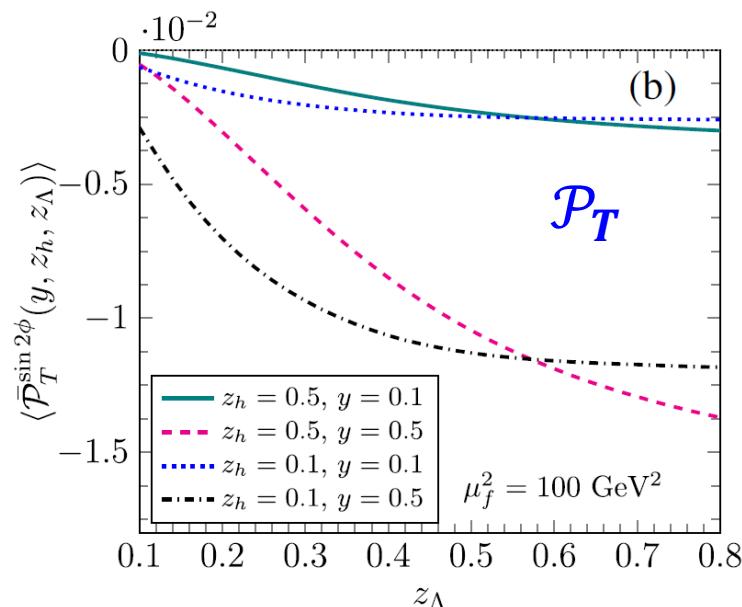
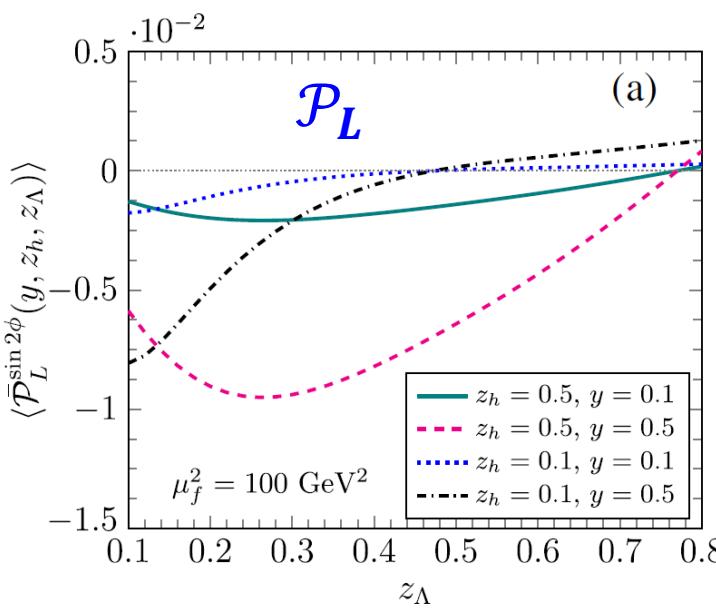
# Weighted polarization $\mathcal{P}_L, \mathcal{P}_T, \mathcal{P}_N$ in $e^+e^- \rightarrow \Lambda\pi^+X$

$$\langle \mathcal{P}_L^{\sin 2\phi} \rangle = \frac{\mathcal{B}(y)\mathcal{I}[\tilde{w}_2 H_1^{\perp\pi} H_{1L}^{\perp\Lambda}]}{2\mathcal{A}(y)\mathcal{I}[D_1^\pi D_1^\Lambda]}$$

$$\langle \mathcal{P}_T^{\sin 2\phi} \rangle = \frac{\mathcal{B}(y)\{-\mathcal{I}[\tilde{w}_1 H_1^{\perp\pi} H_{1T}^\Lambda] + \mathcal{I}[\tilde{w}_{3a} H_1^{\perp\pi} H_{1T}^{\perp\Lambda}]\}}{2\mathcal{A}(y)\mathcal{I}[D_1^\pi D_1^\Lambda]}, \quad \langle \mathcal{P}_N^{\cos 2\phi} \rangle = \frac{\mathcal{B}(y)\{\mathcal{I}[\tilde{w}_1 H_1^{\perp\pi} H_{1T}^\Lambda] + \mathcal{I}[\tilde{w}_{3b} H_1^{\perp\pi} H_{1T}^{\perp\Lambda}]\}}{2\mathcal{A}(y)\mathcal{I}[D_1^\pi D_1^\Lambda]}$$

To estimate their magnitudes, we take  $H_{1L}^{\perp\Lambda} \sim H_{1T}^{\perp\Lambda} \sim D_{1T}^{\perp\Lambda}/z_\Lambda$ ,  $H_{1T}^\Lambda \sim G_{1L}^\Lambda$

$D_{1T}^{\perp\Lambda}$ : CLPSW.  $D_1^\Lambda, G_{1L}^\Lambda$ : DSV.  $D_1^\pi$ : DEHSS.  $H_1^{\perp\pi}$ : *Anselmino et al., PRD 2015.*

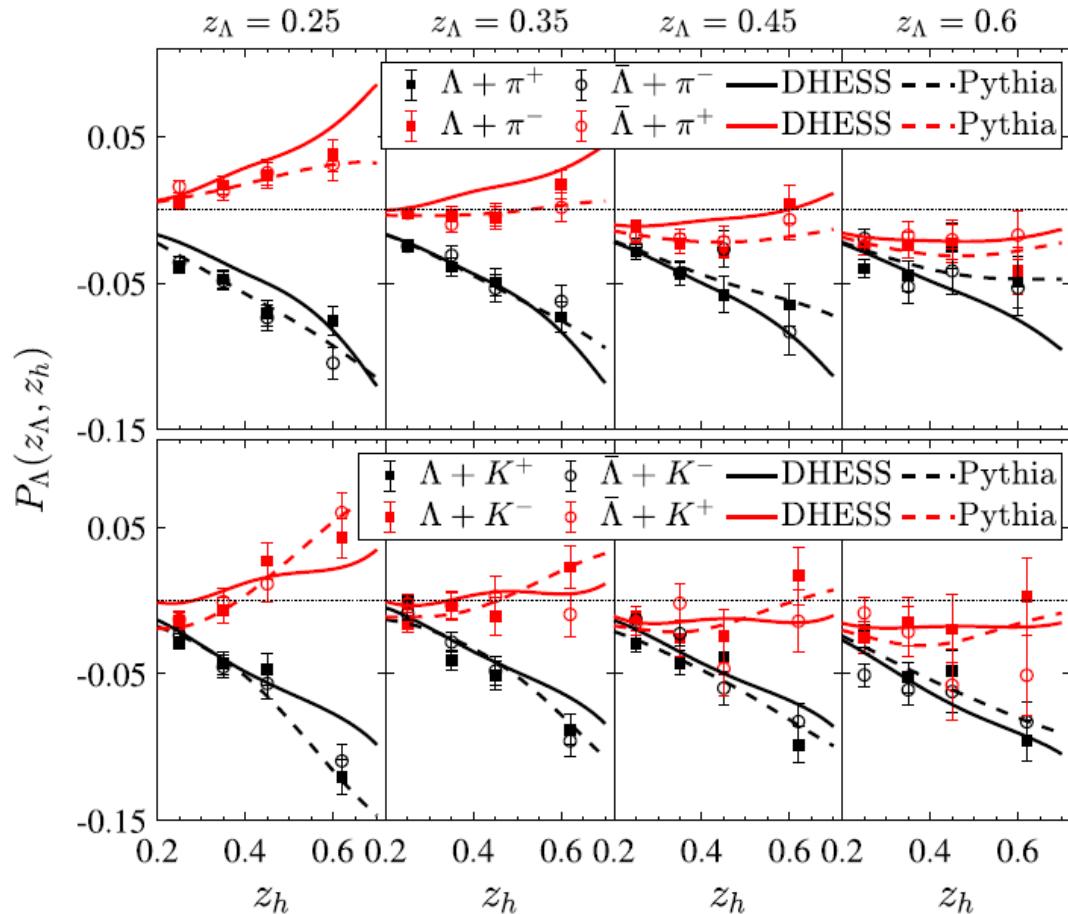


*Chen, Liang, Song, Wei, PRD 2022*

## Conclusions and outlook

- Isospin-symmetric polarized TMD FF  $D_{1T}^\perp$ 
  - Isospin asymmetry violation from EW decay is tiny
  - We can fit Belle data well with isospin symmetric  $D_{1T}^\perp$
- Chiral-odd TMD FFs  $H_{1Tq}^\Lambda$ ,  $H_{1Lq}^{\perp\Lambda}$  and  $H_{1Tq}^{\perp\Lambda}$ 
  - Longitudinal and transverse polarizations  $\vec{\mathcal{P}}_\Lambda = (\mathcal{P}_L, \mathcal{P}_T, \mathcal{P}_N)$  in unpolarized SIDIS and  $e^+e^-$  are estimated, and the polarizations are measurable.
  - eA collision at EIC can be employed to test isospin symmetries in  $D_{1T}^\perp$

# Backup : Fitted parameters for $D_{1T}^\perp$



parameter	$u, d$	$s$	$c$	$\bar{u}, \bar{d}$	$\bar{s}$	$\bar{c}$	
$\frac{\Delta}{M_\Lambda} N_{Tq}$	0.391	-0.391	0.0278	-0.456	-0.430	0.401	← DSV
	0.245	-0.148	0.108	-0.231	0.523	-0.324	← Pythia
$\alpha_q$	1.38	6.91	1.43	1.00	2.64	11.6	
	2.41	1.54	5.14	1.86	1.74	1.02	
$\beta_q$	3.98	0.646	14.3	0.0319	2.77	14.9	
	7.69	0.551	15.0	2.35	14.9	2.41	

138 data points, 19 free parameters in all

$$\chi^2/d.o.f = \begin{cases} 2.215, & (\text{DSV}) \\ 0.978, & (\text{Pythia}) \end{cases}$$

K.B.Chen, Z.T.Liang, Y.L.Pan, YKS, S.Y.Wei, Phys. Lett. B 816 (2021) 136217

# Extension to SIDIS and 3-dimensional polarizations

- In parton model at leading twist

$$\mathcal{P}_L = \frac{B(y)\mathcal{I}[w_2 h_1^\perp H_{1L}^{\perp\Lambda}]\sin 2\phi}{A(y)\mathcal{I}[f_1 D_1^\Lambda] + B(y)\mathcal{I}[w_2 h_1^\perp H_1^{\perp\Lambda}]\cos 2\phi}$$

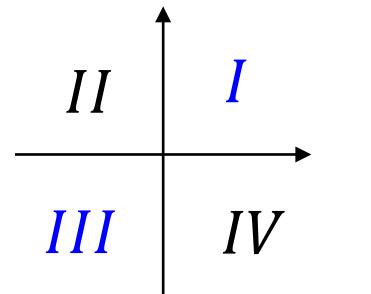
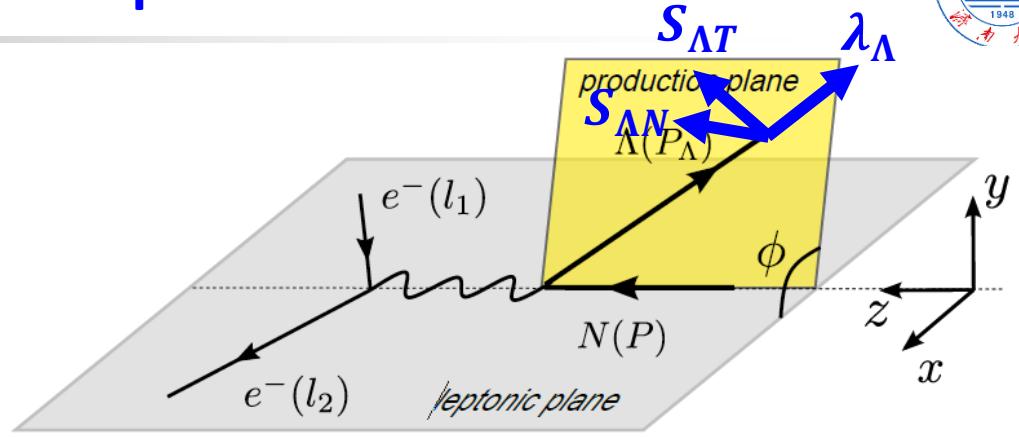
$$\mathcal{P}_T = \frac{B(y)\{-\mathcal{I}[w_1 h_1^\perp H_{1T}^\Lambda] + \mathcal{I}[w_{3a} h_1^\perp H_{1T}^{\perp\Lambda}]\}\sin 2\phi}{A(y)\mathcal{I}[f_1 D_1^\Lambda] + B(y)\mathcal{I}[w_2 h_1^\perp H_1^{\perp\Lambda}]\cos 2\phi}$$

$$\mathcal{P}_N = \frac{A(y)\mathcal{I}[\bar{w}_1 f_1 D_{1T}^{\perp\Lambda}]/z_\Lambda + B(y)\{\mathcal{I}[w_1 h_1^\perp H_{1T}^\Lambda] + \mathcal{I}[w_{3b} h_1^\perp H_{1T}^{\perp\Lambda}]\}\cos 2\phi}{A(y)\mathcal{I}[f_1 D_1^\Lambda] + B(y)\mathcal{I}[w_2 h_1^\perp H_1^{\perp\Lambda}]\cos 2\phi}$$

- Polarization  $\mathcal{P}_i$  in given Quadrants

$$\langle \mathcal{P}_L \rangle_{I+III} = \frac{2}{\pi} \frac{B(y)\mathcal{I}[w_2 h_1^\perp H_{1L}^{\perp\Lambda}]}{A(y)\mathcal{I}[f_1 D_1^\Lambda]}, \quad \langle \mathcal{P}_T \rangle_{I+III} = \frac{2}{\pi} \frac{B(y)\{-\mathcal{I}[w_1 h_1^\perp H_{1T}^\Lambda] + \mathcal{I}[w_{3a} h_1^\perp H_{1T}^{\perp\Lambda}]\}}{A(y)\mathcal{I}[f_1 D_1^\Lambda]}$$

$$\langle \mathcal{P}_N \rangle_{I+IV} = \frac{A(y)\mathcal{I}[\bar{w}_1 f_1 D_{1T}^{\perp\Lambda}]/z_\Lambda}{\mathcal{I}[f_1 D_1^\Lambda]}, \quad \langle \mathcal{P}_N \rangle_{i+iii} = \frac{A(y)\mathcal{I}[\bar{w}_1 f_1 D_{1T}^{\perp\Lambda}]/z_\Lambda + B(y)\{\mathcal{I}[w_1 h_1^\perp H_{1T}^\Lambda] + \mathcal{I}[w_{3b} h_1^\perp H_{1T}^{\perp\Lambda}]\}(2/\pi)}{A(y)\mathcal{I}[f_1 D_1^\Lambda] + B(y)\mathcal{I}[w_2 h_1^\perp H_1^{\perp\Lambda}](2/\pi)}$$



## Extension to SIDIS and 3-dimensional polarizations

- $\sin n\phi$ - and  $\cos n\phi$ -weighted polarizations

$$\langle \mathcal{P}_{L/T/N}^{\sin n\phi} \rangle = \frac{\int d\phi \sigma (\mathcal{P}_{L/T/N} \sin n\phi)}{\int d\phi \sigma},$$

$$\langle \mathcal{P}_{L/T/N}^{\cos n\phi} \rangle = \frac{\int d\phi \sigma (\mathcal{P}_{L/T/N} \cos n\phi)}{\int d\phi \sigma}$$

For example,

$$\langle \mathcal{P}_L^{\sin 2\phi} \rangle = \frac{1}{2} \frac{B(y) \mathcal{I}[w_2 h_1^\perp H_{1L}^{\perp\Lambda}]}{A(y) \mathcal{I}[f_1 D_1^\Lambda]}, \quad \langle \mathcal{P}_T^{\sin 2\phi} \rangle = \frac{1}{2} \frac{B(y) \{-\mathcal{I}[w_1 h_1^\perp H_{1T}^\Lambda] + \mathcal{I}[w_{3a} h_1^\perp H_{1T}^{\perp\Lambda}]\}}{A(y) \mathcal{I}[f_1 D_1^\Lambda]}$$

$$\langle \mathcal{P}_N^{\cos 2\phi} \rangle = \frac{1}{2} \frac{B(y) \{\mathcal{I}[w_1 h_1^\perp H_{1T}^\Lambda] + \mathcal{I}[w_{3b} h_1^\perp H_{1T}^{\perp\Lambda}]\}}{A(y) \mathcal{I}[f_1 D_1^\Lambda]}$$

- Measurements of these polarizations will help to study chiral-odd TMD FFs  $H_1^\perp, H_{1L}^\perp, H_{1T}, H_{1T}^{\perp\Lambda}$
- Quite similar kinematics and polarization observables at  $e^+e^-$  annihilations

