Jet-axis probe of TMD PDFs and phenomenology of time-reversal-odd jet function

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We reinterpret jet clustering as an axis-finding procedure which, along with the proton beam, defines the virtual-photon transverse momentum q_T in deep inelastic scattering (DIS). In this way, we are able to probe the nucleon intrinsic structure using jet axes in a fully inclusive manner, similar to the Drell-Yan process. We present the complete list of azimuthal asymmetries and the associated factorization formulae at leading power for deep-inelastic scattering of a nucleon. The factorization formulae involve both the conventional time-reversal-even (T-even) jet function and the T-odd one, which have access to all transverse-momentum-dependent parton distribution functions (TMD PDFs) at leading twist. Since the factorization holds as long as $q_T \ll Q$, where Q is the photon virtuality, the jet-axis probe into the nucleon structure should be feasible for machines with relatively low energies such as the Electron-Ion Collider in China (EicC). We show that, within the winner-take-all (WTA) axis-finding scheme, the coupling between the T-odd jet function and the quark transversity or the Boer-Mulders function could induce sizable azimuthal asymmetries at the EicC, the EIC, and HERA.

KEYWORDS: T-odd jet function, 3D nucleon tomography, nucleon spin structure

1. Introduction

Jets, originated as a primary technique for studying the strong interactions and having demonstrated its power in probing the fundamental properties of quantum chromodynamics (QCD), are now a rising star as a means to probe the nucleon spin structure. The high luminosity of the current and future colliders have considerably boosted the studies of jets and jet substructure for portraying the full three-dimensional (3D) image of the nucleons. The jet probe into the nucleon structure has been shown to be able to access some of the transverse-momentum-dependent parton distribution functions (TMD PDFs), including the Sivers function of a transversely polarized nucleon in addition to the unpolarized one.

In order to maximize the full outreach of the jet probe into the complete list of the nucleon spin structures, the concept of the time-reversal-odd (T-odd) jet function was proposed recently [1]. The T-odd jet function couples directly to the chiral-odd nucleon parton distributions, such as the quark

transversity and the Boer-Mulders function of the proton. It immediately opens up many unique opportunities for probing the nucleon intrinsic spin dynamics using jets, which were thought to be impossible. Besides, the T-odd jet function is interesting by its own, since it could "film" the QCD nonperturbative dynamics by continuously changing the jet axis from one to another.

In Section 2, we explain how the jet axis is used for measuring the photon q_T in a fully inclusive way and argue why the jet-axis probe of the nucleon TMD PDFs is feasible even for low-energy machines such as the EicC. In Section 3, we review the notion of the T-odd jet function. In Section 4, we show that, including the T-odd jet function, one can access all the eight TMD PDFs of the nucleon at leading twist from the azimuthal asymmetries in jet-axis probes in deep-inelastic scattering (DIS) of a nucleon. We give predictions on the azimuthal asymmetries associated with the couplings of the T-odd jet function with the quark transversity and the Boer-Mulders function at the EIC, the EicC, and HERA. In Section 5, we give a summary and an outlook.

2. Measuring photon q_T in DIS

All conventional probes of the TMDs and the spin structure are more or less equivalent to measuring the virtual-photon transverse momentum q_T with respect to two pre-defined axes. For instance, in the Drell-Yan process, the incoming nucleon beams naturally set up the $\pm z$-axis and the photon transverse momentum is then straightforwardly determined. In DIS, since we only have one nucleon beam, we thus need another direction to define the photon q_T. Tagging a final-state hadron becomes a natural option for this purpose, and in this case, the photon q_T is then measured with respect to the nucleon beam and the tagged hadron momentum P_h. This is nothing but the semi-inclusive deep inelastic scattering (SIDIS).

Finding an axis for measuring the photon q_T in DIS is certainly not limited to tagging hadrons. Many other strategies could also help here, such as the final-state-particle clustering. The procedure follows exactly the jet clustering algorithms, but with a different emphasis. Here the jet clustering procedure is merely a recursive algorithm for us to determine the axes. Once the axes are determined, we measure the photon q_T with respect to one of them and the proton beam to probe the nucleon structure, while totally ignore the jet. Therefore the jet-axis probe is fully differential just like the SIDIS.

Based on what we have described, we can derive the factorization formula for the jet-axis probe in DIS. Formally, the factorization theorem reads

$$d\sigma \propto f_i(x, p_T^2) \otimes \mathcal{J}_{1,i}(z, k_T^2) + g_i(x, p_T^2) \otimes \mathcal{J}_{T,i}(z, k_T^2) \tag{1}$$

where f_i and g_i are the chiral-even and chiral-odd nucleon TMD PDFs for parton flavor i respectively, and $\mathcal{J}_{1,i}(z, k_T^2)$ and $\mathcal{J}_{T,i}(z, k_T^2)$ are both the jet-axis-finding functions (jet functions) which encode the perturbatively calculable jet clustering procedure. The conventional unpolarized jet function $\mathcal{J}_{1,i}(z, k_T^2)$ is induced by an unpolarized quark, while a transversely polarized quark gives rise to the time-reversal odd (T-odd) jet function $\mathcal{J}_{T,i}(z, k_T^2)$. The detailed factorization form and the definition of the T-odd jet function, $\mathcal{J}_{T,i}(z, k_T^2)$, will be given in the following section. Here we note that the factorization theorem holds as long as $Q \gg k_T$, which is the same requirement for the SIDIS factorization to be valid. In this sense, just like the SIDIS, the jet-axis probe will also be low-energy-machine friendly, and could likely be implemented at the EicC.

To adapt the jet-axis finding procedure to low-energy machines, instead of using the usual k_T-type jet algorithms that are widely used at the LHC, in DIS we adopt the energy-type jet algorithms which is more feasible for clustering particles with low transverse momenta and populated in the forward/backward rapidities. For instance, we can adopt the spherically-invariant jet algorithm [2],

defined by

$$d_{ij} = \min(E_i^{-2}, E_j^{-2}) \frac{1 - \cos \theta_{ij}}{1 - \cos R}, \quad d_{iB} = E_i^{-2}, \tag{2}$$

where θ_{ij} is the angle between particles i and j, while E_i and E_j are the energies carried by them. The radius parameter R will be chosen such that $R \sim O(1) \gg q_T/Q$.

3. T-odd jet function

The inclusive photon q_T cross sections with respect to the proton beam and the jet axis can be written as a factorization theorem Eq. (1) derived from the soft-collinear effective theory (SCET). The factorization theorem involves the transverse-momentum-dependent (TMD) correlator

$$\mathcal{J}^{ij}(z, k_T) = \frac{1}{2z} \sum_X \int \frac{dy^+ d^2\boldsymbol{y}_T}{(2\pi)^3} e^{ik \cdot y} \langle 0|\chi_{\bar{n}}^i(y)|JX\rangle\langle JX|\bar{\chi}_{\bar{n}}^j(0)|0\rangle|_{y^-=0}, \tag{3}$$

where $\bar{n}$ is a light-like vector along the direction of the jet, $\chi_{\bar{n}} = W_{\bar{n}}^\dagger \xi_{\bar{n}}$ is the product of the collinear quark field $\xi_{\bar{n}}$ and the collinear Wilson line $W_{\bar{n}}^\dagger$. Here, z is the momentum fraction of the jet with respect to the fragmenting quark which initiates the jet, i.e. $z = P_J^-/k^-$, with P_J being the jet momentum that defines the jet axis, and k the momentum of the fragmenting quark. The jet-clustering algorithm dependence is implicit in Eq. (3), which determines the P_J and hence the jet axis, and can be calculated perturbatively.

Conventionally, only the chiral-even Dirac structure $\bar{\slashed{n}}$ in Eq. (3) was considered. However, as noted in Ref. [1], in the nonperturbative regime in which $k_T \sim \Lambda_{\text{QCD}}$, spontaneous chiral symmetry breaking leads to a nonzero component of the jet which is both time-reversal-odd (T-odd) and chiral-odd, when the jet axis is different from the direction of the fragmenting parton. Therefore, the correlator in Eq. (3) in general is a sum of two structures:

$$\mathcal{J}(z, k_T) = \mathcal{J}_1(z, k_T^2)\frac{\bar{\slashed{n}}}{2} + i\mathcal{J}_T(z, k_T^2)\frac{\slashed{k}_T\bar{\slashed{n}}}{2}, \tag{4}$$

where $\mathcal{J}_1(z, k_T^2)$ is the traditional jet function, and $\mathcal{J}_T(z, k_T^2)$ is the T-odd jet function. Due to its chiral-odd nature, an immediate application of the T-odd jet function is to probe the chiral-odd TMD PDFs of the nucleons in DIS, such as the Boer-Mulders function and the transversity, which were thought to be impossible to access using jets.

The T-odd jet function has the following advantages:

- *Universality*
  Like the traditional jet function, the T-odd jet function is process independent.

- *Flexibility*
  The flexibility of choosing a jet recombination scheme and hence the jet axis allows one to adjust sensitivity of the jet function to different nonperturbative contributions. This provides an opportunity to "film" the QCD nonperturbative dynamics, if one continuously changes the axis from one to another.

- *Perturbative predictability*
  Since a jet contains many hadrons, the jet function has more perturbatively calculable degrees of freedom than the fragmentation function. For instance, in the winner-take-all (WTA) scheme, for $R \sim O(1) \gg |\boldsymbol{q}_T|/E_J$, the z-dependence in the jet function is completely determined [3]:

$$\mathcal{J}(z, k_T, R) = \delta(1 - z)\mathfrak{J}(k_T) + O\left(\frac{k_T^2}{E_J^2 R^2}\right). \tag{5}$$

- *Nonperturbative predictability*
  Similar to the study in Ref. [4], the T-odd jet function can be factorized into a product of a perturbative coefficient and a nonperturbative factor. The nonperturbative factor has an operator definition [5], and as a vacuum matrix element, it can be calculated on the lattice [6, 7]. This is unlike the TMD fragmentation function, which is an operator element with a final-state hadron tagged, making evaluation on the lattice impossible by known techniques.

4. Jet-axis probe in DIS

Consider deep-inelastic scattering of an electron off a polarized nucleon $e^-(l) + N(P) \to e^-(l') + J(P_J) + X$, $(N = p, n)$, in which we tag a jet and specify the jet axis with some recombination scheme. We define the $\boldsymbol{q}_T$ of the virtual photon by going to the so-called factorization frame, in which the proton-beam direction and the jet-axis direction are exactly opposite to each other, as shown in Fig. 1 (a). Alternatively, one can go to the gamma-nucleon system (GNS), a frame in which the virtual photon momentum and the proton beam are head-to-head (including the case of proton being at rest), and define $\boldsymbol{P}_{J\perp}$ of the jet as in Fig. 1 (b). One can show that $\boldsymbol{q}_T = -\boldsymbol{P}_{J/\perp}/z$ up to corrections of order $1/Q^2$. Therefore, measuring $\boldsymbol{P}_{J\perp}$ is equivalent to measuring $\boldsymbol{q}_T$. In the following, we will describe the kinematics in the GNS system, which is a convention commonly used in semi-inclusive hadron production in SIDIS [8].
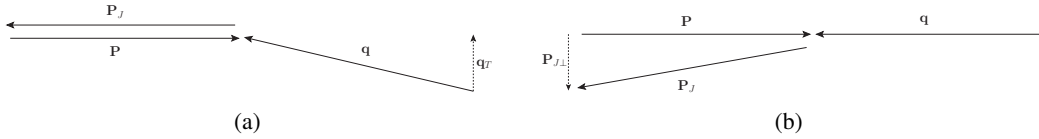

Figure 1. Axes in DIS in different frames: (a) the factorization frame, in which $\boldsymbol{q}_T$ is defined, and (b) the GNS system, in which $\boldsymbol{P}_{J\perp}$ is defined.

The definitions of $P_{J\perp}$, ϕ_J, and ϕ_S in the GNS system are the standard ones, and are depicted pictorially in Ref. [9]. We show the complete list of azimuthal asymmetries and the corresponding factorization formulae at leading power in $1/Q$ in Ref. [9]. At leading order in α_s and leading power in $1/Q$, there are eight nonvanishing asymmetries. Four of them arise from couplings of the traditional jet function with the four chiral-even TMD PDFs, f_1, g_{1L}, $f_{1T}^\perp$, and g_{1T}, while the other four arise from couplings of the T-odd jet function with the four chiral-odd TMD PDFs, $h_1^\perp$, $h_{1L}^\perp$, h_1, and $h_{1T}^\perp$.

As an example, we give predictions for the azimuthal asymmetry

$$A^{\sin(\phi_J+\phi_S)}(|\boldsymbol{P}_{J\perp}|) \equiv \frac{2}{|\boldsymbol{S}_\perp| \int d\sigma \epsilon} \int d\sigma \sin(\phi_J + \phi_S) \tag{6}$$

$$\propto h_1 \otimes \mathcal{J}_T, \tag{7}$$

which arises from the coupling of the T-odd jet function with the quark transversity. We adopt the spherically-invariant jet algorithm Eq. (2) with $R = 1$ and measure the jet charge as proposed in Ref. [10]. Figure 2 (a) shows the predictions for the asymmetry $A^{\sin(\phi_J+\phi_S)}(|\boldsymbol{P}_{J\perp}|)$ at the EIC in the WTA jet-axis scheme according to Eq. (7) (lines) (see Ref. [9] for details) and Eq. (6) (data points) from simulations using Pythia 8.2 [11] with the package StringSpinner [12], which incorporates spin interactions in the event generator. From Fig. 2 (a), we see that the theoretical predictions on the $A^{\sin(\phi_J+\phi_S)}(|\boldsymbol{P}_{J\perp}|)$ distribution from the factorization formula Eq. (7) roughly agree with the event generator simulations. In Fig. 2 (b), we show the predictions for $A^{\sin(\phi_J+\phi_S)}(|\boldsymbol{P}_{J\perp}|)$ from Pythia 8.2

with SMALL STRINGSPINNER in the E-scheme for the jet-axis definition, with the same kinematic setting as Fig. 2 (a). We see that the asymmetry no longer exists in the E-scheme. This is because the asymmetry is nonvanishing only when the direction of the fragmenting parton which initiates the jet differs from that of the jet axis, which hardly is the case in the E-scheme. In this sense, by choosing different jet axes we are able to "film" the nonperturbative dynamics of QCD.
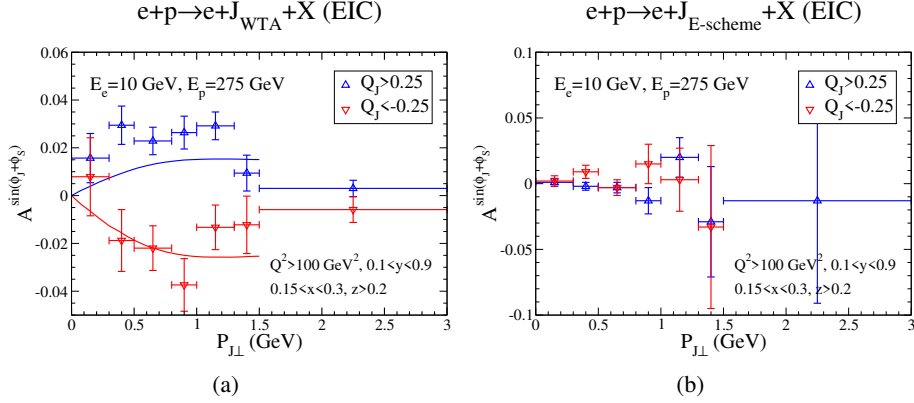

Figure 2. Azimuthal asymmetry $A^{\sin(\phi_J+\phi_S)}$ in jet-axis probes at the EIC in (a) the WTA scheme and (b) the E-scheme. The data points with error bars are from parton shower simulations. The solid lines in (a) are from Eq. (7).

In Ref. [9], we also give predictions on $A^{\sin(\phi_J+\phi_S)}(|\boldsymbol{P}_{J\perp}|)$ at the EicC and HERA, as well as predictions on the azimuthal asymmetry $A^{\cos(2\phi_J)}(|\boldsymbol{P}_{J\perp}|)$ associated with the coupling of the T-odd jet function with the Boer-Mulders function at the EIC, EicC, and HERA, in the WTA scheme. We find that the predicted asymmetries are of size similar to or bigger than those in Fig. 2 (a), and should be measurable.

5. Summary and outlook

In this work, we reinterpreted the jet clustering procedure as a way to define an axis, which together with the proton beam defines the transverse momentum of the vitual photon in DIS. In this way, one can use jet-axis measurements in DIS to probe the TMD PDFs of the nucleons, just like in the Drell Yan process. We provided the complete list of azimuthal asymmetries in the jet-axis probe in DIS at leading power. We showed that, by including the T-odd jet function in addition to the traditional one, all eight TMD PDFs of a nucleon at leading twist can be accessed by the jet-axis probe. As concrete examples, within the WTA axis scheme, we demonstrated that with both event-generator simulations and predictions from the factorization formulae, couplings of T-odd jet function with the quark transversity and the Boer-Mulders function give rise to sizable azimuthal asymmetries at DIS machines of various energy regimes, such as the EIC, the EicC, and HERA. We also demonstrated, with event-generator simulations, how the change of the jet-axis definition induces changes in the asymmetry distributions drastically. The T-odd jet function has opened the door to a fully comprehensive study of nucleon 3D structure with jet probes. Further theoretical and phenomenological studies of the T-odd jet function, such as high-order corrections, evaluations of the soft function on the lattice, and fittings with experimental data, will empower the jet probe as a precision tool which is fully differential for the study of TMD physics.

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