

3D Breit frame distributions as an Abel image of 2D light front EMT distributions

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Outline

- ▶ Energy momentum tensor (EMT) and Gravitational form factors
- ▶ Light front quark diquark model (LFQDM)
- ▶ GFFs in LFQDM
- ▶ 2D light front EMT distributions of proton in LFQDM
- ▶ Abel transformation of 2D EMT distributions

Energy momentum tensor (EMT) $T^{\mu\nu}$

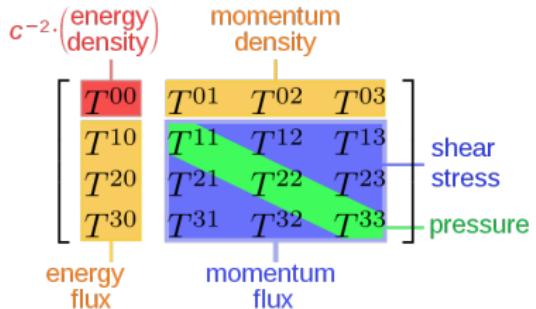
- The EMT arises from the response of the nucleon to a change of the external space-time metric.
- The matrix element of Energy momentum tensor (EMT) operator is parametrized in terms of four GFFs :

$$\langle p' | T_{q,g}^{\mu\nu} | p \rangle = \bar{u}(p') \left[-B_{q,g}(Q^2) \frac{P^\mu P^\nu}{M} + (A_{q,g}(Q^2) + B_{q,g}(Q^2)) \frac{i P^{\{\mu} \sigma^{\nu\}} \alpha \Delta_\alpha}{2M} \right. \\ \left. + \frac{D_{q,g}(Q^2)}{4M} (\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2) + \bar{C}_{q,g}(Q^2) M g_{\mu\nu} \right] u(p),$$

- The GFFs furnish essential information on the internal structure of the nucleon such as the mass, spin and mechanical stability of the nucleon.
- GFFs are related to the second moments of Generalized parton distributions (GPDs) .

EMT general properties

The components of the energy-momentum tensor inform how matter couples to the gravitational field.



- The 00 and 0i component of the static EMT contains the information about the energy and spin distribution inside the hadron respectively.
- The *D – term*, the last unknown global property of the proton, is extracted through the spatial-spatial component of the energy-momentum tensor, is deeply related to the stability of the nucleon.

Gravitational form factors (GFFs) in light front

$$T^{\mu\nu} = \frac{i}{2} [\bar{\psi} \gamma^\mu (\vec{\partial}^\nu \psi) - \bar{\psi} \gamma^\mu \overleftarrow{\partial}^\nu \psi],$$

$$\langle p, S | T_i^{\mu\nu}(0) | p', S' \rangle$$

GFFs	μ	ν	S	S'
A	+	+	\uparrow	\uparrow
B	+	+	\uparrow	\downarrow
A,B,D	-	\perp	\uparrow	\downarrow
A,B, \bar{C} ,D	+	-	\uparrow	\downarrow

- Sum rules

$$A(0) = A_q(0) + A_g(0) = 1, \quad B(0) = B_q(0) + B_g(0) = 0,$$

$$J(Q^2) = \frac{1}{2}(A(Q^2) + B(Q^2)), \quad J(0) = \frac{1}{2}$$

Light front quark diquark model (LFQDM)

- We assume that the virtual incoming photon is interacting with a valence quark and two other valence quarks form a diquark of definite mass with spin 0, called a scalar diquark.
- The proton state $P = |u(ud)\rangle + |d(uu)\rangle$ having momentum P and spin S , can be represented as a two-particle Fock-state as

$$\begin{aligned} |P; \pm\rangle &= \sum_q \int \frac{dx d^2 \mathbf{p}_\perp}{2(2\pi)^3 \sqrt{x(1-x)}} \\ &\times \left[\psi_+^{q\pm}(x, \mathbf{p}_\perp) \left| +\frac{1}{2}, 0; xP^+, \mathbf{p}_\perp \right\rangle + \psi_-^{q\pm}(x, \mathbf{p}_\perp) \left| -\frac{1}{2}, 0; xP^+, \mathbf{p}_\perp \right\rangle \right] \end{aligned}$$

where

$$p \equiv (xP^+, \frac{p^2 + |\mathbf{p}_\perp|^2}{xP^+}, \mathbf{p}_\perp) ; \quad P_X \equiv ((1-x)P^+, P_X^-, -\mathbf{p}_\perp)$$

-T. Gutsche, et al. Phys. Rev. D 92, 019902 (2015)

Light front Wave functions (LFWF)

- The LFWFs with spin-0 diquark

$$\begin{aligned}\psi_+^{q+}(x, \mathbf{p}_\perp) &= \varphi^{q(1)}(x, \mathbf{p}_\perp) \\ \psi_-^{q+}(x, \mathbf{p}_\perp) &= -\frac{p^1 + ip^2}{xM} \varphi^{q(2)}(x, \mathbf{p}_\perp) \\ \psi_+^{q-}(x, \mathbf{p}_\perp) &= \frac{p^1 - ip^2}{xM} \varphi^{q(2)}(x, \mathbf{p}_\perp) \\ \psi_-^{q-}(x, \mathbf{p}_\perp) &= \varphi^{q(1)}(x, \mathbf{p}_\perp)\end{aligned}$$

where wave functions $\varphi_q^{(1)}(x, \mathbf{p}_\perp)$ and $\varphi_q^{(2)}(x, \mathbf{p}_\perp)$ are the modified form of the soft-wall AdS/QCD prediction

$$\varphi^{q(i)}(x, \mathbf{p}_\perp) = N_q^{(i)} \frac{4\pi}{\kappa} \sqrt{\frac{\log(1/x)}{1-x}} x^{a_q^{(i)}} (1-x)^{b_q^{(i)}} \exp \left[-\frac{\mathbf{p}_\perp^2}{2\kappa^2} \frac{\log(1/x)}{(1-x)^2} \right]$$

where $\kappa = 0.4$ GeV is the AdS/QCD scale parameter.

-D. Chakrabarti and C. Mondal, Phys. Rev. D 88, 073006 (2013).

Gravitational form factors in LFQDM

- The GFFs in LFQDM is defined in terms of overlap integrals

$$A^{u+d}(Q^2) = \mathcal{I}_1^{u+d}(Q^2), \quad B^{u+d}(Q^2) = \mathcal{I}_2^{u+d}(Q^2)$$
$$D^{u+d}(Q^2) = -\frac{1}{Q^2} [2M^2 \mathcal{I}_1^{u+d}(Q^2) - Q^2 \mathcal{I}_2^{u+d}(Q^2) - \mathcal{I}_3^{u+d}(Q^2)]$$

where

$$\mathcal{I}_1^q(Q^2) = \int dx x \left[N_1^2 x^{2a_1} (1-x)^{2b_1+1} + N_2^2 x^{2a_2-2} (1-x)^{2b_2+3} \right. \\ \left. \times \frac{1}{M^2} \left(\frac{k^2}{\log(1/x)} - \frac{Q^2}{4} \right) \right] A(x),$$

$$\mathcal{I}_2^q(Q^2) = 2 \int dx N_1 N_2 x^{a_1+a_2} (1-x)^{b_1+b_2+2} A(x),$$

$$\mathcal{I}_3^q(Q^2) = 2 \int dx N_2 N_1 x^{a_1+a_2-2} (1-x)^{b_1+b_2+2} \left[\frac{4(1-x)^2 x^2}{\log(1/x)} + Q^2 (1-x)^2 - 4m^2 \right] A(x),$$

$$A(x) = \exp \left[-\frac{\log(1/x)}{k^2} \frac{Q^2}{4} \right],$$

-D. Chakrabarti, et al. Phys. Rev. D 102, 113011 (2020)

Extraction of GFFs in LFQDM

- $A(Q^2)$ and $B(Q^2)$ are well defined in whole range of energy.
- The $D(Q^2)$ form factor is not well defined at $Q^2 = 0$.
- Use a multipole parametrization to approximate the results of model calculations

$$D^q(Q^2) = \frac{a_q}{(1 + b_q Q^2)^{c_q}}$$

Parameters	$\mu^2(GeV^2)$	a_q	b_q	c_q
D_0^{fit}	initial scale	-18.8359	2.2823	2.7951
D_1^{fit}	[0.32 → 4.00]	-1.521	0.531	3.026

- We have checked the accuracy of our fitting techniques at the initial scale using the multidimensional Monte Carlo integration program **Vegas**.

2D LF EMT distributions

- The form factors appearing in matrix elements of the EMT encode spatial densities via Fourier transforms.
- The 2D light front energy, angular momentum, pressure and shear distributions are related to the GFFs by following relations respectively:

$$\mathcal{E}^{(2D)}(x_\perp) = P^+ \tilde{A}(x_\perp), \quad \rho_J^{(2D)}(x_\perp) = -\frac{1}{2} x_\perp \frac{d}{dx} \tilde{J}(x_\perp)$$

$$\begin{aligned} p^{(2D)}(x_\perp) &= \frac{1}{2x_\perp} \frac{d}{dx_\perp} \left(x_\perp \frac{d}{dx_\perp} \tilde{D}(x_\perp) \right), \\ s^{(2D)}(x_\perp) &= -x_\perp \frac{d}{dx_\perp} \left(\frac{1}{x_\perp} \frac{d}{dx_\perp} \tilde{D}(x_\perp) \right), \end{aligned}$$

where

$$\tilde{F}(x_\perp) = \int \frac{d^2 \Delta}{(2\pi)^2} e^{-i \Delta_\perp \cdot x_\perp} F(-\Delta_\perp^2)$$

x_\perp and Δ_\perp are the position and momentum vectors in the 2D transverse plane.

3D breit frame EMT distributions

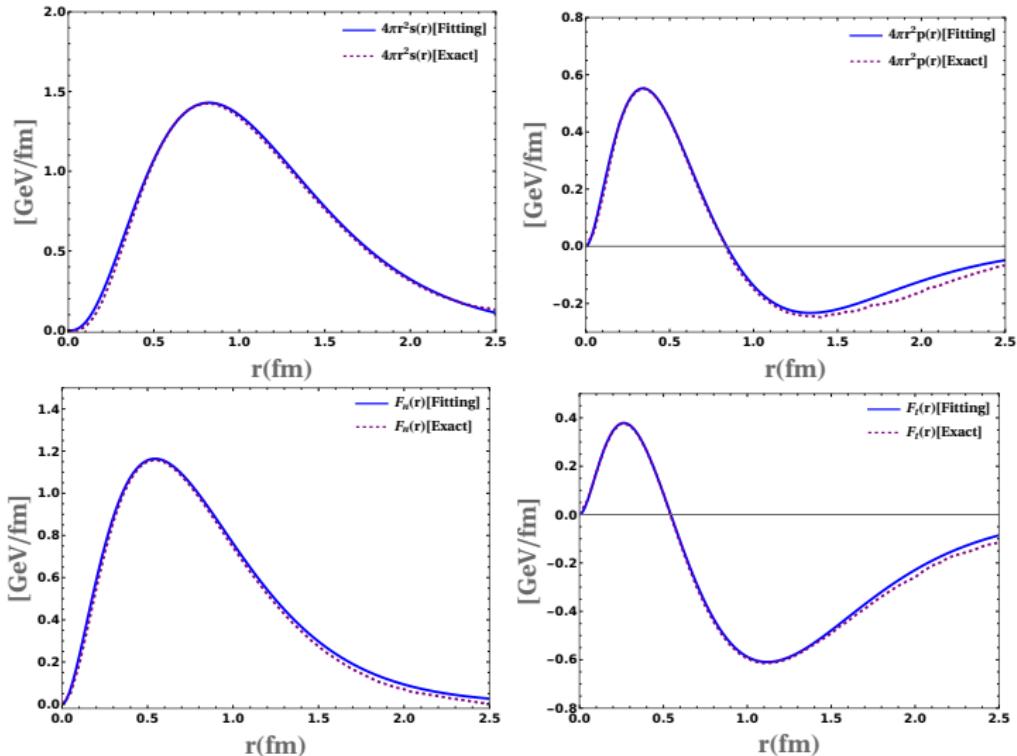
- The densities obtained through three-dimensional Fourier transforms are meaningless in the EMT density literature.
- The 3D breit frame EMT distributions are derived from the 2D light front EMT distributions by using the inverse **Abel transformation**

$$\begin{aligned}\epsilon(r) &= -\frac{1}{\pi} \int_r^\infty \frac{dx_\perp}{x_\perp} (\mathcal{E}(x_\perp)) \frac{1}{\sqrt{x_\perp^2 - r^2}}, \\ \rho_J(r) &= -\frac{2}{\pi} r^2 \int_r^\infty dx_\perp \frac{d}{dx_\perp} \left(\frac{\rho_J(x_\perp)}{x_\perp^2} \right) \frac{1}{\sqrt{x_\perp^2 - r^2}} \\ s(r) &= -\frac{2}{\pi} r^2 \int_r^\infty dx_\perp \frac{d}{dx_\perp} \left(\frac{s(x_\perp)}{x_\perp^2} \right) \frac{1}{\sqrt{x_\perp^2 - r^2}} \\ \frac{2}{3}s(r) + p(r) &= \frac{4}{\pi} \int_r^\infty \frac{dx_\perp}{x_\perp} \mathcal{S}(x_\perp) \frac{1}{\sqrt{x_\perp^2 - r^2}}\end{aligned}$$

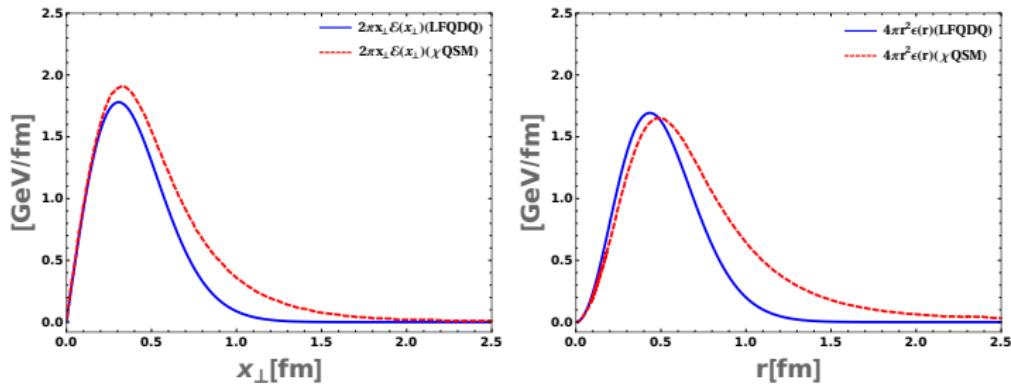
-J. Panteleeva, M. Polyakov Phys.Rev.D 104 (2021)

$\frac{2}{3}s(r) + p(r)$] is the Abel image of the light front shear force distribution $\frac{4}{\pi}\mathcal{S}(x_\perp)$.

Results at initial scale : Exact (Vegas) vs Multipole Fit



Results: Mass distributions in LFQDM



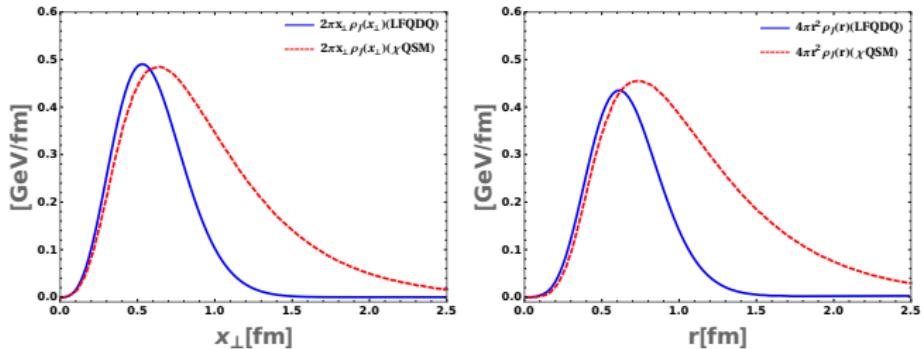
- The 3D mass distribution exhibits a broader shape than the 2D mass distributions.

$$\langle x_{\perp}^2 \rangle_{\text{mass}} = \frac{1}{M} \int d^2 x_{\perp} x_{\perp}^2 \mathcal{E}(x_{\perp}), \quad \langle r^2 \rangle_{\text{mass}} = \frac{\int d^3 r r^2 \epsilon(r)}{\int d^3 r \epsilon(r)},$$

In LFQDM

$$\langle x_{\perp}^2 \rangle_{\text{mass}} = 0.21 \text{ fm}^2, \quad \langle r^2 \rangle_{\text{mass}} = 0.32 \text{ fm}^2, \quad \frac{\langle x_{\perp}^2 \rangle_{\text{mass}}}{\langle r^2 \rangle_{\text{mass}}} = \frac{2}{3},$$

Results: Angular momentum distributions in LFQDM



$$\int d^2x_{\perp} \rho_J^{2D}(x_{\perp}) = \int d^3r \rho_J(r) = J(0) = \frac{1}{2},$$

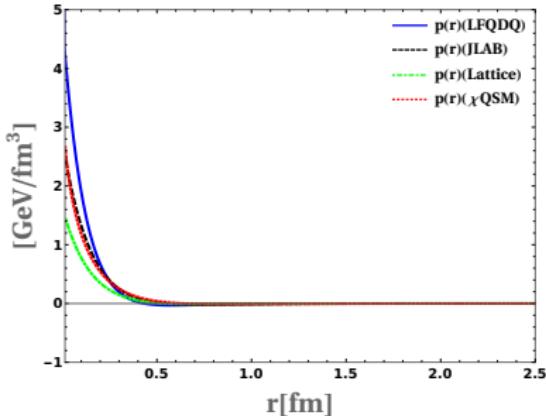
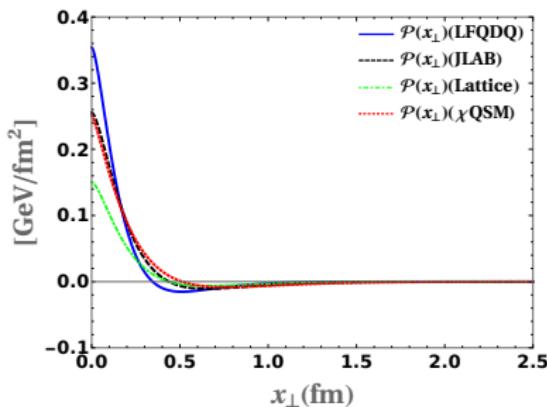
- 2D and 3D angular momentum radii are defined as

$$\langle x_{\perp}^2 \rangle_J = 2 \int d^2x_{\perp} x_{\perp}^2 \rho_J^{(2D)}(x_{\perp}), \quad \text{and} \quad \langle r^2 \rangle_J = \frac{\int d^3r r^2 \rho_J(r)}{\int d^3r \rho_J(r)}.$$

In LFQDM

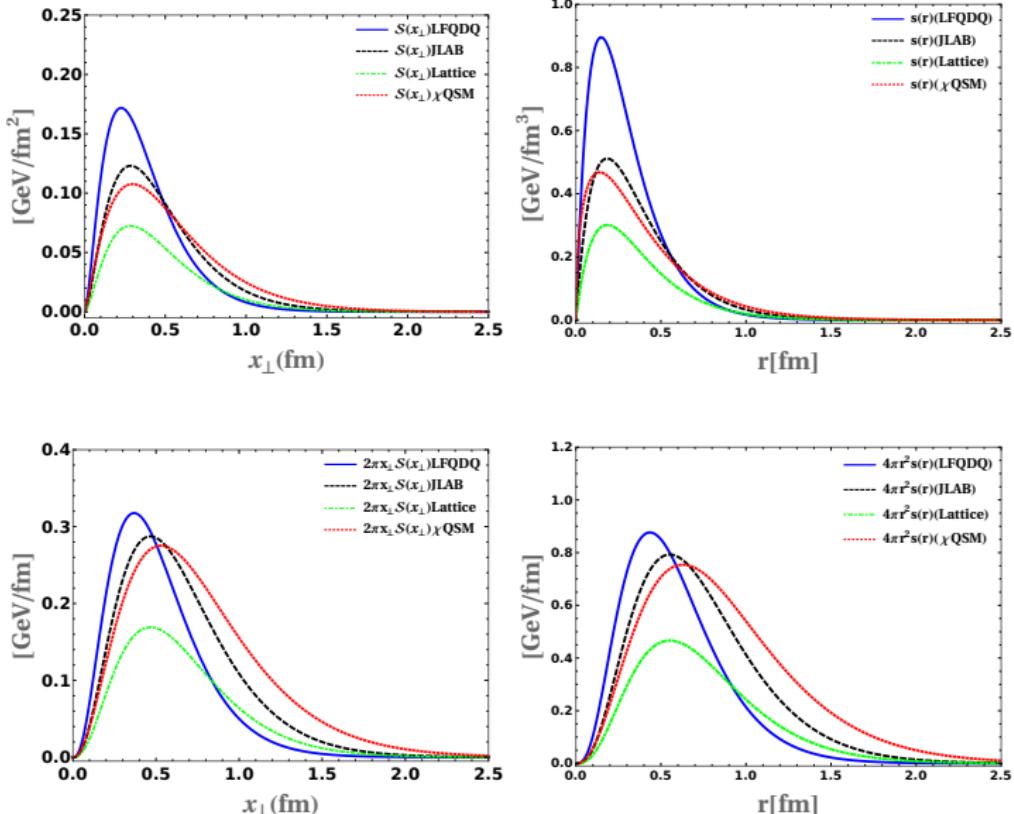
$$\langle x_{\perp}^2 \rangle_J = 0.38 \text{ fm}^2, \quad \langle r^2 \rangle_J = 0.51 \text{ fm}^2, \quad \frac{\langle x_{\perp}^2 \rangle_J}{\langle r^2 \rangle_J} = \frac{4}{5},$$

Results: Pressure distribution in LFQDM



- At $r = 0$ pressure $p(0)=4.76 \text{ GeV}/\text{fm}^3 = 7.60 \times 10^{34} \text{ Pa}$, which is 10-100 times higher than the pressure inside a neutron star.
- The pressure decreases monotonically and becomes zero at the nodal point $r_0 = 0.43 \text{ fm}$.
- The positive sign of the pressure corresponds to repulsion and negative sign is for attraction.

Results: Shear distribution in LFQDM



Stability conditions

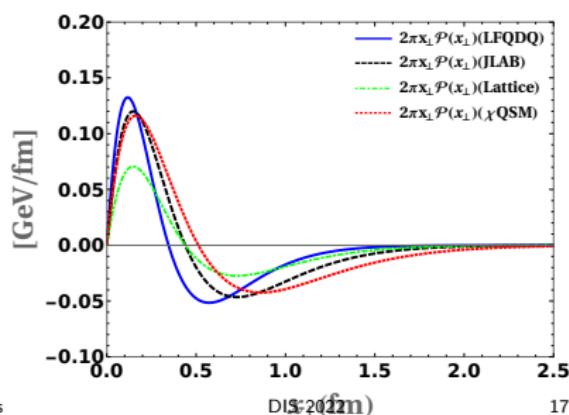
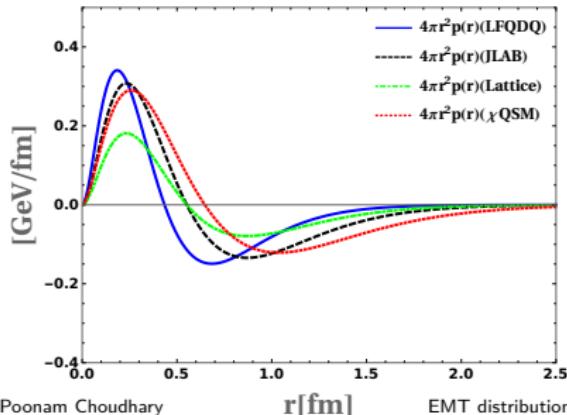
- The EMT conservation provides 3D and 2D stability conditions

$$p'(r) + \frac{2s(r)}{r} + \frac{2}{3}s'(r) = 0 \iff \mathcal{P}'(x_{\perp}) + \frac{\mathcal{S}(x_{\perp})}{x_{\perp}} + \frac{1}{2}\mathcal{S}'(x_{\perp}) = 0$$

Which results Von-Laue stability conditions for the nucleon

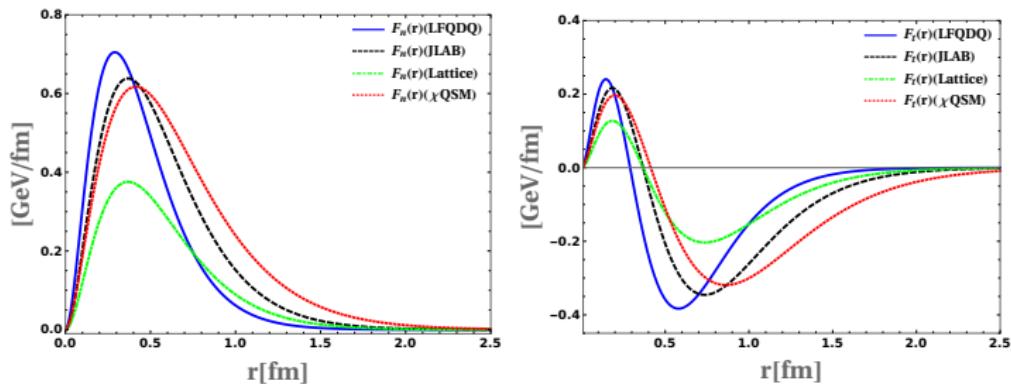
$$\int d^3 r p(r) = 0 \iff \int d^2 x_{\perp} \mathcal{P}(x_{\perp}) = 0$$

$$\int_0^{\infty} dr r \left[p(r) - \frac{1}{3}s(r) \right] = 0 \iff \int_0^{\infty} dx_{\perp} \left[\mathcal{P}(x_{\perp}) - \frac{1}{2}\mathcal{S}(x_{\perp}) \right] = 0$$



3D force distributions

$$F_n(r) = 4\pi r^2 \left[\frac{2}{3}s(r) + p(r) \right], \quad F_t(r) = 4\pi r^2 \left[-\frac{1}{3}s(r) + p(r) \right]$$



- $F_n(r)$ must be a stretching force otherwise the system would squeeze and collapse to the center.
- $F_t(r)$ have at least one nodal point, which tells that the direction of the force field should be reversed at this point.

D-term

Various observables in LFQDM

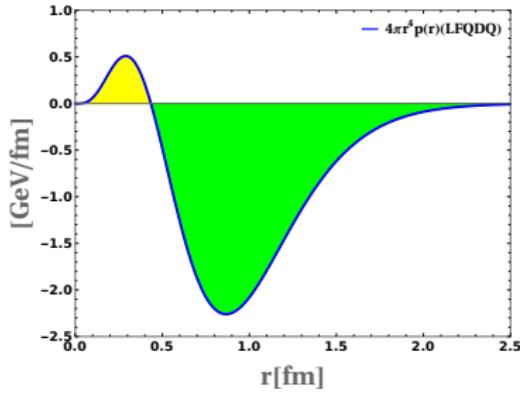
$\mathcal{P}(0)$ (GeV/fm ²)	$(x_{\perp})_0$ (fm)	$\langle x_{\perp}^2 \rangle_{\text{mass}}$ (fm ²)	$\langle x_{\perp}^2 \rangle_J$ (fm ²)	$\langle x_{\perp}^2 \rangle_{\text{mech}}$
0.354	0.34	0.21	0.38	0.167
$p(0)$ (GeV/fm ³)	r_0 (fm)	$\langle r^2 \rangle_{\text{mass}}$ (fm ²)	$\langle r^2 \rangle_J$ (fm ²)	$\langle r^2 \rangle_{\text{mech}}$
4.76	0.43	0.32	0.51	0.251

Approaches	D-term
LFQDM	-1.521
JLAB	-1.688
Lattice	-1.07
LCSR	-2.104

$$D(0) = M \int d^3r r^2 p(r)$$

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EMT distributions



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Summary

- The EMT distributions explain the mechanical properties, mass and angular momentum distribution of the nucleon.
- We evaluated 2D light front EMT distributions in LFQDM and obtain the 3D breit frame EMT distributions using inverse Abel transformation property.
- We compared our results with lattice QCD, χ QSM and JLAB data at scale $\mu^2 = 4 \text{ GeV}^2$.
- We demonstrated that the 3D stability conditions for the force distributions automatically imply the stability of 2D mechanical system and vice-versa.
- The negative value of D-term $D(0) = -1.521$ in LFQDM explains mechanical stability of the nucleon.

Thank you for your attention...