3D EMT distributions as an Abel image of 2D EMT distributions on the light front

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Abstract

The energy-momentum tensor (EMT) and corresponding gravitational form factors (GFFs) provide us information about the internal structure like spin, mass and spatial densities of the nucleon. The Druck gravitational (D-term) form factor is related to the mechanical stability of the nucleon and gives information about the spatial distributions of the forces inside the hadron. In this work, we study the GFFs in the framework of the light-front quark diquark model. The model has been successful to derive various properties of nucleons. We investigate the three dimensional spatial distributions of proton as an Abel image of two dimensional distributions in this model\textsuperscript{[1]}. We explicitly show the global and local stability conditions which are satisfied by both 2D and 3D distributions in our model. We compare our results with chiral quark soliton model, JLab and lattice data.


1 Introduction

Nucleon scattering by gravitational field is described by gravitational form factors (GFFs) which explains mass, spin and force distributions \textsuperscript{[2, 3]} inside the nucleon. Gravitational form factors are parameterized in terms of the matrix element of the energy-momentum tensor between the plane wave states. The components of the energy-momentum tensor inform how matter couples to the gravitational field.

\begin{equation}
\left< p' \right| \hat{\Phi}_{QCD}^{\mu}(0) \left| p \right> = \bar{u} \left( p' \right) \left[ A(t) \frac{P^\mu P^\nu}{M^2} + J(t) \frac{P^\mu P^\nu - \eta^\mu^\nu}{M^2} \Delta_\mu \right. \\
\left. + \frac{D(t)}{3M} \left( \Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2 \right) \right] u(p), \tag{1} \end{equation}

The GFFs furnish essential information on the internal structure of the nucleon and could be extracted through hard exclusive processes like deeply virtual Compton scattering as the second moments of Generalized Parton distribution functions (GPDs) \textsuperscript{[4, 5]}. The GFFs $A(Q^2)$ and $J(Q^2)$ give the mass and angular momentum of the proton and are constrained at $Q^2 = 0$ \textsuperscript{[6]}. While, The D-term, which is the last unknown global property of the proton, is extracted through the spatial-spatial component of the energy-momentum tensor, is deeply related to the stability of the nucleon and is unconstrained at $Q^2 = 0$ \textsuperscript{[7, 8]}.

2 Light front quark diquark model

In this model, We assume that the virtual incoming photon is interacting with a valence quark and two other valence quarks form a diquark of definite mass with spin 0, called a scalar diquark. Therefore the nucleon state $|P, S\rangle$ having momentum $P$ and spin $S$, can be represented as a two-particle Fock-state as
following

\[ |P; \pm \rangle = \sum_q \int \frac{dx d^2p_\perp}{2(2\pi)^3 \sqrt{x(1-x)}} \times \left[ \psi_{q^+}^\pm (x, p_\perp) + \frac{1}{2}; 0; xP^+, p_\perp \right] \psi_{q^\pm} (x, p_\perp) \left[ -\frac{1}{2}; 0; xP^+, p_\perp \right], \quad (2) \]

Here \( \psi_{q^\lambda_N}^\pm \) are the light-front wave functions with nucleon helicities \( \lambda_N = \pm \). The LFWFs at an initial scale \( \mu_0^2 = 0.32 \text{ GeV}^2 \) are given by [9].

\[
\begin{align*}
\psi_{q^+}^+ (x, p_\perp) &= \varphi_{q}^{(1)} (x, p_\perp) \\
\psi_{q^+}^- (x, p_\perp) &= -\frac{p^1 + ip^2}{xM} \varphi_{q}^{(2)} (x, p_\perp) \\
\psi_{q^-}^+ (x, p_\perp) &= \frac{p^1 - ip^2}{xM} \varphi_{q}^{(2)} (x, p_\perp) \\
\psi_{q^-}^- (x, p_\perp) &= \varphi_{q}^{(1)} (x, p_\perp)
\end{align*}
\]

where \( \varphi_{q}^{(1)} (x, p_\perp) \) and \( \varphi_{q}^{(2)} (x, p_\perp) \) are the wave functions predicted by the soft-wall AdS/QCD and can be written as [10]

\[
\varphi_{q}^{(i)} (x, p_\perp) = N_{q(i)} \frac{4\pi}{\kappa} \sqrt{\log(1/x)} x^{a(i)} (1-x)^{b(i)} \exp \left[ -\frac{p^2}{2\kappa^2} \log(1/x) \right]; \quad (4)
\]

where \( \kappa = 0.4 \text{ GeV} \) is the AdS/QCD scale parameter and the quarks are assumed to be massless, whereas all the model parameters are extracted by using the electromagnetic properties of the nucleons and can be found in [11].

### 3 Extraction of GFFs

The Form factors \( A^{u+d}(Q^2) \), \( B^{u+d}(Q^2) \) and \( D^{u+d}(Q^2) \) in the LFQDQ model can be parametrized in terms of structure integrals as [12, 13, 14]

\[
\begin{align*}
A^{u+d}(Q^2) &= \mathcal{I}^{u+d}_1(Q^2), \\
B^{u+d}(Q^2) &= \mathcal{I}^{u+d}_2(Q^2) \\
D^{u+d}(Q^2) &= -\frac{1}{Q^2} \left[ 2M^2\mathcal{I}^{u+d}_1(Q^2) - Q^2\mathcal{I}^{u+d}_2(Q^2) - \mathcal{I}^{u+d}_3(Q^2) \right],
\end{align*}
\]

where the explicit expressions of the structure integrals \( \mathcal{I}^{u+d}_i(Q^2) \) are given in [12, 1] It turns out that the form factor \( D(Q^2) \) can be described by the multipole function as [14],

\[
D(Q^2) = \frac{a}{(1 + bQ^2)c}, \quad (7)
\]

where the parameters \( a, b \) and \( c \) are given in below table at the initial scale as well as at a higher evolved scale. We have used Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations of QCD with next-to-next-to-leading order (NNLO) with the higher-order perturbative parton evolution toolkit (HOPPET) [15] to perform the scale evolution.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D^{0\text{fit}}_0 )</td>
<td>-18.8359</td>
<td>2.2823</td>
<td>2.7951</td>
</tr>
<tr>
<td>( D^{0\text{fit}}_1 )</td>
<td>-1.521</td>
<td>0.531</td>
<td>3.026</td>
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4 EMT distributions

The form factors appearing in matrix elements of the EMT encode spatial densities via Fourier transforms. The two-dimensional light front energy, angular momentum, pressure and shear distributions are related to the GFFs by following relations respectively:

\[ E^{(2D)}(x_\perp) = P^+ \tilde{A}(x_\perp), \quad \rho_j^{(2D)}(x_\perp) = -\frac{1}{2} x_\perp \frac{d}{dx_\perp} \tilde{J}(x_\perp) \]  
\[ p^{(2D)}(x_\perp) = \frac{1}{2 x_\perp} \frac{d}{d x_\perp} \left( x_\perp \frac{d}{d x_\perp} \tilde{D}(x_\perp) \right), \quad s^{(2D)}(x_\perp) = -x_\perp \frac{d}{d x_\perp} \left( \frac{1}{x_\perp} \frac{d}{d x_\perp} \tilde{D}(x_\perp) \right), \]  

where

\[ \tilde{F}(x_\perp) = \int \frac{d^2 \Delta}{(2\pi)^2} e^{-i \Delta \cdot x_\perp} F(-\Delta^2), \]

and \( x_\perp \) and \( \Delta_\perp \) are the position and momentum vectors in the 2D plane transverse to the propagation direction of the nucleon. Similarly the three-dimensional EMT distributions in breit frame are derived from the 3D inverse fourier transform of the GFFs. In this work [1] we calculated the 3D breit frame EMT distributions from the 2D light front EMT distributions using the following inverse Abel transformation [16] relations between them.
In Fig. 1 and Fig. 2 we obtain the 2D and 3D mass and angular momentum distribution in our model and compared them with \( \chi QSM \) model [17], and showed that 2D and 3D distributions are Abel images of each other. Similarly in Fig. 3 and Fig. 4 our results for 2D and 3D pressure and shear distribution are presented and compared with available results for \( \chi QSM \) [17], JLab [4, 18] and Lattice [19], respectively. Corresponding 2D and 3D radii for each distribution are shown in Table 1. Explicit calculation of each observable can be found in [1].

The 3D normal and tangential force fields in the breit frame are defined as [7, 17],

\[
\epsilon(r) = -\frac{1}{\pi} \int_r^\infty \frac{dx_{\perp}}{x_{\perp}} \left( \mathcal{E}(x_{\perp}) \right) \frac{1}{\sqrt{x_{\perp}^2 - r^2}}, \quad \rho_f(r) = -\frac{2}{\pi} \int_r^\infty \frac{dx_{\perp}}{x_{\perp}} \frac{d}{dx_{\perp}} \left( \frac{\rho_f(x_{\perp})}{x_{\perp}} \right) \frac{1}{\sqrt{x_{\perp}^2 - r^2}},
\]

\[
s(r) = -\frac{2}{\pi} \int_r^\infty \frac{dx_{\perp}}{x_{\perp}} \frac{d}{dx_{\perp}} \left( \frac{S(x_{\perp})}{x_{\perp}} \right) \frac{1}{\sqrt{x_{\perp}^2 - r^2}}, \quad \frac{2}{3} s(r) + p(r) = \frac{4}{\pi} \int_r^\infty \frac{dx_{\perp}}{x_{\perp}} S(x_{\perp}) \frac{1}{\sqrt{x_{\perp}^2 - r^2}}.
\]

(11)

In Fig. 5 we present our model results for normal and tangential force fields and compared them with other models. One nodal point in the tangential force shows the mechanical stability of the nucleon, whereas the positively distributed normal force field satisfies von laue stability conditions.

\[
F_n(r) = 4\pi r^2 \left[ \frac{2}{3} s(r) + p(r) \right], \quad F_t(r) = 4\pi r^2 \left[ \frac{1}{3} s(r) + p(r) \right].
\]

(12)
Table 1: Different observable obtained from the EMT distributions for the proton in both 2D LF and 3D BF are listed: the energy distributions at the nucleon center ($E(0), \epsilon(0)$), pressure distribution at the nucleon center ($P(0), p(0)$), nodal points of the pressure ($(x_\perp)_0, r_0$), and the mean square radii of the mass, angular momentum and mechanical ($\langle x^2_\perp \rangle, \langle r^2 \rangle$).

<table>
<thead>
<tr>
<th>$E(0)$ (GeV/fm$^2$)</th>
<th>$P(0)$ (GeV/fm$^2$)</th>
<th>$(x_\perp)_0$ (fm)</th>
<th>$\langle x^2_\perp \rangle_{\text{mass}}$ (fm$^2$)</th>
<th>$\langle x^2_\perp \rangle_{\text{tot}}$ (fm$^2$)</th>
<th>$\langle x^2_\perp \rangle_{\text{mech}}$ (fm$^2$)</th>
</tr>
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<tbody>
<tr>
<td>1.54</td>
<td>0.354</td>
<td>0.34</td>
<td>0.21</td>
<td>0.38</td>
<td>0.167</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\epsilon(0)$ (GeV/fm$^3$)</th>
<th>$p(0)$ (GeV/fm$^3$)</th>
<th>$r_0$ (fm)</th>
<th>$\langle r^2 \rangle_{\text{mass}}$ (fm$^2$)</th>
<th>$\langle r^2 \rangle_{\text{tot}}$ (fm$^2$)</th>
<th>$\langle r^2 \rangle_{\text{mech}}$ (fm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.02</td>
<td>4.76</td>
<td>0.43</td>
<td>0.32</td>
<td>0.51</td>
<td>0.251</td>
</tr>
</tbody>
</table>

Figure 5: Three dimensional normal forces and tangential forces in the left and right panel respectively at evolution scale $\mu^2 = 4$ GeV$^2$.

**Conclusion**

In this paper, the 2D LF distributions are evaluated in a scalar quark-diquark model of proton and then the 3D distributions are obtained in the model using the Abel transformation. Our results are compared with the $\chi QSM$, JLab and lattice predictions. The stability conditions are found to be satisfied with the LFQDQ model. The normal and shear force distributions are also evaluated in the LFQDQ model and are found to be consistent with lattice and other model predictions.

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**References**

