The phenomenological cornucopia of SU(3) exotica

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Exotics Charges Under SU(3)

Some scattered phenomenological examples of fields in various representations

- Scalar Fundamentals: Squarks (supersymmetry)-QCD pair production
- Fermion Octets: Gluions (supersymmetry)-QCD pair production
- Scalar Octets: Monohar Wise(also weak doublets), Sgluons (R-symmetric supersymmetry) Gluon Fusion, QCD pair production
- Sextet Quarks: General Models -QCD pair prodcution

There are many more possible interactions between hypothetical color-charged states and the Standard Model. Some with unusual and distinct collider signatures.

Therefore we attempt a catalogue of possible interactions so that the phenomenology can be systematically explored

- Use EFT interactions to catalog all interactions between SM and Exotic Sector (Later build out with simplified models)

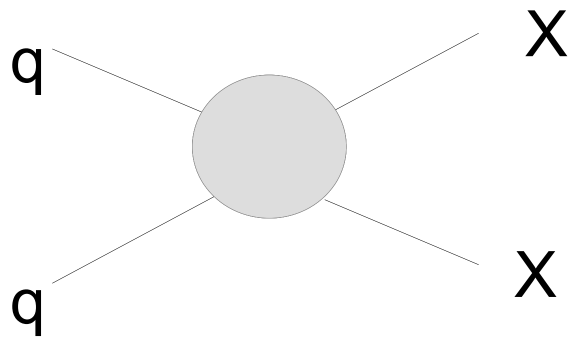
Contains SM fields

Contains Exotic fields

$$\frac{\mathcal{O}_1 \mathcal{O}_2}{\Lambda^n}$$

Appropiate power gives dim 4 operator

Effective Cut-Off



Interaction Term in Lagrangian

Build Operators using new Exotic Fields

ϕ: spin 0, sextet of SU(3), SU(2) singlet

ψ: spin ½ , sextet of SU(3), SU(2) singlet

Write down interaction which preserve all symmetries(Lorentz invariance, gauge symmetries of SM, CPT). For gauge invariance allowvarious possibilities for hypercharge.

Color Structure and the Construction of SU(3) singlets

To construct color invariants we use an iterative method exploiting the known tensor products of SU(3)

$$\begin{aligned}
\mathbf{3} \otimes \mathbf{3} &= \bar{\mathbf{3}}_{\mathrm{a}} \oplus \mathbf{6}_{\mathrm{s}}, \\
\mathbf{3} \otimes \bar{\mathbf{3}} &= \mathbf{1} \oplus \mathbf{8}, \\
\mathbf{6} \otimes \mathbf{3} &= \mathbf{8} \oplus \mathbf{10}, \\
\mathbf{6} \otimes \bar{\mathbf{3}} &= \mathbf{3} \oplus \mathbf{15}, \\
\mathbf{6} \otimes \mathbf{6} &= \bar{\mathbf{6}}_{\mathrm{s}} \oplus \mathbf{15}_{\mathrm{a}} \oplus \mathbf{15}'_{\mathrm{s}}, \\
\mathbf{6} \otimes \bar{\mathbf{6}} &= \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27}, \\
\mathbf{8} \otimes \mathbf{3} &= \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15}, \\
\mathbf{8} \otimes \bar{\mathbf{6}} &= \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15} \oplus \mathbf{24}, \\
\mathbf{8} \otimes \mathbf{8} &= \mathbf{1}_{\mathrm{s}} \oplus \mathbf{8}_{\mathrm{s}} \oplus \mathbf{8}_{\mathrm{a}} \oplus \mathbf{10}_{\mathrm{a}} \oplus \overline{\mathbf{10}}_{\mathrm{a}} \oplus \mathbf{27}_{\mathrm{s}}.
\end{aligned}$$

Interpreting Products

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

quarks/antiquarks

Singlet of SU(3): Higgs,photon,Z,W

Octet of SU(3): gluon

Move all fields to left side of product equation to construct a singlet

$$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{8}$$

This is the invariant interaction term we can write in the Lagrangian

We can use this method to construct all color invariant terms in the Lagrangian that contain the new sextet fields

We use an interactive method to construct color invariants. First we find the operators with three or fewer color-charged fields

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}}_{\mathrm{a}} \oplus \mathbf{6}_{\mathrm{s}},$$
$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8},$$
$$\mathbf{6} \otimes \mathbf{3} = \mathbf{8} \oplus \mathbf{10},$$
$$\mathbf{6} \otimes \bar{\mathbf{3}} = \mathbf{3} \oplus \mathbf{15},$$
$$\mathbf{6} \otimes \mathbf{6} = \bar{\mathbf{6}}_{\mathrm{s}} \oplus \mathbf{15}_{\mathrm{a}} \oplus \mathbf{15}'_{\mathrm{s}},$$
$$\mathbf{6} \otimes \bar{\mathbf{6}} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27},$$
$$\mathbf{8} \otimes \mathbf{3} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15},$$
$$\mathbf{8} \otimes \bar{\mathbf{6}} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15} \oplus \mathbf{24},$$
$$\mathbf{8} \otimes \mathbf{8} = \mathbf{1}_{\mathrm{s}} \oplus \mathbf{8}_{\mathrm{s}} \oplus \mathbf{8}_{\mathrm{a}} \oplus \mathbf{10}_{\mathrm{a}} \oplus \overline{\mathbf{10}}_{\mathrm{a}} \oplus \mathbf{27}_{\mathrm{s}}$$

Identify 6 with sextets
To find all products with the exotic fields

From 7 products we find 5 distinct invariants

We construct singlets using the proper choice of Clebsch-Gordon coefficients

Two field invariant

$$\mathbf{6} \otimes \overline{\mathbf{6}}$$

Three field invariant

$i \leftrightarrow j$ symmetric	$K_s^{\ ij}$	$\longleftarrow$	$\mathbf{3} \otimes \mathbf{3} \otimes \overline{\mathbf{6}},$
	$J^{s\,ia}$	$\longleftarrow$	$\mathbf{3} \otimes \mathbf{6} \otimes \mathbf{8},$
$t \leftrightarrow u$ symmetric	S^{stu}	$\longleftarrow$	$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6},$
Generators of $\mathbf{6}$	$[t^a_{\mathbf{6}}]_s^{\ t}$	$\longleftarrow$	$\mathbf{6} \otimes \overline{\mathbf{6}} \otimes \mathbf{8}.$

We now construct interaction in the Lagrangian

$$\mathbf{6} \otimes \bar{\mathbf{6}}$$

Two field invariant

$$\begin{aligned}
&\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{6}}, \\
&\mathbf{3} \otimes \mathbf{6} \otimes \mathbf{8}, \\
&\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6}, \\
&\mathbf{6} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}.
\end{aligned}$$

Three field invariant

Interpret 3 as quark and 8 as gluon, we make now complete the Lorentz Structure of the operators. We have already found some interesting new operators

All operators with this color structure

$\mathbf{3} \otimes \mathbf{6} \otimes \mathbf{8}$	Singlet (Lorentz + $\mathcal{G}_{\rm SM}$)			L	Y
	Generic	Specific	Coupling		
Scalar Φ_s	$(q\ell)\Phi G$	$J^{s\,ia}\,\Phi_s\,(\overline{q^{\rm c}_{\rm R}}_{Ii}\,\sigma^{\mu\nu}\,\ell_{{\rm R}X})\,G_{\mu\nu\,a}$	$\frac{1}{\Lambda_\Phi^2}\,\lambda_I^X$	-1	$\{\frac{1}{3},\frac{4}{3}\}$
	$(\bar\ell q)\Phi G$	$J^{s\,ia}\,\Phi_s\,(\bar L_{{\rm L}X}\,H\sigma^{\mu\nu}q_{{\rm R}Ii})\,G_{\mu\nu\,a}$	$\frac{1}{\Lambda_\Phi^3}\,\lambda_I^X$	1	$\{-\frac{5}{3},-\frac{2}{3}\}$
Dirac Ψ_s	$(q\Psi)G$	$J^{s\,ia}\,(\overline{q^{\rm c}_{\rm R}}_{Ii}\,\sigma^{\mu\nu}\,\Psi_s)\,G_{\mu\nu\,a}$	$\frac{1}{\Lambda_\Psi}\,\kappa_I$	0	$\{-\frac{2}{3},\frac{1}{3}\}$
		$J^{s\,ia}\,(\overline{q^{\rm c}_{\rm R}}_{Ii}\,\Psi_s)\,B^{\mu\nu}\,G_{\mu\nu\,a}$	$\frac{1}{\Lambda_\Psi^3}\,\kappa_I$		
	$(q\Psi)\vert H\vert^2 G$	$J^{s\,ia}\,(\overline{q^{\rm c}_{\rm R}}_{Ii}\,\sigma^{\mu\nu}\,\Psi_s)\,\vert H\vert^2\,G_{\mu\nu\,a}$			

$$\mathbf{3} \otimes \mathbf{6} \otimes \mathbf{8}$$

We interpret this as an interaction between a quark, gluon and sextet the lowest dimesion operators is

$$\frac{1}{\Lambda_\Psi} \kappa_I \;\; J^{s\,ia} \, (\overline{q^{\mathrm{c}}_{\mathrm{R}}}{}_{I i} \, \sigma^{\mu\nu} \, \Psi_s) \, G_{\mu\nu\, a}$$

$$\{-\tfrac{2}{3}, \tfrac{1}{3}\}$$

Possible hypercahrges of sextet

quark

fermion sextet

gluon

$$\mathbf{3} \otimes \mathbf{6} \otimes \mathbf{8}$$

We interpret this as an interaction between a quark, gluon and sextet the lowest dimesion operators is

Lorentz indicies contracted

$$\frac{1}{\Lambda_\Psi}\kappa_I J^{s\,ia}\left(\overline{q^{c}_{R}}_{Ii}\,\sigma^{\mu\nu}\Psi_s\right)G_{\mu\nu\,a} \qquad \left\{-\frac{2}{3}, \frac{1}{3}\right\}$$

Possible hyper-charges of sextet

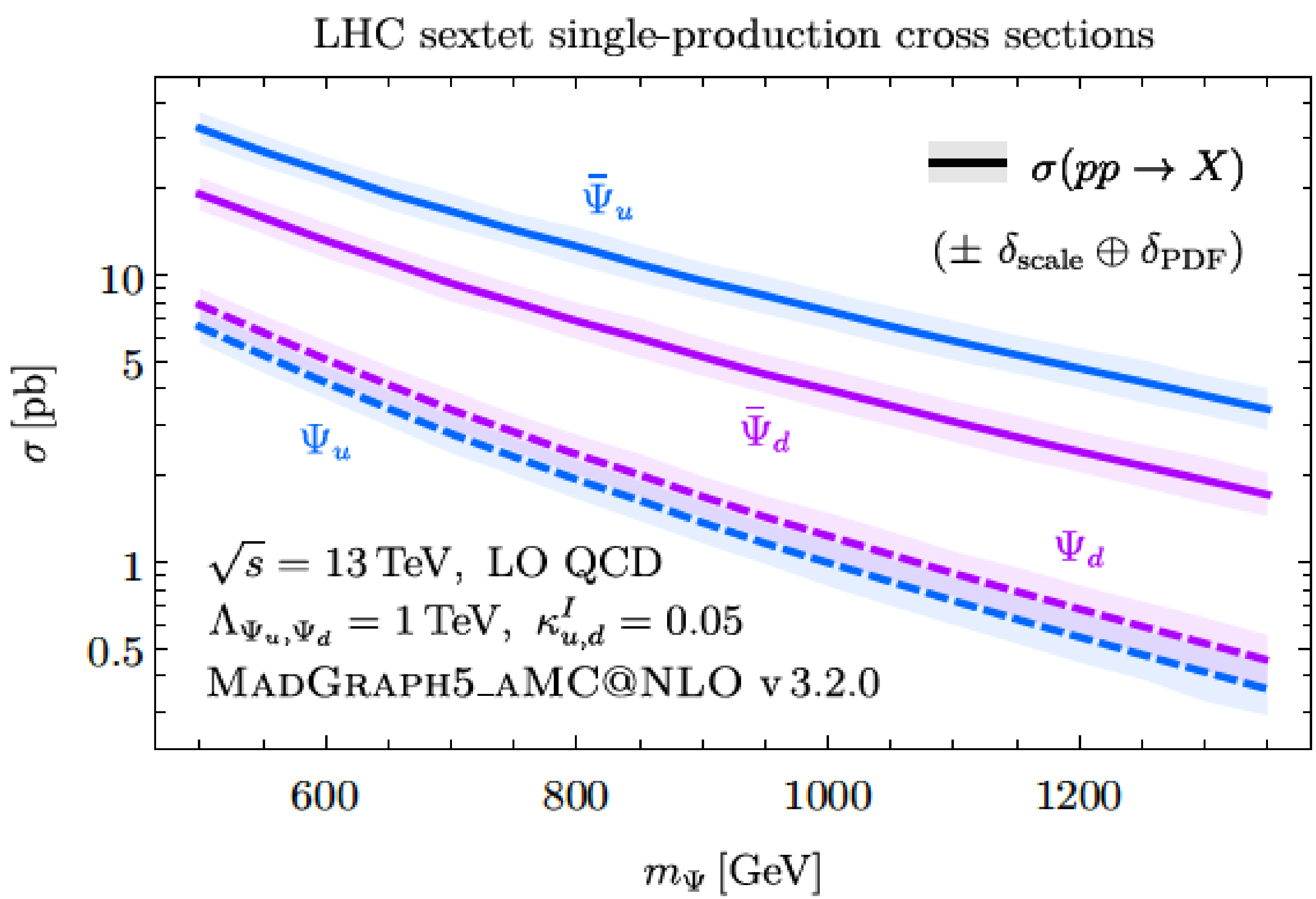
Color indicies contracted

Sextet had opposite hyper-charge as the up/down type quarks

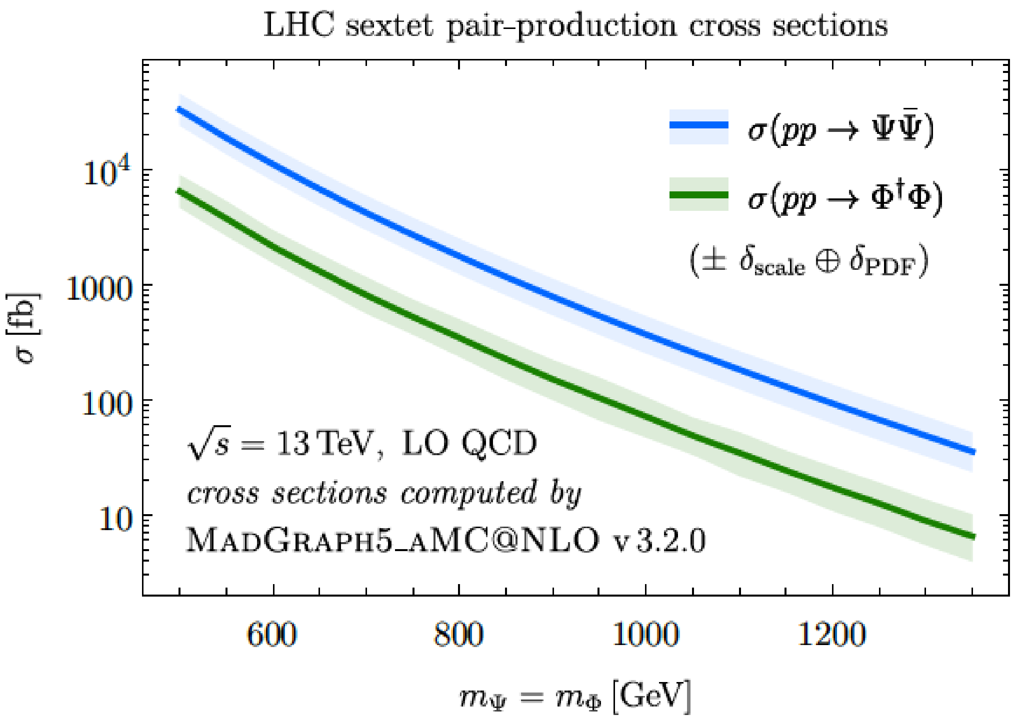
Dimension 5 operator

The coupling constant to various flavors of Quark may be independent

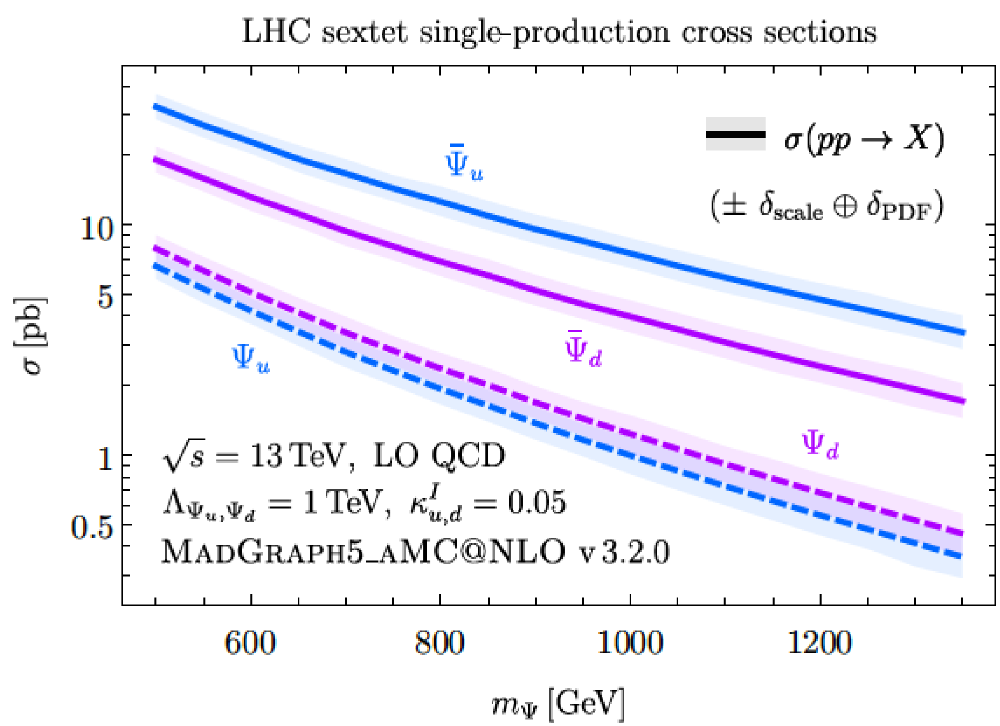
This operator leads to single sextet production through quark-gluon fusion

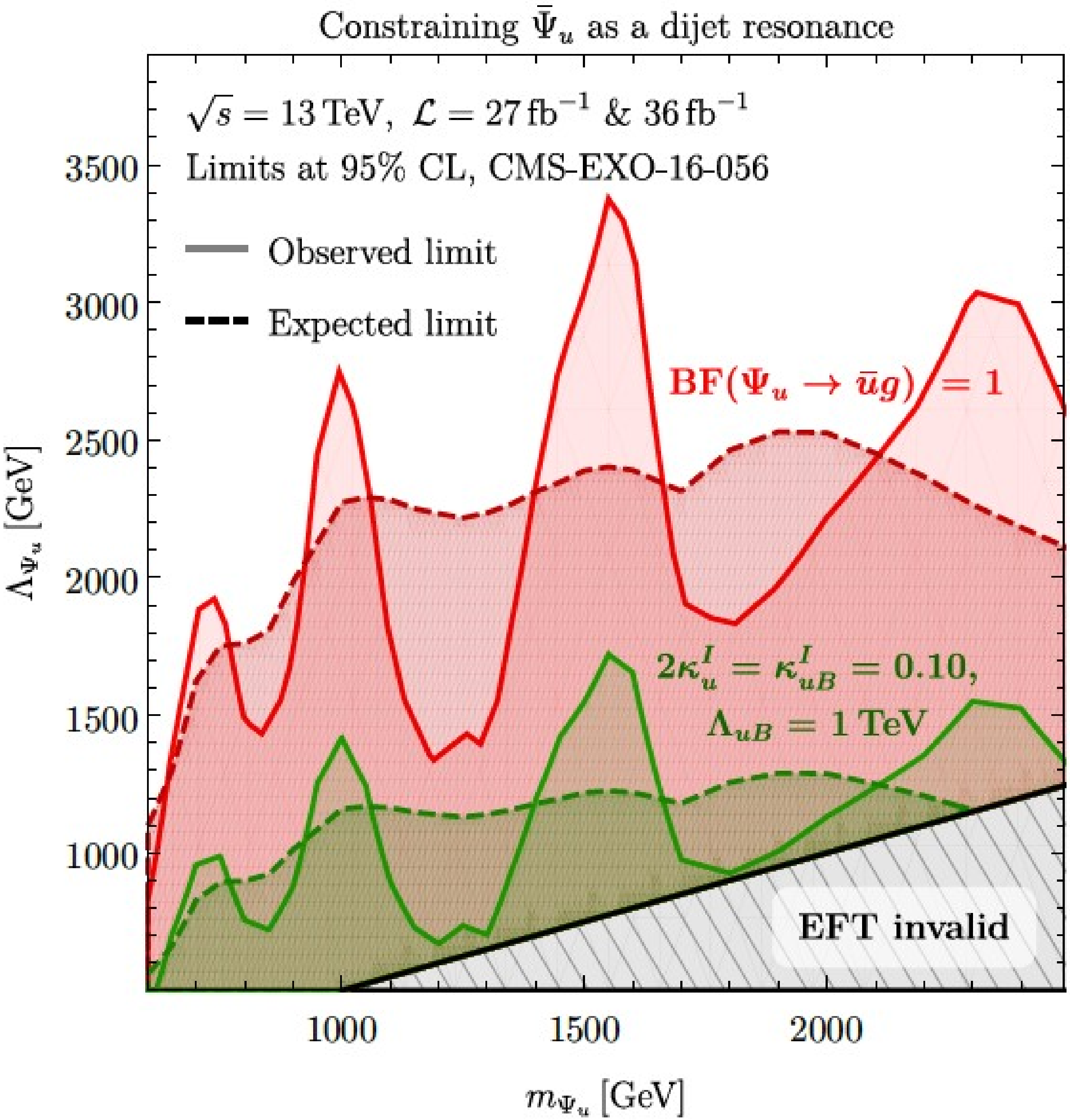


Differential quark-antiquarks PDF's ensure different production cross section for Upand down type sextets between sextets and anti-sextets



Comparing LHC pair production from gluon fusion to single production from the new operator





Constraining $\bar{\Psi}_u$ as a dijet resonance
$\sqrt{s} = 13\,\mathrm{TeV},\ \mathcal{L} = 27\,\mathrm{fb}^{-1}\ \&\ 36\,\mathrm{fb}^{-1}$
Limits at 95% CL, CMS-EXO-16-056
Observed limit
Expected limit
$\mathrm{BF}(\Psi_u \to \bar{u}g) = 1$
$2\kappa_u^I = \kappa_{uB}^I = 0.10,$
$\Lambda_{uB} = 1\,\mathrm{TeV}$
EFT invalid
Λ_{Ψ_u} [GeV]
m_{Ψ_u} [GeV]
3500
3000
2500
2000
1500
1000
1000
1500
2000

Now consider this same color structure but try to build out the operator with a spin 0 sextet

To contract Lorentz indices
We need to add a fermion

$$\frac{1}{\Lambda_\Phi^2}\lambda_I^X \quad J^{s\,ia}\,\Phi_s\,(\overline{q^c_{\mathrm{R}}}_{Ii}\,\sigma^{\mu\nu}\,\ell_{\mathrm{R}X})\,G_{\mu\nu\,a}$$

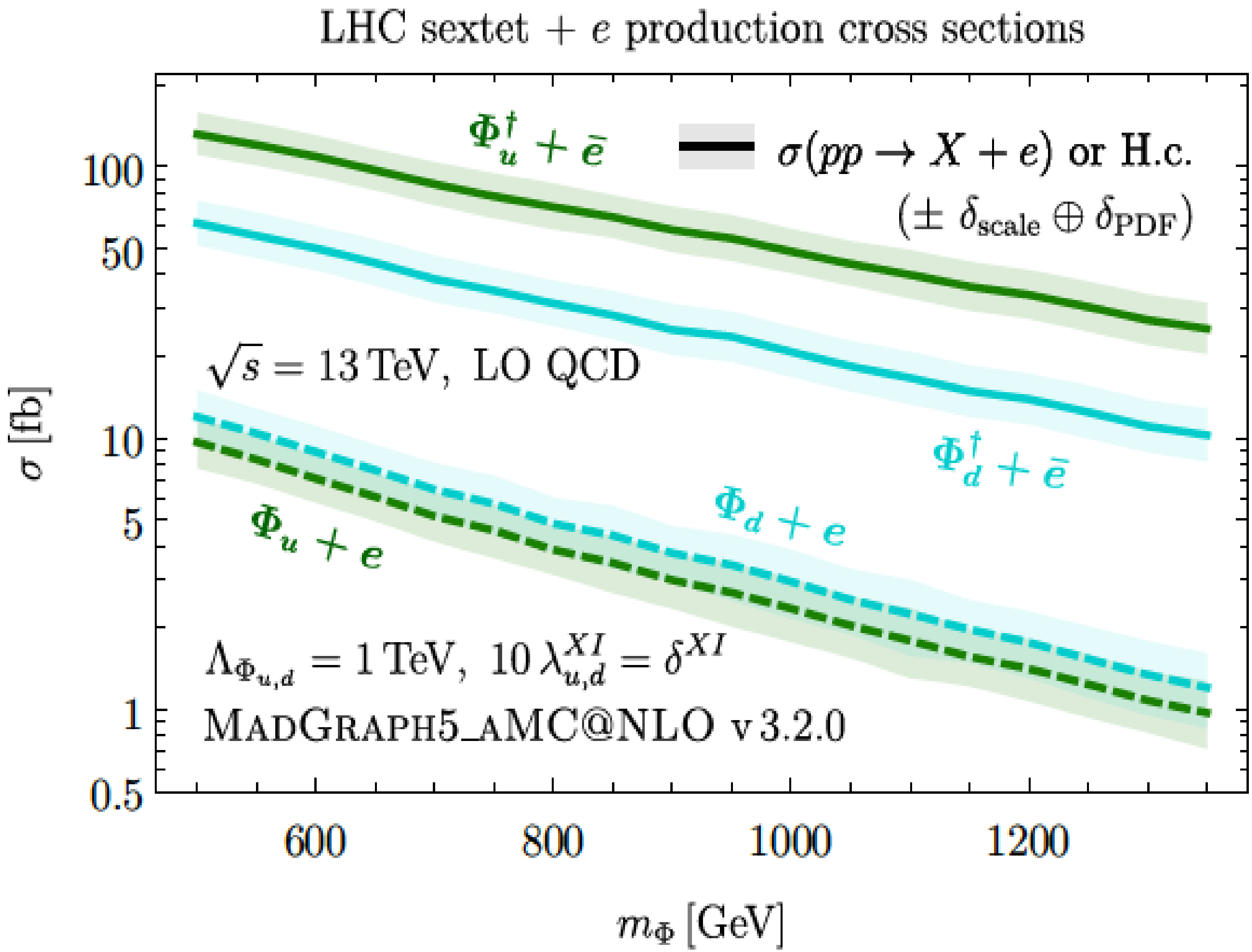
Now this is a dimension 6 operator

To prevent proton decay
The sextet has a lepton
Number -1

$\{\frac{1}{3}, \frac{4}{3}\}$

Now we the sextet must
Have the opposite hypercharge
As the sum of the lepton and quark
charges

Now there is a process where a scalar sextet is produced in association with a lepton



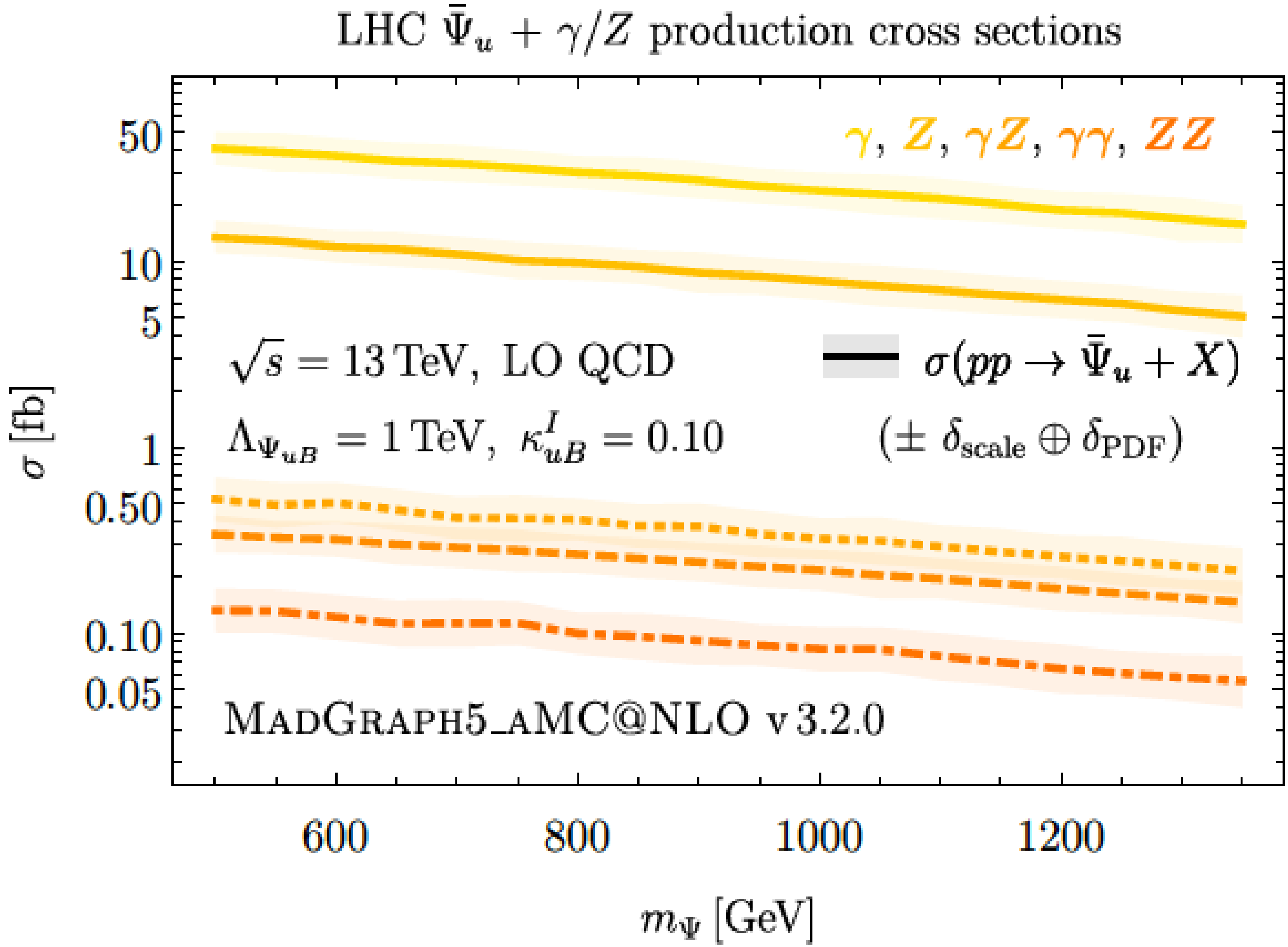
Higher Dimensional Operators

$$\frac{1}{\Lambda_{\Psi}^{3}}\kappa \; J^{s\,ia}\,(\overline{q^{c}_{\mathrm{R}}}_{Ii}\,\Psi_{s})\,B^{\mu\nu}G_{\mu\nu\,a}$$

Lorentz indices contracted with color singlet U(1) field strength tensor

Dimension 7 operator

This operator allows the production of sextets in association with photon or Z



LHC $\bar{\Psi}_u + \gamma/Z$ production cross sections
γ, Z, γZ, $\gamma\gamma$, ZZ
$\sqrt{s} = 13\,\text{TeV}$, LO QCD
$\Lambda_{\Psi_{uB}} = 1\,\text{TeV}$, $\kappa^I_{uB} = 0.10$
$\sigma(pp \to \bar{\Psi}_u + X)$
$(\pm\, \delta_{\text{scale}} \oplus \delta_{\text{PDF}})$
MadGraph5_aMC@NLO v 3.2.0
σ [fb]
50
10
5
1
0.50
0.10
0.05
600
800
1000
1200
m_Ψ [GeV]

$\mathbf{3}\otimes\mathbf{3}\otimes\bar{\mathbf{6}}$	Singlet (Lorentz + $\mathcal{G}_{\text{SM}}$) Generic	Specific	Coupling	L	Y
Scalar Φ_s	$(qq')\Phi^\dagger$	$K_s{}^{ij}\,\Phi^{\dagger s}\,(\overline{q^c_{\mathrm{R}}}_{Ii}\,q_{\mathrm{R}Jj})$	λ_{IJ}	0	$\{-\frac{2}{3},\frac{1}{3},\frac{4}{3}\}$
		$K_s{}^{ij}\,\Phi^{\dagger s}\,(\overline{q^c_{\mathrm{R}}}_{Ii}\,\sigma^{\mu\nu}q_{\mathrm{R}Jj})\,B_{\mu\nu}$	$\frac{1}{\Lambda_\Phi^2}\,\lambda_{IJ}$		
		$K_s{}^{ij}\,\Phi^{\dagger s}\,(\overline{Q^c_{\mathrm{L}}}_{Ii}\,\mathrm{i}\tau^2\,Q_{\mathrm{L}Jj})$	λ_{IJ}		$\frac{1}{3}$
		$K_s{}^{ij}\,\Phi^{\dagger s}\,(\overline{Q^c_{\mathrm{L}}}_{Ii}\,\sigma^{\mu\nu}\mathrm{i}\tau^2\,Q_{\mathrm{L}Jj})\,B_{\mu\nu}$	$\frac{1}{\Lambda_\Phi^2}\,\lambda_{IJ}$		
		$K_s{}^{ij}\,\Phi^{\dagger s}\,(\overline{Q^c_{\mathrm{L}}}_{Ii}HH^\dagger Q_{\mathrm{L}Jj})$	$\frac{1}{\Lambda_\Phi^2}\,\lambda_{IJ}$		
		$K_s{}^{ij}\,\Phi^{\dagger s}\,(\overline{Q^c_{\mathrm{L}}}_{Ii}\,\sigma^{\mu\nu}\,t_2^A\,Q_{\mathrm{L}Jj})\,W_{\mu\nu\,A}$			
	$(qq')\lvert H\rvert^2\Phi^\dagger$	$K_s{}^{ij}\,\Phi^{\dagger s}\,(\overline{q^c_{\mathrm{R}}}_{Ii}\,q_{\mathrm{R}Jj})\,\lvert H\rvert^2$			$\{-\frac{2}{3},\frac{1}{3},\frac{4}{3}\}$
Dirac Ψ_s	$(qq')(\bar\Psi\ell)$	$K_s{}^{ij}\,(\overline{q^c_{\mathrm{R}}}_{Ii}\,q_{\mathrm{R}Jj})(\bar\Psi^s\,\ell_{\mathrm{R}X})$	$\frac{1}{\Lambda_\Psi^2}\,\kappa^X_{IJ}$	1	$\{-\frac{5}{3},-\frac{2}{3},\frac{1}{3}\}$
		$K_s{}^{ij}\,(\overline{q^c_{\mathrm{R}}}_{Ii}\,q_{\mathrm{R}Jj})(\bar\Psi^s H^\dagger L_{\mathrm{L}X})$	$\frac{1}{\Lambda_\Psi^3}\,\kappa^X_{IJ}$		
		$K_s{}^{ij}\,(\overline{q^c_{\mathrm{R}}}_{Ii}\,\sigma^{\mu\nu}q_{\mathrm{R}Jj})(\bar\Psi^s\sigma_{\mu\nu}H^\dagger L_{\mathrm{L}X})$			
		$K_s{}^{ij}\,(\overline{Q^c_{\mathrm{L}}}_{Ii}\,\mathrm{i}\tau^2\,Q_{\mathrm{L}Jj})(\bar\Psi^s\,\ell_{\mathrm{R}X})$	$\frac{1}{\Lambda_\Psi^2}\,\kappa^X_{IJ}$		$\frac{4}{3}$
		$K_s{}^{ij}\,(\overline{Q^c_{\mathrm{L}}}_{Ii}\,\mathrm{i}\tau^2\,Q_{\mathrm{L}Jj})(\bar\Psi^s H^\dagger L_{\mathrm{L}X})$	$\frac{1}{\Lambda_\Psi^3}\,\kappa^X_{IJ}$		$-\frac{2}{3}$
		$K_s{}^{ij}\,(\overline{Q^c_{\mathrm{L}}}_{Ii}H\gamma^\mu q_{\mathrm{R}Jj})(\bar\Psi^s\gamma_\mu\,\ell_{\mathrm{R}X})$			$\{-\frac{2}{3},\frac{1}{3}\}$
	$(\bar\Psi q)(q\ell)$	$K_s{}^{ij}\,(\bar\Psi^s\sigma^{\mu\nu}q_{\mathrm{R}Ii})(\overline{q^c_{\mathrm{R}}}_{Jj}\,\sigma_{\mu\nu}\,\ell_{\mathrm{R}X})$	$\frac{1}{\Lambda_\Psi^3}\,\kappa^X_{IJ}$		$\{-\frac{5}{3},-\frac{2}{3},\frac{1}{3}\}$
		$K_s{}^{ij}\,(\bar\Psi^s\gamma^\mu q_{\mathrm{R}Ii})(\overline{Q^c_{\mathrm{L}}}_{Jj}H\gamma_\mu\,\ell_{\mathrm{R}X})$	$\frac{1}{\Lambda_\Psi^2}\,\kappa^X_{IJ}$		$\{-\frac{2}{3},\frac{1}{3}\}$
	$(\bar\Psi q)(\bar\ell q)$	$K_s{}^{ij}\,(\bar\Psi^s\gamma^\mu q_{\mathrm{R}Ii})(\bar L_{\mathrm{L}X}\gamma_\mu\,\mathrm{i}\tau^2\,Q_{\mathrm{L}Jj})$		-1	$\{\frac{1}{3},\frac{4}{3}\}$
		$K_s{}^{ij}\,(\bar\Psi^s\gamma^\mu q_{\mathrm{R}Ii})(\bar\ell_{\mathrm{R}X}\gamma_\mu q_{\mathrm{R}Jj})$			$\{\frac{1}{3},\frac{4}{3},\frac{7}{3}\}$

We can build out
The operators for
each tensor product

	Scalar sextet Φ only		*Dirac* sextet Ψ only		≥ 1 of each	
$\mathrm{SU}(3)_c$ invariant	d_{min}	Structure	d_{min}	Structure	d_{min}	Structure
$\mathbf{6} \otimes \bar{\mathbf{6}}$	$4^\dagger$	$\Phi^\dagger \Phi$	$5^\dagger$	$(\bar{\Psi}\Psi)$	4	$(\bar{\Psi}\ell)\Phi$
						$(\Psi\ell)\Phi^\dagger$
						$(\bar{\ell}\Psi)\Phi^\dagger$
$\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{6}}$	4	$(qq')\Phi^\dagger$	6	$(qq')(\bar{\Psi}\ell)$		
	6	$(qq')\lvert H\rvert^2\Phi^\dagger$		$(\bar{\Psi}q)(q\ell)$		
				$(\bar{\Psi}q)(\bar{\ell}q)$		
$\mathbf{3} \otimes \mathbf{6} \otimes \mathbf{8}$	6	$(q\ell)\Phi G$	5	$(q\Psi)G$		
		$(\bar{\ell}q)\Phi G$	7	$(q\Psi)\lvert H\rvert^2 G$		
$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6}$	$5^\dagger$	$\Phi\Phi\Phi$	6	$(\Psi\Psi)(\Psi\ell)$	5	$(\Psi\ell)\Phi\Phi$
				$(\Psi\Psi)(\bar{\ell}\Psi)$		$(\bar{\ell}\Psi)\Phi\Phi$
					$6^\dagger$	$(\Psi\Psi)\Phi$
$\mathbf{6} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$	6	$\Phi^\dagger\Phi G B$	5	$(\bar{\Psi}\Psi)G$	6	$(\bar{\Psi}\ell)\Phi G$
			7	$(\bar{\Psi}\Psi)\lvert H\rvert^2 G$		$(\Psi\ell)\Phi^\dagger G$
						$(\bar{\ell}\Psi)\Phi^\dagger G$

Iterating tensor products

Observation. If there exist invariant combinations of $n+1$ and $m+1$ fields transforming in the direct product representations $\mathbf{r}_1 \otimes \cdots \otimes \mathbf{r}_n \otimes \mathbf{p}$ and $\mathbf{q}_1 \otimes \cdots \otimes \mathbf{q}_m \otimes \mathbf{p}$ of SU(3), then there exists an invariant combination of $n+m$ fields in the reducible representation $\mathbf{r}_1 \otimes \cdots \otimes \mathbf{r}_n \otimes \bar{\mathbf{q}}_1 \otimes \cdots \otimes \bar{\mathbf{q}}_m$.

Constructing 4 field invariants

$$\mathbf{3}\otimes\mathbf{3}=\bar{\mathbf{3}}_{\mathrm{a}}\oplus\mathbf{6}_{\mathrm{s}},$$
$$\mathbf{3}\otimes\bar{\mathbf{3}}=\mathbf{1}\oplus\mathbf{8},$$
$$\mathbf{6}\otimes\mathbf{3}=\mathbf{8}\oplus\mathbf{10},$$
$$\mathbf{6}\otimes\bar{\mathbf{3}}=\mathbf{3}\oplus\mathbf{15},$$
$$\mathbf{6}\otimes\mathbf{6}=\bar{\mathbf{6}}_{\mathrm{s}}\oplus\mathbf{15}_{\mathrm{a}}\oplus\mathbf{15}'_{\mathrm{s}},$$
$$\mathbf{6}\otimes\bar{\mathbf{6}}=\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{27},$$
$$\mathbf{8}\otimes\mathbf{3}=\mathbf{3}\oplus\bar{\mathbf{6}}\oplus\mathbf{15},$$
$$\mathbf{8}\otimes\bar{\mathbf{6}}=\mathbf{3}\oplus\bar{\mathbf{6}}\oplus\mathbf{15}\oplus\mathbf{24},$$
$$\mathbf{8}\otimes\mathbf{8}=\mathbf{1}_{\mathrm{s}}\oplus\mathbf{8}_{\mathrm{s}}\oplus\mathbf{8}_{\mathrm{a}}\oplus\mathbf{10}_{\mathrm{a}}\oplus\overline{\mathbf{10}}_{\mathrm{a}}\oplus\mathbf{27}_{\mathrm{s}}$$

By iterating tensor products
We construct new invariant

$$\mathbf{3}\otimes\bar{\mathbf{3}}=\mathbf{8}=\mathbf{6}\otimes\mathbf{3}$$

$$\mathbf{3}\otimes\bar{\mathbf{3}}\otimes\bar{\mathbf{3}}\otimes\bar{\mathbf{6}}$$

With coefficient

$$[t_3^a]_j^{\ i}\,\bar{J}_{s\,ak}$$

Fundamentals contacted into 8

6-3-8 contraction

color-structure matters

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}}_{\mathrm{a}} \oplus \mathbf{6}_{\mathrm{s}},$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8},$$

$$\mathbf{6} \otimes \mathbf{3} = \mathbf{8} \oplus \mathbf{10},$$

$$\mathbf{6} \otimes \bar{\mathbf{3}} = \mathbf{3} \oplus \mathbf{15},$$

$$\mathbf{6} \otimes \mathbf{6} = \bar{\mathbf{6}}_{\mathrm{s}} \oplus \mathbf{15}_{\mathrm{a}} \oplus \mathbf{15}'_{\mathrm{s}},$$

$$\mathbf{6} \otimes \bar{\mathbf{6}} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27},$$

$$\mathbf{8} \otimes \mathbf{3} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15},$$

$$\mathbf{8} \otimes \bar{\mathbf{6}} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15} \oplus \mathbf{24},$$

$$\mathbf{8} \otimes \mathbf{8} = \mathbf{1}_{\mathrm{s}} \oplus \mathbf{8}_{\mathrm{s}} \oplus \mathbf{8}_{\mathrm{a}} \oplus \mathbf{10}_{\mathrm{a}} \oplus \overline{\mathbf{10}}_{\mathrm{a}} \oplus \mathbf{27}_{\mathrm{s}}$$

Iterating we get

$$\mathbf{6} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} = \bar{\mathbf{3}}_{\mathrm{a}}$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{6}}$$

With coefficient

$$\bar{L}_{jkl} \, K_s{}^{li}$$

Fundamentals contacted into 3

3-3-6bar contraction

All four field color invariants

Invariant	Clebsch-Gordan coefficients							Notes
$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{6} \otimes \mathbf{6}$			$K_u{}^{ij}\, S^{\,ust}$					$\ni [\Pi_{\mathbf{3366}}]^{ij\,st}$
$\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$	$L^{ijk} \bar{J}_{s\,ka}$	$K_s{}^{ik}[t^a_{\mathbf{3}}]_k{}^j$	$K_r{}^{ij}[t^a_{\mathbf{6}}]_s{}^r$			$Q^{q\,i}{}_s V_q{}^{ja}$		$\ni [\Pi_{\mathbf{33\bar{6}8}}]^{ij}{}_s{}^a$
$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{6}}$	$\bar{L}_{jkl}\, K_s{}^{li}$			$[t^a_{\mathbf{3}}]_j{}^i\, \bar{J}_{s\,ak}$				$\ni [\Pi_{\mathbf{3\bar{3}\bar{3}\bar{6}}}]^i{}_{jks}$
$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{6} \otimes \bar{\mathbf{6}}$	$\delta_j{}^i\, \delta_t{}^s$			$[t^a_{\mathbf{3}}]_j{}^i [t^a_{\mathbf{6}}]_t{}^s$				$\ni [\Pi_{\mathbf{3\bar{3}6\bar{6}}}]^i{}_j{}^s{}_t$
$\mathbf{3} \otimes \mathbf{6} \otimes \mathbf{6} \otimes \bar{\mathbf{6}}$				$J^{\,s\,ia}[t^a_{\mathbf{6}}]_u{}^t$		$Q^{q\,i}{}_u W_q{}^{st}$		$\ni [\Pi_{\mathbf{366\bar{6}}}]^{i\,st}{}_u$
$\mathbf{3} \otimes \mathbf{6} \otimes \mathbf{8} \otimes \mathbf{8}$	$[t^a_{\mathbf{3}}]_j{}^i J^{\,s\,jb}$		$J^{\,t\,ia}[t^b_{\mathbf{6}}]_t{}^s$	$J^{\,s\,ic}\{f,d\}^{abc}$	$E_x{}^{is} G^{\,x\,ab}$	$V_q{}^{ia} X^{q\,sb}$		$\ni [\Pi_{\mathbf{3688}}]^{i\,s\,ab}$
$\mathbf{3} \otimes \bar{\mathbf{6}} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$	$K_s{}^{ij}\, \bar{J}_{s\,ai}$		$J^{\,s\,ia} \bar{S}_{stu}$					$\ni [\Pi_{\mathbf{3\bar{6}\bar{6}8}}]^i{}_{st}{}^a$
$\mathbf{6} \otimes \mathbf{6} \otimes \bar{\mathbf{6}} \otimes \bar{\mathbf{6}}$	$\delta_u{}^s\, \delta_v{}^t$			$[t^a_{\mathbf{6}}]_u{}^s [t^a_{\mathbf{6}}]_v{}^t$				$\ni [\Pi_{\mathbf{66\bar{6}\bar{6}}}]^{st}{}_{uv}$
$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} \otimes \mathbf{8}$				$S^{\,str}[t^a_{\mathbf{6}}]_r{}^u$		$W_q{}^{st} X^{q\,ua}$		$\ni [\Pi_{\mathbf{6668}}]^{stu\,a}$
$\mathbf{6} \otimes \bar{\mathbf{6}} \otimes \mathbf{8} \otimes \mathbf{8}$	$\delta_t{}^s\, \delta_b{}^a$			$[t^c_{\mathbf{6}}]_t{}^s [t^c_{\mathbf{8}}]_b{}^a$			$F^{\,n}{}_t{}^s H^{\,n\,ab}$	$\ni [\Pi_{\mathbf{6\bar{6}88}}]^s{}_t{}^a{}_b$

Operators built from 4 field color-invariants

	Scalar sextet Φ only		*Dirac* sextet Ψ only		≥ 1 of each	
$\mathrm{SU}(3)_c$ invariant	$d_{\min}$	Structure	$d_{\min}$	Structure	$d_{\min}$	Structure
$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{6} \otimes \mathbf{6}$	5	$(qq')\Phi\Phi$	6	$(qq')(\Psi\Psi)$	7	$(qq')(\Psi\ell)\Phi$
	7	$(qq')\Phi\|H\|^2\Phi$		$(q\Psi)(q'\Psi)$		$(q\ell)(q'\Psi)\Phi$
$\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$	6	$(qq')\Phi^\dagger G$				
$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{6}}$	7	$(\bar{q}q')(\bar{q}''\ell)\Phi$	6	$(\bar{q}q')(\bar{q}''\Psi)$		
		$(qq')^\dagger(q''\ell)\Phi$		$(qq')^\dagger(\bar{q}''\Psi)$		
$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{6} \otimes \bar{\mathbf{6}}$	5	$(\bar{q}q')\Phi^\dagger\Phi$	6	$(\bar{q}q')(\bar{\Psi}\Psi)$	7^*	$(\bar{q}q')(\bar{\Psi}\ell)\Phi$
	7	$(\bar{q}q')\Phi^\dagger\|H\|^2\Phi$		$(\bar{q}\Psi)(\bar{\Psi}q')$		$(\bar{q}\Psi)(q'\ell)\Phi^\dagger$
$\mathbf{3} \otimes \mathbf{6} \otimes \mathbf{6} \otimes \bar{\mathbf{6}}$	6	$(q\ell)\|\Phi\|^2\Phi$	6	$(q\Psi)(\bar{\Psi}\Psi)$	5	$(q\Psi)\Phi^\dagger\Phi$
		$(\bar{\ell}q)\|\Phi\|^2\Phi$		$(\bar{\Psi}q)(\Psi\Psi)$		$(\bar{\Psi}q)\Phi\Phi$
					7^*	$(q\Psi)(\Psi\ell)\Phi^\dagger$
						$(q\ell)(\bar{\Psi}\Psi)\Phi$
$\mathbf{3} \otimes \mathbf{6} \otimes \mathbf{8} \otimes \mathbf{8}$			7	$(q\Psi)GG$		
$\mathbf{3} \otimes \bar{\mathbf{6}} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$	7	$(q\ell)\Phi^\dagger\Phi^\dagger G$			6	$(\bar{\Psi}q)\Phi^\dagger G$
		$(\bar{\ell}q)\Phi^\dagger\Phi^\dagger G$				
$\mathbf{6} \otimes \mathbf{6} \otimes \bar{\mathbf{6}} \otimes \bar{\mathbf{6}}$	$6^\dagger$	$\|\Phi\|^4$			6	$(\bar{\Psi}\ell)\|\Phi\|^2\Phi$
						$(\Psi\ell)\|\Phi\|^2\Phi^\dagger$
						$(\bar{\ell}\Psi)\|\Phi\|^2\Phi^\dagger$
					7	$(\bar{\Psi}\Psi)\|\Phi\|^2\|H\|^2$
$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} \otimes \mathbf{8}$	7	$\Phi\Phi\Phi GB$			6	$(\Psi\Psi)\Phi G$
					7	$(\Psi\ell)\Phi\Phi G$
						$(\bar{\ell}\Psi)\Phi\Phi G$
$\mathbf{6} \otimes \bar{\mathbf{6}} \otimes \mathbf{8} \otimes \mathbf{8}$	6	$\|\Phi\|^2 GG$	7	$(\bar{\Psi}\Psi)GG$		

More interesting processes

	Scalar sextet Φ only		*Dirac* sextet Ψ only		≥ 1 of each	
$\mathrm{SU}(3)_c$ invariant	$d_{\min}$	Structure	$d_{\min}$	Structure	$d_{\min}$	Structure
$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{6} \otimes \mathbf{6}$	5	$(qq')\Phi\Phi$	6	$(qq')(\Psi\Psi)$	7	$(qq')(\Psi\ell)\Phi$
	7	$(qq')\Phi\lvert H\rvert^2\Phi$		$(q\Psi)(q'\Psi)$		$(q\ell)(q'\Psi)\Phi$
$\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$	6	$(qq')\Phi^\dagger G$				
$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{6}}$	7	$(\bar{q}q')(\bar{q}''\ell)\Phi$	6	$(\bar{q}q')(\bar{q}''\Psi)$		
		$(qq')^\dagger(q''\ell)\Phi$		$(qq')^\dagger(\bar{q}''\Psi)$		
$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{6} \otimes \bar{\mathbf{6}}$	5	$(\bar{q}q')\Phi^\dagger\Phi$	6	$(\bar{q}q')(\bar{\Psi}\Psi)$	7*	$(\bar{q}q')(\bar{\Psi}\ell)\Phi$
	7	$(\bar{q}q')\Phi^\dagger\lvert H\rvert^2\Phi$		$(\bar{q}\Psi)(\bar{\Psi}q')$		$(\bar{q}\Psi)(q'\ell)\Phi^\dagger$
$\mathbf{3} \otimes \mathbf{6} \otimes \mathbf{6} \otimes \bar{\mathbf{6}}$	6	$(q\ell)\lvert\Phi\rvert^2\Phi$	6	$(q\Psi)(\bar{\Psi}\Psi)$	5	$(q\Psi)\Phi^\dagger\Phi$
		$(\bar{\ell}q)\lvert\Phi\rvert^2\Phi$		$(\bar{\Psi}q)(\Psi\Psi)$		$(\bar{\Psi}q)\Phi\Phi$
					7*	$(q\Psi)(\Psi\ell)\Phi^\dagger$
						$(q\ell)(\bar{\Psi}\Psi)\Phi$
$\mathbf{3} \otimes \mathbf{6} \otimes \mathbf{8} \otimes \mathbf{8}$			7	$(q\Psi)GG$		
$\mathbf{3} \otimes \bar{\mathbf{6}} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$	7	$(q\ell)\Phi^\dagger\Phi^\dagger G$			6	$(\bar{\Psi}q)\Phi^\dagger G$
		$(\bar{\ell}q)\Phi^\dagger\Phi^\dagger G$				
$\mathbf{6} \otimes \mathbf{6} \otimes \bar{\mathbf{6}} \otimes \bar{\mathbf{6}}$	$6^\dagger$	$\lvert\Phi\rvert^4$			6	$(\bar{\Psi}\ell)\lvert\Phi\rvert^2\Phi$
						$(\Psi\ell)\lvert\Phi\rvert^2\Phi^\dagger$
						$(\bar{\ell}\Psi)\lvert\Phi\rvert^2\Phi^\dagger$
					7	$(\bar{\Psi}\Psi)\lvert\Phi\rvert^2\lvert H\rvert^2$
$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} \otimes \mathbf{8}$	7	$\Phi\Phi\Phi GB$			6	$(\Psi\Psi)\Phi G$
					7	$(\Psi\ell)\Phi\Phi G$
						$(\bar{\ell}\Psi)\Phi\Phi G$
$\mathbf{6} \otimes \bar{\mathbf{6}} \otimes \mathbf{8} \otimes \mathbf{8}$	6	$\lvert\Phi\rvert^2 GG$	7	$(\bar{\Psi}\Psi)GG$		

Associated production with quark
From quark-gluon fusion

Three body decay of sextet

Iterate again

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}}_{\mathrm{a}} \oplus \mathbf{6}_{\mathrm{s}},$$
$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8},$$
$$\mathbf{6} \otimes \mathbf{3} = \mathbf{8} \oplus \mathbf{10},$$
$$\mathbf{6} \otimes \bar{\mathbf{3}} = \mathbf{3} \oplus \mathbf{15},$$
$$\mathbf{6} \otimes \mathbf{6} = \bar{\mathbf{6}}_{\mathrm{s}} \oplus \mathbf{15}_{\mathrm{a}} \oplus \mathbf{15}'_{\mathrm{s}},$$
$$\mathbf{6} \otimes \bar{\mathbf{6}} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27},$$
$$\mathbf{8} \otimes \mathbf{3} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15},$$
$$\mathbf{8} \otimes \bar{\mathbf{6}} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15} \oplus \mathbf{24},$$
$$\mathbf{8} \otimes \mathbf{8} = \mathbf{1}_{\mathrm{s}} \oplus \mathbf{8}_{\mathrm{s}} \oplus \mathbf{8}_{\mathrm{a}} \oplus \mathbf{10}_{\mathrm{a}} \oplus \overline{\mathbf{10}}_{\mathrm{a}} \oplus \mathbf{27}_{\mathrm{s}}$$

$$\mathbf{6} \otimes \bar{\mathbf{3}}$$

$$\mathbf{6} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} = \bar{\mathbf{3}}_{\mathrm{a}}$$

$$\mathbf{6} \otimes \bar{\mathbf{3}} \otimes \mathbf{6} \otimes \bar{\mathbf{3}} = \bar{\mathbf{3}}_{\mathrm{a}}$$

Five field invariant

	Scalar sextet Φ only		≥ 1 of each	
$\mathrm{SU}(3)_c$ invariant	$d_{\min}$	Structure	$d_{\min}$	Structure
$\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{6}$	7	$(qq')(q''q''')\Phi$		
$\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}\otimes\bar{\mathbf{3}}\otimes\bar{\mathbf{6}}$	7	$(qq')(\bar{q}''q''')\Phi^\dagger$		
$\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{6}\otimes\bar{\mathbf{6}}\otimes\bar{\mathbf{6}}$	6	$(qq')\lvert\Phi\rvert^2\Phi^\dagger$	7*	$(qq')(\bar{\Psi}\Psi)\Phi^\dagger$
				$(\bar{\Psi}q)(q'\Psi)\Phi^\dagger$
$\mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{6}\otimes\mathbf{6}\otimes\mathbf{6}$	6	$(\bar{q}q')\Phi\Phi\Phi$	7	$(\bar{q}q')(\Psi\Psi)\Phi$
				$(\bar{q}\Psi)(q'\Psi)\Phi$
$\mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{6}\otimes\bar{\mathbf{6}}\otimes\mathbf{8}$	7	$(\bar{q}q')\lvert\Phi\rvert^2G$		
$\mathbf{3}\otimes\mathbf{6}\otimes\mathbf{6}\otimes\mathbf{6}\otimes\mathbf{6}$	7	$(q\ell)\Phi\Phi\Phi\Phi$		
		$(\bar{\ell}q)\Phi\Phi\Phi\Phi$		
$\mathbf{3}\otimes\mathbf{6}\otimes\bar{\mathbf{6}}\otimes\bar{\mathbf{6}}\otimes\bar{\mathbf{6}}$	7	$(q\ell)\lvert\Phi\rvert^2\Phi^\dagger\Phi^\dagger$		
		$(\bar{\ell}q)\lvert\Phi\rvert^2\Phi^\dagger\Phi^\dagger$		
$\mathbf{6}\otimes\mathbf{6}\otimes\bar{\mathbf{6}}\otimes\bar{\mathbf{6}}\otimes\mathbf{8}$			7	$(\bar{\Psi}\Psi)\Phi^\dagger\Phi G$
$\mathbf{6}\otimes\mathbf{6}\otimes\mathbf{6}\otimes\mathbf{8}\otimes\mathbf{8}$	7	$\Phi\Phi\Phi GG$		
$\mathbf{6}\otimes\bar{\mathbf{6}}\otimes\bar{\mathbf{6}}\otimes\bar{\mathbf{6}}\otimes\bar{\mathbf{6}}$	7	$\Phi\Phi^\dagger\Phi^\dagger\Phi^\dagger\Phi^\dagger\lvert H\rvert^2$		

Conclusion and Future Directions

Systematic exploration of color invariants leads to a complete catalogue of BSM operators with new and interesting collider phenomenology

Future Directions Include

- In depth phenomenological studies for sextet models including future colliders
- Building out UV completions
- Completing catalogue for other representations, including fields with nontrivial representations underSU(3)xSU(2)